

# Buckling Verification of Laminated Glass Elements in Compression

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**Abstract-** Because of the characteristic of high slenderness ratios, monolithic and laminated glass elements are frequently subjected to buckling phenomena. As regards laminated glass beams and panels, in particular, the effects of possible temperature or time-load variations represent only some aspects that make complex their global structural response. In this context, the paper focuses on the load-carrying behavior of in-plane compressed laminated glass elements. In it, some analytical formulations are presented to describe realistically their typical behavior. As shown, the proposed formulations are in good agreement with experimental and numerical data available in literature. At the same time, they allow to perform a rational buckling verification of such brittle load-bearing elements. Finally, according to the suggestions the Eurocodes give for the verification of traditional structural elements, a series of buckling curves opportunely calibrated are proposed to guarantee the requisites of resistance, serviceability, and durability typically imposed in the design of conventional structural systems.

**Keywords-** *Laminated Glass; In-plane Compressed Beams and Panels; Sandwich Elements; Equivalent Thickness; Buckling Curves*

## I. INTRODUCTION

The use of monolithic or laminate glass (LG) elements in modern and innovative architectural applications showed a strong increase in the last years. Because of aesthetic, lighting, and architectural advantages, glass elements are frequently used as structural components able to sustain loads. However, the real capabilities of such innovative bearing components are currently not well known and several aspects related to their typical load-carrying behavior are very complex to evaluate. The load-carrying capacity of LG beams or panels, for example, strongly depends on the degradation of the mechanical properties of the interlayer, as well as on the presence and the amplitude of possible imperfections, or the presence of additional external loads.

In this context, several authors observed that temperature variations could strongly influence the mechanical properties of the thermoplastic materials usually adopted to bond together the glass panes [1, 2, and 3]. Recently, numerous authors focused on the buckled response of structural glass elements in several boundary or loading conditions. Luible [4], for example, investigated the load-carrying behavior of LG beams or panels and performed experimental tests to analyze their typical response. Belis [5] studied the lateral-torsional buckling of LG beams. In [6, 7], the authors proposed an exact analytical approach for the buckling verification of LG beams in compression or in out-of-plane bending. The behavior of LG panels under in-plane

compression or shear has been deeply examined in [8, 9, and 10].

Rationally, a realistic buckling verification of LG compressed beams and panels should be performed by contemporarily satisfying a series of requisites concerning the resistance, the serviceability, and the durability of such brittle elements. At the same time, the effective connection offered by the interlayer should be precisely estimated, since strongly time-load and temperature dependent. Nevertheless, consolidate verification criteria available in literature for the buckling verification of traditional structural elements (steel, for example [11]) cannot be directly applied to LG elements. Because of this reasons, in the paper some analytical formulations are proposed for the buckling verification of in-plane compressed LG beams and panels. The aim is to derive simple and consistent design rules for pane-like glass columns with laminated sections subjected to axial compressive loads. As shown, these analytical models are in good agreement with numerical and experimental data.

In addition, according to the suggestions of Eurocodes 3, 4, 5 [11, 12, and 13], a series of buckling curves opportunely calibrated are proposed to guarantee the requisites of resistance, serviceability and durability typically required in the design of conventional structural systems. As proposed in the following sections, these buckling curves are in good agreement with experimental data collected in literature, as well as with numerical results.

## II. ANALYTICAL MODEL FOR COMPRESSED LAMINATED GLASS BEAMS

The analysis of compressed laminated glass beams is generally performed by using the elastic theory of sandwich elements [14]. Recently, based on these sandwich formulations, an equivalent thickness approach has been proposed to study the behavior of a monolithic compressed element characterized by a rectangular cross section of thickness  $t_{eff}$  [4]. As a result, the flexural behavior of the composite beam is described through the classical theory of deflection. This analytical formulation undoubtedly constitutes a suitable design method for compressed LG elements, but does not allow understanding how, depending on variations of temperature and load duration, the mechanical properties of the interlayer can influence the global response of the layered element.

In this last years, also Blaauwendraad [15] proposed a simplified formulation able to easily control the transition

between the *layered limit* (*abs*, absence of connection between the glass sheets) and the *monolithic limit* (*full*, presence of a rigid connection between the glass sheets). By means of a dimensionless parameter representative of the

effective level of connection between the glass panes, Blaauwendraad suggests also a unity check for tensile stresses. Also in this circumstance, the formulation is simple, but provides only approximate results.

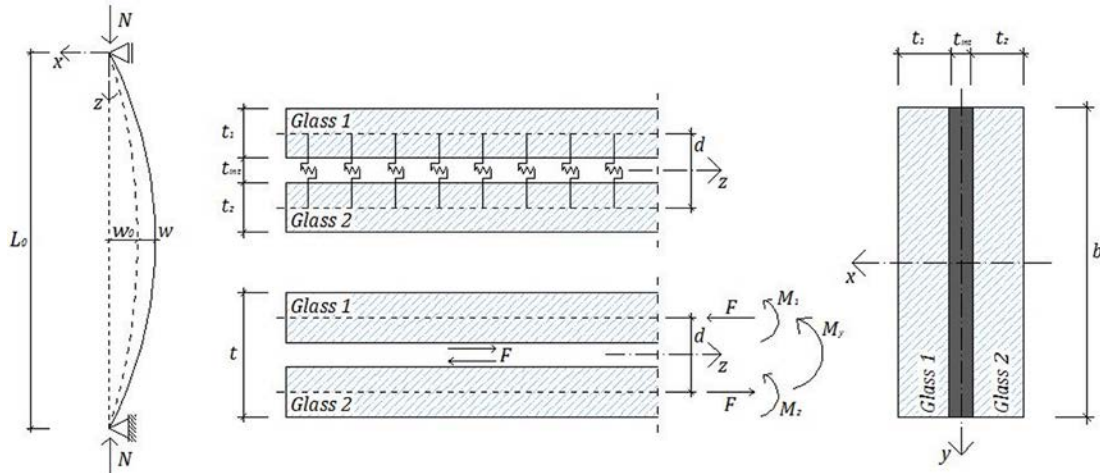


Fig. 1 Analytical model for compressed LG beams

The alternative [6], consists in an analytical model based on the original elastic theory of Newmark et al. [16], concerning the flexural behavior of 2-layer composite beams with deformable connection. As shown in [6], the advantage of this new formulation consists in the possibility of taking into account the real thickness of the layers constituting the LG beam. At the same time, the proposed formulation allows taking into account the effective level of connection offered by the interlayer, thus it can be used to estimate the buckled response of a compressed LG beam in a well-defined condition of temperature and load duration. Although the model applies only to 2-layer structural systems, that is to beams consisting of two interacting glass sheets, tied together by a shear connection able to transfer the horizontal shear from one element to the other, it provides accurate results.

In this context, let us consider a 2-layer LG beam, pinned at the ends of its buckling length  $L_0$ , subjected to an axial compressive load  $N$  (Fig. 1). The beam, having a rectangular cross section (width  $b$ ), is assumed to be constituted by two external glass layers (thicknesses  $t_1$ ,  $t_2$ , elastic Young's modulus  $E$ , shear modulus  $G$ ) and a middle interlayer (thickness  $t_{int}$ , shear modulus  $G_{int}$ ), and affected by a sinusoidal imperfection of maximum amplitude  $w_0$ . An extended experimental campaign performed by Belis et al. [17] on 312 glass beams with variable length, height, thickness, glass type, recently highlighted that the sinusoidal shape describes the initial imperfection in monolithic or laminated glass beams with a good level of accuracy.

