

Aftershocks Distributions in a Modified Olami-Feder-Christensen Model

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Abstract- We introduce a stress decay process into a continuous cellular automaton version of the Olami-Feder-Christensen model of earthquakes. The most important model parameter is the decay level of non-conservation during local stress redistribution. In particular, the Gutenberg-Richter and Omori laws for aftershocks sequences can be reproduced synchronously by this model simulation. The simple toppling mechanism of this modified model is sufficient to account for the clustering of real aftershocks. The self-organized criticality properties of the model are discussed. The high correspondence of the simulated results to observations supports the hypothesis that fault systems are in a state of self-organized criticality. At the same time, our work suggests that the occurrence of self-organized criticality depends, at least in part, on the decay dynamics of the non-conservation.

Keywords- Aftershock; Self-organized Criticality; Gutenberg-Richter Law; Omori Law; OFC Earthquake Model

I. INTRODUCTION

Several self-similar properties of temporal variations of aftershocks have been found [1]. Among these relations, the Gutenberg-Richter law is a well-established empirical law in seismology [2]. The law shows earthquake energy (magnitude) follows a power-law distribution, namely $\log N(M) = a - bM$, where $N(M)$ is the cumulative number of earthquakes with a magnitude greater than M . The exponent b varies over a wide range of 0.8-1.2 [3] and close to 1 in many cases [4]. The Omori law for aftershocks is another well-established empirical law in seismology, describing the rate of decay of aftershocks with time possessing a power-law scaling [5]. It has the form $n(t) = K(c+t)^{-p}$, where $n(t)$ is the number of aftershocks events following the main shock at the time t . The decay exponent p has received most attention in studies of aftershock sequences. Utsu *et al* report that published values of p vary between 0.6 and 2.5 [5]. For an oceanic environment, present p of 1.74 and 2.37 for two aftershock sequences is associated with the spreading of the mid-Atlantic ridge, and p in the range of 0.94-1.29 for aftershock sequences is associated with transform faults offsetting the ridge [6]. These distribution patterns indicate that the occurrence of aftershocks is complex, but not random.

In order to reproduce the observed distribution patterns, some simplified models for complex fault systems have been studied. A one-dimensional slider block model can reproduce the Gutenberg-Richter law [7]. Several modifications of this model have been analyzed by different authors [8-10]. In these models, the Olami-Feder-Christensen (OFC) model is perhaps the most prominent representative [11]. The OFC model can explain the proper b value and its empirically observed variability. Moreover, several mechanisms were introduced to simulate the Omori Law: viscoelastic relaxation in the fault zone [12], fault weakening after a block slips [13], pore fluid flow [14], rate-state friction [15], and damage rheology [16]. Although these models can yield the Omori law, none of these models can explain the Gutenberg-Richter law synchronously. It was recently found that the Omori law of aftershock sequences is reproduced by the OFC model at least qualitatively [17]. However, the theoretical exponents predicted by the model are lower than observed in nature. These results make the OFC model attractive for application in seismology, but some other results raise doubts concerning its applicability to real earthquakes.

Upon failure, each aftershock will modify the time to failure of its near surroundings through the redistribution of stresses, eventually resulting in the occurrence of the next aftershock. The modification of the time to failure should be time-dependent. So we suppose that the force transmission parameter in OFC model algorithm maybe depends on the time. In this paper, we introduce a stress transmission attenuation process in the classical OFC model. In our model, the force transmission parameter decays with time exponentially. The results indicate that such modified OFC model can reproduce the features of the Gutenberg-Richter and Omori laws in the realistic aftershock sequences. We note that the assumed attenuated force transmission parameter can provide an effective way of parameterizing macroscopically the temporal variation of some physical processes including viscous relaxation, rate-state friction and fluid migration during aftershocks process.

II. THE MODEL

The classical OFC earthquake model is successful in the explanation of Gutenberg-Richter law in observed earthquake

sequences, but not the empirical Omori law. It cannot explain the observed p value and its empirically observed variability. We consider that aftershock sequences in the realistic process reflect the gradual decaying process of the total earthquake energy relaxation, which is important and not described by the classical OFC model algorithm. For this, we put forward a continuous, non-conservative, attenuated, isotropic, generalized cellular automaton modeling the realistic aftershock.

The modified OFC earthquake model is implemented in the form of continuous cellular automaton by defining a two-dimensional $L \times L$ array of blocks (i, j) with open boundaries, where i, j are integers, $1 \leq i, j \leq L$. These blocks are connected with each other and with a driving plate moving at a predefined velocity. The force $F_{i,j}$ acting on any given block (i, j) exceeds a fixed threshold F_{th} , the block becomes unstable and moves instantaneously to a new equilibrium position where the forces acting on it relaxes to zero. Only a part of the force acting on the block is transferred to its neighbors owing to energy dissipation, so that some of its neighbors may become unstable. This process may lead to avalanches. The gradual weakening process of the total earthquake energy relaxation implies attenuation of the strengths of the springs among blocks with time during aftershock process. For this, at last, we presume that the elastic ratios among blocks will decay with time in the aftershock process following the first level of attenuation kinetics.

