Structural Identification of Static Processes with Hysteresis Nonlinearities in Civil Engineering

Nikolay N. Karabutov

Department of Problems Control, Moscow State Engineering University of Radio Engineering, Electronics and Automation Moscow, Russia

kn22@yandex.ru

***Abstract*- Methods of structural identification of processes in systems of civil engineering with the nonlinearities which are belonging to the class of the hysteresis are considered. Methods are based on the multiple-informational approach which is based on the analysis of static structures, reflecting a condition of nonlinear system of identification. The criterion is introduced, allowing making the solution on level of linearity (nonlinearity) of processes in parametrical space on set of secants. Algorithms of definition of structural parameters hysteresis nonlinearities on the basis of the analysis of properties of special static structures are offered. The mode of construction of functions of saturation and hysteresis on a class of random inputs is developed.**

Keywords*- Structure; Identification; Saturation; Hysteresis; Model; Secant

I. INTRODUCTION

The processes that flow past in plants of civil engineering have nonlinear character. Their mathematical formulation becomes complicated action of various exterior and interior perturbations which often have stochastic character. All it is reflected in procedure of a choice of structure of the mathematical model describing researched processes. Here it is necessary to refer to a priori uncertainty rather than both ranges of parameters, and operating perturbations. Therefore, exposition of processes in plants of civil engineering very often applies parametrical methods of estimation. So in [1], the method of identification of parameters of Prony series according to trials on creep of a linear viscoelastic material is offered. Algorithm Nelder-Mead is applied to an estimation of parameters. For study of nonlinear properties of the materials applied in plants of civil engineering, the dependences described by functions of saturation and a hysteresis [2] can be applied. In [3], properties of systems with a hysteresis that can arise in the process of wood handling are study. In paper [4] influence on properties of wood of various factors, which can be described the nonlinear differential equations c saturation, is studied. Influence on properties of soil of such parameters as can be described permeability and a water stream power by means of the mathematical models also containing function of saturation [5]. In [6], influence of impurity polluting water for the purpose of an estimation of degree of arising risks is research. The predicting model described by a polynomial of the second order is for this purpose applied. For an estimation of parameters of model, the method of least-squares method is used. In [7], the mathematical model for an estimation of influence organic pollutant, and also climatic conditions is offered. The model is static and is used for study of influence of seasonal oscillations of temperature and humidity of the soil caused by actual climatic conditions in an aeration zone. The models based on methodology of estimation NAPL at building of water pools and the analysis of level of reactive impurity, were considered in [8]. In [9] the model of an estimation of a condition of steel designs in which elements there can be hysteresis effect is offered. The review of existing approaches to construction hysteresis models is spent and their generalisation is offered. Modes that approximation of parameters of a loop of a hysteresis by means of dissipation functions are considered.

In [10] the application that reviews wavelet transformations to systems of identification of the processes flowing past in plants of civil engineering is reduced. Most effectively, this method is applicable to identification to the structures subject to vibrational actions. Frequency methods of identification are considered. The various models describing processes of saturation of soil by water are studied in [11]. They are based on development of model Leverett-Cassie. The analysis of methods of the identification applied in civil engineering, on the basis of handling of experimental data is fulfilled in [12]. The basic attention is given the stochastic methods of identification based on regression analysis, frequency procedures. For exposition of plants of civil engineering, the monography [13] is devoted application of models of autoregression moving average models. The procedures of identification based on use of a finite element method, are described in [14]. The review of genetic algorithms, artificial intellect procedures in systems of identification of plants of civil engineering is given in [15].

From the analysis set forth above follows that in most cases parametrical methods of identification are applied to construction of mathematical models of processes in plants of civil engineering. Thus the model structure is set a priori. The problem of structural identification of static plants is complicated and in many respects depends on the available a priori information [16-18]. Very often, as appears from stated above, plants can contain complicated nonlinear elements. To such nonlinearities the hysteresis and saturation belong. With a hysteresis, the set of publications [19, 20] is devoted identification of dynamic systems. Basically, the dynamic model is applied to hysteresis description the Bouc-Wen. In the majority of considered works, the problem of structural identification is shown to the parametrical. Input signals are assumed by the monotone. Other approaches to hysteresis definition are considered in [21]. In many systems and plants of civil engineering, input signals have irregular (random) character that essentially complicates process of structural identification with the specified nonlinearities. In particular, it is fair for static systems. The problem of structural identification of such class of systems with static models of a hysteresis was not studied. Domination of the parametrical approach can explain it. The nonlinearity analysis leads to introduction of dependent variables (a problem of collinearity) which for improvement of quality of a parametrical estimation are usually excluded. New variables allow describing nonlinear parts of system, but existing methods do not allow using them effectively.

The approach to structural identification of static system with nonlinearities of type saturation and a hysteresis is lower stated at an action of irregular inputs in the conditions of uncertainty. It is based on the development of functionally multiple approaches offered in [18, 22]. The basis of considered algorithms and methods is made by decision making procedures on a class of linear functions of a special form (secants) and structures. Procedure of an estimation of parameters of nonlinearity is described. Results of modelling are bring.

II. PROBLEM STATEMENT

Consider plant

$$y_n = A^T U_n + bf(u_{i,n}) + \xi_n, \tag{1}$$

where $y_n \in R$, $U_n \in R^k$ is exit and input of system, $A \in R^k$ is a vector of parameters, $b \in R$, n is discrete time, $\xi_n \in R$ is a perturbation, $|\xi_n| \leq \infty$, $f(u_{i,n}) \in R$ is a function set on a class hysteresis of models $\mathcal{F}_h$, described by static dependence. Elements $u_{i,n}$ of vector U_n represent limited irregular (nonmonotone) extremely nondegenerate functions.

