

On the Negative Index Perfect Lens with Loss

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Abstract- We revisit Pendry's theory of the "perfect lens" made of a negative index material (NIM) slab and explicitly treat the situation of a lossy NIM. We show that the usual boundary conditions prohibit the situation of ideal lossless NIM where $\epsilon = -1$ and $\mu = -1$. We further demonstrate that at the boundary of air and lossy NIM, all input evanescent waves will be refracted into decaying propagating waves inside the NIM, with different spatial frequency components having different propagation directions. Due to the different and non-zero real part of the propagation constant inside the lossy NIM, these high spatial frequency components will not all contribute constructively to the image formed inside the lossy NIM. Accordingly, the information contained in these high spatial frequency components will be lost, making sub-diffraction-limited imaging impossible inside the lossy NIM and in turn for the lossy NIM slab lens' external image plane.

Keywords- Negative Refraction; Perfect Lens; Electro-Magnetic Boundary Conditions

I. INTRODUCTION

Pendry's seminal paper [1] on the perfect lens made of a slab of negative index medium with $\epsilon = -1$ and $\mu = -1$ sparked an exciting new research field in negative index materials (NIM) and, more generally, metamaterials (MM). These unnatural materials have great promise in many breakthrough applications such as super-resolution imaging and invisibility cloaking. In particular, low-loss NIM with $\epsilon = -1$ and $\mu = -1$ have been heavily researched in the hope of developing a far field optical imaging system that beats the diffraction limit. Initially, Pendry's surprising result that a passive slab of NIM can amplify all evanescent waves was hotly debated [2–10]. 't Hooft commented that Pendry might have used incorrect arguments to arrive at the otherwise correct conclusions [3]. 't Hooft did, however, point out a puzzling fact of Pendry's conclusion that, due to the amplification of the evanescent waves, the amplitude of the electric field can grow extremely large and, it "can easily reach values beyond the breakdown of any material". In his reply [6], Pendry argued that the approach he took in his original paper is "in accord with multiple scattering theory as documented in standard textbooks". As for the puzzling point of a possibly infinitely large energy density, Pendry argued that since evanescent waves do not transport energy, energy conservation is not violated by amplification of evanescent waves. Although there is no recent record showing this conversation continues, Pendry's reply to the diverging energy density issue is worthy of closer scrutiny. This is because, although evanescent waves do not contribute to energy flow, they do store electromagnetic field energy. In the ideal case with no losses then the electro-magnetic field energy density due to the presence of these evanescent waves does grow exponentially. The fact that evanescent waves do store energy was also pointed out by Williams [2]. Williams has several other issues with the Pendry's theory but all have been well-addressed by Pendry's reply [6]. Garcia and Nieto-Vesperinas [4, 5, 10] argued that although there is amplification of evanescent waves in the ideal lossless, dispersion-less NIM, it still does not make a perfect lens since the effect as a result of the finite thickness of the slab prevents image forming. Furthermore, Garcia and Nieto-Vesperinas [4, 10] pointed out that any loss would dramatically diminish the evanescent wave amplification effect and instead change it to decay, which is now widely accepted of course. Pendry himself addressed the lossy NIM issue and concluded that the loss may indeed pose a severe limitation to the image resolution and hence also to the thickness of the NIM slab lens, limiting it to only wavelength scale with any practical material loss [11]. Much of the substance of these early debates was pushed into the background, especially after experimental results, notably by Liu, et al. [12, 13], indicated support for the amplification of evanescent waves in a silver slab as originally suggested in Pendry's paper. In 2008, we raised another concern with Pendry's theory [14]. The issue was centred on the (usual) electromagnetic boundary conditions, namely, the tangential components of both the electric and magnetic field have to be continuous across the interface. We can show that, while these normal boundary conditions are applied in all NIM discussions, they are self-contradictory when evanescent waves are refracted at the interface between air and an NIM with $\epsilon = -1$ and $\mu = -1$. Admittedly, discussing a pure theoretical case of an ideal NIM may have limited practical value, but nevertheless, a satisfactory resolution to this self-contradictory boundary condition issue could be expected to provide critical insights for practical cases where ϵ and μ deviate from the ideal value of -1.

In this paper, we first elaborate on this inconsistency in Pendry's theory where normal boundary conditions can lead to a set of self-contradictory conditions in the limit as ϵ and μ tend to -1. Next we reconsider the transfer of evanescent waves at an interface of air and NIM under the assumption that the NIM is necessarily dispersive and lossy [15]. We argue that the

different wave solutions in air and NIM must maintain the continuity of the reflection and transmission coefficients when the tangential component of the k vector (i.e. k_x) approaches the propagation constant $k_0 = (\epsilon\mu)^{1/2} \omega / c$ from either $k_x - k_0 < 0$ or $k_x - k_0 > 0$. We show that under these conditions, all evanescent waves in air will be refracted into decaying *propagating* waves inside the lossy NIM, with different spatial frequency components having different propagation directions. Accordingly, a finite size slab will not be able to support infinite multi-path interference in the slab and there will be no significant transport of the original evanescent waves through the NIM slab. As a result, high spatial frequency information contained in evanescent waves is lost and no sub-diffraction-limited imaging would be expected. In light of this conclusion, we maintain that previous experimental verifications of Pendry's theory for sub-diffraction-limited optical imaging can only be explained by alternative theories other than Pendry's perfect lens theory, such as the theory put forward by Zhou et al [16].

