Clustering Accuracies on Concepts of Nursing

Jui-Chiao Tseng

Min-Hwei College of Health Care Management

jctseng@mail.mhchcm.edu.tw

Abstract- **The purpose of this study is to integrate fuzzy clustering algorithm based on common Mahalanobis distance. Fuzzy clustering could distinguish characteristics of concept structures on nursing. Finally, some limitations and suggestions of this study are discussed. It shows that knowledge structures will be feasible for remedial instruction and help students get more chance to get professional certification on nurses. Based on the findings and results, combined with fuzzy clustering algorithm based on normalized Mahalanobis distance could be very feasible for cognition diagnosis in the future.**

Keywords- Fuzzy Clustering Algorithm; Nursing

I. INTRODUCTION

The algorithm provided by Bezdek [1] is used in this study. Fuzzy c-mean algorithm based on Euclidean distance function converges to a local minimum of the objective function, which can only be used to detect spherical structural clusters. Gustafson-Kessel clustering algorithm and Gath-Geva clustering algorithm were developed to detect non-spherical structural clusters. Gustafson-Kessel clustering algorithm needs added constraint of fuzzy covariance matrix, whereas Gath-Geva clustering algorithm can only be used for the data with multivariate Gaussian distribution. In GK-algorithm, modified Mahalanobis distance with preserved volume was used. However, the added fuzzy covariance matrices in their distance measure were not directly derived from the objective function. The fuzzy covariance matrices in the Mahalanobis distance can be directly derived by minimizing the objective function.

II. LITERATURE REVIEW

FCM can only work well for spherical shaped clusters. In the objective function the distances between data points to the centers of the clusters are calculated by Euclidian distances. To overcome the above drawback, we could try to extend the distance measure to Mahalanobis distance (MD). However, Krishnapuram and Kim (1999) pointed out that the Mahalanobis distance can not be used directly in clustering algorithm. Gustafson and Kessel (1979) extended the Euclidian distances of the standard FCM by employing an adaptive norm, in order to detect clusters of different geometrical shape without changing the clusters' sizes in one data set. Gath-Geva (GG) fuzzy clustering algorithm is an extension of Gustafson-Kessel (GK) fuzzy clustering algorithm, and also takes the size and density of clusters for classification. Hence, it has better behaviors for irregular features. For improving the limitation of GK algorithm and GG algorithm, we added a regulating factor of covariance matrix to each class in the objective function, and deleted the constraint of the determinant of covariance matrices. We can obtain the Fuzzy C-Means based on adaptive Mahalanobis distance (FCM-M) as following [2, 3]. For improving the stability of the clustering results, we replace all of the covariance matrices with the same common covariance matrix in the objective function in the FCM-M algorithm, and then, an improved fuzzy clustering method, called the Fuzzy C-Means algorithm based on common Mahalanobis distance (FCM-CM) is proposed.

We can obtain the objective function of FCM-CM. Minimizing the objective function respect to all parameters with some constraints, we can obtain the following FCM-CM algorithm. The steps of the FCM-CM are also determining the number of cluster; c and m-value (let m=2), given converge error, randomly choose the initial membership.

A. FCM-CM Algorithm

In this paper, not only z-score is normalizing for each feature in the objective function in the FCM-CM algorithm, but also replacing the threshold D:

$$D=\sum_{i=1}^{c}\sum_{j=1}^{n}\left[\mu_{ij}^{(0)}\right]^{m}\left[\left(\underline{x}_j-\underline{a}_i^{(0)}\right)'\left(\underline{x}_j-\underline{a}_i^{(0)}\right)\right]>0$$

The new fuzzy clustering method, called the Fuzzy C-Means algorithm based on normalized Mahalanobis distance (FCM-NM) is proposed. We can obtain the objective function of FCM-NM as following:

$$J_{FCM-NM}^{m}\left(U,A,R,Z\right)=\sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^{m}d^{2}\left(z_j,\underline{a}_i\right) \tag{1}$$

Where

$$X=\left[\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_n\right],\ \underline{x}_j\in R^{p},\ j=1,2,\ldots,n \tag{2}$$

DOI:10.5963/PHF0203002

$$\underline{z}_j = \left(z_{1j}, z_{2j}, \ldots, z_{pj}\right)', \; z_{tj} = \frac{x_{tj} - \overline{x}_t}{s_t}, \; j = 1, 2, \ldots, n, \; t = 1, 2, \ldots, p \tag{3}$$

$$\overline{x}_t = \frac{1}{n}\sum_{j=1}^{n} x_{tj}, \; s_t = \frac{1}{n}\sum_{j=1}^{n}\left(x_{tj} - \overline{x}_t\right)^2, \quad t = 1, 2, \ldots, p \tag{4}$$

