A Direct Algorithm for Computing Nash Equilibriums

Lunshan Gao

THALES CANADA, 105 Moatfield Drive, Toronto, ON., Canada

lunshan.gao@gmail.com

***Abstract*- This paper describes a relationship between the expected average payoffs of a two-person general-sum game and the fuzzy average of two linguistic values. It is shown that the expected average payoff is identical to the fuzzy average. A new algorithm for calculating mixed Nash equilibriums is introduced by using this concept. The new algorithm simplifies the process of finding mixed Nash equilibriums of two-person general-sum games to solving linear equations.**

Keywords- Two-Person General-Sum Game; Mixed Nash Equilibrium; Expected Average Payoff; Linguistic Variable; The Fuzzy Average; Triangular Fuzzy Number; Consequence Matrix

I. INTRODUCTION

Game theory is a branch of applied mathematics which helps scientists and engineers to analyze decision making in conflict situations. It has been used primarily in economics, in order to model competition between companies. Recently, game theory has been applied to computer networks to solve routing and resource allocation issues also in a competitive situation, especially, for wireless network [3,5,11,12,20]. The purpose of these researches is to find a useful algorithm which let network systems have high performance under limited energy consumption. These previous studies usually involved the computation of mixed Nash equilibriums; this is within the research field of computer science and applied mathematics. According to C. Daskalak is et al [4], the answer to the question *Is there an efficient algorithm for computing a mixed Nash equilibrium?* is negative. However, this paper introduces a new algorithm that simplifies the process of computing Nash equilibriums of two-person general sum games.

If a game is zero-sum, it can be solved efficiently by using linear programming. However, the linear programming approach cannot be easily extended to general-sum games because the analysis of two-person games is significantly more complex for general-sum games than for zero-sum games. When the sum of the payoffs is no longer zero (or constant), maximizing one's own payoff is no longer equivalent to minimizing the opponent's payoff. The minimax theorem does not apply to bimatrix games. One cannot expect to play "optimally" by simply looking at one's payoff matrix and guarding themselves against the worst case. Clearly, one must take into account the opponent's matrix and the reasonable strategy options of the opponent.

For a general-sum game in normal form, computing Nash equilibriums is a fundamental problem in the algorithmic game theory. In two-person game theory, the expected average payoff is defined as $p^T Aq$ for a player who has game matrix A. The expected average payoff is a function of p and q, and so we denote it as $f(p,q)$. Finding mixed Nash equilibriums is to find mixed strategies p and q where the function $f(p,q)$ reaches its maximum value. If each element in p and q is described with a probability density function (PDF for short), and the PDFs are concave functions in R, finding the maximum values of function $f(p,q)$ is equivalent to solve the following equations.

$$\partial f(p,q)/\partial p = 0, \ \partial f(p,q)/\partial q = 0.$$

However, it is usually complicated to solve the two equations, when p and q are a set of PDFs respectively because of the complexity of the differential of its function. It has been proven that finding mixed Nash equilibriums with traditional algorithms is intractable [4,14].

In this paper, instead of discussing the complexity of computing Nash equilibriums, we extend the algorithm of computing mixed Nash equilibriums for two-person zero-sum game, which was introduced in [7], to two-person general-sum games. The idea is to find the maximum value of function $f(p,q)$. However, instead of using PDFs, p and q are each represented by a set of triangular fuzzy numbers (TFNs for short). This paper includes the following sections: Section 2 gives brief reviews of the fuzzy average; Section 3 describes that an expected average payoff is identical to the fuzzy average; Section 4 describes the algorithm; Section 5 shows examples; Section 6 gives the conclusion.

II. THE FUZZY AVERAGE

The fuzzy average [8] is defined as the average of values of a linguistic variable [21]. A linguistic variable is usually defined as (x, T(x), U, G, M), where x denotes the symbolic name; T(x) is a set of linguistic values that x can take; U is the physical domain that defines certain value; G is a syntactic rule which generates the values in T(x); and M is a mapping from the set T(x) to a set of fuzzy sets which are defined on U.