The transversal displacement  $w(z)$  of the simply supported beam [ $w(0) = w(L_0) = 0$  and  $w''(0) = w''(L_0) = 0$ ] due to the axial compression  $N$  is [6]:

$$w(z) = \frac{(\alpha^2 EJ_{abs} L_0^2 + EJ_{full} \pi^2) L_0^2 N w_0 \sin(\pi z / L_0)}{\alpha^2 EJ_{abs} L_0^2 (EJ_{full} \pi^2 - NL_0^2) + EJ_{full} \pi^2 (EJ_{abs} \pi^2 - NL_0^2)} \quad (1)$$

with:

$$\alpha^2 = \frac{K}{EA^*} \frac{EJ_{full}}{EJ_{abs}}, \quad (2)$$

$$K = \frac{G_{int} b}{t_{int}} \quad (3)$$

$$EA^* = \frac{(EA_1)(EA_2)}{EA_1 + EA_2} = \frac{Ebt_1 t_2}{t_1 + t_2} \quad (4)$$

the equivalent axial stiffness,

$$EJ_{full} = EJ_{abs} + Eb \left[ t_1 \left( \frac{t_1 + t_{int}}{2} \right)^2 + t_2 \left( \frac{t_2 + t_{int}}{2} \right)^2 \right] \quad (5)$$

the *monolithic* flexural stiffness,

$$EJ_{abs} = \frac{Eb}{12} (t_1^3 + t_2^3) \quad (6)$$

the *layered* flexural stiffness.

Assuming for the initial sine-shape imperfection of the composite beam a maximum amplitude  $w_0$ , its total maximum deflection is:

$$w_{max} = w_0 + w \left( \frac{L_0}{2} \right), \quad (7)$$

and the corresponding tensile stress  $\sigma_{max}$  can be estimated as:

$$\sigma_{max} = -\frac{N}{A} + \frac{Nw_{max}}{W_y} + \frac{F}{A_{min}}. \quad (8)$$

In Eq. (8),  $F$  is the axial load acting in each glass pane, due to the flexural deflection of the beam:

$$F = \frac{Nw_{max} + \chi_{max} EJ_{abs}}{d}; \quad (9)$$

$d$  represents the distance between the centroidal axis of each glass pane (Fig. 1);  $\chi_{max}$  is the midspan curvature of the beam corresponding to the deflection  $w_{max}$ :

$$\chi_{\max} = -\frac{\pi^2}{L_0^2} w_{\max}; \quad (10)$$

$W_y$  is the elastic resistant modulus of the total cross section (thickness  $t = t_1 + t_{int} + t_2$ ):

$$W_y = \frac{bt^2}{6}; \quad (11)$$

and  $A_{\min}$ , with  $t_{\min} = \min(t_1, t_2)$ , is the minimum transversal area of each glass pane:

$$A_{\min} = bt_{\min} \quad (12)$$

Eq. (7) represents useful information, since it allows describing the load  $N$ -transversal displacement  $w$  of a generic LG beam in compression, by taking into account the effective shear stiffness  $G_{int}$  of the interlayer and the presence of possible sinusoidal imperfections  $w(z) = w_0 \sin(\pi z/L_0)$ . If the compression  $N$  gradually increases, the loss of stability of the beam typically shows in the form of an abrupt and non-proportional increase of the corresponding displacement  $w_{\max}$ .

Consequently, Eq.(7) can be used to express the critical buckling load  $N_{cr}^{(E)}$  of the LG beam, assumed as the asymptotical value  $N$  to which the growing displacement  $w_{\max}$  tends:

$$N_{cr}^{(E)} = \frac{\pi^2 EJ_{abs} EJ_{full}}{L_0^2} \left( \frac{\alpha^2 L_0^2 + \pi^2}{\alpha^2 EJ_{abs} L_0^2 + EJ_{full} \pi^2} \right). \quad (13)$$

Depending on the shear modulus  $G_{int}$  of the material constituting the interlayer, the critical load  $N_{cr}^{(E)}$  of a generic LG beam is always comprised between the well-known limit values  $N_{cr,full}^{(E)}$  (*monolithic limit*,  $G_{int} \rightarrow \infty$ , that is  $\alpha \rightarrow \infty$ ) and  $N_{cr,abs}^{(E)}$  (*layered limit*,  $G_{int} \rightarrow 0$ , that is  $\alpha \rightarrow 0$ ):

$$N_{cr,full}^{(E)} = \frac{\pi^2 EJ_{full}}{L_0^2}, \quad (14)$$

$$N_{cr,abs}^{(E)} = \frac{\pi^2 EJ_{abs}}{L_0^2}. \quad (15)$$

This finding constitutes an important aspect in the analysis of LG elements, sine the material commonly used to bond together the glass sheets (PVB Butacite®, SG®, EVA, etc.) consists in thermoplastic materials strongly temperature and load time-dependent. PVB-films, in particular, have good mechanical properties if subjected to room temperatures or short-term loads, but present a strong degradation of shear stiffness with high temperatures and long-term loads [18].

In Fig. 2, for example, the effects of stiffness degradation on the value of the critical buckling load  $N_{cr}^{(E)}$  are proposed for a PVB-laminated glass beam (5/1.52/5mm) having dimensions  $b=200\text{mm} \times L_0=3000\text{mm}$ . In particular,  $N_{cr}^{(E)}$  is evaluated by means of Eq. 13 by assuming  $G_{int}$  a value comprised in the range  $10^{-4} \text{ N/mm}^2 < G_{int} < 10^4 \text{ N/mm}^2$ . In

the figure, also three specific values of  $N_{cr}^{(E)}$  are highlighted. These critical loads are evaluated in presence of short-term loads (3 seconds) and room/medium temperatures ( $T=20^\circ\text{C}$ ,  $G_{int}=8.06\text{N/mm}^2$ ;  $T=30^\circ\text{C}$ ,  $G_{int}=0.971\text{N/mm}^2$  [18]), or long-term loads (1 year) and high temperatures ( $T=50^\circ\text{C}$ ,  $G_{int}=0.052\text{N/mm}^2$  [18]).

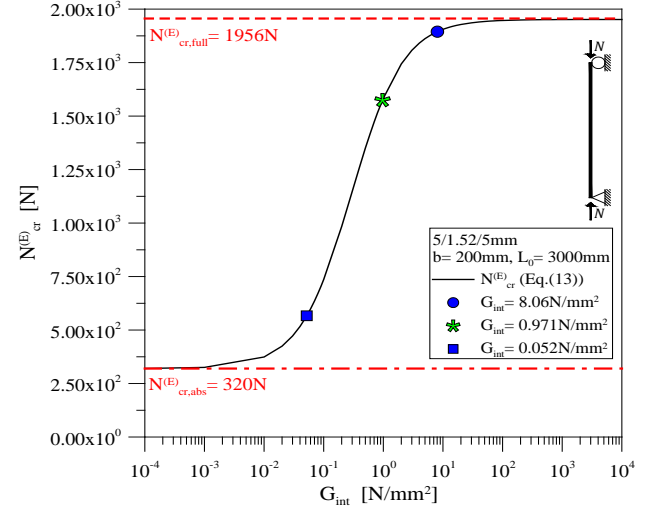
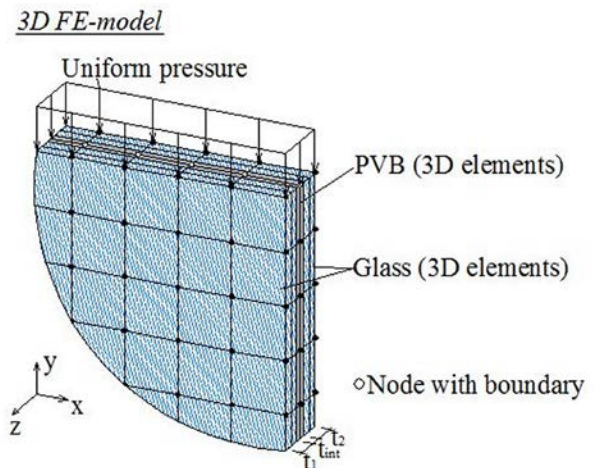


Fig. 2 Effects of temperature and time-load conditions on the critical buckling load  $N_{cr}^{(E)}$  of a compressed LG beam (5/1.52/5mm,  $b=200\text{mm} \times L_0=3000\text{mm}$ ) [18]

### III. NUMERICAL VALIDATION

To validate the analytical approach proposed for the buckling analysis of compressed LG beams, two different nonlinear finite-element (FE) models were constructed with the commercial nonlinear code ABAQUS [19]. In the first three-dimensional (3D) FE-model, the glass panes and the middle PVB-film have been described by means of 3D eight-node elements. In similar models, it is important to define a sufficiently accurate mesh for the elements, since the convergence of simulations as well as the accuracy of numerical results may be seriously compromised. Because of this reason, two elements over the depth of each glass sheet and the interlayer have been used. At the same time, an opportune mesh has been applied in the width of the examined LG beams (Fig. 3).



