

Rarefied Gas Flow around Flat Plates

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Abstract- By joint solution of the Boltzmann and Navier-Stokes equations, flows of rarefied gas about a single flat plate as well as a lattice of the plates are numerically studied. The flows with different Mach numbers and angles of attack are considered. Global characteristics of gas flows and the aerodynamic reactions at the surfaces of the plates are obtained.

Keywords- Boltzmann Equation; Navier-Stokes Equations; Flat Plate; The Lattice of Flat Plates; Transonic Flow; Supersonic Flow

I. INTRODUCTION

The relevance of the study of different flows of rarefied gas through constructions composed of flat plates is due to their practical value in a number of applications (movement of a gas in porous bodies, capillaries, the membranes in the devices of division and gas cooling, etc.). General characteristics of the flow are determined by interaction of individual streams generated by the elements of the investigated structure. This structure is usually a three-dimensional construction, consisting of cylindrical channels or of a set of limited plates, located along or across the stream.

Are also of interest the installations intended to control or to create gas flows with given properties, without changing the device geometry, which can be achieved by means of a certain variation of temperature and other parameters of gas at the streamlined surfaces. This work is devoted to the analysis of such systems consisting of flat plates. The subsonic regime of the rarefied gas flows is typical for the considered constructions. Supersonic regime arises in a flow around elements of antennas of solar panels of space engines.

Let's note some works that marked the beginning of the research in this area. Rarefied gas flows in the capillary cite in the range of Knudsen numbers $0.0001 < K_n < 0.1$ at different temperatures for the first time are studied experimentally for a number of inert gases in [1], where the dependence of flow rate from the geometry of the flow and the characteristics of the interaction of gas molecules with the walls of the capillary was determined. Supersonic flow around perforated screens in a range of Mach numbers from 2 to 3 at large Reynolds numbers is studied in [2]. The results of numerical experiments on the basis of direct statistical simulation of one-dimensional flow of rarefied gas through permeable planar surface are given in [3] under the assumption that with probability P a gas molecule passes through the surface without any interaction, and with probability $1-P$ dissipates. The stationary flow in the range of Mach number from 3 to 10 at different temperatures of the barriers and parameters of gas-surface accommodation was studied. In particular, there were determined regularities of changes of the flow parameters in the compression zone formed in front of the obstacle and the differences of its structure from the structure of a shock wave were determined.

In this work the flow of non-equilibrium rarefied gas is described by the Boltzmann kinetic equation. Numerical solution of the Boltzmann equation requires large amounts of computing, as the determined distribution function in addition to the spatial variables contains a three-dimensional velocity space. Hence, even in a fairly simple two-dimensional case, the number of nodes of the distribution functions counts up to several tens of million and the calculation of the kinetic equation in the whole region is very intensive. To a large extent, this is manifested in the cases when the studied flow occupies a large space and the process of stabilization of the solution is slow. These features are typical for subsonic regimes where the adequate setting of boundary conditions requires a large area of the calculation. To overcome these difficulties we have developed the combined method in which essentially non equilibrium flow are computed with the non-stationary Boltzmann equation, and the non-stationary Navier-Stokes equations are used in the area where the condition of the gas are close to thermodynamic equilibrium. The method is especially designed for small Knudsen numbers where gas flows are frequently accompanied by formation of narrow highly non equilibrium zones, such as shock waves and Knudsen layers near an obstacle, in which flow parameters sharply vary at a scale of a molecular mean free path. Outside of the indicated zones the flow is in hydrodynamic regime. For such kind of flows the application of kinetic numerical methods in a whole area might be not efficient, and hybrid methods that combine solution of kinetic and continuum fluid dynamic equations in a framework of domain decomposition approach should be preferred.