For (1) the measuring information is known

$$\mathrm{I}_o(N) = \{y_n, U_n, \; n \in J_N = [0, N], \; N < \infty\} \tag{2}$$

and map $\Gamma_o : \{U_n\} \times \{y_n\} \;\; \forall n \in J_N$ corresponding to it describing an observable informational portrait [18].

Consider at first a case when $f(u_{i,n}) = f_s(u_{i,n}) = \mathrm{sat}(u_{i,n})$ there is a function of saturation ($f_s(u_{i,n}) \in \mathcal{F}_{sat} \subset \mathcal{F}_h$)

$$\mathrm{sat}(u_{i,n}) = \begin{cases} \beta, & u_{i,n} \geq \beta, \\ u_{i,n}, & \alpha < u_{i,n} < \beta, \\ \alpha, & u_{i,n} \leq \alpha, \end{cases} \tag{3}$$

where α, β there are some numbers.

The problem is put: on the basis of analysis I_o, Γ_o to estimate structure of plant (1), (3). It means that it is necessary to estimate degree of linearity and form of function $f_s(u_{i,n})$.

III. STRUCTURAL IDENTIFICATION OF PLANT (1) WITH $f_s(u_{i,n}) \in \mathcal{F}_{sat}$

A. *Estimation of Degree of Linearity of Plant (1), (3)*

Consider a contraction of observable informational portrait $\Gamma_o^{u_i} \subset \Gamma_o\big|_{u_i \in U}$ $\forall i = \overline{1,k}$ and for every $\Gamma_o^{u_i}$ constructs a secant

$$\bar{\gamma}(y, u_i) = a_i u_{i,n},$$

where a_i is some real number.

Introduce set on (2) set of secants for Γ_o

$$\mathrm{S}(U, y) = \{\bar{\gamma}(y, u_i) \quad \forall i = \overline{1,k}\}.$$

Definition 1: [23] A field of structures $\mathcal{S}_S$ of system (1) name a set of maps $\bar{\gamma}(y,u_i) \subset \{u_i\} \times \{y\}$ $\forall i = \overline{1,k}$ on Euclidean plane E $\mathcal{S}_S = \mathrm{S}(U, y)$.

Designate $\Phi_n = [\bar{\gamma}(y_n, u_{1,n}), \bar{\gamma}(y_n, u_{2,n}), \ldots, \bar{\gamma}(y_n, u_{k,n})]^T$ and consider the equation

$$\hat{y}_n = \Psi^T \Phi_n, \tag{4}$$

where vector $\Psi \in R^k$ is defined by means of a method of least squares so that to minimize an error of forecasting y_n. Estimation Ψ exists owing to the suppositions made in Section 2 concerning input U_n.

Theorem 1: [23] Consider a vector of informative variables $U_n \in R^k$ and a field of structures $\mathcal{S}_S$ for (1), (3), set on set of secants $\mathrm{S}(U, y)$. Then the field of structures $\mathcal{S}_S$ of System (1), (3) is full, if

$$\sum_{i=1}^{k} \psi_i = 1, \tag{5}$$

where ψ_i is i-th element of vector Ψ in (4).

The Theorem 1 gives linearity sufficient conditions (nonlinearities, collinearity) systems (1), (3) on the set field of structures $\mathcal{S}_S$. If the condition (4) is fulfilled, that field $\mathcal{S}_S$ is full and, therefore, $\mathcal{S}_S$ is a linear span of an exit of system (1). Otherwise, the solution is made on presence of nonlinearity or collinearity (autocorrelations) in system (1).

Let

$$\sum_{i=1}^{k} \psi_i = \chi.$$

Then magnitude $\Delta(\Psi) = \chi - 1$ name level of nonlinearity of system (1), (3) in parametrical space $\mathcal{P}_{\mathcal{P}} = (A, B)$.

B. *Set for Structure $f_s(u_{i,n})$ Estimation*

Consider model

$$\hat{y}_{s,n} = \hat{a}_s I^T U_n,$$

where $\hat{a}_s \in R$, $I \in R^m$ is an unit vector.

Designate through $e_n = \hat{y}_{s,n} - y_n$ a variable which contains the data about $f_s(u_{i,n})$. Choice $u_{i,n}$ realize so as to ensure maximum value of factor of determination r^2_{e,u_i} between e_n and $u_{i,n}$.

Generate set $\mathrm{I}_e = \{e_n, U_n, n \in N\}$. Introduce factor

$$k_{e,u_i,n} = k_s(e,u_i,n) = \frac{e_n}{u_{i,n}}.$$

J_N will order $k_{e,u_i,n}$ increase. Receive set $\{k_q^v\}$, where $k_q^v = k_s(e,u_i,q)$, $q \in J_N^v = [0,N]$. To every k_q^v there corresponds value e_q^v . Receive $\mathrm{I}_k^v = \{e_q^v, k_q^v\}$. Consider on I_k^v map $\Gamma_{e,k}^v : \{k_q^v\} \times \{e_q^v\}$ and structure $\mathcal{S}_{k,e}^v$ corresponding to it. Structure $\mathcal{S}_{k,e}^v$ has been introduced in [22] that describe processes in nonlinear system of identification. The analysis of properties $\mathcal{S}_{k,e}^v$ is given in [22, 24].

Now the problem consists in an estimation of structural parameters of function $f_s(u_{i,n})$ on the basis of the analysis of properties $\mathcal{S}_{k,e}^v$.

Remark 1: In spite of the fact that structure $\mathcal{S}_{k,e}^v$ has a general view for a wide class of nonlinearities, it reflects features, characteristic for each class of nonlinearities.