### *MShell* FE-model

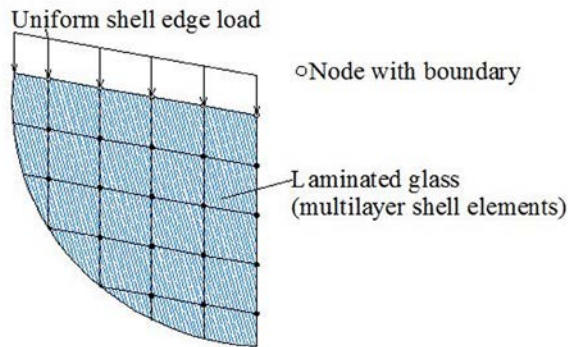


Fig. 3 3D and *Mshell* numerical models for LG beams in compression (ABAQUS)

The external glass sheets and the PVB-film were connected together by using the same nodes. To avoid possible eccentricities, boundaries were applied at the central nodes of the PVB-film, at both the ends of each simply supported LG beam. The compressive axial load was introduced in the FE-model in the form of uniformly distributed pressure acting on the lower and upper surfaces of 3D elements.

The second and simplest FE-model (*MShell*) consists in multilayer composite shell elements (S4R) able to describe the real flexural stiffness and the effective thicknesses of the layers constituting the analyzed LG beams (Fig. 3). In this specific circumstance, the axial compression was described in terms of uniform compressive shell edge loads acting on the lower and upper edges of each LG beam.

Concerning the materials, in both the FE-models glass has been modeled as an isotropic, linear-elastic material characterized by Young's modulus  $E = 70000 \text{ N/mm}^2$  and Poisson's ratio  $\nu = 0.23$ . Also PVB has been described as a linear elastic material, characterized by "equivalent" mechanical properties able to take into account for the degradation of its shear stiffness  $G_{int}$  due to temperature or load-time variations [6, 18]. In all the performed simulations, Poisson's ratio for PVB was fixed equal to  $\nu_{int} = 0.498$  [18].

Firstly, parametric buckling analyses were performed with the 3D and *Mshell* FE-models to investigate the effects of mechanical (PVB stiffness) or geometrical properties (slenderness of the beam, ratio between the thicknesses of glass and interlayer) in the buckling response of LG beams in compression. In this simulation phase, also the accuracy of the simplest *Mshell* FE-model was checked.

The main results are proposed in Fig. 4, in the form of critical loads  $N_{cr}^{(E)}$  of a 5/1.52/5mm beam ( $b = 200$ ,  $L_0 = 3000 \text{ mm}$ ) characterized by PVB-interlayers of various stiffness ( $10^{-4} \text{ N/mm}^2 < G_{int} < 10^4 \text{ N/mm}^2$ ). Analytical results (Eq. (13)) are compared with numerical predictions.

As shown, the 3D FE-model provides results in good agree with analytical calculations. Nevertheless, if the mesh of the 3D FE-model is not sufficiently detailed, the obtained critical load  $N_{cr}^{(E)}$  could be overestimated, especially in presence of soft interlayers ( $G_{int} < 10 \text{ N/mm}^2$ ). Contrarily,

buckling analyses performed with the *Mshell* FE-model do not require long processing time, but the accuracy of results is comparable to that of the 3D FE-model only in presence of sufficiently rigid interlayers ( $G_{int} > 10 \text{ N/mm}^2$ ). As shown in Fig. 4, the *MShell* FE-model clearly tends to underestimate the effective critical load  $N_{cr}^{(E)}$  of LG beams if the interlayer is soft. This aspect should be taken into account in the analysis of LG beams assembled with PVB-films, which are strongly time and load-time dependent.

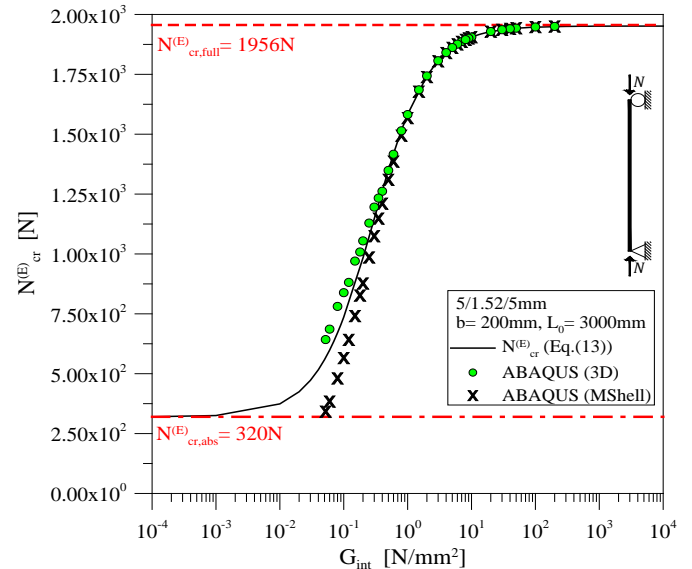


Fig. 4 Comparison between analytical and numerical (ABAQUS, 3D and *Mshell* FE-models) critical loads of LG beams in compression

Further comparisons were performed to validate the proposed analytical procedure [6]. Results proposed in Fig. 5, for example, refer to a LG beam obtained by assembling two 5mm thick glass sheets and 1.52mm thick PVB film.

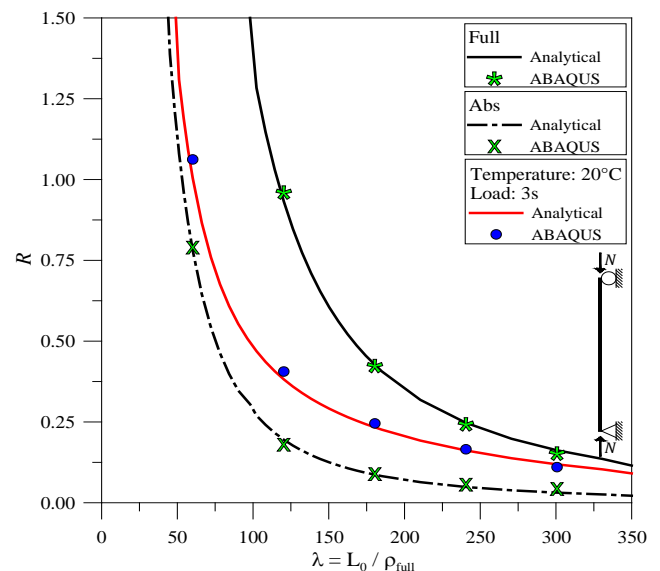


Fig. 5 Comparison between analytical and numerical (ABAQUS) results for LG beams in compression ( $w_0 = L_0/500$ ).

