Inventory Control of New Products with Price Change Anticipation under Influence of Innovation Diffusion Process

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Abstract-The new product diffusion models such as Bass model and their revised and extended forms have played a major role in marketing literature because those models have been widely used for forecasting dynamic demand growth in retail service, industrial technology, consumer durable goods, etc. However, the demand pattern of these models has not been recognized by the models developed in the inventory field. The approach of this paper is to develop an inventory model in which the demand of the product follows extended Bass diffusion model by incorporating price in a multiplicative way. The model is illustrated with a numerical example and a comprehensive sensitivity analysis of the optimal solution with respect to different parameters was performed to make the model more effective.

Keywords- Innovation Diffusion; EOQ; Potential Market Size; Price Dependent Demand

I. INTRODUCTION

The models developed under every field of management, whether it is of "marketing" or "inventory", are of paramount interest to the researchers as well as users. The researchers use the models of their field and make new and more innovative models based on the previous models, which are of a great help as far as innovative researches are concerned while users use the models in decision making to maximize gains and minimize loss. It has been commonly observed that models developed in marketing field especially new product diffusion models are of immense use while developing inventory models. The new product diffusion models in marketing have been developed to estimate new product sales. Because of the dynamic behaviour of new products, it becomes an essential attribute to understand its proper diffusion pattern. Rogers [1] had observed that when a new innovation is introduced in the market, consumers show variable buying behaviour and categorize it according to their time to purchase. Bemhardt and Mackenzie [2] stated that to use the theory of diffusion as an aid in planning new product introductions, the marketing manager must have a model that represents the process of diffusion for the adoption of his new product. The researchers in marketing field have studied various diffusion patterns that explain the behaviour of new products. Rogers ([3, 4]) defines diffusion as the process by which an innovation is communicated through certain channels over time among members of a social system. Fourt and Woodlock [5] give pure innovative model, whereas Fisher and Fry's [6] is purely a substitution model. The Fourt and Woodlock model [5] explains the diffusion process in terms of number of customers who have bought the product by time 't' by a modified exponential curve (Fig. 1). A mixed influence model that captures both the innovation and imitation aspects of product adoption was proposed by Bass [7]. The Bass Model [7] explained above has been widely used in marketing by Mahajan and Muller [8], Mahajan and Yoram [9], Parker [10] and Mahajan et al. ([11, 12]).

The Bass diffusion model basically describes the process of how new products get adopted as an interaction between users and potential users. It has been described as one of the most famous empirical generalizations in marketing and has also been regarded as most influential in marketing and management science. In forecasting field especially product forecasting and technology forecasting, the Bass model has been widely used. The widely used Bass model has not been paid much attention by the researchers in the field of inventory management. It is really unfortunate for those researchers engaged in the inventory field who are not taking into account the effect of innovation diffusion process that has been well explained by the Bass model. Chern, Teng and Yang [13] have formulated the economic order quantity model in which the demand follows innovation diffusion criterion as considered by Bass model [7]. There are also some inventory models that have considered the effect of innovation diffusion process as stated below. Chanda and Kumar [14] have explained the economic order quantity model with demand influenced by dynamic innovation effect. Chanda and Kumar [15] explain the EOQ model having demand influenced by innovation diffusion criterion under inflationary condition. There are few models in marketing and inventory fields that have considered the dynamic nature of potential market size in an innovation diffusion environment. Sharif and Ramanathan [16] presented a modified binomial innovation diffusion model that incorporates dynamic potential adopter populations. Sharif and Ramanathan [17] considered the application of dynamic potential adopter diffusion model through their study of diffusion of oral contraceptives in Thailand. The inventory model that explains the dynamic behaviour of potential market size through economic order quantity model in an innovation diffusion environment was developed by Aggarwal, Jaggi and Kumar [18]. In

a similar way, recently Aggarwal, Jaggi and Kumar [19] developed an EOQ model where demand follows innovation diffusion criterion having dynamic potential market size using fuzzy criterion.

Now, the diffusion model incorporating price variable as a determining factor assumes that price affects both the adoption rate and the potential market. Feichtinger [20] and Jorgensen [21] focus on optimal pricing policies. Jorgensen [21] assumes that potential market is a linear decreasing function of price. Feichtinger [20] assumes that the potential market is a concave decreasing function of price. Kalish and Lilien ([22, 23]) assume a dynamic potential market by testing their model on a new durable product and suggested that the potential market decreases with a price increase. Jain and Rao [24] extended the diffusion model by incorporating price. Robinson and Lakhani [25] extended the Bass model by incorporating price in a multiplicative way. Subramanyam and Kumaraswamy [26] considered the effect of price elasticity as well as frequency of advertisement on the demand assuming a linear relationship. There are few inventory models discussed above that have incorporated the innovation diffusion concept in their demand model but unfortunately have not considered the joint effect of price with time in an innovation diffusion environment.