C. Decision-making on Structure of Function $f_s(u_{i,n})$

Results of modelling show that structure $\mathcal{S}_{k,e}^v$ for the functions belonging to class $\mathcal{F}_h$, has the same form (Fig. 1). Therefore, for decision-making on a form of function $f(u_{i,n}) \in R$ work in some criterion,

Introduce an index [22]

$$\mu_{k,q} = \frac{\Delta k_q^v}{k_q^v} = \frac{k_{q+1}^v - k_q^v}{k_q^v}, \tag{6}$$

which define a relative rate of change of factor k_q^v on set I_k^v . The analysis of modification $\mu_{k_{s,q}}$ allows defining the moment of change of derivative k_q^v . In [22, 24] $\mu_{k,q}$ it was applied to definition of the moment of transition of function $f_s(u_{i,n})$ in a saturation state. By analogy introduce variable $\mu_{e,q}$.

Consider structure $\mathcal{S}_{\mu_k,\mu_e}$ defined on set $\{\mu_{k,q}\} \times \{\mu_{e,q}\}$.

Theorem 2: If $\mathcal{S}_{\mu_k,\mu_e}$ contains fragment $\mathcal{PL}_{qst} \subset \mathcal{S}_{\mu_k,\mu_e}$ on which modification $\mu_{e,q}$ has quasistationary character, then $f(u_{i,n}) \in \mathcal{F}_{sat}$.

In Fig. 1, examples of structures $\mathcal{S}_{k,e,s}^v$, $\mathcal{S}_{k,e,\mathrm{sgn}}^v$ for $f_s(u_{1,n})$ and signum function $f_{\mathrm{sgn}}(u_{1,n})$ which gain from (3) as a special case are shown. From Fig. 1 it is visible that $\mathcal{S}_{k,e,s}^v$, $\mathcal{S}_{k,e,\mathrm{sgn}}^v$ have almost identical structure.

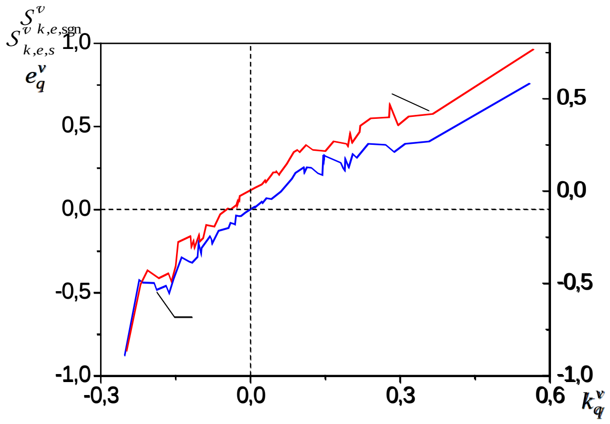

Fig. 1 $\mathcal{S}_{k,e}^v$ -structures for system (1) with $f_s(u_{1,n})$, $f_{\mathrm{sgn}}(u_{1,n})$

In Fig. 2, structure $\mathcal{S}_{\mu_k,\mu_e,\mathrm{sgn}}$, which characterizes a system status of identification with function $f_{\mathrm{sgn}}(u_{1,n})$, is shown. The area about coordinate origin O_s corresponds to transition of function $f_s(u_{1,n})$ in saturation. Despite the fact that, structure $\mathcal{S}_{\mu_k,\mu_e}$ is presented in other space, it is possible to observe some correspondence with structure $\mathcal{S}_{k,e}^v$. It confirms legitimacy of introduction of structure $\mathcal{S}_{\mu_k,\mu_e}$.

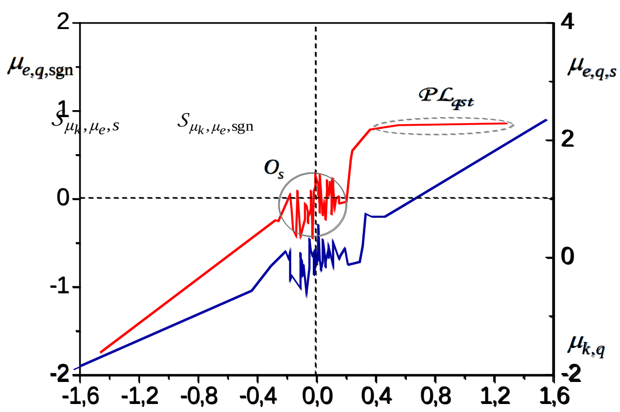

Fig. 2 $\mathcal{S}_{\mu_k,\mu_e}$ -structures of system of identification with $f_s(u_{1,n})$, $f_{\mathrm{sgn}}(u_{1,n})$

Consider now a fragment about coordinate origin $O_{sat} \subset \mathcal{S}_{\mu_k,\mu_e,s}$, defined on interval $\mu_{k,q} \in [-\mu_{k,r}; \mu_{k,l}]$.

Statement 1: Consider a neighbourhood of zero O_s of structure $\mathcal{S}_{\mu_k,\mu_e,s}$, defined on interval $[-\mu_{k,r}; \mu_{k,l}]$. Then

$$f_s(u_{i,n}) = u_{i,n}\ \forall u_{i,n} \in (\alpha,\beta),\ \text{если} \begin{cases} \mu^v_{e,q} > 0 & \forall \mu^v_{e,q} \in [-\mu_{e,r};0], \\ \mu^v_{e,q} < 0 & \forall \mu^v_{e,q} \in [0;\mu_{e,l}]. \end{cases}$$