The width  $b$  of the beam was fixed equal to 20mm and its buckling length  $L_0$  was modified in a pre-established range ( $L_0 = 200, 400, 600, 800, 1000 \text{ mm}$ ). The maximum amplitude  $w_0 = L_0/500$  of the initial imperfection was

introduced in the model as an initial sine-shape imposed displacement. In this specific circumstance,  $G_{int}$  was assumed equal to  $G_{int} = 8.06 \text{ N/mm}^2$ , as suggested by Bennison ( $T = 20^\circ\text{C}$ , load duration: 3 seconds [18]). At the same time, to simulate the presence of an extremely rigid interlayer (*monolithic limit, full*) as well as a soft interlayer (*layered limit, abs*) the shear moduli  $G_{int,full} = 500 \text{ N/mm}^2$  and  $G_{int,abs} = 0.0001 \text{ N/mm}^2$  were considered.

Static incremental analyses were performed with 3D FE-model to investigate the load-carrying behavior of imperfect LG beams subjected to an increasing compression  $N$ . Geometrical nonlinearity was taken into account to simulate the buckling response of the examined LG elements. As the axial compression growth, the maximum tensile stress occurring in the glass surface was monitored. Analyses were stopped at the reaching of a prefixed value  $\sigma_{Rk}$  of characteristic tensile strength for glass. In the specific example, a characteristic strength  $\sigma_{Rk} = 17 \text{ N/mm}^2$  was taken into account, as suggested in [20] for the verification of float glass elements subjected to long-term loads. For each LG beam, the value of the compressive load  $N^*$  associated to a maximum tensile stress  $\sigma_{\max}(N^*) = \sigma_{Rk}$  was collected. Analytical and numerical results are presented in Fig. 5 in terms of reduction factor  $R$ :

$$R = \frac{N^*}{\sigma_{Rk} A}, \quad (16)$$

with  $A = bt$  the total cross-sectional area, and full slenderness ratio  $\lambda$ :

$$\lambda = \frac{L_0}{\rho_{full}}, \quad (17)$$

with  $\rho_{full}$  the radius of gyration of the total cross section (hypotheses of rigid connection between the glass panes):

$$\rho_{full} = \sqrt{\frac{J}{A}}, \quad (18)$$

and  $J = bt^3/12$  the moment of inertia.

Also in this circumstance, analytical and numerical results are in good agreement.

#### IV. ADDITIONAL LOADS

Clearly, the proposed analytical model is able to provide accurate results, agreeing well with the nonlinear numerical results. Moreover, although applicable only to 2-layer simple structural systems, the same formulation can be easily applied to the analysis of LG beams subjected to compressive loads  $N$  and to simultaneous transversal loads  $q$ . As discussed in [6], in fact, the maximum deflection of the LG beam can be estimated as:

$$w_{\max} = \frac{(\alpha^2 EJ_{abs} L_0^2 + EJ_{full} \pi^2)(L_0^2 q + 8Nw_0)}{8[\alpha^2 EJ_{abs} L_0^2 (EJ_{full} \pi^2 - NL_0^2) + EJ_{full} \pi^2 (EJ_{abs} \pi^2 - NL_0^2)]} + w_0 \quad (19)$$

in which  $q$  is the maximum amplitude of the transversal sinusoidal load affecting the beam;  $w_0$  represents the

maximum amplitude of the sine-shape imperfection;  $\alpha$ ,  $EJ_{full}$ ,  $EJ_{abs}$  are given by Eqs. 2, 5, and 6.

In this specific circumstance, the maximum tensile stresses associate to the deflection  $w_{\max}$  is:

$$\sigma_{\max} = -\frac{N}{A} + \frac{Nw_{\max}}{W_y} + \frac{qL_0^2}{8W_y} \frac{F}{A_{\min}}, \quad (20)$$

with:

$$F = \frac{(Nw_{\max} + qL_0^2) + \chi_{\max} EJ_{abs}}{d}, \quad (21)$$

whereas the critical buckling load  $N_{cr}^{(E)}$  of the LG beam can be still evaluated by means of Eq. 13.

#### V. DESIGN CRITERIA FOR COMPRESSED LAMINATED GLASS BEAMS

In the previous sections, it was shown that the maximum transversal displacement  $w_{\max}$  of a compressed LG beam caused by an axial load  $N$  can be evaluated by means of Eq. (7). At the same time, the corresponding maximum tensile stress can be estimated by means of Eq. (8). If additional external loads perpendicular to the plane of the beam are present (sinusoidal loads of maximum amplitude  $q$  or, equivalently, uniformly distributed loads of amplitude  $q$ ), the corresponding deflection and mid-span tensile stress are given by Eqs. (19) and (20).

In this context, in accordance with the Limit State approach, the buckling verification of compressed LG beams should be developed by contemporarily satisfying three different conditions, respectively referred to requirements of structural resistance, serviceability, and durability. Moreover, to perform a reasonable verification, the presence of an initial sinusoidal imperfection of maximum amplitude  $w_0$  should always be taken into account, to represent the possible effects of geometrical deformations (residual stresses, geometrical imperfections due to fabrication) or eccentricities (of load or boundary, or a combination of them) in the beam. Rationally, as suggested by Belis et al. [17], the amplitude of the initial sine-shape imperfection  $w_0$  should be at least assumed equal to 1/400 of the structural span.

In this context, the maximum tensile stress  $\sigma_{Ed}$  in each cross section (Eq. (8) or Eq. (20)), if additional transversal loads are present) should be compared with the design tensile resistance of glass  $\sigma_{Rd}$ :

$$\sigma_{Ed} \leq \sigma_{Rd} \quad (22)$$

At the same time, the maximum deflection  $\delta_{\max}$  of the LG beam (Eq. (7) or Eq. (19)) might not exceed a specific value, defined as a ratio of its buckling length  $L_0$ , as for example:

$$\delta_{\max} \leq \frac{L_0}{k} \quad (23)$$

with  $k = 120$ .

Finally, the column buckling verification should require the comparison of the design compressive load  $N_{Ed}$  and the buckling resistance  $N_{b,Rd}$  of the compressed LG beam:

$$N_{Ed} \leq N_{b,Rd} = \frac{N_{cr}^{(E)}}{\gamma_{M1}}, \quad (24)$$

with  $N_{cr}^{(E)}$  given by Eq. (13) and  $\gamma_{M1} = 1.40$ , for example, a buckling safety factor.

#### A. Alternative Verification Criteria for Compressed Laminated Glass Beams

For the buckling verification of traditional structural elements, constructed with conventional materials as steel or concrete, consolidate verification criteria are available in literature. The Ayrton-Perry formulation [21], for example, was originally formulated for the analysis of geometrically imperfect columns loaded by uniform compressive loads. In accordance with this analytical approach, the initial imperfections, as well as other effects (residual stresses, possible eccentricities) can be efficiently described through a generalized imperfection factor.

Specifically, the maximum second order lateral displacement of a compressed LG beam affected by a sinusoidal imperfection of maximum amplitude  $w_0$  can be expressed as:

$$w_{\max} = \frac{w_0}{1 - \frac{N}{N_{cr}^{(E)}}} \quad (25)$$

with  $N_{cr}^{(E)}$  the elastic critical load (Eq. (13)) and  $N$  the applied compression.

If  $\sigma_{Rk}$  is the characteristic tensile strength of glass, the failure of the beam occurs when:

$$-\frac{N}{A} + \frac{Nw_{\max}}{W_y} = \sigma_{Rk}, \quad (26)$$

with  $A$  the cross-sectional area and  $W_y$  the elastic resistant modulus (Eq. (11)).

The substitution of Eq.(25) into Eq. (26) provides the well-known expression [21]:

$$(\sigma_{Rk} - \sigma_{\max})(\sigma_{cr}^{(E)} - \sigma_{\max}) = \sigma_{\max} \sigma_{cr}^{(E)} \eta, \quad (27)$$

where:

$\sigma_{\max}$  is the maximum tensile stress due to the applied compression  $N$ ,

$$\sigma_{cr}^{(E)} = \frac{N_{cr}^{(E)}}{A} \quad (28)$$

is the critical stress of the beam, with  $N_{cr}^{(E)}$  given by Eq.(13), and

$$\eta = w_0 \frac{A}{W_y} \quad (29)$$

is a generalized non-dimensional imperfection factor.