DOI:10.5963/IJCSAI0302006

Let us consider the average of two values of a linguistic variable. Suppose that

$$(x, T_1(x), U_x, G_1, M_1) \text{ and } (y, T_2(y), U_y, G_2, M_2)$$

are two values of a linguistic variable

$$(x, T(x), U, G, M).\ \ T_1(x) = \{x_1, x_2, ... x_m\},\ T_2(y) = \{y_1, y_2, ... y_n\}.\ \ M_1(x): T_1(x) \to \{C_1, C_2, ..., C_m\},$$
$$M_2(y): T_2(y) \to \{D_1, D_2, ... D_n\}.$$

Where $C_i (i = 1, ... m)$, $D_j (j = 1, ..., n)$ are TFNs which are defined on U_x, U_y respectively. Without loss of generality, we suppose $U_x = U_y = [0,1]$.

The fuzzy average is defined as follows,

$$f(x, y) = \sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{C_i}(x) \times \mu_{D_j}(y) \times r_{ij} \tag{2.1}$$

where $x \in$ Ux and $y \in$ Uy; $\mu_{C_i}(x)$ and $\mu_{D_j}(y)$ are the membership functions of C_i and D_j respectively; $r_{ij} \in R$ is the element of the consequence matrix.

For given x and y, $\mu_{C_i}(x)$ and $\mu_{D_j}(y)$ are required to satisfy (2.2) and (2.3).

$$\mu_{C_i}(x) \ge 0 (i = 1, ... m),\ \sum_{i=1}^{m} \mu_{C_i}(x) = 1, \tag{2.2}$$

$$\mu_{D_j}(y) \ge 0 (j = 1, ... n),\ \sum_{j=1}^{n} \mu_{D_j}(y) = 1. \tag{2.3}$$

It has been proven that (2.2) and (2.3) are satisfied when the mapping of $M_1(x)$ and $M_2(y)$ are fuzzy uniform mapping [7].

III. THE FUZZY AVERAGE AND EXPECTED AVERAGE PAYOFFS

Consider a two-person general-sum game with a finite number of pure strategies with $m \times n$ payoff matrices $A = (a_{ij})$ and $B = (b_{ij})$ for player I and II, respectively. Let $m \times 1$ probability vector p be a mixed strategy of player I, $n \times 1$ probability vector q be a mixed strategy of player II. Nash equilibrium for such a game is a point (p_0, q_0) that satisfies the following relations [6]:

$$p_0^T A q_0 = \max_p \{p^T A q \mid \sum_{i=1}^{m} p_i = 1; p_i \ge 0\}$$

$$p_0^T B q_0 = \max_q \{p^T B q \mid \sum_{i=1}^{n} q_i = 1; q_i \ge 0\}$$

where

$$p^T A q = \sum_{i=1}^{m}\sum_{j=1}^{n} p_i a_{ij} q_j \tag{3.1}$$

$$p^T B q = \sum_{i=1}^{m}\sum_{j=1}^{n} p_i b_{ij} q_j \tag{3.2}$$

are expected average payoffs of Player I and Player II, respectively.

Theorem 3.1. For a two-person general-sum game, the fuzzy average (2.1) is identical to the expected average payoffs (3.1) or (3.2), if and only if the following is true.

(1) payoff matrix ***A*** or ***B*** is replaced with the consequence matrix (r_{ij}),

(2) p and q are replaced with $\mu_{C_i}(x)$ and $\mu_{D_j}(y)$, respectively,

(3) $\mu_{C_i}(x)$ and $\mu_{D_j}(y)$ satisfy (2.2) and (2.3), respectively.