The construction of similar methods, mostly with the use of DSMC method matched with Euler equations, was discussed in a number of papers, ex. [4, 5]. In [6] the DSMC solution is matched with the solution of Navier-Stokes equations that permits to reduce the area of the stochastic simulation. Note, that sticking of stochastic solution with the smooth solution of continuum equations causes some specific problems. In some papers, ex. [7, 8] the DSMC simulation in hybrid methods is replaced by deterministic solution of BGK kinetic equation by finite-difference schemes.

II. GENERAL STRUCTURE OF THE METHOD

In our approach the kinetic Boltzmann equation is solved on a fixed grid in velocity and configuration space by a finite-difference method. One solve time dependent problem and apply the symmetrical splitting on a free molecular and relaxation stages at each time interval. The free molecular advection operator is approximated by the second order explicit finite-difference flux conservative scheme of SHASTA type [9]. To evaluate the collision integral, the conservative projection method [10] is applied. This method ensures rigorous fulfillment of mass, momentum, and energy conservation laws, together with zero value of the collision integral from a local Maxwellian distribution function. The later property considerable improves the accuracy of calculations at small Knudsen numbers and in near equilibrium zones [11, 12] where the solutions are matched. A cubature formula for the collision integral uses multidimensional grid of uniformly distributed nodes. At each time step of the algorithm a new integration grid is being generated, and then applied in all grid points of the configuration space. This procedure sharply reduces the amount of arithmetic operations, and smoothes the solution as well. The collision integral is evaluated for arbitrary molecular potential [13].

The Navier-Stokes solver applies the splitting on a convective (Euler) and viscous subsystems of equations. Viscosity and heat conduction coefficients are determined by molecular potential chosen for the Boltzmann equation. Viscous subsystem is approximated by a second order implicit scheme, and for the Euler part the SHASTA scheme is used. At each time step of the algorithm the solutions of Boltzmann and Navier-Stokes equations are matched at interfaces of their domains that provide new boundary conditions for the next step. A steady regime of the flow, if it exists, is obtained as a limit of the unsteady solution.

III. MATCHING OF BOLTZMANN AND NAVIER-STOKES SOLUTIONS

Suppose that the solutions are matched on a rectangular grid x_k, y_l for a fixed $k = i$ on a boundary $x_{i+1/2}$ that separates the Navier-Stokes domain at the left side from the kinetic domain at the right side, as is shown in Fig. 1. The matching points are marked by circles. To compute the Boltzmann equation in the cell layer $(i+1, l)$, one must have a distribution function $f_{i+1/2, l}^+$ defined at all positive molecular velocities. It is chosen in a form of the Chapman-Enskog function, in which the values of variables n, u, v, T are defined in the layer (i, l) , whereas gradients $du/dx, dv/dx, dT/dx$ are calculated by the values in layers with the first indices $i-1, i, i+1/2$ by following the approximation rules used for derivatives in Navier-Stokes equations. In these calculations the values of $n_{i+1/2, i}, u_{i+1/2, i}, v_{i+1/2, i}, T_{i+1/2, i}$ are taken from the lower time layer. Note that the leading term of the Chapman-Enskog function-the Maxwellian function is determined by flow parameters in the Navier-Stokes area.