In the present study, an attempt was made to formulate an inventory model, in which demand of the product is assumed to follow innovation diffusion process as defined by Bass [7] with the effect of selling price of an item, which is dependent on time, to obtain optimal price and optimal cycle length jointly, which maximizes the net profit function. Basically, this paper develops an inventory model that uses demand function as an extension of fundamental Bass diffusion model by incorporating the time dependent selling price of the product. A comprehensive sensitivity analysis was performed to highlight the impact of parameters associated with the model on the economic ordering policies. The article is divided into different sections such as model development, special cases and managerial implications. Finally, the article concludes with a discussion on the application, extension and limitations of the model.
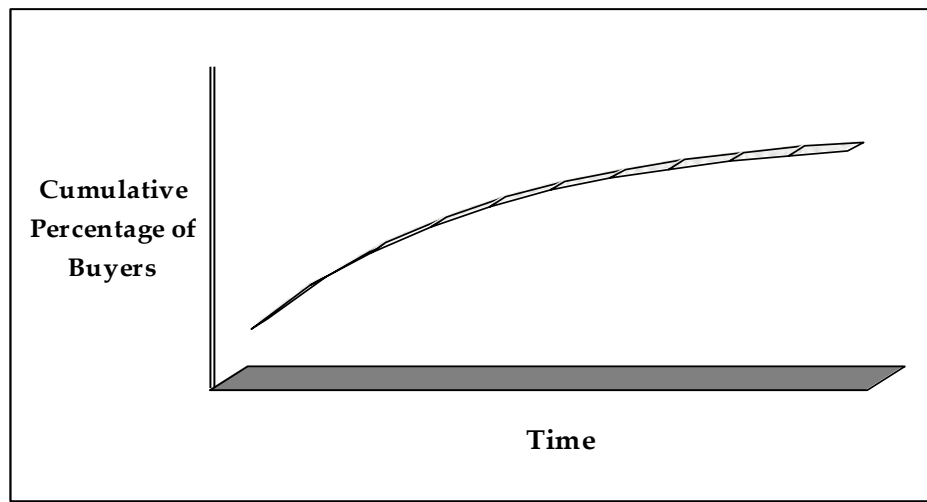

Fig. 1 Fourt and Woodlock Curve (Source: Lilien, Kotler and Moorthy [27])

II. ASSUMPTIONS AND NOTATIONS

The mathematical model developed is based on certain relevant assumptions and makes use of certain defined notations, which are well described here. This model is concerned with the diffusion of new product acceptance, having the objective to represent the level of spread of an innovation and imitation among a given set of prospective buyers with the effect of dynamic behaviour of the selling price. Here, the model uses the dynamic demand model, which is time dependent, and incorporates the effect of some marketing variables such as innovation and imitation parameters, pricing factor, etc. The demand model used here assumes that the market clear is achieved at each point of time and is based on Robinson and Lakhani [25] model having

the functional form as $n(t) = \lambda(t) = \dfrac{dN(t)}{dt} = [m - N(t)]\left[p + q\dfrac{N(t)}{m}\right]e^{-dP(t)}$, where m is the potential market size of

total number of adopters and it is assumed as constant, p and q are the coefficient of innovation and coefficient of imitation respectively, $N(t), t \geq 0$ is the cumulative adoption of a product in a targeted population by time 't', $P(t)$ is the price at time t and d is the coefficient of price sensitivity. The coefficient of innovation (p) which is supposed to be constant throughout the cycle reflects the extent of a consumer's intrinsic propensity to purchase the product, it is also stated as the likelihood that somebody who is not yet using the product will start using it because of mass media coverage or other external factors. Whereas the coefficient of imitation (q) reflects the adoption behaviour of a product by imitating from others, and is assumed as constant throughout the cycle. It is also assumed here that the time interval ' T ' is given and we are to plan an inventory policy for a certain commodity of new products during this specified time interval ' T '. There are certain significant

assumptions on which the model works stated as follows. The replenishment rate is infinite, implying that the replenishments are instantaneous. Lead time is zero and shortages are not allowed. There is only one product bought per new adopter. The innovation's sales are confined to a single geographical area and it is assumed that there is no seasonality in sales of the new product. It is considered that the impacts of marketing strategies by the innovator are adequately captured by the model's parameters.

In addition, the following notations were used in developing the proposed model.

A : Ordering cost per order

C : Unit cost

I : Inventory carrying charge

IC : Inventory carrying cost

T : Length of the replenishment cycle

Q : Number of items received at the beginning of the period

$K(T, P_0)$: The total cost of the system per unit time

$I(t)$: On hand inventory at any time t

$n(t) = \lambda(t) = S(t)$: The number of adoptions at time t, i.e. Demand at time t

$f(t)$: The likelihood of purchase at time t

$F(t)$: The cumulative fraction of adopters at time t

$R(T, P_0)$: Total Revenue of the system per unit time

$Z(T, P_0)$: Total profit of the system per unit time

III. MATHEMATICAL MODEL

The basic assumption considered by different researchers in marketing literature for a fundamental diffusion model is that the rate of diffusion or the number of adopters at any given point in time is directly proportional to the number of remaining potential adopters at that moment. Mathematically, this can be represented as follows:

$$n(t) = \frac{dN(t)}{dt} = g(t)\{m - N(t)\} ,\tag{1}$$

where $g(t)$ is known as the rate of adoption or individual probability of adoption.

It has also been assumed that $g(t)$ depends on time through a linear function of $N(t)$ (Mahajan and Peterson [28]). Hence,

$$n(t) = \frac{dN(t)}{dt} = (a + bN(t))\{m - N(t)\} , a \geq 0, \ b \geq 0 \tag{2}$$

Depending on the importance of each source of influence, different versions can be derived from the fundamental diffusion model (Mahajan and Peterson [28]). When b=0, the model only considers external influence; when a=0, the model only considers internal influence; when $a \neq 0$ and $b \neq 0$, the resulting model is called a mixed influence diffusion model (Conde et al. [29]).