Conditions for FCM-CM are

$$\begin{gathered} m \in [1, \infty); U = \left[\mu_{ij}\right]_{c \times n}; \mu_{ij} \in [0,1], \; i = 1, 2, \ldots, c, \; j = 1, 2, \ldots, n \\ \sum_{i=1}^{c} \mu_{ij} = 1, \; j = 1, 2, \ldots, n, \; 0 < \sum_{j=1}^{n} \mu_{ij} < n, \; i = 1, 2, \ldots, c \end{gathered} \tag{5}$$

$$d^2\left(\underline{z}_j, \underline{a}_i\right) = \begin{cases} \left(\underline{z}_j - \underline{a}_i\right)' R^{-1}\left(\underline{z}_j - \underline{a}_i\right) - \ln\left|\Sigma^{-1}\right| & \text{if } \left(\underline{z}_j - \underline{a}_i\right)' R^{-1}\left(\underline{z}_j - \underline{a}_i\right) - \ln\left|R^{-1}\right| \geq 0 \\ 0 & \text{if } \left(\underline{z}_j - \underline{a}_i\right)' R^{-1}\left(\underline{z}_j - \underline{a}_i\right) - \ln\left|R^{-1}\right| < 0 \end{cases} \tag{6}$$

Minimizing (1), we can obtain the updating functions as follows and the steps of the FCM-NM are listed as follows:

Step 1: Determining the number of cluster; c, m-value (let m=2), and the threshold as follows:

$$|R| = \left|\frac{1}{n}\sum_{j=1}^{n}\left(\underline{z}_j - \underline{a}_t\right)\left(\underline{z}_j - \underline{a}_t\right)'\right|$$

Where

$$0 \leq |R| \leq 1, \; And \qquad \underline{a}_t = \frac{1}{n}\sum_{j=1}^{n} \underline{z}_j \tag{7}$$

Randomly choose the initial membership $u^{(0)}{}_{ij}, i = 1, 2, \ldots, c, \; j = 1, 2, \ldots, n$ such that

$$\sum_{1 \leq i \leq c} u^{(0)}{}_{ij} = 1, \; j = 1, 2, \ldots, n \tag{8}$$

$$\underline{a}_i^{(0)} = \left[\sum_{j=1}^{n} \mu_{ij}^{(0)}\right]^{-1} \sum_{j=1}^{n} \mu_{ij}^{(0)} \underline{z}_j, \quad i = 1, 2, \ldots, c, \quad k = 1, 2, .. \tag{9}$$

$$R^{(0)} = \left[\sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^{(0)}\right]^{-1} \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^{(0)}\left(\underline{z}_j - \underline{a}_i^{(0)}\right)\left(\underline{z}_j - \underline{a}_i^{(0)}\right)' \quad if \qquad \left|R^{(0)}\right| < |R| \quad then \;\; R^{(0)} = I \tag{10}$$

Step 2: Find

$$\underline{a}_i^{(k)} = \left[\sum_{j=1}^{n}\left(\mu_{ij}^{(k-1)}\right)^m\right]^{-1} \sum_{j=1}^{n}\left(\mu_{ij}^{(k-1)}\right)^m \underline{z}_j, \;\; i = 1, 2, \ldots, c, \;\; k = 1, 2, .. \tag{11}$$

$$R^{(k)} = \left[\sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^{(k-1)}\right]^{-1} \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^{(k-1)}\left(\underline{z}_j - \underline{a}_i^{(k)}\right)\left(\underline{z}_j - \underline{a}_i^{(k)}\right)' \quad if \qquad \left|R^{(k)}\right| < |R| \quad then \;\; R^{(k)} = I \tag{12}$$

$$\mu_{ij}^{(k)} = \left[\sum_{s=1}^{c}\left[\frac{\left(\underline{z}_j - \underline{a}_i^{(k)}\right)'\left[R^{-1}\right]^{(k)}\left(\underline{z}_j - \underline{a}_i^{(k)}\right) - \ln\left\|\left[R^{-1}\right]\right\|^{(k)}}{\left(\underline{z}_j - \underline{a}_s^{(k)}\right)'\left[R^{-1}\right]^{(k)}\left(\underline{z}_j - \underline{a}_s^{(k)}\right) - \ln\left\|\left[R^{-1}\right]\right\|^{(k)}}\right]^{\frac{1}{m-1}}\right]^{-1} \tag{13}$$