Proof: According to the commutative and associative properties, (3.1) becomes

DOI:10.5963/IJCSAI0302006

$$\sum_{i=1}^{m}\sum_{j=1}^{n} p_i a_{ij} q_j = \sum_{i=1}^{m}\sum_{j=1}^{n} (p_i \times q_j) \times a_{ij} \tag{3.3}$$

For $\mu_{Ci}(x)$ and $\mu_{Dj}(y)$ satisfy (2.2) and (2.3), respectively, if we replace p_i with $\mu_{Ci}(x)$, q_j with $\mu_{Dj}(y)$ and matrix A with consequence matrix (r_{ij}) respectively, then (3.3) is identical to the fuzzy average (2.1). Therefore, (3.1) is equivalent to (2.1) under the conditions in this theorem. Similarly, (3.2) is identical to (2.1).

To distinguish the fuzzy averages of Player I and Player II in this paper, we use $f_A(x, y)$ and $f_B(x, y)$.

Theorem 3.2. If the following conditions are satisfied, then the fuzzy average $f_A(x, y)$ ($f_B(x, y)$) has at least one maximum value in $U_x \times U_y$.

(1) function $f_A(x, y)$ ($f_B(x, y)$) is partial differentiable in $U_x \times U_y$,

(2) there exist $x_1 \in U_x, y_1 \in U_y$ which satisfy (3.4) ((3.5)).

$$\begin{cases} \partial f_A(x, y)/\partial x = 0 \\ \partial f_A(x, y)/\partial y = 0 \end{cases} \tag{3.4}$$

$$\begin{cases} \partial f_B(x, y)/\partial x = 0 \\ \partial f_B(x, y)/\partial y = 0 \end{cases} \tag{3.5}$$

Proof: If (x_1, y_1) is the solution of (3.4), $f_A(x_1, y_1)$ has an extremum value at (x_1, y_1). On the other hand, since $\mu_{Ci}(x)$ is a continuously concave function in U_x, for any $x_1 \in U_x, x_2 \in U_x$ and $t \in [0,1]$, we have,

$$\mu_{Ci}(tx_1 + (1-t)x_2) \geq t\mu_{Ci}(x_1) + (1-t)\mu_{Ci}(x_2)$$

$$f_A(tx_1 + (1-t)x_2, y) = \sum_{i=1}^{n}\sum_{j=1}^{m} \mu_{Ci}(tx_1 + (1-t)x_2) \times \mu_{Dj}(y) \times r_{ij} \geq \sum_{i=1}^{m}\sum_{j=1}^{n} (t\mu_{Ci}(x_1) + (1-t)\mu_{Ci}(x_2)) \times \mu_{Dj}(y) \times r_{ij} =$$
$$t f_A(x_1, y) + (1-t) f_A(x_2, y).$$

Therefore, $f_A(x, y)$ is x concave. Similarly, one can prove that $f_A(x, y)$ is y concave as well. That is, $f_A(x, y)$ is a concave function in $U_x \times U_y$. Thus, $f_A(x, y)$ has a maximum value at (x_1, y_1).

The Condition (2) in Theorem 3.2 guarantees that (3.4) and (3.5) have a solution. Even $f_A(x, y)$ and $f_B(x, y)$ are partial differentiable in $U_x \times U_y$, (3.4) or (3.5) may not have a solution. The discussion of the existence of solutions of (3.4) and (3.5) exceeds the scope of this paper; thus it is not discussed here.

Theorem 3.3. If (x_0, y_0) $(x_0 \in U_x, y_0 \in U_y)$ is a solution of (3.4), (x_1, y_1) $(x_1 \in U_x, y_1 \in U_y)$ is a solution of (3.5), and (2.2) and (2.3) are satisfied at (x_0, y_0) and (x_1, y_1), respectively. Then, $(\mu_C(x_0), \mu_D(y_0))$ and $(\mu_C(x_1), \mu_D(y_1))$ are the mixed Nash equilibriums of Player I and Player II ,

where :