The basic assumption used in the Bass Model is that the adoption of a new product spreads through a population primarily due to contact with prior adopters. Hence, the probability that an initial purchase occurs at time t, given that no purchase has occurred, is a linear function of the number of previous buyers, i.e.

$$\frac{f(t)}{1 - F(t)} = p + qF(t).\tag{3}$$

If we define $n(t) = mf(t)$ and $N(t) = mF(t)$, Eq. (3) can be expressed as follows:

$$n(t) = \frac{dN(t)}{dt} = \left(p + q\frac{N(t)}{m}\right)\{m - N(t)\} .\tag{4}$$

Eq. (4) depicts the mathematical formulation of Bass [7]. The pictorial representation of Eq. (4), as referred by Kumar and Aggarwal [30], is described in Fig. 2.
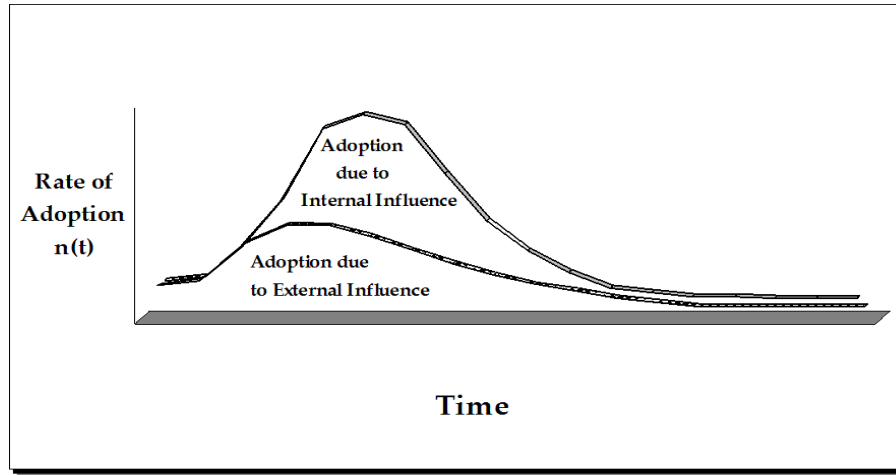
Fig. 2 Bass New Product Diffusion Model (Source: Mahajan, Muller and Bass [11])

Robinson and Lakhani [25] extended the Bass model [7] represented in Eq (4) by incorporating price in a multiplicative way as follows:

$$\frac{dN(t)}{dt} = [m - N(t)]\left[p + q\frac{N(t)}{m}\right]e^{-dP(t)},$$

(5)

where $P(t)$ is the price at time t and d is the coefficient of price sensitivity.

Here, $P(t)$ is the price at time 't' and the model emphasizes the diffusion of new products in the market. Chang et al. [31] have considered time dependent selling price with increasing trend under the influence of inflation. Yang [32] has discussed a partial backlogging model for deteriorating items with fluctuating selling price and purchase cost. It is commonly observed that the price of new products may increase or decrease or remain constant as time passes for a given time period. It all depends on the type of product introduced in the market. The price of various electronic goods having same configuration and same features generally decreases with time for a specified time period due to various reasons, such as competitive environment in the market, introduction of substitute products in the market because of technological advancement, reaching old-fashioned levels among products, etc. Therefore, effective inventory management for such products plays an important role for smooth functioning and growth of an organization. Hence, keeping in view the above situation and to match with the reality, we constructed the following expression of time dependent decreasing price phenomenon.

$$\text{Taking, } P(t) = P_0(1 - \beta t), \ 0 \le \beta < 1$$

(6)

P_0 is the unit selling price at time zero.

Yang [32] has incorporated in his article that price varies linearly with time.

Therefore, the demand function for this model is described as follows:

$$\frac{dN(t)}{dt} = [m - N(t)]\left[p + q\frac{N(t)}{m}\right]e^{-dP_0(1-\beta t)}$$

(7)

$$\text{Taking } n(t) = \lambda(t) = \frac{dN(t)}{dt},$$

(8)

$$\lambda(t) = [m - N(t)]\left[p + q\frac{N(t)}{m}\right]e^{-dP_0(1-\beta t)}$$

(9)

The demand usage $\lambda(t)$, which is a function of time, plays pivotal role to shrink the inventory size over a period of time. If in the time interval $(t, t+dt)$ the inventory size is dipping at the rate $\lambda(t)dt$, then the total reduction in the inventory size during the time interval dt can be given by $-dI(t) = \lambda(t)dt$. Thus, the differential equation describing the instantaneous state of the inventory level at any time t, $I(t)$, in the interval $(0, T)$ is given by:

$$\frac{dI(t)}{dt} = -\lambda(t), \ 0 \le t \le T$$

(10)

$$\Rightarrow \quad \frac{dI(t)}{dt} = -\big[m - N(t)\big]\left[p + q\,\frac{N(t)}{m}\right]e^{-dP_0(1-\beta t)} \quad 0 \le t \le T \tag{11}$$

The solution to the above differential Eq. (11) after applying the boundary conditions $N(0) = 0$ and $I(t) = I(0)$ at $t = 0$ is given by:

$$I(t) = \frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} - \frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)t}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)t}-1\right)}} \tag{12}$$

According to the model assumptions, replenishment is instantaneous and shortages are not allowed. Thus, the inventory level at the initial point of the planning horizon can be assumed to be the cumulative adoption of the product during the cycle time T. Hence,

$$I(0) = Q = \int_0^T \lambda(t)dt \tag{13}$$

$$\Rightarrow \quad I(0) = Q = \frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} \tag{14}$$

$$\text{Now,} \int_0^T I(t)dt = T\,\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} - \left(\frac{m}{(p+q)e^{-dP_0}}\,LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right)$$
$$- \frac{mp}{q(p+q)e^{-dP_0}}\,LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right] \tag{15}$$

The different cost elements involved in the inventory system per unit time were defined as follows:

$$\text{Ordering cost per unit time} = \frac{A}{T} \tag{16}$$

Material cost per unit time $= \dfrac{QC}{T}$

$$= \left(\frac{C}{T}\right)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\phi P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} \tag{17}$$

Cost of carrying inventory per unit time $= \dfrac{IC}{T}\displaystyle\int_0^T I(t)dt$

$$= (IC)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} - \left(\frac{mIC}{T(p+q)e^{-dP_0}}\,LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right)$$
$$- \frac{mpIC}{Tq(p+q)e^{-dP_0}}\,LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right] \tag{18}$$

Using Eqs. (16), (17) and (18), the total cost per unit time $K(T, P_0)$ is given by:

$$K(T, P_0) = \frac{A}{T} + \left(\frac{C}{T}\right)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} - \left(\frac{mIC}{T(p+q)e^{-dP_0}} \; LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right)$$

$$+ (IC)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} - \frac{mpIC}{Tq(p+q)e^{-dP_0}} \; LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right]$$