Step 3: Increment k;

$$\sum_{i=1}^{c}\left\|\underline{a}_i^{(k-1)} - \underline{a}_i^{(k-2)}\right\|^2 \geq \sum_{i=1}^{c}\left\|\underline{a}_i^{(k)} - \underline{a}_i^{(k-1)}\right\|^2 \geq \ldots \geq \sum_{i=1}^{c}\left\|\underline{a}_i^{(k+9)} - \underline{a}_i^{(k+8)}\right\|^2 \tag{14}$$

Step 4: Classification strategy; If $\arg\max_{1 \leq i \leq c} u_{ij}^{(k)} = t$ then x_j is assigned to cluster t.

DOI:10.5963/PHF0203002

B. Clustering Accuracy

C. Ding, T. Li, and W. Ping, use the clustering accuracy, as follows,

$$A_c = \frac{1}{n} \max \sum_{C_s, L_t} T\left(C_s, L_t\right) \tag{15}$$

Where n is the number of objects in the data set, C_s is the s-th cluster and L_t is the t-th class, $T\left(C_s, L_t\right)$ is the number of objects which belong to class t and are assigned to cluster s. Accuracy computes the maximum sum of $T\left(C_s, L_t\right)$ for all pairs of clusters and these pairs have no overlaps. Accuracy, A_c, is the percentage of the points that were correctly recovered in a clustering result. Generally, the grater the accuracy values the better the cluster performance.

III. EMPIRICAL ANALYSIS

The result of FCM is obtained by applying the Matlab toolbox developed by [4]. The balance Iris data [5] has a sample size of 150. The features of the Iris data contain length of sepal, width of sepal, length of petal, and width of petal. The samples were assigned the original 3 clusters based on the clustering analysis. The characteristics of 3 clusters for Iris data were shown in Table 1 [6-8].

TABLE 1 THE CHARACTERISTICS OF 3 CLUSTERS FOR IRIS DATA

Cluster	***Samples size***	***Species***
1	50	Setosa
2	50	Versicolor
3	50	Virginica

The clustering performances of Iris data set is calculated by applying three fuzzy clustering algorithms, as mentioned above, with the same fuzzier m=2. In these experiments, the mean clustering accuracies of 100 different initial value sets were calculated and compared for this data set. From Table 2, we can find that FCM Algorithm with Mahalanobis Distances has the better performance. We know that FCM Algorithms with Mahalanobis Distances contain FCM-M and FCM-CM [9-10].

TABLE 2 THE ACCURACIES OF DIFFERENCE ALGORITHMS

Algorithms	***Accuracies(%)***
FCM	89.33%
FCM-M	90.00%
FCM-CM	92.79%

The total sample used in this study consists of 315 graduated students who studied in Min-Hwei College of Health Care Management School in Taiwan, and we collected their information about results of national examinations of Registered Nurse and Licensed Practical Nurse. According to their results, students can be separated into following four groups: None of Both, Registered Nurse, Licensed Practical Nurse, and All of Both. They took an achievement test on medicine and technology of nursing while they were in school, we tried to find out that students' knowledge structures of each group looks like. The structure of the achievement test is reported in Table 3.

TABLE 3 THE CHARACTERISTICS OF ACHIEVEMENT TEST

Concepts	***Description of attributes***	***Measured items***
1	Clinical technology of nursing	2, 3, 4, 6, 8, 10, 12, 13, 15
2	General biological medicine	5, 7, 9, 11, 14, 16
3	Accomplishment of ethics	17
4	Responsibility	1
5	Life long learning	18, 19, 20

Applying fuzzy clustering algorithms as we mentioned before, the performances of each algorithm are calculated with same fuzzier m=2 and the clustering accuracies and mean scores are shown in Table 4 and Table 5 respectively. The result shows that FCM-NM has the most performance than other algorithms [11-12].