Using non-dimensional variables, its rules can be written in the form:

(1) Initialize all sites (i, j) to a random value between 0 and F_{th} . $F_{th} = 1$.

(2) Add $F_{th} - F_{max}$ to all sites. F_{max} is the largest force acting on all blocks.

(3) If any $F_{i,j} \geq F_{th}$, instantaneous relaxation of unstable blocks is

$$F_{i\pm 1,j} \rightarrow F_{i\pm 1,j} + \alpha F_{i,j}, \quad F_{i,j\pm 1} \rightarrow F_{i,j\pm 1} + \alpha F_{i,j}, \quad F_{i,j} \rightarrow 0.$$

The transmission parameter α expresses the elastic ratios among blocks.

(4) α , which depends on the strengths of the springs between blocks, will decay to e^{-k} of the original level.

$$\alpha = \alpha \times e^{-k},$$

The k value is stress decay coefficient.

(5) Repeat Step 2 until all sites are stable.

After this, we measure the probability distribution function $P(s)$ of avalanche sizes (s) of the earthquakes, which is proportional to the energy released during aftershock process. At the same time, the waiting time series of simulated earthquakes $D(t)$ are measured. The waiting time of earthquake is defined as the computer time interval for computer to complete all avalanches every adding $F_{th} - F_{max}$ to all sites.

III. RESULTS AND DISCUSSION

A. Magnitude Distribution of Aftershocks in the Model

We show results obtained by fixing $k = 1.0 \times 10^{-9}$, which is small enough to lead to steady distribution of aftershocks. To be confident that stationary regime has been reached, the first 10^6 avalanches are skipped. The lattice size ranges from $L = 16$ to $L = 128$, and statistical distributions are obtained averaging over 10^7 avalanches.

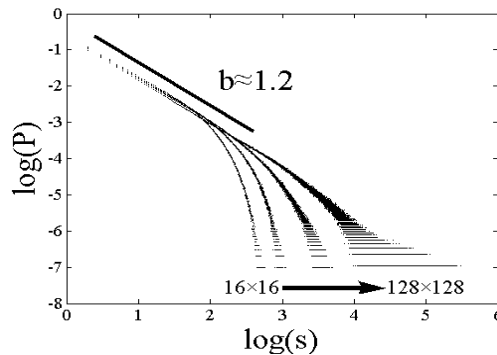


Fig. 1 The probability density function $P(S)$ of earthquake energy for $k = 1.0 \times 10^{-9}$. The different curves refer to different system

In the Fig. 1, one can see that probability distribution function $P(s)$ of avalanche sizes (s) obeys the relationship $P(S_0 > S) \propto S^{-b}$ with $b \approx 1.2$. The theoretical prediction is in accordance with the actual result. And for $L = 16, 20, 32, 128$, the power law relations are robust. Further, we analyze the effect of k values on SOC behavior.

Fig. 2 shows the relation between the calculated power-law exponent b and decay coefficient k . The b values are almost not changed near 1.2. Here, stress decay coefficient k also does not destroy SOC behavior. Similar results can be found in our previous study.

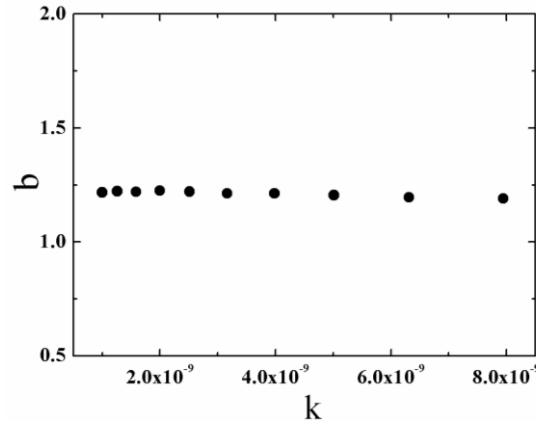


Fig. 2 The relation between the simulated b value and decay coefficient k

B. Temporal Distribution of Aftershocks in the Model

The rupture of some blocks in fault region is immeasurably small, which results in some aftershocks with smaller magnitude range than measured in the actual situation. So it is appropriate to ignore some smaller avalanches in our simulation. For this, we need to determine some threshold values S_{th} based on original avalanches sizes series (S) firstly. A sequential occurrence of aftershocks with energy size above S_{th} can be obtained based on original avalanches series. Thus, the waiting time of simulated aftershocks is defined as the computer time between consecutive events. Next, the total waiting time is divided by fixed time window. The length of fixed time window is $\sum D(t)/n$, where n is the total number of time windows. Finally we measure the occurrence number of aftershocks with energy size above S_{th} in each time window and complete the statistical analysis of Omori law.