Eq. (25) is the analytical expression able to represent the relationship between the applied compressive load  $N$ , the Euler critical load  $N_{cr}^{(E)}$  and the equivalent initial deflection of maximum amplitude  $w_0$  for the compressed LG beam. Eq. (25) can also be written in the standard form:

$$\chi^2 + \chi \left( -1 - \frac{1}{\bar{\lambda}^2} - \frac{1}{\bar{\lambda}^2} \eta \right) + \frac{1}{\bar{\lambda}^2} = 0, \quad (30)$$

where:

$$\bar{\lambda} = \sqrt{\frac{A \sigma_{Rk}}{N_{cr}^{(E)}}} \text{ is a normalized slenderness and} \quad (31)$$

$$\chi = \frac{N}{A \sigma_{Rk}} \text{ is a buckling reduction factor.} \quad (32)$$

The main advantage of the analytical approach proposed by Ayrton and Perry consists in the definition of an equivalent initial sine-shape imperfection representative of geometrical imperfections, as well as residual stresses or possible eccentricities. Because of this reason, it actually constitutes the theoretical background of the column design curves proposed by several codes for the verification of compressed steel members.

With reference to Eq. (30), for example, the Eurocode 3 [11] estimates the buckling resistance of compressed steel members by taking into account a series of appropriate buckling curves, opportunely calibrates to take into account the effects in their buckled response of possible initial imperfections of different amplitude, as well as the different cross-section class. Nevertheless, this method cannot be directly applied to LG beams.

In this context, the buckling verification of a compressed LG beam can be still performed by satisfying the condition given by Eq. (24), in which the design buckling resistance  $N_{b,Rd}$  of the layered beam is:

$$N_{b,Rd} = \chi A \sigma_{Rd}, \quad (33)$$

with  $\chi$  an opportune reduction factor.

In Eq. (33), the reduction factor  $\chi$  can be estimated by means of the expression obtained by Eq. (30) and suggested by the Eurocode 3 [11]:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \quad \text{with } \chi \leq 1 \quad (34)$$

where:

$$\Phi = 0.5 [1 + \alpha_{\text{imp}} (\bar{\lambda} - \alpha_0) + \bar{\lambda}^2], \quad (35)$$

$\bar{\lambda}$  is the slenderness of Eq. (31),

$$\alpha_{\text{imp}} = 0.71,$$

$$\alpha_0 = 0.60.$$

In this case, the imperfection coefficients  $\alpha_{\text{imp}}$  and  $\alpha_0$  have been opportunely calibrated on the basis of numerical and experimental data available in literature for compressed monolithic or laminated glass beams [4, 22, 23]. In the specific, the coefficient  $\alpha_0$  individuates the values of slenderness  $\bar{\lambda}$  associated to a reduction factor equal to  $\chi = 1$ . The value of the coefficient  $\alpha_{\text{imp}}$  individuates the maximum allowable imperfection for the compressed member.



In Fig. 6, several numerical and experimental data are presented to validate the proposed verification approach. In particular, experimental data refer to buckling tests recently performed by Luible [4] and Aiello et al. [22] on monolithic or laminated glass beams in compression. Additional experimental data are collected in [23]. Glass beams present a typical brittle-elastic behavior, and due to the absence of post-critical resistance in them, an abrupt failure generally occurs. Because of this reason, the safety factors should be carefully calibrated.

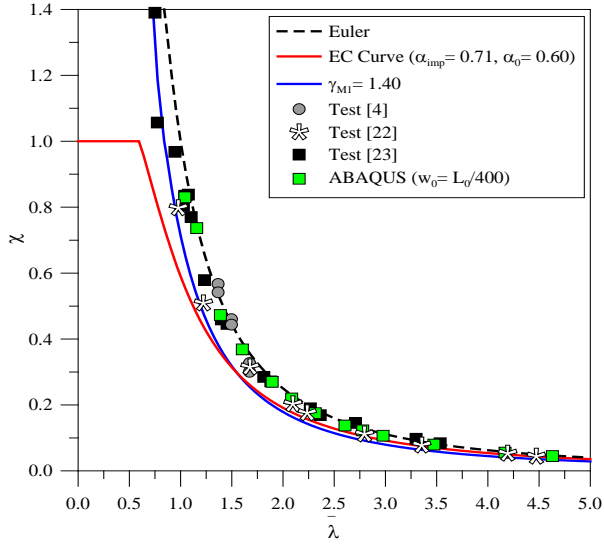


Fig. 6 Buckling curves for the verification of LG beams in compression ( $e=0$ ), comparisons with numerical (ABAQUS) and experimental results [4, 22, 23]

In the same figure, also numerical results are reported, obtained by performing in ABAQUS (3D FE-model) a series of static incremental analysis on compressed glass beams characterized by different geometrical properties. As suggested by Belis et al. [17], in this specific circumstance an initial sine-shape imperfection of maximum amplitude  $w_0 = L_0/400$  was taken into account. As shown in Fig. 6, the buckling curve proposed for the verification of compressed beams is in good agreement with numerical and experimental data. Additional analytical comparisons allowed to notice that a similar limitation (Eq. (34), with  $\alpha_{imp} = 0.71$  and  $\alpha_0 = 0.60$ ) approximately coincides, for  $\bar{\lambda} > 1.10$ , with the assumption of  $k = 120$  in Eq.(23) or  $\gamma_{M1} = 1.40$  in Eq.(24).

In Fig. 7, additional numerical results are proposed for compressed LG beams affected by an initial sine-shape imperfection of maximum amplitude  $w_0 = L_0/400$  and an accidental eccentricity  $e = t/6$ , with  $t$  the total thickness of the beam. In this circumstance, is interesting to notice that although the applied eccentricity has negligible amplitude, it strongly reduces the buckling resistance of the examined beams. As a result, the optimal buckling curve (which can be called “EC curve”), to be taken into account for their verification is characterized by imperfection factors equal to  $\alpha_{imp} = 1.80$  and  $\alpha_0 = 0.40$  (Fig. 7). Undoubtedly, a similar verification approach can be applied only to purely compressed glass beams, but it could constitute a useful design method.

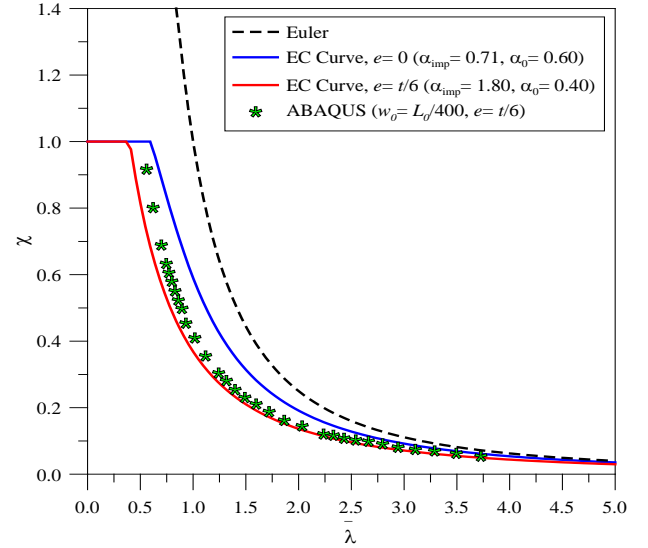


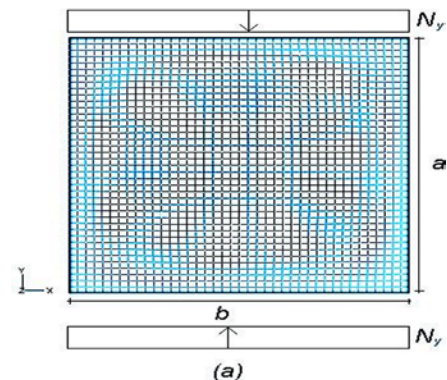
Fig. 7 Buckling curves for the verification of LG beams in compression ( $e = t/6$ ), comparisons with numerical results (ABAQUS)

## VI. ANALYTICAL MODEL FOR IN-PLANE COMPRESSED LAMINATED GLASS PANELS

Laminated glass panels, as well as beams, are largely used in the realization of façades, roofs, stiffeners, etc. The LG panels are mainly associated to the realization of futuristic and innovative architectures in modern buildings. However, due to their typical slenderness, they can be affected by buckling problems. Consequently an appropriate verification criterion should be adopted to prevent possible failure mechanisms.