D. Structure $f_s(u_{i,n})$ Identification

Analyse $\mathcal{S}^v_{k,e}$ for an estimation of structural parameters of function $f_s(u_{i,q}) \in \mathcal{F}_{sat}$. For decision-making on the nonlinear dependence describing structure $\mathcal{S}^v_{k,e}$, construct model (secant)

$$\bar{\gamma}_q = \bar{\gamma}(e^v_q, k^v_q) = a^v k^v_q + b^v ,$$

where a^v, b^v are from a condition

$$\arg\min_{a^v,b^v}(e^v_q - \bar{\gamma}_q)^2 = a^v_*, b^v_* .$$

Select fragment $\mathcal{S}_\alpha \in \mathcal{S}^v_{k,e}$ which passes parallel $\bar{\gamma}_q$. To it there corresponds set $\mathrm{I}_\alpha = \{e^v_q,\ k^v_q,\ q \in J_\alpha \subset J^v_N\}$ on which parameter α is estimated. Owing to construction of set I^v_k fragment $\mathcal{S}_\alpha$ is arranged in right parts $\mathcal{S}^v_{k,e}$ [22]. On set I_α, applying regression model $\bar{\gamma}_{\alpha,q} = \bar{\gamma}_\alpha(k^v_q) = a_\alpha k^v_q + b_\alpha$, by means of least squares method defines parameters a_α, b_α and suppose $\hat{\alpha} = a_\alpha$. So, the estimation of parameters α in (3) is received.

More thin approach is demanded by an estimation of parameter β in (3). Direct allocation of a corresponding subset (fragment $\mathcal{S}_\beta$) from structure $\mathcal{S}^v_{k,e}$ is interfaced to certain difficulties. It is connected by that map $\Gamma^v_{e,k}$ often has irregular character and on some interval $J_\beta \subset J$ $u_{i,q}$ and $f_s(u_{i,q})$ coincide. Therefore, for shaping the informational set containing the demanded data, it is necessary to attract indirect criteria.

Apply indicator $\mu_{k,q}$ (6). The analysis of modification $\mu_{k,q}$ allows defining the moment of change of derivative k^v_q that is the beginning of transition of system in saturation.

For definition of the interval containing the information on parameter β, apply the following approach [22, 24]. On the basis of a modification of indicator $\mu_{k,q}$, generate set $\mathrm{I}_\beta = \{e^v_q,\ k^v_q,\ q \in J_\beta \subset J^v_N\}$ on which $\mu_{k,q}$ has stationary character. Using a least squares method on I_β find the secant $\mathcal{S}^v_{k,e}$

$$\bar{\gamma}_{\beta,q} = \bar{\gamma}(e^v_q, k^v_q) = a_\beta k^v_q + b_\beta \tag{7}$$

and name interval $J_\beta \subset J$ identifying if on J_β the coefficient of determination satisfies to condition $r^2_{e^v,k^v} \geq \varepsilon_\beta$, where $\varepsilon_\beta > 0$ there is some set number. If the specified condition is not fulfilled, change boundaries of set J_β. Considering that a_β is an estimation of input $u_{i,n}$, receive $\hat{\beta} = a_\beta$.

The problem of a membership of the received structure of function $f_s(u_{i,n})$ to class $\mathcal{F}_{sat} \subset \mathcal{F}_h$ is not less important. For description, hysteresis curves introduce following parameters [21]:

1) diameter $f_s(u_{i,n})$

$$\operatorname{diam} f_s(u_{i,n}) = \sup_{(n_j,n_l)\in J_N} |\, f_s(u_{i,n_j}) - f_s(u_{i,n_l}) \,|, \quad j \neq l ;$$

2) distance

$$d(u_{i,n_j}, u_{i,n_l}) = \min_{(n_j,n_l)\in J_N} |\, u_{i,n_j} - u_{i,n_l} \,|. \tag{8}$$

$\operatorname{diam} f(u_{i,n})$ defines a function range. On the basis of the analysis of distance $d(u_{i,n_j}, u_{i,n_l})$ it is possible to establish a class to which research function belongs. For diameter it is received

$$\operatorname{diam} f(u_{i,n}) = \hat{\beta} - \hat{\alpha} .$$

As elements, $u_{i,n} \in U_n$ are assumed irregular. The evaluation of a distance on the basis of (8) can state an incorrect estimation. Apply for this purpose the analysis of set I^v_k. The theorem [22, 24] is fair.

Theorem 3: Let $(k^v_{q_j}, k^v_{q_l}) \in \Omega^L_k \subseteq \Omega_k \subset R$, where $k^v_{q_j}$ there is monotonically varying function, $j \neq l$, Ω^L_k is an interval corresponding to growth of function $f_s(u_{i,n})$ on plane (k^v_q, e^v_q) . Then

$$d(u_{i,q_j}, u_{i,q_l}) \geq \eta \min_{(q_j,q_l)\in J^v_N} |\, \Delta k^v_{q_{jl}} \,|,$$

where

$$\eta = \frac{\min_{q_l \in J^v_N} |\, u_{i,q_l} - \pi \,|}{\max_{k^v_{q_j} \in \Omega^L_k} |\, k^v_{q_j} \,|}, \quad \pi > 0 .$$

If $d(u_{i,q_j}, u_{i,q_l}) \leq \varepsilon$, $\varepsilon \geq 0$ and $\Delta e^v_q \geq 0$ for almost $\forall q \geq 0$, then $f_s(u_{i,n}) \in \mathcal{F}_{sat}$.

Remark 2: For function $f_s(u_{i,n})$ it is possible to construct sector, that is area to which can belong $f_s(u_{i,n})$. For this purpose, apply the approach offered in [22].

Remark 3: Using received parameters α, β , on the basis of the Theorem 2 and Statements 1 it is possible to construct function $f_s(u_{i,n})$ set on $\mu_{k,q}$. The example of the received structure of Function (3) in space $(\mu_{k,q}, f_s)$ is shown in Fig.3. Input $u_{1,q}$ is here too reduced.