TABLE 4 THE ACCURACIES OF EACH FUZZY CLUSTERING ALGORITHMS

Algorithms	**FCM**	**FCM-M**	**FCM-CM**
Accuracies(%)	**31.72**	**31.72**	**32.04**

TABLE 5 MEAN SCORES OF ORIGINAL GROUP

Groups	**1**	**2**	**3**	**4**	**5**
1	5.661	3.252	0.426	0.852	1.174
2	6.138	3.448	0.448	0.966	1.552
3	6.462	3.692	0.308	0.923	1.692
4	**6.943**	**4.000**	**0.595**	**0.949**	**1.665**

DOI:10.5963/PHF0203002

IV. CONCLUSIONS

The well-known FCM is based on Euclidean distance function, which can only be used to detect spherical structural clusters. FCM-M and FCMCM are the fuzzy clustering algorithms that are based on different Mahalanob distances and can be better than FCM algorithm. The experimental results of real data set show that FCM-CM algorithm has the best performance. The mean clustering accuracies of 100 different initial value [13-15] sets of FCM and FCM-CM for the datasets were shown in Appendix 1. From this appendix, we can find that the performance of our proposed FCM-CM algorithm is simultaneously better than which of FCM algorithm in the datasets. In other words, which of FCM-M is better than FCM algorithm and the new algorithm FCM-CM has the best performance. In a word, FCM-CM algorithm is better than others.

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APPENDIX 1 : ACCURACIES OF 100 DIFFERENT INITIAL VALUE OF IRIS DATA SET

FCM	FCM-M	FCM-CM
89.33%	90.00%	91.33%
89.33%	90.00%	96.67%
89.33%	90.00%	96.67%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	92.67%
89.33%	90.00%	92.00%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	92.00%
89.33%	90.00%	90.00%
89.33%	90.00%	92.67%
89.33%	90.00%	92.67%
FCM	**FCM-M**	**FCM-CM**
89.33%	90.00%	92.00%
89.33%	90.00%	91.33%
89.33%	90.00%	92.67%
89.33%	90.00%	92.67%

DOI:10.5963/PHF0203002

89.33%	90.00%	92.67%
89.33%	90.00%	92.67%
89.33%	90.00%	91.33%
89.33%	90.00%	96.67%
89.33%	90.00%	92.67%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	96.67%
89.33%	90.00%	96.67%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	92.67%
89.33%	90.00%	92.00%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	92.00%
89.33%	90.00%	90.00%
89.33%	90.00%	92.67%
89.33%	90.00%	92.67%
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89.33%	90.00%	92.67%
89.33%	90.00%	92.67%
89.33%	90.00%	91.33%
89.33%	90.00%	96.67%
89.33%	90.00%	92.67%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%

FCM	**FCM-M**	**FCM-CM**
89.33%	90.00%	91.33%
89.33%	90.00%	96.67%
89.33%	90.00%	96.67%
89.33%	90.00%	91.33%
89.33%	90.00%	91.33%
89.33%	90.00%	92.67%
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89.33%	90.00%	92.67%
89.33%	90.00%	92.67%
89.33%	90.00%	91.33%
89.33%	90.00%	92.00%
89.33%	90.00%	96.67%
89.33%	90.00%	92.67%
FCM	**FCM-M**	**FCM-CM**
89.33%	90.00%	91.33%
89.33%	90.00%	92.67%
89.33%	90.00%	90.67%
89.33%	90.00%	96.67%
89.33%	90.00%	92.00%
89.33%	90.00%	96.67%
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89.33%	90.00%	92.00%
89.33%	90.00%	96.67%
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89.33%	90.00%	92.67%
89.33%	90.00%	91.33%