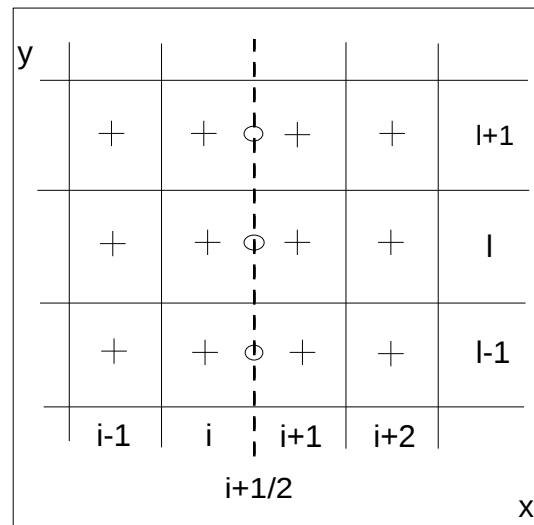


Fig. 1 Scheme of Matching of Boltzmann and Navier-Stokes Solutions

Next, the composite distribution function $f_{i+1/2, l} = f_{i+1/2, l}^+ + f_{i+1/2, l}^-$ is constructed in the layer $(i+1/2, l)$ where $f_{i+1/2, l}^-$ is equal to the solution to the Boltzmann equation at the node $(i+1, l)$ for negatives molecular velocities. This function is used to calculate the new values of $n_{i+1/2, l}, u_{i+1/2, l}, v_{i+1/2, l}, T_{i+1/2, l}$, which determine the boundary conditions for the Navier-Stokes equations. Following the sequence of calculations used in the splitting method applied here, these values are used for to obtain the fluxes of flow variables along the x -axis, which are required to compute the Euler equations, and for to find the values of partial derivatives and mixed partial derivatives in Navier-Stokes equations, including corresponding boundary conditions.

The emergence of not monotonic at the interface of two regions, if occurred, is not a desirable feature. It can be caused by two main reasons: 1) an improper choice of the matching interface in substantially non-equilibrium region, where the Navier-Stokes equation are far to be correct; 2) different description of the structure of a shock wave by the kinetic equation and by the Navier-Stokes equations for $M > 1.5$, resulting in the distortion of the form of shock waves at the interface boundary, despite that the velocity of propagation and the hydrodynamic parameters behind the shock wave are correctly determined. In

this paper we don't use any criteria to determine the matching interface, as it is done for example in [8], and find it by making some trials to exclude the first reason of not monotonic.

The details of the matching procedure are reported in [9, 14] and its efficiency for more complex flows is demonstrated in [15, 16].

IV. RAREFIED GAS FLOW THROUGH A PERIODIC GRID

Let a periodic lattice, whose elements are infinitely thin flat plates of infinite length and a width L spaced l apart, be immersed in a uniform rarefied flow. The plates are located normally to the incoming flux. The periodicity length being $L+l$, we denote $L/l = S$. The incident flow is determined by density, temperature, and velocity with $\gamma = 5/3$. The length end time scales are the molecular mean free path and mean inter collision time in the incident flow, while the density and temperature are referred to their free stream values. In the domain in which the Boltzmann equation is solved (in the figures it is located inside the rectangle indicated by broken lines), a power-law intermolecular potential with exponent equal to -12 was taken. The interaction of gas molecules with the surface is described by a reflected Maxwellian distribution function at a wall temperature which is equal to the temperature of the incident flow. Since the flow is periodic in Y , the calculation were carried out on the range $0 < y < y_m$. At $y = y_m$ and $y = 0$, the plane of symmetry passing through the middle of the plate, symmetry conditions are posed. In the considered example the Boltzmann equation is solved on a spatially uniform 60×20 grid, with about 2000 grid points in velocity space. The same uniform coordinate grid was used to solve the Navier-Stokes equations; where the number of grid points varied from 300×200 to 300×25 depending on the particular problem (here, the first and second cofactors relate to the x and y coordinates).

The problems under consideration are characterized by large number of parameters and, accordingly, many different types of flow can be realized. The calculations were carried out for various parameters of the lattice of the plates and different speed of the incoming flow. The scheme of calculations allows not only to identify the general characteristics of the flow, but also to analyze at the kinetic level the processes on the plates. It gives the possibility to calculate the drag force, friction, heat flux, temperature jump, and the slip velocity at the plates.