$$\mu_C(x_0) = (\mu_{C1}(x_0), \mu_{C2}(x_0) ... \mu_{Cm}(x_0)),$$

$$\mu_D(y_0) = (\mu_{D1}(y_0), \mu_{D2}(y_0) ... \mu_{Dn}(y_0));$$

$$\mu_C(x_1) = (\mu_{C1}(x_1), \mu_{C2}(x_1) ... \mu_{Cm}(x_1)),$$

$$\mu_D(y_1) = (\mu_{D1}(y_1), \mu_{D2}(y_1), ... \mu_{Dn}(y_1)).$$

Proof: Since (x_0, y_0) satisfies (3.4), according to Theorem 3.2, $f_A(x_0, y_0)$ is a maximum value of $f_A(x, y)$. Similarly, (x_1, y_1) satisfies (3.5); $f_B(x_1, y_1)$ is a maximum value of $f_B(x, y)$. Based on the definition of Nash equilibriums, $(\mu_A(x_0), \mu_B(y_0))$ and $(\mu_A(x_1), \mu_B(y_1))$ are the Nash equilibriums of Player I and Player II , respectively.

Some games only have pure Nash equilibriums. For example, Prisoner's Dilemma game only has a pure Nash equilibrium, in which both players defect [4]. As we mentioned before, (3.4) or (3.5) may not have a solution in $U_x \times U_y$. Thus, we

DOI:10.5963/IJCSAI0302006

introduce the following theorem.

Theorem 3.4. For a two-person general-sum game in strategic form, if the Equations (3.4) and (3.5) in Theorem 3.2 do not have solutions, then the game has at least one pure Nash equilibrium.

Proof: The method of reductio ad absurdum is used to prove this theorem. Suppose the game does not have a pure Nash equilibrium; since (3.4) and (3.5) do not have solutions, $f_A(x, y)$ and $f_B(x, y)$ do not have maximum values, which means that the game does not have mixed Nash equilibriums. A finite strategic form game neither has a pure Nash equilibrium nor a mixed Nash equilibrium, which is against Nash's theorem: every finite n-person game in strategic form has at least one mixed strategic equilibrium (Note: a pure Nash equilibrium is considered as a special mixed Nash equilibrium).

IV. THE ALGORITHM

The algorithm in this paper is the extension of the fuzzy average applying to two-person zero-sum games [7]. As mentioned in previous section, each action which is taken by a player can be mapped into a possible range in the real number set. TFNs are defined on the possible range. That is, each action which is taken by a player is mapped into a TFN. The mean values of TFNs for Player I divide the domain U_x into m-1 partitions; the mean values of TFNs for Player II divide the domain U_y into n-1 partitions.

In order to calculate (3.4) and (3.5), the fuzzy average, such as the average payoff function, is required to be differentiable in $U_x \times U_y$. However it is clear that the fuzzy average is not differentiable at the mean value of each TFN, but it can be differentiable within each partition which is divided by the mean values of TFNs.

The algorithm is as follows:

4.1. Define appropriate mappings $M_1(x)$ and $M_2(y)$.

4.2 Calculate the expected average payoff for each player.

4.3. Solve (3.4) and (3.5) in each partition and verify the correctness of each solution.

4.4. Find a maximum value in each partition.

4.5. Find the maximum value by comparing all the local maximum values.

One may have the following questions: (1) Does Nash equilibrium depend on the mapping? Namely, does the maximum value of $f_A(x, y)$ depend on the mapping? (2) Does $f_A(x, y)$ not have a maximum value at a point (c, d) where c, d is a mean value of a TFN in U_x, U_y respectively?

Firstly, the mapping M is from the set of strategies to a set of TFNs which are defined on U. The mapping only divides the domain of independent variables; it does not affect the value of expected average payoff, or the function of the independent variables. Secondly, if $f_A(x, y)$ just has a maximum value at a divided point (c, d), it is possible to miss a maximum value. However, that can be easily verified by calculating $f_A(x, y)$ at the divided point.

V. EXAMPLES

Example 1. Consider a game with the following bimatrix.