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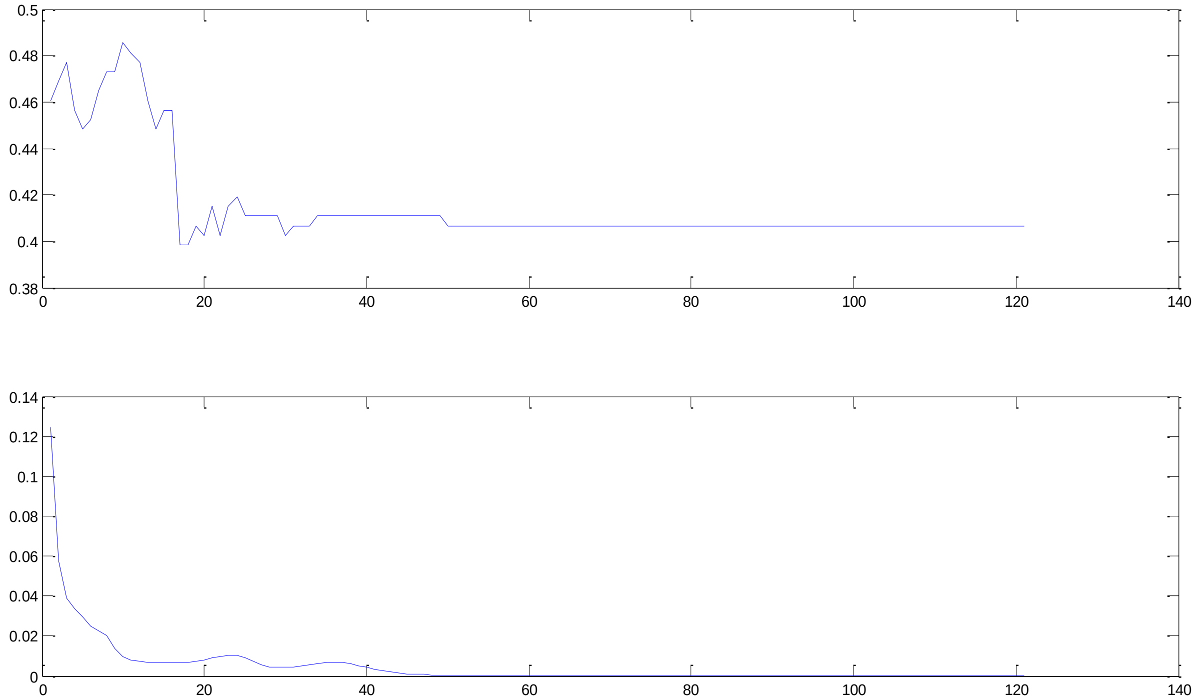
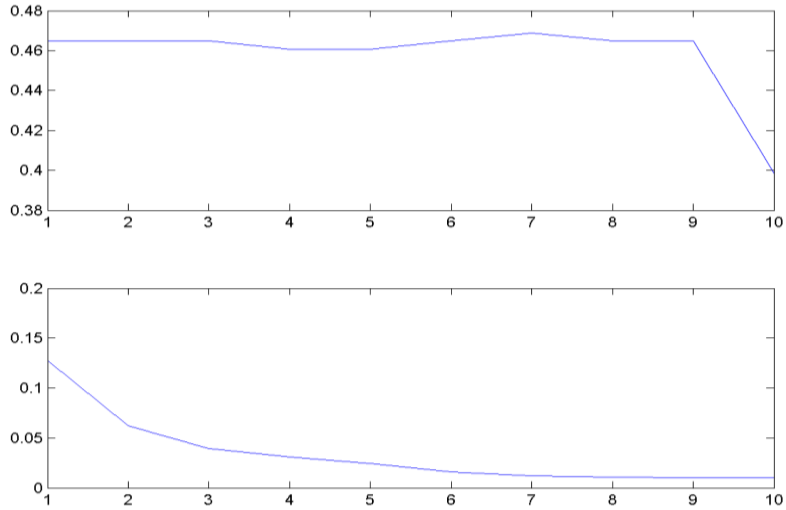
APPENDIX 2: THE ALGORITHM DOES NOT CONVERGE TO THE CORRESPONDING DATA



APPENDIX 3: THE ALGORITHM DOES NOT CONVERGE TO THE CORRESPONDING DATA



APPENDIX 4: THE ALGORITHM OF GK ALGORITHM AND GATH-GEVA ALGORITHM

Gath-Geva (GG) fuzzy clustering algorithm is an extension of Gustafson-Kessel (GK) fuzzy clustering algorithm, and also takes the size and density of clusters for classification.

Hence, it has better behaviors for irregular features.

Probabilistic interpretation of GG clustering is shown by Equation A.

$$P(X|\ \eta)=\sum_{i=1}^{c}P(X,\eta_i)=\sum_{i=1}^{c}P(\eta_i)P(X\mid \eta_i) \tag{A}$$

Gath and Geva assumed that the normal distribution N_i with expected value $\underline{a}_i$ and covariance matrix Σ_i is chosen for generating a datum with prior probability. P_i, satisfying

DOI:10.5963/PHF0203002

$$P\left(\underline{x}_j \mid \eta\right) = \frac{P_i}{(2\pi)^{\frac{P}{2}}\sqrt{|\Sigma_i|}} \exp\left[-\frac{1}{2}\left(\underline{x}_j - \underline{a}_i\right)' \Sigma_i^{-1} \left(\underline{x}_j - \underline{a}_i\right)\right] \tag{B}$$