Subsonic flow. The plate length L is fixed and is equal to 10 molecular mean free paths in the incoming flow. The parameter S was selected equal to 0.77. The flow is essentially a two-dimensional only near the plates and at a distance of several hundreds of molecular mean free paths from the lattice it became similar to the one-dimensional flow. This makes it easier to study essentially subsonic regimes requiring the setting of boundary conditions at large distances before the lattice. For the incoming flow with $M_0 = 2.5$, a supersonic flow with local Mach number $M_c = 1.5$ behind the lattice emerges. When reducing M_0 , the configuration of the supersonic zone behind the lattice changes, and at $M_0 \sim 1.1$ the formation of local supersonic zones in the gaps between the plates is seen. With a further decrease in the speed of incoming flow the dimensions of the supersonic zones are reduced. The case of $M_0 = 0.6$ that is analyzed in this paper corresponds to the intermediate regime between fully subsonic flows and the flows with local supersonic zones.

The overall picture of the flow with a local supersonic zone is shown in Fig. 2 where the results of computations with the incoming flow parameters $M_0 = 0.6$, $T_0 = 1$, $L = 10$, $S = 0.77$ are presented.

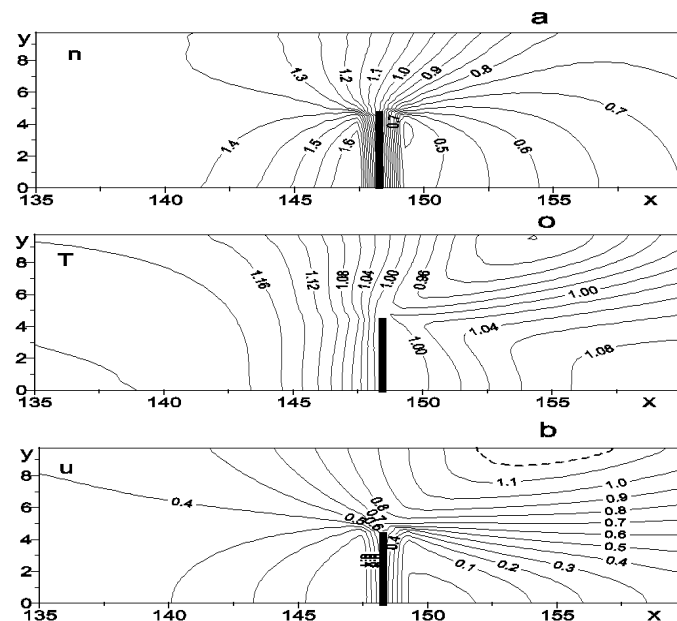


Fig. 2 Flow structure near the lattice for $M_0 = 0.6$, $T_0 = 1$, $L = 10$, $S = 0.77$: a) density, b) temperature, c) longitudinal velocity

The interaction of the incoming flow with the lattice leads to the appearance of the reflected shock wave. With time this shock wave goes upstream and near the plate and downstream of it nearly stationary flow with some average by y-coordinate parameters is installed.

The process of development of the considered flows is described by several characteristic times. The first of them corresponds to the formation of the reflected shock waves and formation in the first approximation of the flows in the gaps between the plates. This happens at the time of about 100-150 units. During 200-300 time units a stabilization of the fields of density and temperature near the lattice take place, however, the flow rate (that is defined as the integral of gas density multiplied by longitudinal velocity) differs from its permanent values by 3-5%. This difference disappears after 1000 time units. The independence of the obtained solutions from the location of the downstream boundary conditions was established empirically if the latter was at a distance of not less than 150-200. At the indicated boundary the so-called soft conditions – the absence of the flows was imposed. On the left border the boundary conditions corresponding to the unperturbed incoming flow were posed. At the approach of the shock wave the boundary with the values n_1 , u_1 , T_1 is moved by 10 in the direction of the rack.

In Fig. 2 the fields of density, temperature and longitudinal velocity near the lattice are presented. The Boltzmann kinetic equation was solved using the steps $dx = 0.85$, $dy = 0.5$, $dt = 0.1$. At the considered moment the shock wave went far to the left, forming a subsonic flow incident to the lattice with parameters $n_1 = 1.32$, $T_1 = 1.21$, $u_1 = 0.39$, $M_1 = 0.27$.