Player II

Player I $\begin{pmatrix} (4,1) & (0,3) & (2,1) \\ (1,2) & (3, 1/2) & (3/2, 2) \end{pmatrix}$

Figure 5.1 Bimatrix of Example 1

By using traditional algorithm [6], the Nash equilibrium (p^*, q^*) is $p^* = (3/7, 4/7)$ and $q^* = (x, \frac{5x}{7} + \frac{1}{7}, \frac{6}{7} - \frac{12x}{7})$ for $\forall x \in [0, \frac{1}{2}]$, the corresponding expected payoffs are,

$$v_1^* = 12/7 + 4x/7 \text{ , } v_2^* = 11/7 \text{ .}$$

When the new algorithm is used, the domain for player I is [0, 1]. The domains for player II are [0, 0.5] and [0.5, 1]. By solving (3.4), one can find the two Nash equilibriums of player I as follows,

DOI:10.5963/IJCSAI0302006

$$\begin{cases} (\mu C1(\tfrac{2}{3}), \mu C2(\tfrac{2}{3})) = (\tfrac{1}{3}, \tfrac{2}{3}) \\ (\mu D1(\tfrac{1}{4}), \mu D2(\tfrac{1}{4}), \mu D3(\tfrac{1}{4})) = (\tfrac{1}{2}, \tfrac{1}{2}, 0) \end{cases} \text{and} \begin{cases} (\mu C1(\tfrac{4}{7}), \mu C2(\tfrac{4}{7})) = (\tfrac{3}{7}, \tfrac{4}{7}) \\ (\mu D1(\tfrac{13}{14}), \mu D2(\tfrac{13}{14}), \mu D3(\tfrac{13}{14})) = (0, \tfrac{1}{7}, \tfrac{6}{7}) \end{cases}$$

By solving (3.5), one can find the two Nash equilibriums of Player II as follows,

$$\begin{cases} (\mu C1(\tfrac{4}{7}), \mu C2(\tfrac{4}{7})) = (\tfrac{3}{7}, \tfrac{4}{7}) \\ (\mu D1(\tfrac{1}{7}), \mu D2(\tfrac{1}{7}), \mu D3(\tfrac{1}{7})) = (\tfrac{5}{7}, \tfrac{2}{7}, 0) \end{cases} \text{and} \begin{cases} (\mu C1(\tfrac{4}{7}), \mu C2(\tfrac{4}{7})) = (\tfrac{3}{7}, \tfrac{4}{7}) \\ (\mu D1(\tfrac{6}{7}), \mu D2(\tfrac{6}{7}), \mu D3(\tfrac{6}{7})) = (0, \tfrac{2}{7}, \tfrac{5}{7}) \end{cases}$$

The expected payoffs of Player I and Player II are as follows.

$$fA(\tfrac{2}{3}, \tfrac{1}{4}) = 2,\ fA(\tfrac{4}{7}, \tfrac{13}{14}) = \tfrac{12}{7},\ fB(\tfrac{4}{7}, \tfrac{1}{7}) = \tfrac{11}{7},\ fB(\tfrac{4}{7}, \tfrac{6}{7}) = \tfrac{11}{7}, \text{ respectively.}$$

When solving (3.4) and (3.5) by using the new algorithm, one may encounter that (3.4) or (3.5) does not have a solution. In this case, Player I or Player II only has pure Nash equilibriums. Please refer to the following example.

Example 2. This is the Forwarder's Dilemma game.

$$\begin{array}{cc} & \text{Player II} \\ & \begin{array}{cc} F & D \end{array} \\ \text{Player I} \begin{array}{c} F \\ D \end{array} & \begin{pmatrix} (1-C, 1-C) & (-C, 1) \\ (1, -C) & (0, 0) \end{pmatrix} \end{array}$$

Figure 5.2 Bimatrix of Forwarder's Dilemma game

Where F represents forwarding and D represents to drop the packet of the other player; C is a constant which represents the energy and computation spent for the forwarding action.