Recently, numerous authors focused on the buckled response of glass panels subjected to in-plane compression [4, 8] or in-plane shear [9], providing useful experimental data, sophisticated numerical simulations, and interesting analytical considerations. In [10], an equivalent thickness approach has been proposed to study the buckling response of LG panels in several boundary conditions, subjected to in-plane compression or in-plane shear. Nevertheless, the knowledge on compressed LG panels behavior is still limited and with constrained applications.

Let us consider, for example, a LG panel simply supported on the four edges (length  $a$ , width  $b$ ), obtained by assembling two monolithic glass sheets and a middle interlayer (Fig. 8), subjected to in-plane compression (pressure force per unit length  $N_y$ , in  $y$ -direction).



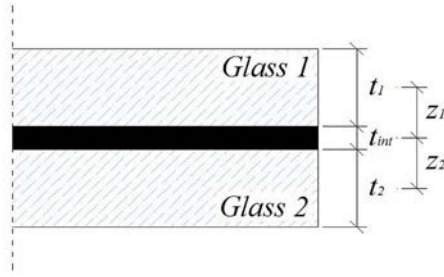


Fig. 8 Simply supported LG panel subjected to in-plane compression: geometry and cross section

The critical buckling load  $N_{y,cr,lam}^{(E)}$  of the LG panel is commonly estimated by means of the linear elastic theory of sandwich elements. In accordance with Zenkert's formulation, in particular, the critical load  $N_{y,cr,lam}^{(E)}$  is given by [24]:

$$N_{y,cr,lam}^{(E)} = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2 \frac{\frac{D_1 + D_2}{D} \left[ \left( \frac{mb}{a} \right)^2 + 1 \right] + \frac{A_s b^2}{\pi^2 D_s}}{\left[ \left( \frac{mb}{a} \right)^2 + 1 \right] + \frac{A_s}{\pi^2 D_s}} \frac{\pi^2 D}{b^2}, \quad (36)$$

with:

$$D = D_1 + D_s + D_2, \quad (37)$$

$$D_i = \frac{Et_i^3}{12(1-\nu^2)}, \quad (38)$$

$$D_s = \frac{(Et_1 z_1^2 + Et_2 z_2^2)}{(1-\nu^2)}, \quad (39)$$

$$A_s = \frac{G_{int} (z_1 + z_2)^2}{t_{int}}, \quad (40)$$

$$z_i = 0.5(t_i + t_{int}). \quad (41)$$

Numerical and analytical comparisons performed by Luible [4] demonstrated that Eq. (36) predicts with a good level of accuracy the bifurcation load  $N_{y,cr,lam}^{(E)}$  of simply supported LG panels, for well-defined values of  $G_{int}$ .

In particular, Luible showed that the mean ratio between analytical and numerical critical loads of 200 LG panels characterized by various geometrical (aspect ratio  $\alpha = a/b$ , thicknesses of glass sheets and interlayer) and mechanical properties (shear modulus  $G_{int}$  of the interlayer) resulted equal to 1.05. Moreover, the so obtained value of critical load is always comprised between the two well-known *monolithic* ( $G_{int} \rightarrow \infty, full$ ) and *layered* ( $G_{int} \rightarrow 0, abs$ ) limit values:

$$N_{y,cr,full}^{(E)} = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2 \frac{\pi^2 D}{b^2}, \quad (42)$$

$$N_{y,cr,abs}^{(E)} = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2 \frac{\pi^2 (D_1 + D_2)}{b^2}. \quad (43)$$

Nevertheless, the estimation of the critical buckling load  $N_{y,cr,lam}^{(E)}$  does not constitute a useful criterion to study in a realistic manner the stability problem of a compressed LG

panel and to define its ultimate strength. The panel, due to the membrane effects that typically characterize its behavior in the post-buckled regime, is in fact able to sustain greater loads than  $N_{y,cr,lam}^{(E)}$ . In addition, no sandwich-based formulations are available to describe the axial compressive load  $N$ -transversal displacement  $w$  of layered panels. Therefore, sophisticated numerical simulations should be performed to realistically investigate their typical behavior.

Because of this reason, in [10] an equivalent thickness approach has been presented to precisely investigate the buckled response of in-plane compressed LG panels. In accordance with this simplified but accurate formulation, the behavior of the LG panel can be described by means of the classical theory of monolithic plates, by assuming:

$$t_{eq,w} = \sqrt[3]{t_1^3 + t_2^3 + 12\Gamma J_s}, \quad (44)$$

with:

$$\Gamma = \frac{1}{1 + 9.6\beta \frac{EJ_s t_{int}}{G_{int} t_i^2 \lambda^2}}, \quad (45)$$

$$\lambda = \min(a, b), \quad (46)$$

$$J_s = t_1 t_{s2}^2 + t_2 t_{s1}^2, \quad (47)$$

$$t_{s1} = \frac{t_s t_1}{(t_1 + t_2)}, \quad (48)$$

$$t_{s2} = \frac{t_s t_2}{(t_1 + t_2)}, \quad (49)$$

$$t_s = 0.5(t_1 + t_2) + t_{int}, \quad (50)$$

and  $\beta$  a coefficient defined in function of the boundary and loading conditions of the panel.

In [10], it was shown that the coefficient  $\beta$  should be assumed equal to:

$$\beta = \frac{1.09}{\alpha^2} + 1.09. \quad (51)$$

In this manner, the critical buckling load  $N_{y,cr,lam}^{(E)}$  can be estimated as:

$$N_{y,cr,lam}^{(E)} = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2 \frac{\pi^2 D_{eq}}{b^2} = k_\sigma \frac{\pi^2 D_{eq}}{b^2}, \quad (52)$$

where:

$$D_{eq} = \frac{Et_{eq}^3}{12(1-\nu^2)} \quad (53)$$

is the equivalent flexural stiffness of the composite panel.

Due to the correction factor  $\beta$  (Eq. (51)) the critical buckling load given by Eq. (52) coincides with the solution of Eq. (36).

At the same time, the load  $N$ -total transversal displacement  $w$  relationship of the in-plane compressed LG panel can be described as [10]:



$$N = N_y b =$$

$$Eb \left( \frac{t_{eq,w}}{a} \right)^2 \left\{ \frac{\pi^2}{3(1-\nu^2)} + \frac{\pi^2}{8} \left[ \left( \frac{w}{t_{eq,w}} \right)^2 + 3 \left( \frac{w}{t_{eq,w}} \right) \left( \frac{w_0}{t_{eq,w}} \right) + 2 \left( \frac{w_0}{t_{eq,w}} \right)^2 \right] \right\} \frac{w}{w+w_0} \quad (54)$$

that is:

$$w = \frac{\sqrt[3]{2} p_1 + 1}{p_2(p_3 + \sqrt{4p_1^3 + p_3^2})^3} - w_0, \quad (55)$$

with:

$$p_1 = 9\sqrt[3]{2} b E \pi^2 (\nu^2 - 1) \left[ -24 a^2 N (\nu^2 - 1) - 8 b E \pi^2 t_{eq,w}^2 - 3 b E \pi^2 w_0^2 (\nu^2 - 1) \right] \quad (56)$$

$$p_2 = 9 b E \pi^2 (\nu^2 - 1), \quad (57)$$

$$p_3 = 1944 (b E \pi^2)^3 t_{eq,w}^2 w_0 (\nu^2 - 1)^2, \quad (58)$$

and  $w_0$  the maximum amplitude of the initial imperfection of the LG panel.