II. REFRACTION AT AIR AND NIM INTERFACE

Fig. 1 shows the refraction of propagating waves between normal media (Fig. 1a) and between normal media and NIM (Fig. 1b). Snell's law of refraction requires that the tangential component of the propagation vector $\mathbf{k}$ is continuous at the interface. Assuming that the interface between air ($z < 0$) and NIM ($z > 0$) is parallel to the x axis and the $(x-z)$ plane is the principal plane that contains the propagation vector $\mathbf{k} = (k_x, k_z)$ and a surface normal to the interface. When $k_x < |\mathbf{k}|$, k_z is a real number and that plane waves $e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$ are propagating waves. When $k_x > |\mathbf{k}|$, k_z is an imaginary number and plane waves $e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$ are evanescent waves. In general, when k_z is a complex number, plane waves $e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$ are non-uniform waves with non-orthogonal equal phase and equal amplitude planes.

Without loss of generality, we will limit our discussion to the case of S polarization. The treatment of P polarization is similar and straightforward and the conclusion is the same. We start with the input evanescent waves in air, whose electric field is given by,

$$E_{0S+} = [0, 1, 0] \exp(ik_z z + ik_x x - i\omega t), \quad (1)$$

where the wave vector

$$k_z = +i\sqrt{k_x^2 - k_0^2}, \quad k_0^2 = \omega^2 c^{-2} < k_x^2. \quad (2)$$

The electric field of the reflected light, following Pendry's notation [1], is given by,

$$E_{0S-} = r[0, 1, 0] \exp(-ik_z z + ik_x x - i\omega t), \quad (3)$$

where r is the reflection coefficient. The transmitted electric field is given by,

$$E_{1S+} = t[0, 1, 0] \exp(ik'_z z + ik_x x - i\omega t), \quad (4)$$

where

$$k_x^2 + k'^2_z = (\epsilon' + i\epsilon'')(\mu' + i\mu'')k_0^2 \quad (5)$$

and we assume the NIM is necessarily dispersive and lossy ($\epsilon'' \neq 0$ and $\mu'' \neq 0$). Note that for time dependence of the form $\exp(-i\omega t)$ chosen here, the imaginary part of k'_z needs to be positive for waves decaying away from the interface. Namely, if we denote

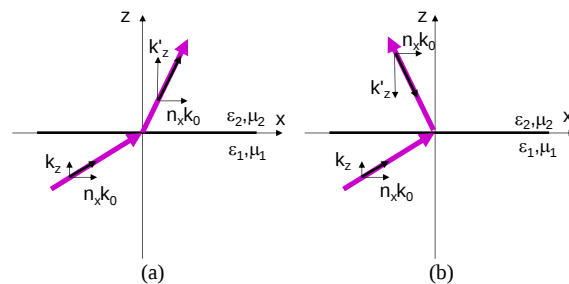


Fig. 1 Refraction of propagating waves between normal media (a), and between normal media and NIM (b)

$$k'_z = (n_z + i\alpha_z)k_0, \quad (6)$$

where n_z and α_z are both real numbers, one has $\alpha_z \geq 0$ for passive materials.

On the other hand, one has from Eq. 5,

$$2n_z\alpha_z = \epsilon'\mu'' + \epsilon''\mu' \quad (7)$$

for NIM, $n_z \leq 0$. Accordingly, for NIM, $\epsilon'\mu'' + \epsilon''\mu' \leq 0$. Given for NIM $\epsilon' < 0$ and $\mu' < 0$, one has for passive NIM, $\epsilon'' \geq 0$ and $\mu'' \geq 0$. Similarly for normal passive positive index materials, $\epsilon'' \geq 0$ and $\mu'' \geq 0$.

The usual boundary conditions at the air and NIM interface are $E_{0y} = E_{1y}$ and $B_{0x} = B_{1x} / (\mu' + i\mu'')$, where E and B are the electric and magnetic fields, related by the Maxwell's equation $\nabla \times \mathbf{E} = i\omega\mathbf{B}$. Accordingly, one has at the interface, respectively,

$$1 + r = t \quad (8)$$

and,

$$k_z(1 - r) = tk'_z / (\mu' + i\mu'') \quad (9)$$

Obviously, if k_z is not zero, then t cannot be zero. Since if $t = 0$, one has from Eq. 8, $1 + r = 0$, or $r = -1$. But this results in $2k_z = 0$ from Eq. 9, which conflicts with the condition that k_z is not zero.

For lossless a NIM in the limit $\epsilon \rightarrow -1$ and $\mu \rightarrow -1$, one has that $k'_z \rightarrow k_z$. As a result, boundary conditions Eq. 8 and Eq. 9 become self-contradictory when the input field is not zero, reducing to $1 + r = t$ and $1 - r = -t$. This self-contradiction of the usual physical boundary conditions can be avoided if ϵ or μ has a non-vanishing imaginary part, for example, as is the case for a lossy NIM.