The domains for Player I and Player II are [0, 1]. By using the new algorithm, one can realize that either (3.4) or (3.5) do not have a solution. As result, this game does not have a mixed Nash equilibrium. One can find a pure Nash equilibrium for this game by using the method of iterated dominance [6]. This example demonstrates the accuracy of Theorem 3.4.

Example 3. This example is cited from [11] with minor modification, which is originally represented in [20].

$$\begin{array}{cc} & \text{Player II} \\ & \begin{array}{ccc} T & L & S \end{array} \\ \text{Player I} \begin{array}{c} T \\ L \\ S \end{array} & \begin{pmatrix} (Pf, \overline{P}f) & (Ps, \overline{Ps}) & (Pf, \overline{P}i) \\ (Ps, \overline{P}s) & (Pi, \overline{P}i) & (Pi, \overline{P}w) \\ (Pi, \overline{P}f) & (Pw, \overline{P}i) & (Pw, \overline{P}w) \end{pmatrix} \end{array}$$

Figure 5.3 Bimatrix of Wireless Sensor Network

T, L, and S represent Transmitting, Listening and Sleeping, respectively; Pi and $\overline{P}i$ are the payoffs for Players I and II when they are listening; Ps and $\overline{P}s$ when they are transmitting a data packet successfully, Pf and $\overline{P}f$ when they have failed to transmit a data packet, and Pw and $\overline{P}w$ when they are in sleep mode, respectively. In this example, we assume that all the elements of the bimatrix are constants, which are defined as follows.

$$Pf(-4) < Pi(1) < Pw(2) < Ps(4) \text{ and } \overline{P}f(-4) < \overline{P}i(1) < \overline{P}w(2) < \overline{P}s(4).$$

Then the matrices for Player I and Player II are as follows.

$$A = \begin{pmatrix} -4 & 4 & -4 \\ 4 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix};\ B = \begin{pmatrix} -4 & 4 & 1 \\ 4 & 1 & 2 \\ -4 & 1 & 2 \end{pmatrix}$$

DOI:10.5963/IJCSAI0302006

The domain of Player Ⅰ same as Player Ⅱ is [0, 0.5] and [0.5, 1]. By solving (3.4) with the new algorithm in each domain, one can find the following Nash equilibriums.

$$\begin{cases}(\mu_{C1}(4/11), \mu_{C2}(4/11), \mu_{C3}(4/11)) = (3/11, 8/11, 0)\\(\mu_{D1}(4/11), \mu_{D2}(4/11), \mu_{D3}(4/11)) = (3/11, 8/11, 0)\end{cases} \tag{1}$$

$$\begin{cases}(\mu_{C1}(x), \mu_{C2}(x), \mu_{C3}(x)); \forall x \in [0, 0.5]\\(\mu_{D1}(11/16), \mu_{D2}(11/16), \mu_{D3}(11/16)) = (0, 5/8, 3/8)\end{cases} \tag{2}$$

$$\begin{cases}(\mu_{C1}(7/8), \mu_{C2}(7/8), \mu_{C3}(7/8)) = (0, 1/4, 3/4)\\(\mu_{D1}(3/8), \mu_{D2}(3/8), \mu_{D3}(3/8)) = (1/4, 3/4, 0)\end{cases} \tag{3}$$

The average payoff of Player Ⅰ on the above points are $f_A(4/11, 4/11) = 1.82$, $f_A(x, 11/16) = 1.0; (\forall x \in [0, 0.5])$, and $f_A(7/8, 3/8) = 1.75$ respectively. There are three Nash equilibriums of Player Ⅰ. They can be interpreted as the following in this case.

Regarding (1), when all players have their transmission probability as $3/11$, and their listen probability as $8/11$, Player Ⅰ can reach a maximum payoff.

Regarding (2), when Player Ⅱ's (other nodes) listen probability is $5/8$, and sleep probability is $3/8$, Player Ⅰ either transmits or listens, and thus Player Ⅰ can get a maximum payoff.