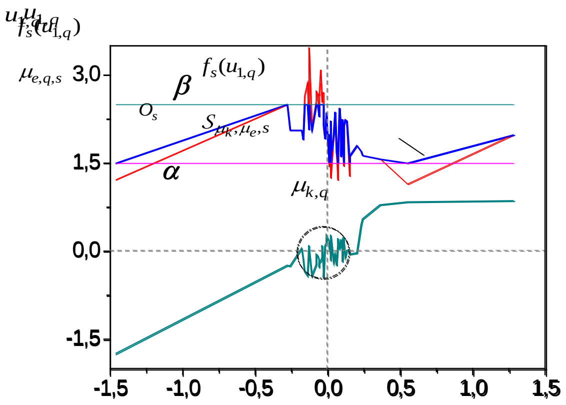

Fig. 3 Identification $f_s(u_{1,q})$ on the basis of structure $S_{\mu_k,\mu_e,s}$ analysis

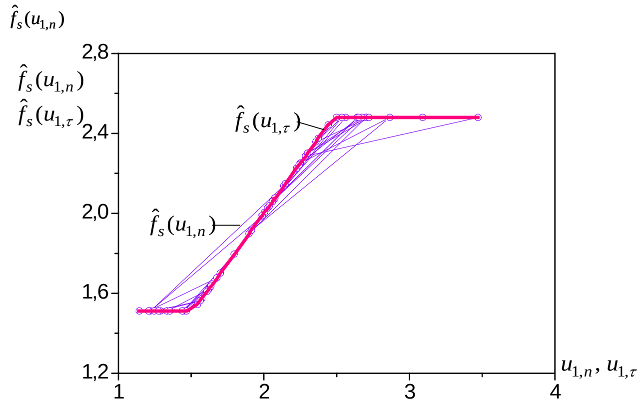

Fig. 4 Function $\hat{f}_s(u_{1,n})$ before and after application of operation of ordering $u_{1,n}$

So, the method of an estimation of structure of function $f_s(u_{i,n})$ in the conditions of uncertainty on the basis of analysis $S^{v}_{k,e}$ is offered.

Construction of an estimation of function $f_s(u_{1,q})$ on the basis of results of identification on a class of irregular inputs represents a problem. In this case, apply the method of ordering, offered in [22].

E. Example

Consider System (1) with $A=[0.4\ 1.25\ 1.5]^T$, $b=0.35$, $|\xi_n|\le 0.3$. Elements of vector U_n represent the stochastic variables having a final variance and expectation. Parameters of nonlinearity $f_s(u_{1,n})$ in (3): $\alpha=1.5$, $\beta=2.5$. Structure $S^{v}_{k,e}$ is shown on Fig.1. Results of decision-making on form of function $f(u_{i,n})$ are shown on Fig.2. For estimation α select structure $S^{v}_{k,e}$ fragment on interval [0.36; 0.5] of modification k^{v}_{q} and define secant $\bar{\gamma}_{\alpha,q}=\bar{\gamma}_{\alpha}(k^{v}_{q})=a_\alpha k^{v}_{q}+b_\alpha$, where $a_\alpha=\hat{\alpha}=1.51$, distance $d=0.007$. Application of the Theorem1 gives that $\Delta(\Psi)\ne 0$. Hence, system is nonlinear.

For determination of parameter β calculate indicator $\mu_{k,q}$ according to (6). On the basis of modification $\mu_{k,q}$ select an interval on k^{v}_{q}, equal $[-0.158;-0.022]$, and receive a secant (7) with parameters $a_\beta=\hat{\beta}=2.48$, $b_\beta=0$.

On the basis of the identified parameters and application of a method of ordering of function of saturation on a class of irregular inputs [22], estimation $\hat{f}_s(u_{i,n})$ of function $f_s(u_{i,n})$ is received. $\hat{f}_s(u_{i,n})$ is shown in Fig. 4.

Through $\hat{f}_s(u_{i,n})$ and $\hat{f}_s(u_{i,\tau})$ estimations $f_s(u_{i,n})$ before and after application of a method of ordering are designated. The results of applications of the Theorem 2 and Statements 1 are show in Fig. 3.

Remark 4: Definitive estimations on parameters of function $\hat{f}_s(u_{1,n})$ receive only at a stage of parametrical identification of System (1), (3).

Will pass now to the more general case when $f(u_{i,n})\in\mathcal{F}_h$

$$f_h(u_{i,n})=\begin{cases}\left.\begin{array}{l}\beta,\ u_{i,n}\ge\beta_t,\\ u_{i,n},\ \alpha_t<u_{i,n}<\beta_t,\\ \alpha,\ u_{i,n}\le\alpha_t,\end{array}\right\}\ \Delta u_{i,n}>0,\\ \left.\begin{array}{l}\beta,\ u_{i,n}+d\ge\beta_b,\\ u_{i,n},\ \alpha_b<u_{i,n}+d<\beta_b,\\ \alpha,\ u_{i,n}+d\le\alpha_b,\end{array}\right\}\ \Delta u_{i,n}\le 0,\end{cases}\tag{9}$$

where $(\alpha_b,\alpha_t)<\infty$, $(\beta_b,\beta_t)<\infty$, $d>0$.

IV. STRUCTURAL IDENTIFICATION OF PLANT (1) WITH $f_h(u_{i,n})\in\mathcal{F}_h$

F. Structure $SK^{v}_{k,\pm}$

Consider System (1), (9). Generate set I_e. The problem consists in definition of set of parameters $(\alpha_b,\alpha_t,\beta_b,\beta_t,d)$ of function $f_h(u_{i,n})$ on the basis of analysis I_e.