For a lossy NIM, from Eq.7 one can see that for $\epsilon'' \neq 0$ and $\mu'' \neq 0$, one has $n_z\alpha_z \neq 0$ unless the real parts of ϵ and μ are both zero. When the real parts of ϵ and μ are not simultaneously zero, the real and imaginary part of k'_z are both non-zero. As a result, the evanescent fields in air will be transferred inside the NIM into decaying non-uniform waves. Eq. 5 also suggests that evanescent fields with different spatial frequency k_x will have different n_z and, accordingly, be transferred into decaying propagating waves having different directions of propagation inside the NIM. The output angle θ_t of the refracted wave is given by,

$$\tan \theta_t = \frac{n_x}{n_z} \quad (10)$$

where $n_x = k_x / k_0 > 1$ for evanescent waves. For NIM we have that $n_z < 0$ and $\theta_t < 0$, which is characteristic of negative refraction.

Fig. 2 shows the refracted angle inside the NIM for the special case of a passive NIM where $\epsilon = -1 + i\delta$, $\delta > 0$, and $\mu = -1$. Different curves in Fig. 2 represent different values of δ of 0.01 (dashed curve), 0.1 (solid curve), and 0.2 (dash-dot curve) respectively. When $n_x < 1$, the input waves are propagating plane waves in air. When $n_x > 1$, the input waves are evanescent waves in air. It is obvious that all input waves in air, including all evanescent waves, will be transferred into propagating waves with the negative refraction angle $\theta_t < 0$. However, due to the non-zero n_z inside lossy NIM for all input evanescent waves, they will reach the image plane inside the NIM all with different phases, confusing their contribution to the image inside the NIM.

For the case of a lossy NIM slab lens, the previous treatment given by Smith et. al. [11] has shown that the loss will significantly impair the performance of the perfect lens, practically limiting the thickness of the NIM slab to a small fraction of a wavelength. While this conclusion is theoretically true, it does not take into account the finite size of the NIM slab lens. We have shown here, given the large negative refraction angles for the high spatial frequency evanescent waves, the implied assumption of multi-path interference inside the NIM slab as used in Ref. [11] may not be valid. This is similar to the situation where a Fabry-Parot cavity can not contain all of the waves that are slanted at oblique angles to the interfaces. Accordingly,

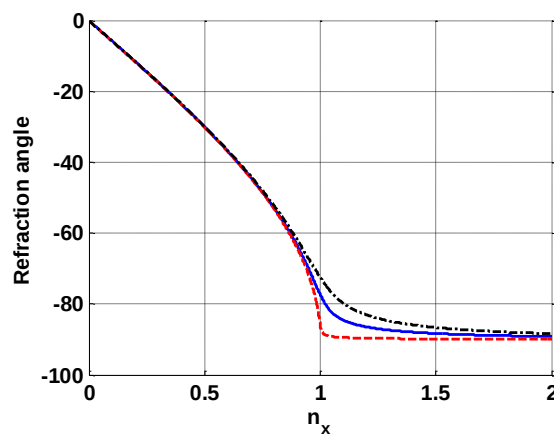


Fig. 2 Output angle of the refraction of propagating waves from air into a passive and lossy NIM. Different curves are for different loss tangent δ of the relative permittivity $\epsilon = -1 + i\delta$ and $\mu = -1$. dashed curve: $\delta = 0.01$; solid curve: $\delta = 0.1$; dash-dot curve: $\delta = 0.2$

given the fixed thickness and finite size of an NIM slab, the non-zero real part of the propagation constant inside the NIM will result in different optical phase accumulation for waves having different spatial frequencies across the NIM slab. Since evanescent waves in air do not acquire optical phase along the z direction, these components will also necessarily reach the image plane behind the NIM slab with different phases. As a result, the lossy NIM slab lenses are not expected to refocus these evanescent waves to a perfect image as the information contained in these evanescent waves is most likely to be lost. For those spatial frequencies that are not lost, there is the possibility of quantifying the evanescent wave transfer into these decaying but propagating waves. This might make a computational reconstruction of an image which retains some sub-wavelength scale features still possible.

III. CONCLUSIONS

In conclusion, we have argued that under the assumption that an NIM is necessarily dispersive and lossy, all evanescent fields in air that impinge on an NIM boundary will be transferred into decaying propagating waves inside the NIM. Furthermore, evanescent waves with different spatial frequencies will also have different propagation directions inside the NIM. Accordingly, under these circumstances, the lossy NIM slab does not refocus the evanescent waves to the image plane inside the NIM or behind the NIM slab lens. As a result, much information about the higher resolution features in the object contained in the evanescent waves will be lost in the image plane and there will be limited super-resolution imaging effects. Previously reported experimental observations of sub-diffraction-limit imaging employing silver or other metallic material slabs need to be interpreted by a different theory, for example, involving the coupling between surface plasmonic states of the front and back metallic surfaces [16], [17], or, since the lens is thin, thinner than the skin depth, through a transfer of the wavefront across the slab as suggested in [16].

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