Regarding (3), when Player Ⅱ's (other nodes) transmission probability is 0.25, and listen probability is 0.75, Player Ⅰ's listen probability is 0.25, and sleep probability is 0.75; thus Player Ⅰ reaches a maximum payoff.

Similarly, by using the new algorithm to solve (3.5) in each domain, one can get the following Nash equilibriums.

$$\begin{cases}(\mu_{C1}(4/11), \mu_{C2}(4/11), \mu_{C3}(4/11)) = (3/11, 8/11, 0)\\(\mu_{D1}(4/11), \mu_{D2}(4/11), \mu_{D3}(4/11)) = (3/11, 8/11, 0)\end{cases} \tag{4}$$

$$\begin{cases}(\mu_{C1}(3/8), \mu_{C2}(3/8), \mu_{C3}(3/8)) = (1/4, 3/4, 0)\\(\mu_{D1}(7/8), \mu_{D2}(7/8), \mu_{D3}(7/8)) = (0, 1/4, 3/4)\end{cases} \tag{5}$$

$$\begin{cases}(\mu_{C1}(11/16), \mu_{C2}(11/16), \mu_{C3}(11/16)) = (0, 5/8, 3/8)\\(\mu_{D1}(1/2), \mu_{D2}(1/2), \mu_{D3}(1/2)) = (0, 1, 0)\end{cases} \tag{6}$$

The average payoff of Player Ⅱ on the above points are $f_B(4/11, 4/11) = 1.82$, $f_B(3/8, 7/8) = 1.75$, and $f_B(11/16, 1/2) = 1.0$. The three Nash equilibriums can be interpreted as follows.

(4) is same as (1). Regarding (5), when Player Ⅰ's transmission probability is 0.25, and listen probability is 0.75, Player Ⅱ's listen probability is 0.25, and sleep probability is 0.75, then Player Ⅱ reaches a maximum payoff. Regarding (6), when layer Ⅰ's listen probability is $5/8$, and sleep probability is $3/8$, Player Ⅱ's listening probability is 1, and then Player Ⅱ can reach a maximum payoff.

In this example, (3) and (5) tell us that when an opponent player's transmission probability 0.25, and listen probability is 0.75, the player himself shall have listen probability 0.25, and sleep probability 0.75.

The expected average payoff is used as the utility function in this example. The conditional collision probability of Player Ⅱ, the throughput and the transmit power are not explicitly represent in the payoff in this example, because the purpose of this example is to describe how the new algorithm works.

VI. CONCLUSION

This paper has shown that the expected average payoff for each player is equivalent to the fuzzy average under the conditions in Theorem 3.1. A new algorithm for calculating mixed Nash equilibriums for two-person general-sum games has been introduced. From the examples in Section 5, one can perceive that instead of using PDFs to represent distribution profiles, when the distribution profiles are expressed with sets of TFNs, the new algorithm can be used as it is more efficient and simpler. Therefore, the new algorithm has exchanged the problem of finding Nash equilibriums to simply solving linear equations in partition domains.

The application of the fuzzy average to two-person general-sum games can be extended to n-person non-cooperative games.

DOI:10.5963/IJCSAI0302006

This will be discussed in future studies. By using computers, the new algorithm can solve large scale problems in game theory. This will be discussed in future studies as well. Furthermore, one study to be conducted in the future is to prove that the new algorithm is P-complete.

REFERENCES

[1] J.P. Aubin *Mathematical Methods of Game and Economic Theory*, North-Holland, Amsterdam, 1979.

[2] B. Arfi, "Linguistic Fuzzy Logic Methods in Social Sciences", STUDFUZZ 253, pp. 63-103.

[3] T. Dam, and K. Langendoen, "An Adaptive Energy-Efficient MAC Protocol for Wireless Sensor Networks", SenSys, CA, 2003.

[4] C. Daskalakis, P. Goldberg and, C. Papadimitriou, "The Complexity of Computing a Nash Equilibrium", SICOMP, 2008.