## VII. NUMERICAL AND EXPERIMENTAL VALIDATION

To check the accuracy of the proposed analytical approach for compressed LG panels, also in this circumstance a series of FE-models were developed with the code ABAQUS [19].

Firstly, an accurate three-dimensional numerical FE model was developed. The *3D+shell* FE model consists in glass sheets (thicknesses  $t_1 = t_2 = 8\text{mm}$ ) described by means of *shell* elements (S4R) and in a PVB-interlayer (thickness  $t_{int} = 1.52\text{mm}$ ) modeled in the form of 3D-8 node elements (C3D8H, *hybrid formulation, incompatible modes*). The examined LG panel, simply supported along the four edges, was assumed to have dimensions  $a = 1\text{m} \times b = 1\text{m}$ . To ensure the accuracy of numerical results, an accurate mesh was used in the model (20 x 20mm module). Moreover, over the depth of the PVB-film, two 3D elements have been used (Fig. 9).

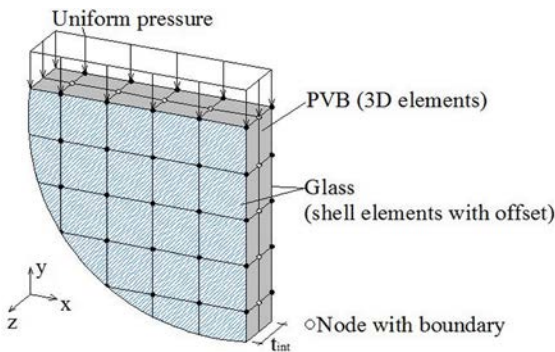


Fig. 9 *3D+shell* numerical model for simply supported in-plane compressed LG panels detail (ABAQUS)

3D elements and shell elements were connected together by using the same nodes. In addition, to describe the effective geometry of the LG panel, a section offset  $t_{offset} = 4\text{mm}$  from the centroidal axis of each glass pane was applied to shell elements. In-plane compression was introduced in

the model in the form of uniformly distributed pressure acting on the upper and lower surfaces of 3D elements. To avoid possible eccentricities in the model, boundaries were applied to the central nodes of the PVB-interlayer.

The second *Mshell* FE-model, as discussed for LG beams, consists in composite shell elements (S4R) able to take into account the effective thicknesses of each layer constituting the LG panel. The third *Teq* FE-model, finally, was constructed with monolithic glass shell elements (S4R) having an equivalent thickness given by Eq. (44). In all these FE-models, glass was described as an isotropic, linear elastic material ( $E = 70000\text{N/mm}^2$ ,  $\nu = 0.23$ ). Similarly, PVB was considered to behave linear-elastically ( $\nu_{int} = 0.498$ ). Since the aim of numerical simulations consisted in validating the equivalent thickness analytical approach, a series of buckling analysis were performed in ABAQUS. The critical buckling load  $N_{y,cr,lam}^{(E)}$  was predicted for the examined LG panel by assuming in each analysis a different value of shear modulus  $G_{int}$  ( $10^4\text{N/mm}^2 < G_{int} < 10^4\text{N/mm}^2$ ).

The results proposed in Fig. 10 concern the critical load  $N_{y,cr,lam}^{(E)}$ , numerically and analytically evaluated, compared as a function of  $G_{int}$ . In the specific, five different solutions are compared, that is the analytical results of the sandwich-based classical theory (Eq. (36)), the analytical results given by the proposed equivalent thickness approach (Eq. (52)) and the numerical results obtained by the buckling analyses performed in ABAQUS with the *3D+shell*, *Mshell* and *Teq* FE-models. As shown, due to the calibrated correction factor  $\beta$  (Eq. (51)), the analytical results coincide with each other.

At the same time, they are in good agreement with numerical data obtained by ABAQUS with the *3D+shell* FE-model. Clearly, the *Mshell* FE-model does not require long processing time to perform buckling analyses but does not agree with the analytical predictions, thus it should be used with attention. In contrary, the *Teq* FE-model rapidly converges and provides accurate results, thus it could be used in practice to avoid 3D sophisticated simulations.

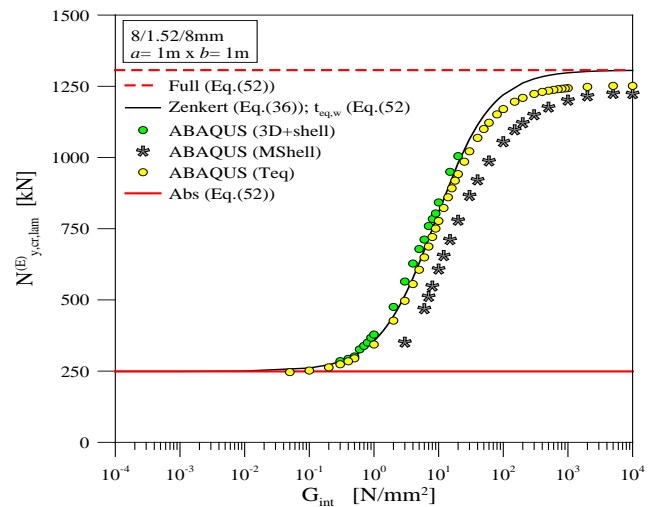


Fig. 10 Critical buckling load  $N_{y,cr,lam}^{(E)}$  for in-plane compressed LG simply supported along the edges: analytical and numerical comparisons

The second proposed comparison concerns the typical load  $N$ -transversal displacement  $w$  relationship characterizing the buckled response of in-plane compressed LG panels. For this purpose, some experimental data available in literature for out-of-plane displacements of simply supported LG panels under compression were taken into account [4]. Specifically, the experimental data proposed in Fig.11 and summarized in Table 1 refer to three squared 8/1.52/8mm LG panels ( $a = 1\text{ m} \times b = 1\text{ m}$ ) tested by Luible. Analytical results, in particular, have been defined for each LG panel by means of Eq. (55), having estimated the corresponding equivalent thickness  $t_{eq,w}$  (Eq.(44)). As shown, experimental and analytical data are in good agreement.

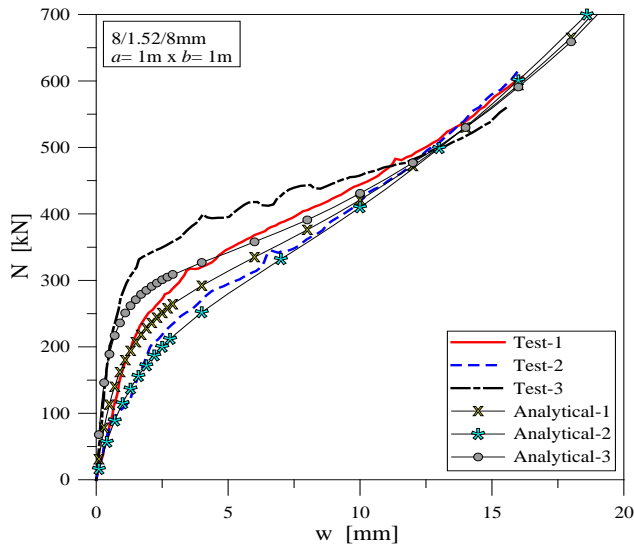


Fig. 11 Load  $N$ -transversal displacement  $w$  relationship for in-plane compressed LG panels. Experimental [4] and analytical results (Eq. (44))