Find the first difference $\Delta u_{i,n}=u_{i,n+1}-u_{i,n}$ and divide I_e into two subsets

$$\mathrm{I}_e=\mathrm{I}_{e,+}\cup\mathrm{I}_{e,-},$$

where

$$(e_n,u_{i,n})\in\mathrm{I}_{e,+}=\{e_n\in R,u_{i,n}\in R\,|\,\Delta u_{i,n}>0\},$$

$$(e_n,u_{i,n})\in\mathrm{I}_{e,-}=\{e_n\in R,u_{i,n}\in R\,|\,\Delta u_{i,n}\le 0\},$$

$$\# \mathrm{I}_{e,+} + \# \mathrm{I}_{e,-} = \# \mathrm{I}_e .$$

Generate sets $\mathrm{I}_{k,+}^{v}(\mathrm{I}_{e,+})$, $\mathrm{I}_{k,-}^{v}(\mathrm{I}_{e,-})$ ordered on $k_{q,+}^{v}$, $k_{q,-}^{v}$. Consider structure $\mathcal{S}_{k,e}^{v}$ set by map $\Gamma_{e,k}^{v}:\{k_q^v\}\times\{e_q^v\}$. In [22], it is shown that analysis $\mathcal{S}_{k,e}^{v}$ can appear insufficiently for structure $f_h(u_{i,n})$ estimation. Therefore consider map $\Gamma_{k,\pm}^{v}:\{k_{q,-}^{v}\}\times\{k_{q,+}^{v}\}$ and structure $\mathcal{SK}_{k,\pm}^{v}$ corresponding to it.

Designate through $J_{k_{q,-}^{v}}$ an interval of modification $k_{q,-}^{v}$. Let $\bar{J}_{k_{q,-}^{v}} \subset J_{k_{q,-}^{v}}$ there is some subinterval $J_{k_{q,-}^{v}}$.

Definition: Structure $\mathcal{SK}_{k,\pm}^{v}$ contains an islet of level η if for $\forall k_{q,-}^{v} \in \bar{J}_{k_{q,-}^{v}}$ $k_{q,+}^{v}$ is quasistationary.

Designate an islet of level η through $\mathcal{IS}_{\eta} \subset \mathcal{SK}_{k,\pm}^{v}$.

Theorem 4: [22] If function $f_h(u_{i,n})$ is described by the Equation (9), structure $\mathcal{SK}_{k,\pm}^{v}$ contains islets $\mathcal{IS}_{\eta_\rho}$, $\rho \geq 1$.

Breadth $\mathcal{IS}_{\eta}$ (range of definition $\mathrm{rng}(\mathcal{IS}_{\eta})$) is a basis for an estimation of a distance of function $f_h(u_{i,n})$. As $\mathrm{rng}(\mathcal{IS}_{\eta})$ is defined on a range of irregular input $u_{i,n}$ directly to define a distance on the basis of (8), analyzing $\mathcal{IS}_{\eta}$, it is impossible. Apply the following algorithm AD of finding of a distance.

Algorithm AD:

1. Select islet $\mathcal{IS}_{\eta}$ on $\mathcal{SK}_{k,\pm}^{v}$ and to define $\mathrm{rng}(\mathcal{IS}_{\eta})$.

2. On the basis of handling $\mathrm{I}_{k,+}^{v}$, $\mathrm{I}_{k,-}^{v}$ receive corresponding $\mathcal{IS}_{\eta}$ subsets of inputs $\{u_{i,q_1+}\}$, $\{u_{i,q_2-}\}$.

3. Find distance estimation

$$\hat{d}(u_{i,q_1+}, u_{i,q_2-}) = \min_{q_1,q_2} |u_{i,q_1+} - u_{i,q_2-}|. \qquad (10)$$

Remark 5: Indexes +, - specify in sets $\mathrm{I}_{k,+}^{v}$, $\mathrm{I}_{k,-}^{v}$ on which basis values of input $u_{i,q}$ are received.

Remark 6: Indexes q_1, q_2 specify in diversity on time of values $u_{i,q}$.

To distance estimation, apply the Theorem 3.

Remark 7: Islets $\mathcal{IS}_{\mathcal{G}}$ can contain and structure $\mathcal{S}_{k,e}^{v}$, but their allocation and interpretation demands carrying out of additional research. $\mathcal{IS}_{\mathcal{G}_1}$ (See Fig. 5).

G. Definition of Parameters $\alpha_b, \alpha_t, \beta_b, \beta_t$

As the distance estimation is received, be limited to determination of estimations only one of the pairs of parameters (α_b, β_b) or (α_t, β_t).

The problem of estimation (α_t, β_t) on a class of irregular inputs is uneasy. Therefore consider one of possible procedures of determination of parameters. It is based on the following algorithm $A_{\alpha,\beta}$ of decision-making which allows decreasing influence of an irregularity of input $u_{i,q+}$.

***Algorithm** $A_{\alpha,\beta}$*

1. Construct structure $\mathcal{S}_{k,e}^{v}$ set by map $\Gamma_{e,k}^{v}:\{k_q^v\}\times\{e_q^v\}$.

2. Add to $\mathcal{S}_{k,e}^{v}$ secants $\bar{\gamma}_{q+}, \bar{\gamma}_{q-}$ corresponding to structures $\mathcal{S}_{k,e+}^{v}$, $\mathcal{S}_{k,e-}^{v}$, set by maps

$$\Gamma_{e,k-}^{v}:\{k_{q-}^{v}\}\times\{e_{q-}^{v}\},\ \Gamma_{e,k+}^{v}:\{k_{q+}^{v}\}\times\{e_{q+}^{v}\},$$

where

$$\bar{\gamma}_{q+} = \bar{\gamma}(e_{q+}^{v}, k_{q+}^{v}) = a_{+}^{v} k_{q+}^{v} + b_{+}^{v},\ \bar{\gamma}_{q-} = \bar{\gamma}(e_{q-}^{v}, k_{q-}^{v}) = a_{-}^{v} k_{q-}^{v} + b_{-}^{v}.$$

3. Select fragment $\mathcal{P}_{k,e+}^{v} \subset \mathcal{S}_{k,e+}^{v}$ (accordingly $\mathcal{P}_{k,e-}^{v} \subset \mathcal{S}_{k,e-}^{v}$) which corresponds to growth $\mathcal{S}_{k,e+}^{v}$, and generate set $U_{i+}(\mathcal{P}_{k,e}^{v}) = \{u_{i,q+}\}$.