$d^2\left(\underline{x}_j, \underline{a}_i\right)$ for GG algorithm is chosen to be indirectly proportional to Equation B, which is the posterior probability (likelihood) function. A small distance means a high probability, and a large distance means a low probability for membership. GG algorithm is based on minimization of the sum of weighted square distances between the data and the cluster centers of the objective function in Equation C

$$\begin{aligned} J_{GG}^m(U, A, \Sigma, X) &= \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^m d^2\left(\underline{x}_j, \underline{a}_i\right) \\ d^2\left(\underline{x}_j, \underline{a}_i\right) &= \frac{1}{P\left(\underline{x}_j \mid \eta\right)} = \frac{(2\pi)^{\frac{P}{2}}\sqrt{|\Sigma_i|}}{P_i} \exp\left[-\frac{1}{2}\left(\underline{x}_j - \underline{a}_i\right)' \Sigma_i^{-1} \left(\underline{x}_j - \underline{a}_i\right)\right] \end{aligned} \tag{C}$$

Replace all of the covariance matrices with the same common covariance matrix to get a more stable fuzzy clustering algorithm, called Fuzzy C-Means algorithm based on common Mahalanobis distance (FCM-CM), its object function is listed as following:

$$J_{FCM-CM}^m\left(U, A, \Sigma, X\right) = \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^m \left[\left(\underline{x}_j - \underline{a}_i\right)' \Sigma^{-1} \left(\underline{x}_j - \underline{a}_i\right) - \ln\left|\Sigma^{-1}\right|\right] \tag{D}$$

Minimizing the objective function Eq. D, we can obtain the FCM-CM algorithm as follows:

Step 1: Choose the result membership matrix of FCM algorithm as the initial one $\mu_{ij}^{(0)}, i = 1,2,\ldots,c; j = 1,2,\ldots,n$ such that

$$\sum_{1\le i\le c} u^{(0)}{}_{ij} = 1,\ j = 1, 2, \ldots, n,\ \underline{a}_i^{(0)} = \frac{\sum_{j=1}^{n}\mu_{ij}^{(0)}\underline{x}_j}{\sum_{j=1}^{n}\mu_{ij}^{(0)}}, i = 1, 2, \ldots, c \tag{E}$$

$$D = \sum_{i=1}^{c}\sum_{j=1}^{n}\left[\mu_{ij}^{(0)}\right]^m \left(\underline{x}_j - \underline{a}_i^{(0)}\right)'\left(\underline{x}_j - \underline{a}_i^{(0)}\right) > 0,\ \ \Sigma^{(0)} = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^{(0)}\left(\underline{x}_j - \underline{a}_i^{(0)}\right)'\left(\underline{x}_j - \underline{a}_i^{(0)}\right)}{\sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^{(0)}} \tag{F}$$

$$if\ \ \left|\Sigma^{(0)}\right| > D,\ \ or \left|\Sigma^{(0)}\right| < \frac{1}{D},\ \ then\ \ \Sigma^{(0)} = I \tag{G}$$

Step 2:

$$\underline{a}_i^{(k)} = \frac{\sum_{j=1}^{n}\left[\mu_{ij}^{(k-1)}\right]^m \underline{x}_j}{\sum_{j=1}^{n}\left[\mu_{ij}^{(k-1)}\right]^m}, i = 1, 2, \ldots, c,\quad \Sigma^{(k)} = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^{(k-1)}\left(\underline{x}_j - \underline{a}_i^{(k)}\right)'\left(\underline{x}_j - \underline{a}_i^{(k)}\right)}{\sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^{(k-1)}} \tag{H}$$

$$if\ \ \left|\Sigma^{(k)}\right| > D,\ \ or \left|\Sigma^{(k)}\right| < \frac{1}{D},\ \ then\ \ \Sigma^{(k)} = I \tag{I}$$

$$\mu_{ij}^{(k)} = \left[\sum_{s=1}^{c}\left[\frac{\left(\underline{x}_j - \underline{a}_i^{(k)}\right)'\left[\Sigma^{-1}\right]^{(k)}\left(\underline{x}_j - \underline{a}_i^{(k)}\right) - \ln\left\|\left[\Sigma^{-1}\right]\right\|^{(k)}}{\left(\underline{x}_j - \underline{a}_s^{(k)}\right)'\left[\Sigma^{-1}\right]^{(k)}\left(\underline{x}_j - \underline{a}_s^{(k)}\right) - \ln\left\|\left[\Sigma^{-1}\right]\right\|^{(k)}}\right]^{\frac{1}{m-1}}\right]^{-1} \tag{J}$$

Step 3: Increment k; until $\max_{1\le\le c}\left\|\underline{a}_i^{(k)} - \underline{a}_i^{(k-1)}\right\| < \varepsilon$.

DOI:10.5963/PHF0203002