TABLE 1 EXPERIMENTAL DATA AND CORRESPONDING EQUIVALENT THICKNESSES FOR IN-PLANE COMPRESSED LAMINATED GLASS PANELS

$a = 1\text{ m} \times b = 1\text{ m}$	$T$ [4]	$G_{int}$ [4]	$t_{eq,w}$ (Eq.(44))
8/1.52/8mm	[°C]	[N/mm <sup>2</sup> ]	[mm]
Test-1	20.5	0.81	11.14
Test-2	21.0	0.76	11.09
Test-3	20.0	0.84	11.18

#### VIII. DESIGN CRITERIA FOR IN-PLANE COMPRESSED LAMINATED GLASS PANELS

As proposed for LG beams in compression, the buckling verification of simply supported in-plane compressed LG panels should be performed by contemporarily satisfying requisites of deformability and durability. In accordance with the Limit State approach, the maximum transversal displacement  $w_{max}$  of the panel, given by Eq. (55), should be opportunely limited, as a function of the length  $a$ , by posing for example the condition:

$$w_{max} \leq \frac{a}{k} \quad (59)$$

Rationally, a reasonable check of the maximum deformation should be carried out taking into account an initial sinusoidal imperfection, proportional to the first modal shape of the panel, of maximum amplitude  $w_0$ . In this context, Englhardt [7] experimentally investigated monolithic float and toughened glass panels having a maximum amplitude of geometrical imperfection equal to  $w_0 = a/2000$ . Tests performed by Luible and Crisinel [25] highlighted that initial imperfections in non-tempered annealed flat glass panels have small amplitude ( $w_0 < a/2500$ ), whereas heat-strengthened and fully toughened glass panels can have sinusoidal imperfections up to  $w_0 = a/300$ . Reasonably, in the buckling verification of LG panels a minimum amplitude of imperfection  $w_0 = a/1000$  should be considered. Moreover, Englhardt suggests for the coefficient  $k$  a value equal to  $k = 300$  [7].

At the same time, the design compressive load  $N_{y,Ed}$  should be compared with the design buckling resistance  $N_{y,b,Rd}$  of the panel:

$$N_{y,Ed} \leq N_{y,b,Rd} = \frac{N_{y,cr,lam}^{(E)}}{\gamma_{M1}} \quad (60)$$

where  $N_{y,cr,lam}^{(E)}$  is given by Eq. (52). In this context, Englhardt suggests for the buckling safety factor  $\gamma_{M1}$  a value of 1.40 [7].

Also in this circumstance, an alternative verification criterion for in-plane compressed LG panels can be derived from buckling curves suggested by the Eurocode 3 for the verification of traditional structural elements [11]. In the specific case of compressed panels, the buckling verification could be still performed by satisfying the condition given by Eq. (60), in which the design buckling resistance  $N_{y,b,Rd}$  of the LG panel can be estimated as:

$$N_{y,b,Rd} = \chi A \sigma_{Rd} \quad (61)$$

with  $\chi$  an opportune reduction factor and  $A$  the total cross-section area.

In Eq. (61), the reduction factor  $\chi$  can be estimated by means of Eq. (34), where  $\Phi$  is given by Eq. (35), with  $\alpha_{imp} = 0.49$  and  $\alpha_0 = 0.60$ . In addition, the non-dimensional slenderness  $\bar{\lambda}$  (Eq. (31)) should be expressed as a function of the critical buckling load  $N_{y,cr,lam}^{(E)}$  of the examined LG panel, given by Eq. (53).

Also in this circumstance, the imperfection factors  $\alpha_{imp} = 0.49$  and  $\alpha_0 = 0.60$  have been opportunely calibrated on the basis of experimental and numerical data available in literature. As proposed in Fig. 12, for example, the limitation provided by the suggested EC curve ( $\alpha_{imp} = 0.49$  and  $\alpha_0 = 0.60$ ) is approximately equal to the conditions expressed by Eq. (59), with  $k = 300$  [7], and by Eq.(60), with  $\gamma_{M1} = 1.40$  [7].

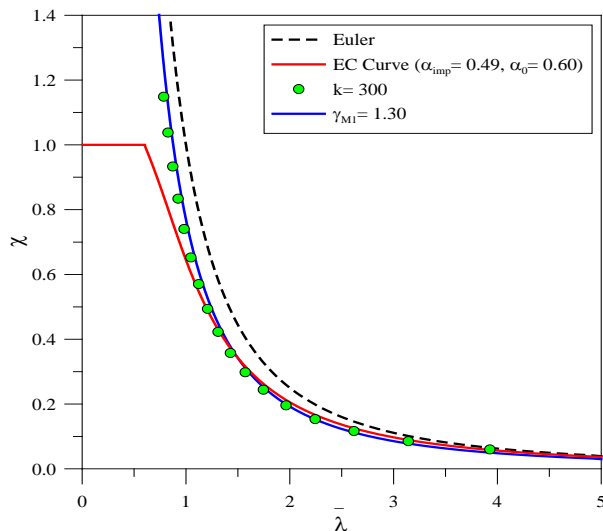


Fig. 12 Buckling curve for the verification of LG panels under in-plane compression: comparisons with analytical results

In Fig. 13, some experimental and numerical data are also presented to validate the proposed verification procedure. The experimental data, in particular, refer to buckling tests performed by Luible [4] on in-plane compressed LG panels simply supported along the edges, having different geometrical properties. As shown in Figs. 11 and 13, a significant post-critical resistance can be observed in the typical behavior of LG panels under compression. A similar behavior is also confirmed by numerical and experimental results obtained by Englhardt [7] and Luible [4]. However, it is possible to notice that, ignoring the post-critical resistance of the examined LG panels, the proposed solution provides adequate safety requirements.

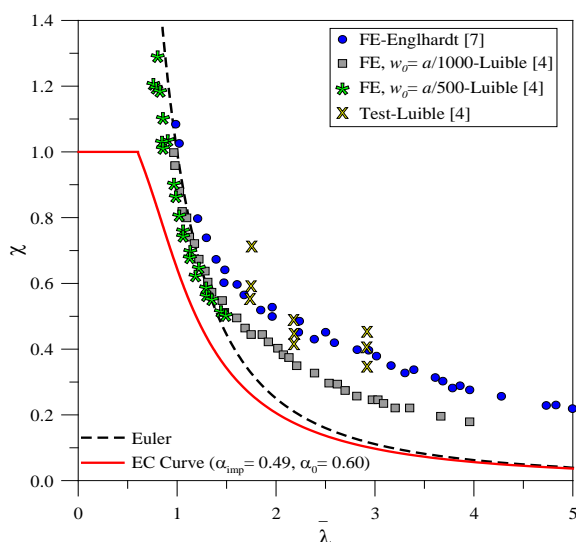


Fig. 13 Buckling curve for the verification of LG panels under in-plane compression: comparisons with numerical [4, 7] and experimental results [4]

## IX. CONCLUSIONS

Laminated glass beams and panels subjected axial compression are frequently subjected to stability problems. In literature, several analytical formulations derived from the

classical theory of sandwich elements under compression are available, but often they can be used only to predict the value of their critical buckling load. In the paper, exact analytical formulations are proposed for the buckling verification of compressed LG beams and panels. For LG beams, a model developed on the basis of Newmark's theory of composite beams with partial interaction is proposed. At the same time, a simplified but accurate equivalent thickness formulation is proposed for the verification of in-plane compressed LG panels. Comparisons with numerical and experimental data are presented to validate the accuracy of analytical formulations. In both the circumstances, the presented models allow to take into account the effective level of connection offered by the adopted interlayer. At the same time, a criterion based on the buckling curves of Eurocodes is suggested for a rational buckling verification of compressed LG elements.

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