4. Exclude from U_+ elements, using procedure

$$\{u_{i,q+}\} \in \tilde{U}_{i+},\ \tilde{U}_{i+} \subset U_{i+},\ \tilde{U}_{i+} = \{u_{i,q+} \mid e_q^v \in \mathrm{Sec}(\bar{\gamma}_{q+}, \bar{\gamma}_{q-})\},$$

where $\mathrm{Sec}(\bar{\gamma}_{q+}, \bar{\gamma}_{q-})$ there is a sector limited to straight lines $\bar{\gamma}_{q+}, \bar{\gamma}_{q-}$.

5. Designate

$$\hat{\alpha}_t = \min_{u_{i,q+} \in \tilde{U}_{i+}} u_{i,q+},\quad \hat{\beta}_t = \max_{u_{i,q+} \in \tilde{U}_{i+}} u_{i,q+}.$$

So, the structure of function $f_h(u_{i,n})$ is defined. The received estimations of parameters of structure $f_h(u_{i,n})$ are preliminary and are subject to an improvement at a stage of parametrical identification of system (1), (9). It is necessary to apply the methods stated in [22] to an estimation of adequacy of the received structure of function $f_h(u_{1,n})$.

H. Example

Consider System (1), (9) with $A = [0.9\ 2.5\ 2]^T$, $b = 0.35$, $|\xi_n| \leq 0.3$. Elements of vector U_n are the stochastic variables having a final variance and expectation. Parameters of nonlinearity $f_h(u_{1,n})$ in (9): $\alpha = 1.6$, $\beta = 2.2$, $d = 0.3$, $\alpha_t = 1.6$, $\beta_t = 2.2$. Choice of parameters α, β realize on the basis of the analysis of structure $\mathcal{SK}^{v}_{k,\pm}$ which is shown on Fig.5. Interval estimations for α, β

$$\hat{\alpha} = [1.47; 1.61], \ \hat{\beta} = [2.18; 2.2].$$

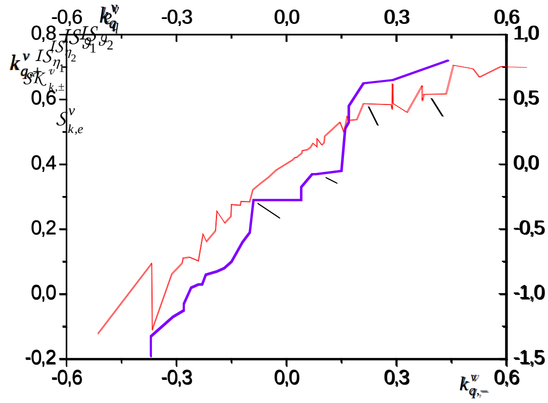

Fig. 5 Structure $\mathcal{SK}^{v}_{k,\pm}$ example

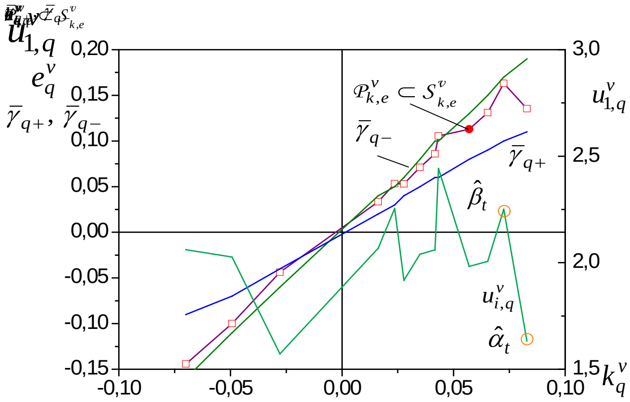

Fig. 6 Estimation of parameters $\hat{\alpha}_t$, $\hat{\beta}_t$

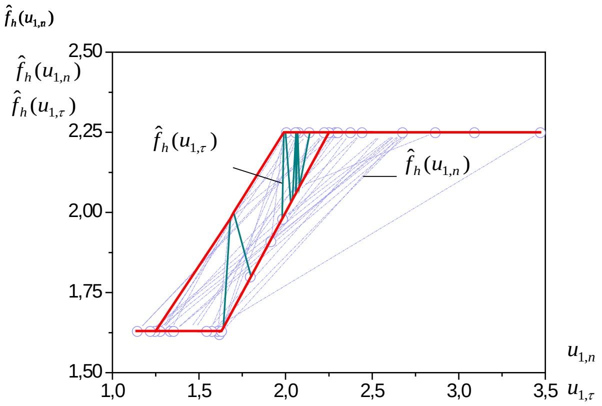

Fig. 7 Results of identification of function $f_h(u_{1,n})$

Distance estimation fulfil by means of the Theorem 3 on the basis of the analysis of values u_{i,q_1+}, u_{i,q_1-} corresponding to islet IS_1. $\hat{d} = 0.44$. Apply algorithm $A_{\alpha,\beta}$ (Fig. 6 see) and receive following estimations $\hat{\alpha}_t = 1.63$, $\hat{\beta}_t = 2.24$, $\mathrm{diam} = 0.57$. In Fig.7, the form of Function (9) after identification and application of a method of ordering [22] is shown. From Fig.7, it follows that before application of operation of ordering by the form functions $\hat{f}(u_{1,n})$ are difficult for making an inference about its structure. More regular form carries $\hat{f}(u_{1,\tau})$.