(19)

Now, total revenue per cycle $= \int_0^T \lambda(t)P(t)dt$

$$= P_0(1-\beta T)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} + \left(\frac{\beta mP_0}{(p+q)e^{-dP_0}} \; LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right)$$

$$+ \frac{P_0 mp\beta}{q(p+q)e^{-dP_0}} \; LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right]$$

(20)

Therefore, total revenue per unit time $R(T)$ is given by

$$R(T, P_0) = \left(\frac{P_0}{T}\right)(1-\beta T)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}}$$

$$+ \left(\frac{\beta mP_0}{T(p+q)e^{-dP_0}} \; LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right) + \frac{P_0 mp\beta}{qT(p+q)e^{-dP_0}} \; LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right]$$

(21)

Total profit per unit time $Z(T, P_0)$ is given by

$$Z(T, P_0) = R(T, P_0) - K(T, P_0)$$

(22)

$$\Rightarrow$$

$$Z(T, P_0) = \left(\frac{P_0}{T}\right)(1-\beta T)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}} - \left(\frac{C}{T}\right)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}}$$

$$+ \left(\frac{\beta mP_0}{T(p+q)e^{-dP_0}} \; LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right) - (IC)\frac{mpe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)} - mp}{q + pe^{\frac{(p+q)}{dP_0\beta}e^{-(dP_0)}\left(e^{(d\beta P_0)T}-1\right)}}$$

(23)

$$+ \frac{P_0 mp\beta}{qT(p+q)e^{-dP_0}} \; LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right] + \left(\frac{mIC}{T(p+q)e^{-dP_0}} \; LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right)$$

$$+ \frac{mpIC}{Tq(p+q)e^{-dP_0}} \; LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right] - \frac{A}{T}$$

Here, the objective is to jointly optimize the cycle length and the unit selling price in order to obtain the optimum order quantity and hence the optimum profit.

Since the objective is to maximize the profit function $Z(T, P_0)$, the necessary conditions for maximizing $Z(T, P_0)$ are

$$\frac{\partial Z(T, P_0)}{\partial T} = 0 \tag{24}$$

$$\text{and } \frac{\partial Z(T, P_0)}{\partial T} = 0 \tag{25}$$

The sufficient conditions for maximizing $Z(T, P_0)$ are

$$\frac{\partial^2 Z(T, P_0)}{\partial T^2} < 0, \ \frac{\partial^2 Z(T, P_0)}{\partial P_0^2} < 0 \tag{26}$$

$$\text{and } \frac{\partial^2 Z(T, P_0)}{\partial T^2} \frac{\partial^2 Z(T, P_0)}{\partial P_0^2} - \left(\frac{\partial^2 Z(T, P_0)}{\partial T \partial P_0} \right)^2 > 0 \tag{27}$$

at $P_0 = P_0^*$ and $T = T^*$

Since Eqs. (24) and (25) are highly nonlinear, the problem was solved numerically for given parameter values. Now, the solutions to Eqs. (24) and (25) give the optimal values of 'T' and 'P_0', i.e. T^* and P_0^*, respectively. The optimal value of $Z(T, P_0)$, i.e. $Z(T^*, P_0^*)$ and the optimal value of Q, i.e. Q^* will be obtained from Eqs. (23) and (14), respectively, provided that it satisfies Eqs. (26) and (27). The numerical solution for the given base value was obtained by using software packages Excel-Solver and LINGO, and the surface graph to show the concavity of the profit function was drawn by using the software package Mathematica -7.

A. Special Case: Constant Price

When

$$\beta = 0, \ P(t) = P_0. \tag{28}$$

This implies that the price of the product is constant throughout the cycle. Here, the diffusion of those new products is taken into account, and the selling price of the new products generally do not change for a given time period. In this case, the total cost of the system per unit time $K_1(T, P_0)$ is given by:

$$K_1(T, P_0) = \frac{A}{T} + \left(\frac{C}{T} \right) \frac{mpe^{(p+q)e^{-dP_0}T} - mp}{q + pe^{(p+q)e^{-dP_0}T}} - \frac{mpIC}{Tq(p+q)e^{-dP_0}} LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)} \right]$$
$$- \left(\frac{mIC}{T(p+q)e^{-dP_0}} LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)} \right] \right) + \frac{ICmpe^{(p+q)e^{-dP_0}T} - ICmp}{q + pe^{(p+q)e^{-dP_0}T}} \tag{29}$$

The number of items received at the beginning of the period, Q_1, is given by

$$Q_1 = I(0) = \int_0^T \lambda(t) dt = \frac{mpe^{(p+q)e^{-dP_0}T} - mp}{q + pe^{(p+q)e^{-dP_0}T}} \tag{30}$$

Total Revenue per unit time $R_1(T, P_0)$ is given by

$$R_1(T, P_0) = \left(\frac{P_0}{T} \right) \frac{mpe^{(p+q)e^{-dP_0}T} - mp}{q + pe^{(p+q)e^{-dP_0}T}} \tag{31}$$

Therefore, total profit per unit time $Z_1(T, P_0)$ is given by

$$Z_1(T,P_0) = \left(\frac{P_0}{T}\right)\frac{mpe^{(p+q)e^{-dP_0}T} - mp}{q + pe^{(p+q)e^{-dP_0}T}} - \frac{A}{T} - \frac{ICmpe^{(p+q)e^{-dP_0}T} - ICmp}{q + pe^{(p+q)e^{-dP_0}T}}$$

$$+ \frac{mpIC}{Tq(p+q)e^{-dP_0}}LOG\left[\frac{p + qe^{-(p+q)e^{-dP_0}T}}{(p+q)}\right] - \left(\frac{C}{T}\right)\frac{mpe^{(p+q)e^{-dP_0}T} - mp}{q + pe^{(p+q)e^{-dP_0}T}} \tag{32}$$

$$+ \left(\frac{mIC}{T(p+q)e^{-dP_0}}LOG\left[\frac{q + pe^{(p+q)e^{-dP_0}T}}{(p+q)}\right]\right)$$