[5] M. Felegyhazi and J.P. Hubaux, "Game Theory in Wireless Networks: A Tutorial", EPFL Technical Report: LCA-REPORT-2006-002, 2006.

[6] T.S. Ferguson, *Mathematical Statistics- A Decision-Theoretic Approach*, Academic Press, New York, 1967.

[7] L. Gao, "The discussion of Applications of the Fuzzy Average to Matrix Game Theory", The proceeding of CCECE2012, 2012.

[8] L. Gao, "The Fuzzy Arithmetic Mean", *Fuzzy Sets and Systems*, 107, pp. 335-348, 1999.

[9] L. Gao, "Studies on Application of Fuzzy Averaging in Numerical Analysis", PhD thesis, Chiba University, Japan, 1995.

[10] L. Gao and H. Kawarada, "Application of Fuzzy Average to Curve and Surface Fitting", Proceeding of FUZZ-IEEE/IFES'1995.

[11] S. Mehta and K.S. Kwak, "Application of Game Theory to Wireless Networks", *Convergence and Hybrid Information Technologies, Marius Crisan (Ed.), ISBN: 978-953-307-068-1*, pp361, 2010.

[12] F. Meshkati, M. Chiang, H.V. Poor and S.C. Schwartz, "A Game-Theoretic Approach to Energy-Efficient Power Control in Multicarrier CDMA System", *IEEE. J. Sel. Areas Commun., vol.24, no.6, pp1115-1129*, 2006.

[13] J. Nash, "Noncooperative Games", Annals of Mathematics, 54, pp289-295, 1951.

[14] N. Nisan, T. Roughgarden, E. Tardos and V.V. Vazirani, *Algorithmic Game Theory*, Cambridge University Press, 2007.

[15] S.A. Orlovski, "Fuzzy Goals and Sets of Choices in Two-Person Games", *Multiperson Decision Making Models using Fuzzy Sets and Possibility Theory*, Kluwer Academic Publishers, Dordrecht, pp. 288-297, 1990.

[16] F. Pelschus and E.K. Zavadskas, "Fuzzy Matrix Games Multi-Criteria Model for Decision-Making in Engineering", INFORMATICA, Vol. 16, No. 1, pp107–120, 2005.

[17] S.H. Wu and V.W. Soo, "Fuzzy Game Theoretic Approach to Multi-Agent Coordination", T. Ishida: PRIMA'98, LNAI1599, pp. 76-87, 1999.

[18] W. Ye, K. Heidemann and D. Estrin, "An Energy-Efficient MAC Protocol for Wireless Sensor Networks", INFOCOM, pp.1567-1576, 2002.

[19] J. Zhao, "The Equilibria of a Multiple Objective game", *Int. J. Game Theory*, 20, pp. 171-182, 1991.

[20] L. Zhao, H. Zhang and J. Zhang, "Using Incompletely Cooperative Game Theory in Wireless Sensor Networks", In proceeding of IEEE WCNC 2008, pp. 1483-1488, 2008.

[21] H.J. Zimmermann, *Fuzzy Sets, Decision Making, and Expert Systems*, Kluwer Academic, Boston, 1986.

[22] H.J. Zimmermann, *Fuzzy Set Theory and Its Application,* 2nd, Kluwer Academic Publishers, Boston, 1991.

Lunshan GAO, received a Mathematics diploma from LuDong University, Yantai, China in 1980. He received a Master's degree in Electronic and Computer Engineering from Yokohama National University, Yokohama, Japan in 1989, and a Ph.D. in Applied Mathematics from Chiba University, Chiba, Japan in 1995. From 1995 to 2003, he worked at Fujitsu Inc, HITACHI Inc., and WindRiver Japan Inc. He immigrated to Canada in 2004. Since 2007, he has been working at THALES CANADA Inc. His research interests include: mathematical modeling, algorithms, artificial intelligent and software engineering. He is an IEEE member.

DOI:10.5963/IJCSAI0302006