On Fig.7 the same labels, as on Fig.4, are used.

V. CONCLUSION

The structural space is selected, allowing estimating structure of nonlinear processes of civil engineering. The criterion for decision-making on degree of linearity (nonlinearity) of processes is offered.

The method of structural identification of static processes with saturation and hysteresis functions in systems of civil engineering is developed. The method is based on the analysis of properties of the special structure describing a state of nonlinear system of identification. The criterion for classification hysteresis nonlinearities is offered.

Algorithms of an estimation of structural parameters of a hysteresis in the conditions of uncertainty are developed. The estimation for diameter and a distance of function of saturation and a hysteresis is received at irregular inputs. The theorem allowing on the basis of an estimation of diameter to refer nonlinearity to one of subclasses $\mathcal{F}_h$ is proved.

The mode of construction of functions of saturation and hysteresis on a class of random inputs is developed. Results of modelling confirm efficiency of the offered methods.

REFERENCES

[1] E.I. Koroyd, “Identification of Prony series parameters using creep results obtained for linear viscoelastic materials,” *International Journal for Computational Civil and Structural Engineering*, vol.4, is. 2. pp. 67-68. 2008.

[2] A.A. Lukychev, V.V. Ilyin and G. B. Shchekina, Simple function of saturation of the polarization, considering time lag of dielectrics. [Online]. Available: http://rubingoods.ru/877/0.pl

[3] R.O. Foschi, Modeling the hysteretic response of mechanical connections for wood structures. [Online] Available: http:// timber.ce.wsu.edu/Resources/papers/7-1-2.pdf.

[4] K. Krabbenhoft, *Moisture Transport in Wood. A Study of Physical of Mathematical Models and their Numerical Implementation*., Copenhagen, Denmark: Printed by Eurographic A/S Department of Civil Engineering Technical University of Denmark. 2003.

[5] D.G. Fredlund, Xing Anqing, M.D. Fredlund and S.L. Barbour, “The relationship of the unsaturated soil shear strength function to the soil-water characteristic curve,” *Canadian Geotechnical Journal*, vol. 32, pp. 40-448, 1996.

[6] L. R. Chevalier, “Use of optimization to develop a correlation model for predicting residual NAPL saturation,” *Civil Engineering and Environmental Systems*, vol. 23, is. 2, .pp. 65-72. 2006.

[7] A. A. Keller and M. Chen, Seasonal variation in bioavailability of residual NAPL in the vadose zone. [Online]. Available:

http://fiesta.bren.ucsb.edu/~mchen/research/phd/PIGS2002.3.pdf

[8] Evaluation of tracer techniques for characterization of napl volumes and distribution in heterogeneous aquifers. [Online]. Available: http://cesep.mines.edu/papers/dai.pdf

[9] A.M. Amade and A. Mirmiran, "A new hysteresis model for steel members," *International journal for numerical methods in engineering*, vol. 45, pp. 1007-1023. 1999.

[10] T. Kijewski and A. Kareem, "Wavelet transforms for system identification in civil engineering," *Computer-Aided Civil and Infrastructure Engineering*, vol. 18, pp. 339-355. 2003.

[11] D. M. O'Carroll, L. M. Abriola, C. A. Polityka, S. A. Bradford and A. H. Demond, "Prediction of two-phase capillary pressure–saturation relationships in fractional wettability systems," *Journal of Contaminant Hydrology*, vol. 77, pp. 247– 270. 2005.

[12] A. Cunha, E. Caetano, F. Magalhães and C. Moutinho, "From input-output to output-only modal identification of civil engineering structures," in *SAMCO Final Report*. 2006. 22 p.

[13] P. Andersen, *Identification of civil engineering structures using vector ARMA models*, Denmark: Aalborg University. 1998.

[14] Q. Pan, *System identification of constructed civil engineering structures and uncertainty*, Drexel University. 2007.

[15] L. Mao. [Online]. Available: http://www.erp.ac.uk/cee/Posters/R-oster_Lei%20Mao_Jan09.pdf

[16] N. S. Raybman and V. M. Chadeev, *Construction of Models of Production Processes*, Moscow, Russian: Energiya, 1975.

[17] L. Luang, *Identification of Systems. Theory for Users*, Moscow, Russian: Nauka, 1991.

[18] N. N. Karabutov, *Structural Identification of Systems. The Analysis of Information Structures*, Moscow, Russian: Librokom, 2009. (in Russia)

[19] S. Xue H. Tang and J. Zhou, "Identification of structural systems using particle swarm optimization," *Journal of Asian Architecture and Building Engineering*, no. 524, pp. 517-524. 2009.

[20] F. Ikhouane and J. Rodellar, *Systems with hysteresis: analysis, identification and control using the Bouc-Wen model*. England: Wiley, 2007.

[21] M. A. Krasnoselskii and A. V. Pokrovskii, Systems with Hysteresis, Moscow, Russian: Nauka, 1983.

[22] N.N.Karabutov, *Structural identification of static plants: Fields, structures, methods*. Moscow, Russia: Librokom, 2011. (in Russia)

[23] N.N. Karabutov, "Decision-making on structure of univalent nonlinearities in system of structural identification of static systems," *Int. J. Sensing, Computing & Control*, vol. 1, no. 2, pp. 103-110, 2011.

[24] N.N. Karabutov, "Structural identification of static systems with saturation," *Measurement Techniques*, vol. 54, no. 2, May, pp. 135-142. 2011.