Since the objective is to maximize the profit function $Z_1(T,P_0)$, the necessary conditions for maximizing $Z_1(T,P_0)$ are

$$\frac{\partial Z_1(T,P_0)}{\partial T} = 0 \tag{33}$$

$$\text{and } \frac{\partial Z_1(T,P_0)}{\partial P_0} = 0 \tag{34}$$

The sufficient conditions for maximizing $Z_1(T,P_0)$ are

$$\frac{\partial^2 Z_1(T,P_0)}{\partial T^2} < 0, \ \frac{\partial^2 Z_1(T,P_0)}{\partial P_0^2} < 0 \tag{35}$$

$$\text{and } \frac{\partial^2 Z_1(T,P_0)}{\partial T^2}\frac{\partial^2 Z_1(T,P_0)}{\partial P_0^2} - \left(\frac{\partial^2 Z_1(T,P_0)}{\partial T\partial P_0}\right)^2 > 0 \tag{36}$$

$$\text{at } P_0 = P_0^* \text{ and } T = T^*$$

Since Eqs. (33) and (34) are highly nonlinear, the problem was solved numerically for given parameter values. Now, the solutions to Eqs. (33) and (34) give the optimal values of 'T' and 'P_0', i.e. T^* and P_0^*, respectively. The optimal value of $Z_1(T,P_0)$, i.e. $Z_1(T^*,P_0^*)$ and the optimal value of Q, i.e. Q_1^* will be obtained from Eqs. (32) and (30), respectively, provided that it satisfies Eqs. (35) and (36). The numerical solution for the given base value was obtained by using software packages Excel-Solver and LINGO, and the surface graph to show the concavity of the profit function was drawn by using the software package Mathematica -7.

B. *Solution Procedure*

The solution procedure is summarized as below:

Step 1: Input all parameter values such as different cost parameters, coefficient of innovation, coefficient of imitation, potential market size, etc.

Step 2: Compute all possible values of 'T' and 'P_0' jointly using Eqs. (24) and (25) and Eqs. (33) and (34) for the main model and the special case respectively.

Step 3: Select the appropriate values of 'T' and 'P_0' using Eqs. (23) and Eq (32) for the Main model and the special case, respectively, on satisfying Eqs. (26) & (27) for the Main model and Eqs. (35), (36) for the special case.

Step 4: Compute $Z(T,P_0)$ for the main model and $Z_1(T,P_0)$ for the special case by using the appropriate values of 'T' and 'P_0' as computed in step-(3). The above steps are used for all replenishment schedules using appropriate parameter values. In order to obtain the values of 'T' and 'P_0' jointly, Eqs. (24), (25), (33) and (34) were solved for the main model and the special case, respectively, using LINGO and EXCEL-Solver software packages.

IV. NUMERICAL EXAMPLES

In this section, a numerical example is presented to illustrate the effectiveness of the proposed model. Consider a hypothetical example in an inventory system with the following parameters in appropriate units as follows:

A=\$1500/order, $C = \$\,1100$/unit, I= 25%, $d = 0.001, \beta = 0.1, m = 50000, p = 0.005, q = 0.35,$

The following results obtained for this set of parameters were obtained jointly for the values of P_0 and T by following the above solution procedure and using the software packages Excel-Solver and LINGO.

$$P_0 = \$2254,\ T = 0.44,\ Z(T, P_0) = \$23834 \text{ and } Q = 12 \text{ units.}$$

The effect of changes in the parameters of the inventory model is shown numerically in the following tables using the above stated solution procedure. To prove the validity of the model numerically and to get the appropriate parameter values, references were taken from Sharif and Ramanathan [16], Chanda and Kumar [14], Chandrasekaran and Tellis [33], Aggarwal, Jaggi and Kumar [18], ultan, Farley and Lehmann [34], Talukdar, Sudhir and Ainslie [35], Chanda and Kumar [15].

TABLE 1 SENSITIVITY ANALYSIS ON COEFFICIENT OF INNOVATION ' p '

A=\$1500/order, $C = \$\,1100$/unit, $I = 25\%$, d=0.001, $\beta = 0.1$, $m = 50000$, $q = 0.35$

p	P_0^*	T^*	$Z(T^*, P_0^*)$	Q^*
0.001	2480	1.02	3121	5
0.002	2355	0.71	7977	8
0.003	2304	0.57	13129	10
0.004	2274	0.49	18432	11
0.005	2254	0.44	23834	12
0.006	2240	0.40	29304	14
0.007	2229	0.37	34827	15
0.008	2220	0.35	40391	16
0.009	2213	0.33	45989	17
0.01	2207	0.31	51615	18
0.02	2174	0.21	108827	26

TABLE 2 SENSITIVITY ANALYSIS ON COEFFICIENT OF IMITATION 'q'

A=\$1500/order, $C = \$\,1100$/unit, $I = 25\%$, d=0.001, $\beta = 0.1$, $m = 50000$, p=0.005

q	P_0^*	T^*	$Z(T^*, P_0^*)$	Q^*
0.30	2255.12	0.443	23801	12.32
0.32	2255.12	0.444	23814	12.35
0.34	2255.12	0.445	23827	12.38
0.36	2255.12	0.446	23840	12.41
0.38	2255.12	0.446	23854	12.43
0.40	2255.12	0.447	23867	12.46
0.42	2255.12	0.448	23880	12.49
0.44	2253.79	0.448	23893	12.52
0.46	2253.79	0.449	23907	12.55
0.48	2253.79	0.450	23920	12.58
0.50	2253.06	0.451	23933	12.61