Specifically, when $n = 200$, $S_{th} = 55819$, $k = 10^{-8.9}$ and $L = 128$ in the model, the waiting time of simulated aftershocks is shown in Fig. 3. And the Omori analysis of simulated data is shown in Fig. 4. Fitted result indicates that the decay of seismicity rate follows Omori law with the exponent $p = 2.0 \pm 0.12$.

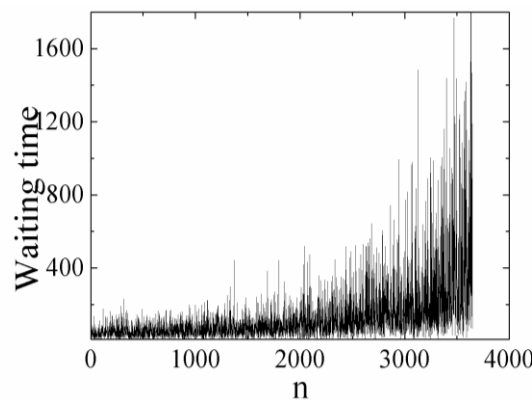


Fig. 3 The waiting time of simulated aftershocks when $n = 200$, $S_{th} = 55819$, $k = 10^{-8.9}$ and $L = 128$ in the model

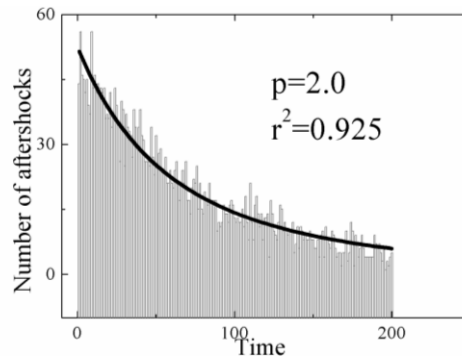


Fig. 4 The simulated seismicity rate as a function of time when $n=200$, $S_{th}=55819$, $k=10^{-8.9}$ and $L=128$ in the model.

The solid lines give estimates of the Omori law with $p=2.0$

We found that the value of decay exponent p depends on S_{th} , L and k . In order to obtain the certain value of p , S_{th} , L and k must meet certain relationship. Firstly, we consider k acts as invariable one. The area of a fault is represented by the lattice of $L \times L$ blocks in the simulation. Thus, because of finite size effect of SOC systems, S_{th} will vary with the system size L . We studied the effect of increasing the system size L on the selection of S_{th} . When $k=10^{-8.9}$, In order to obtain the certain value of p , the relation between L and S_{th} is a simple linear rule, i.e. $L \propto h \times S_{th}$. The results are shown in Fig. 5. For $p=0.5, 1.0, 1.5, 2.0$, $h=0.01015, 0.00639, 0.00639, 0.00177$ respectively. Secondly, when L is fixed at certain value, i.e. $L=128$, we studied the effect of variation of k on the selection of S_{th} . In order to obtain the certain value of p , the relation between k and S_{th} is power law, i.e. $k \propto S_{th}^g$. The results are shown in Fig. 6. For $p=0.5, 1.0, 1.5, 2.0$, $g=0.01015, 0.00639, 0.00639, 0.00177$ respectively. Thus, different values of p can be generated by our model, which is in agreement with what is observed in nature.

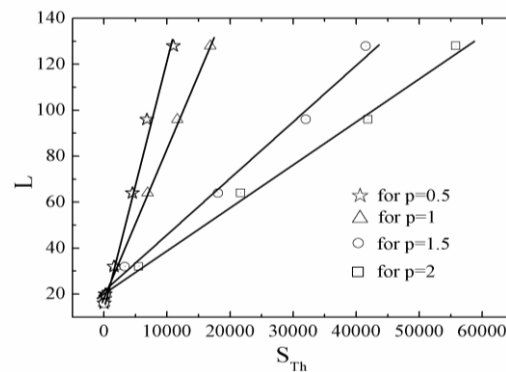


Fig. 5 When k is fixed at certain value, i.e. $k=10^{-8.9}$, in order to obtain the certain value of p , the effect of variation of L on the selection of S_{th}

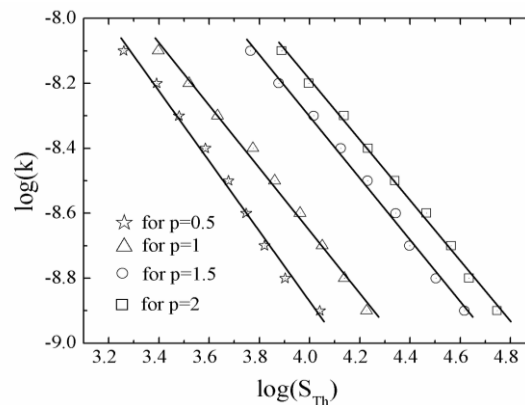


Fig. 6 When L is fixed at certain value, i.e. $L=128$, in order to obtain the certain value of p , the effect of variation of k on the selection of S_{th}