TABLE 3 SENSITIVITY ANALYSIS ON INVENTORY CARRYING CHARGE ' I '

A=\$1500/order, $C = \$ 1100$/unit, d=0.001, $\beta = 0.1$, m =50000, $p = 0.005$, $q = 0.35$

I	P_0^*	T^*	$Z(T^*,P_0^*)$	Q^*
0.20	2246	0.483	24401	13.63
0.21	2248	0.475	24283	13.35
0.22	2250	0.467	24168	13.09
0.23	2251	0.459	24054	12.85
0.24	2253	0.452	23943	12.62
0.25	2254	0.445	23834	12.39
0.26	2256	0.439	23726	12.18
0.27	2258	0.432	23621	11.98
0.28	2259	0.427	23517	11.79
0.29	2261	0.421	23415	11.61
0.30	2262	0.415	23314	11.41

A. *Special Case: Constant Price Situation i.e.* $\beta = 0$

A hypothetical example with the following parameter values in appropriate units was used.

A=\$1500/order, $C = \$1100$/unit, I = 2.5%, $d = 0.001$, m =50000, $\beta = 0$.

TABLE 4 SENSITIVITY ANALYSIS ON COEFFICIENT OF INNOVATION ' p '

A=\$1500/order, $C = \$ 1100$/unit, $I = 25$%, d=0.001, m =50000, $q = 0.35$

p	P_0^*	T^*	$Z_1(T^*,P_0^*)$	Q_1^*
0.001	2347	1.24	3573	6
0.002	2268	0.84	8586	9
0.003	2234	0.68	13859	11
0.004	2215	0.58	19265	13
0.005	2202	0.52	24756	15
0.006	2193	0.47	30307	16
0.007	2186	0.43	35905	17
0.008	2180	0.40	41538	19
0.009	2175	0.38	47201	20
0.01	2171	0.36	52889	21
0.02	2150	0.25	110598	30

TABLE 5 SENSITIVITY ANALYSIS ON COEFFICIENT OF IMITATION 'q'

A=\$1500/order, $C = \$ 1100$/unit, $I = 25$%, d=0.001, m =50000, p=0.005

q	P_0^*	T^*	$Z_1(T^*,P_0^*)$	Q_1^*
0.30	2203.67	0.519	24716	14.46
0.32	2203.67	0.521	24732	14.51
0.34	2203.01	0.522	24748	14.56
0.36	2203.01	0.523	24764	14.60
0.38	2202.35	0.524	24780	14.65
0.40	2202.02	0.525	24796	14.70
0.42	2202.02	0.527	24812	14.74
0.44	2202.02	0.528	24828	14.79
0.46	2201.03	0.529	24844	14.84
0.48	2201.03	0.530	24860	14.89
0.50	2201.03	0.532	24876	14.93

TABLE 6 SENSITIVITY ANALYSIS ON INVENTORY CARRYING CHARGE ' I '

A=$1500/order, $C = \$ 1100$/unit, d=0.001, m =50000, $p = 0.005$, $q = 0.35$

I	P_0^*	T^*	$Z_1\left(T^*, P_0^*\right)$	Q_1^*
0.20	2190	0.58	25426	16.63
0.21	2192	0.57	25284	16.15
0.22	2195	0.55	25147	15.71
0.23	2197	0.54	25013	15.31
0.24	2200	0.53	24883	14.93
0.25	2202	0.52	24756	14.58
0.26	2205	0.51	24632	14.25
0.27	2207	0.50	24510	13.94
0.28	2209	0.49	24391	13.65
0.29	2212	0.48	24275	13.38
0.30	2214	0.47	24160	13.12

B. Graphical Representation

The nature of the profit functions discussed in section-(3) is highly non-linear. Therefore, to prove its concave nature, the graphical representation is provided in Figs. 3 and 4:
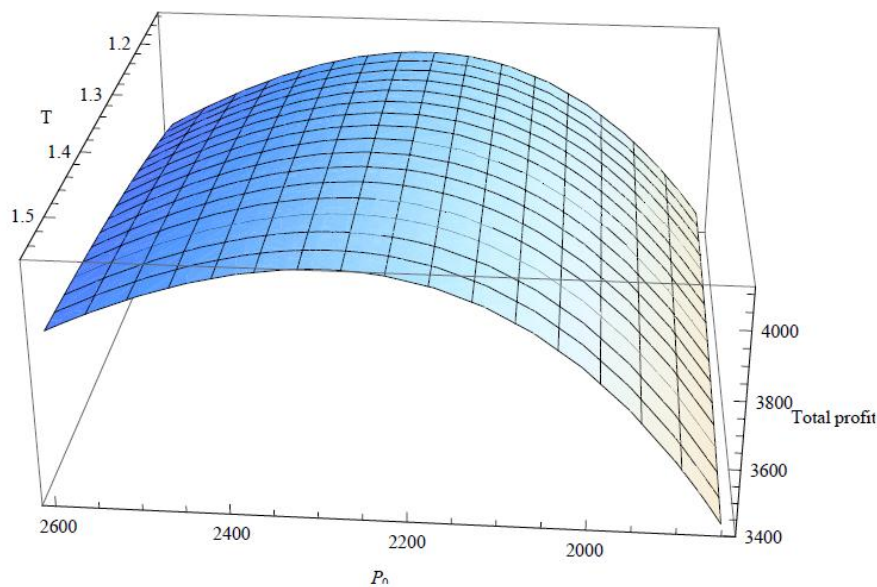

Fig. 3 Graphical representation to show concavity for p=0.001 (Total average profit vs. P_0 and T)
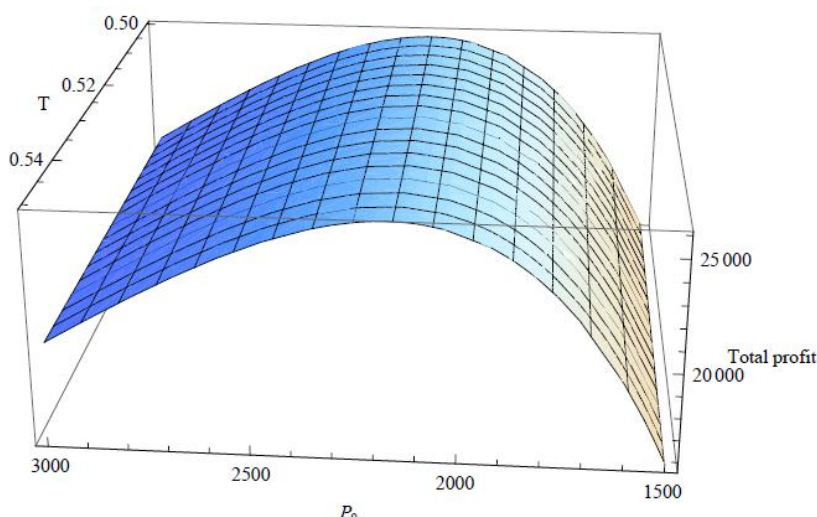
Fig. 4 Graphical representation to show concavity for p=0.005 (Total average profit vs. P_0 and T)

V. OBSERVATIONS

The numerical results obtained from different numerical tables in the numerical example section explain the effect of changes in the system parameters on different optimal values of the model, and the following relationship was observed during the numerical exercise. The sensitivity analysis with respect to different parameters showed the following:

- As the value of 'p' increases with other parameters kept constant, the values of T^* and P_0^* decrease while the optimal net profit and the optimal lot size increase significantly, as depicted in Table 1) and Table 4. This is consistent with the reality as more investment on promotion will increase the diffusion of a product with decreasing trend of price of the item in the market, resulting in shrinkage of the optimal reorder cycle time and addition in the optimal lot size, and as a result the optimal profit is increased.

- As the value of 'q' increases with other parameters kept constant, there is a marginal increasing trend in the value of T^*, the optimal net profit and the optimal lot size, while there is a marginal decreasing trend or constant trend in the value of P_0^*, as depicted in Table 2 and Table 5. This shows that the change in the coefficient of imitation results in little change in the diffusion of products for this model and hence there is a moderate change in all the optimal values of the system.

- As the value of 'I' increases with other parameters kept constant, the value of P_0^* increases while T^*, the optimal lot size and the optimal net profit decrease, as depicted in Table 3 and Table 6. This is also consistent with the reality that as inventory carrying charge increases, the selling price of the product is forced to be increased and also it forces the inventory manager to keep the inventory for shorter time period, and as a result the cycle length decreases. As the selling price of the product increases, the demand lowers, resulting in diminishing of lot size, and as a result the optimum net profit decreases.

VI. CONCLUDING REMARKS WITH MANAGERIAL IMPLICATIONS

The optimal decision-making is a key factor of any successful business operation and it becomes more crucial when the associated parameters are of uncertain nature. The one way to address this problem is to develop realistic models as far as possible. When the nature of the product is dynamic, the problem of its scheduling and managing is aggravated more. In this era of constant innovation, new products are entered into the market because of rapid product development, which leads to reduction in the life cycle of the products. The dynamic behaviour of the new products makes the management to manage its inventories carefully by studying the uncertain nature of parameters associated with it. Here, this paper is concerned with the inventory model of those products, which are newly introduced into the market, and diffusion pattern of such products has been well explained through different models by several researchers in the marketing field. This model incorporates the demand function as an extension of fundamental Bass diffusion model and its diffusion pattern has been explained by Robinson and Lakhani [25]. The approach of this paper is to develop an inventory model for new product diffusion influenced by its dynamic pricing behaviour. The utility of this paper is how to make an optimal inventory policy in a situation where the nature of its products possesses the above-explained characteristics. Therefore, this model will help immensely to an inventory manager while scheduling and managing the inventories of such kind of products. The effect of joint optimization of price and

time was experienced to explain the nature of the model. Here, the optimal cycle length and the optimal price were jointly optimized, which is highly desired when taking decisions regarding procurement policy of inventories of new products based on innovation diffusion process. A numerical example followed by sensitivity analyses on the model parameters were provided to verify the results obtained in the real life situation. The numerical results provided in the different numerical tables play a crucial role when taking optimal inventory decisions under the conditions well stated above. A simple solution procedure in the form of algorithm is presented to determine the optimal cycle time, optimal price and optimal order quantity of the average profit function. A special case was formulated to understand the nature of the model in different situation. The limitation of the model suggests that the existence and the uniqueness of the profit function were shown numerically because of its highly non-linear nature. Thus, research on some alternative approach to get the optimal analytical solution of the problem is important. The research in future can be further extended to develop the inventory models for multiple generation case, shortages and backlogging, quantity discount, partial lost sales and by taking some dynamic nature of the parameters involved in the model.

REFERENCES

[1] E. M. Rogers, "Diffusion of innovations," *The Free Press*, New York, 1962.

[2] L. Bemhardt, and K. E. MacKenzie, "Some problems in using diffusion models for new products," *Management Science*, vol. 18, pp. 187-200, 1972.

[3] E. M. Rogers, "Diffusion of innovations," *The Free Press*, New York, 1983.

[4] E. M. Rogers, "Diffusion of innovations," *The Free Press*, New York, 2003.

[5] L. A. Fourt, and J. W. WoodLock, "Early prediction of market success for new grocery products," *Journal of Marketing*, vol. 25, iss. 2, pp. 31-38, 1960.

[6] J. C. Fisher, and R. H. Pry, "A simple substitution model of technological change," *Technological Forecasting and Social Change*, vol. 3, iss. 2, pp. 75-88, 1971.