C. Mechanisms for Aftershocks in the Modified OFC Model

The most important difference between our model and the classical OFC earthquake model is that nonconservation of the relaxation will increase with time. Thus, α , which determines the force redistributed to the nearest neighbors, will decay with time rather than being a fixed amount. Since the parameter k is decay coefficient of α , the physical meaning of k requires some explanation. Generally, after mainshock, stress will be transferred from the rupturing cells into the creeping parts of the fault. The near vertical creep barriers (margins of the fault segments) do not fail in rapid brittle fashion, and these parts store stress and become highly loaded. After the mainshock these parts creep, thereby transferring stress back to the entire fault region. During the stress relaxation and redistribution, the local stress may grow rapidly, exceed the static friction and eventually result in the occurrence of an aftershock event. Upon failure, each aftershock will modify the failure of its near surroundings through the redistribution of stresses, eventually resulting in the occurrence of the next aftershock, which occurs preferentially close to creeping parts and in high stress fault patches that have not ruptured during the large event. In this process, the total stored energy in the fault region relaxes step by step. So mean stress in the fault region as a whole will show a decreasing trend as time goes on, though high stress maybe emerges into local fault region. The modification of the failure must be time-dependent and nonlinear decaying. We introduce a comprehensive parameter k to show variation intensity rate of the mean stress during whole aftershocks process. In this sense, the k value is called stress decay coefficient. The success of the introduction of k is measured simply on the high correspondence of the simulated results to observations.

In seismology, the widespread applicability of the spatio-temporal power laws is considered as the primary argument supporting the hypothesis that the Earth's crust is in a state of self-organized criticality (SOC) [18]. However, none of previous SOC models can explain the Gutenberg-Richter and Omori laws synchronously. We have shown that a network array of blocks can evolve to an SOC state with striking similarities to observations in real fault systems: a power-law distribution of event sizes and power-law decay of aftershocks with time. In our model, the overall dynamics is dominated by the elastic properties of model. In spite of the fact that we neglect heterogeneities, long range interactions and more realistic friction laws, there are striking similarities to empirical findings concerning the size distribution as well as the spatio-temporal clustering accompanying aftershocks. Since the Gutenberg-Richter and Omori laws can be generated by this simple model, we believe this model elucidates some fundamental mechanisms of the actual aftershocks. Our investigations support the hypothesis that fault systems are in an SOC state. Self-organized critical behavior is characterized by steady inputs, and outputs that are a series of 'events' that satisfy power-law frequency-size statistics. The systems fluctuate about a state of marginal stability. Thus, a fairly accurate forecast of where and when large aftershock events may happen is impossible. The idea behind this concept is that a dissipative system with extended spatial degrees of freedom tends to evolve in a self-organized critical state spontaneously.

The previous works on OFC model demonstrates the non-conservation plays important roles on the existence of SOC [11]. Our work suggests that the occurrence of SOC also depends, at least in part, on the decay dynamics of the non-conservation. In simulation, we consider that α will decay in the exponential way arbitrarily. Obviously, in some concrete cases, this relation may be not appropriate and more realistic micro-dynamics are need.

Although our general aftershock model is merely conceptual, our results have some practical implications. If an actual aftershock phenomenon is an example of an SOC process, one should not attempt to give a fairly accurate forecast of where and when large aftershocks may happen. More appropriately, one should consider that the measured frequency of occurrence of small aftershock events can be used to estimate the frequency of occurrence of large aftershock events. For example, the recurrence interval for large aftershocks can be estimated from the frequency of smaller aftershocks. In the concrete application, considering actual seismic tomography, our previous study about Wenchuan aftershocks shows Wenchuan aftershock is an example of an SOC process [19]. New insights into the Wenchuan aftershocks maybe contribute to guiding post-disaster reconstruction and reducing the hazard of large aftershocks in seismic events.

IV. CONCLUSION

In summary, we have developed a general aftershock model on the bases of the classical OFC model. The modified model is a continuous, non-conservative, attenuated, isotropic cellular automation modeling, which shows robust SOC behavior. We have shown that a simplest possible mechanism for the generation of aftershocks is sufficient to recover essentially the Gutenberg-Richter and Omori laws documented in seismic catalogs. The mechanism includes slow tectonic loading, sudden stress relaxation with local stress redistribution, and decay of mean stress during redistribution. The high correspondence of the results to observations indicates that the model provides an effective parameterization of the key physical process that governs aftershocks on strike-slip faults.

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