[7] F. M. Bass, "A new product growth model for consumer durables," *Management Science*, vol. 15, iss. 5, pp. 215-227, 1969.

[8] V. Mahajan, and E. Muller, "Innovation diffusion and new-product growth models in marketing," *Journal of Marketing Fall*, vol. 43, iss. 4, pp. 55-68, 1979.

[9] V. Mahajan, and W. Yoram, *Innovation Diffusion Models of New Product Acceptance*, Cambridge, MA, Ballinger publishing company, pp. 235-279, 1986.

[10] P. M. Parker, "Aggregate diffusion forecasting models in marketing: A critical review," *International Journal of Forecasting*, vol. 10, iss. 2, pp. 353-380, 1994.

[11] V. Mahajan, E. Muller, and F. M. Bass, "New product diffusion models in marketing: A review and directions for research," *Journal of Marketing*, vol. 54, iss. 1, pp. 1-26, 1990.

[12] V. Mahajan, E. Muller, and Y. Wind, *New-Product Diffusion Models*, Kluwer Academic Publisher , Boston , MA, 2000.

[13] M. S. Chern, J. T. Teng, and H. L. Yang, "Inventory lot-size policies for the bass diffusion demand models of new durable products," *Journal of the Chinese Institute of Engineers*, vol. 24, iss. 2, pp. 237-244, 2001.

[14] U. Chanda, and A. Kumar, "Economic order quantity model with demand influenced by dynamic innovation effect," *International Journal Of Operational Research*, vol. 11, iss. 2, pp. 193-215, 2011.

[15] U. Chanda, and A. Kumar, "Economic order quantity model on inflationary conditions with demand Influenced by innovation diffusion criterion," *International Journal of Procurement Management*, vol. 5, iss. 2, pp. 160-177, 2012.

[16] M. N. Sharif, and K. Ramanathan, "Binomial innovation models with dynamic potential adopter population," *Technological Forecasting and Social Change*, vol. 20, iss. 1, pp. 63-87, 1981.

[17] M. N. Sharif, and K. Ramanathan, "Polynomial innovation diffusion models," *Technological Forcasting and Social Change*, vol. 21, iss. 4, pp. 301-323, 1982.

[18] K. K. Aggarwal, C. K. Jaggi, and A. Kumar, "Economic order quantity model with innovation diffusion criterion having dynamic potential market size," *International Journal of Applied Decision Sciences*, vol. 4, iss. 3, pp. 280-303, 2011.

[19] K. K. Aggarwal, C. K. Jaggi, and A. Kumar, "Economic order quantity model under fuzzy sense when demand follows innovation diffusion process having dynamic potential market size," *International Journal of Services and Operations Management*, vol. 13, iss. 3, pp. 361-391, 2012.

[20] G. Feichtinger, "Optimal pricing in a diffusion model with concave price dependent market potential," *Operations Research Letters*, vol. 1, iss. 6, pp. 236-240, 1982.

[21] S. Jorgensen, "Optimal control of a diffusion model of new products acceptance with price dependent total potential market," *Optimal control Applications and Methods*, vol. 4, pp. 269-276, 1983.

[22] S. Kalish, and G. L. Lilien, "A market entry timing model for new technologies," *Management Science*, vol. 32, iss. 2, pp. 194-205, 1986a.

[23] S. Kalish, and G. L. Lilien, "Applications of innovations diffusion models in marketing," In: V. Mahajan, and W. Yoram *Innovation Diffusion Models of New Product Acceptance*, 1986, Cambridge, MA, Ballinger, pp. 235-279, 1986b.

[24] D. Jain, and R. C. Rao, "Effect of price on the demand for durables," *Journal of Business Economics and Statistics*, vol. 8, pp. 163-170, 1990.

[25] B. Robinson, and C. Lakhani, "Dynamic price models for new-product planning," *Management Science*, vol. 21, iss. 10, pp. 1113-1122, 1975.

[26] E. S. Subramanyam, and S. Kumaraswamy, "EOQ formula under varying market policies and conditions," *AIIE Transactions*, vol. 13, pp. 312-314, 1981.

[27] G. L. Lilien, P. Kotler, and K. S. Moorthy, *Marketing Models*, Prentice-Hall of India Private Limited, New Delhi, 1999.

[28] V. Mahajan, and R. A. Peterson, *Models of innovation diffusion*, Beverly Hills et al., Sage Publications, 1985.

[29] E. R. Conde, P. S. H. Leeflang, and J. E. Wieringa, "Marketing variables in macro-level diffusion models," *State- Of- The- Art- Artikel*, vol. 56, pp. 155-183, 2006.

[30] A. Kumar, and K. K. Aggarwal, "Optimal replenishment policy for an inventory model with demand follows innovation diffusion process under permissible delay in payments," *International Journal of Information and Management Sciences*, vol. 23, iss. 2, pp. 177-197, 2012.

[31] C-T. Chang, S-J. Wu, and L-C. Chen, "Optimal payment time with deteriorating items under inflation and permissible delay in payments," *International Journal of System Science*, vol. 40, iss. 10, pp. 985-993, 2009.

[32] H-L. Yang, "A partial backlogging inventory model for deteriorating items with fluctuating selling price and purchase cost," *Advances in Operations Research*, vol. 2012, Article ID 385371, 15 pages, 2012.

[33] D. Chandrasekaran, and G. J. Tellis, "A critical review of marketing research on diffusion of new products," *Review of Marketing Research*, vol. 3, pp. 39-80, 2007.

[34] F. Sultan, J. U. Farley, and D. R. Lehmann, "A meta-analysis of diffusion models," *Journal of marketing research*, vol. 27, iss. 1, pp. 70-77, 1990.

[35] D. Talukdar, K. Sudhir, and A. Ainslie, "Investigating new product diffusion across products and countries," *Marketing Science*, vol. 21, iss. 1, pp. 97-114, 2002.