Define the specific flow rate K as the ratio of the sum of the values nu at all the grid nodes in y-direction divided by number of these nodes. Then in the inlet section the flow rate is equal to 0.77, behind the shock wave it takes the value $K_1 = 0.51$. Behind the lattice in the section xc the average values of density, temperature and longitudinal velocity are equal to $n_c = 0.80$, $T_c = 1.08$, $u_c = 0.65$, $M_c = 0.48$, and accordingly the relative rate is $K_c = 0.52$. The proximity of values of K_c and K_1 indicates the reaching of practically stationary mode.

Supersonic flow. There is currently interest in the numerical investigation of interaction between rarefied gas flows and systems of plates constituting channels or periodic lattices. At hypersonic free stream velocities, this process can simulate, for example, the flow past space borne wire antennas, while at low subsonic velocities it models the flow over filtering or gas absorbing surfaces. The basic element of these models is the problem of rarefied gas flow past a thin flat plate, which can be oriented in parallel or normally to the incoming flux. When such elements are assembled in a periodic structure, the flow pattern became more complex because of interference of fluxes issued from each of them. In this case one of the main computational difficulties is related with flows at intermediate Knudsen numbers, where the gas state is near to thermodynamic equilibrium, but differs from it. We consider two examples of numerical modeling of 2D periodic structures formed by the plates posed normally to the main direction of the flow.

In series of computations with $M_0 = 2.5$, $n_0 = 1$, $T_0 = 1$, $S = 0.14$ and $10 < L < 40$ the steady flow with an attached shock wave was obtained. In Fig. 3 the density and temperature fields for $L = 10$ at time $t = 150$ are shown. The shock is attached to the lattice and is curved. It is located at a distance L from the plate. The maximum value of the density near the plate is equal to 5.5 and is due to the compression in the shock and additional gas flow towards the cold plate equalizing the pressure in this zone. In the shadow region behind the plate the maximum density is equal to 0.1. The interaction of the shocks results in the straightening of their common front and its displacement into the space between the plates. The flow behind the shock can be subdivided into two zones. In the first zone, the subsonic flow turns around the upper edge of the plate, which is accompanied by the formation of an expansion wave, temperature decrease, and gas acceleration. This zone lies within the region in which the Boltzmann equation is solved. The second zone is related with the shock interaction. It occupies the entire region in which the Navier-Stokes equations are solved, and is determined by the distance between the plates rather than by their dimensions. The equalization of the flow behind the lattice takes place over a distance of 60 to 80. In each section the flow rate is equal to that of the inflow.

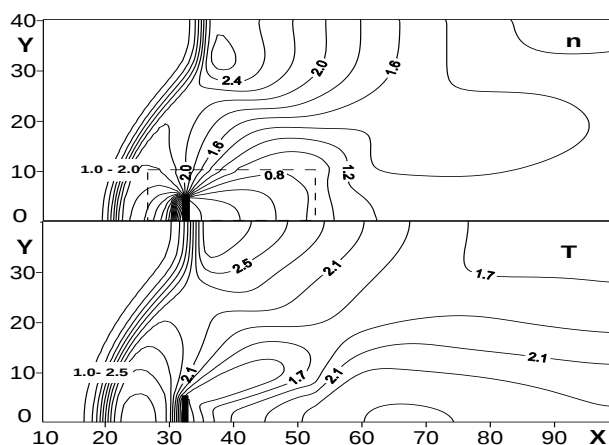


Fig. 3 Steady-State Density and Temperature Distributions for $L = 10$, $S = 0.14$, $M_0 = 2.5$

One of the reasons of the study of flows with Knudsen numbers ranging from 0.1 to 0.001 is to determine the geometric parameters at which the kinetic effects should be taken into account in a certain part of the flow or, on the contrary, one can solve the Navier-Stokes equations inside the entire flow region. From this standpoint, a significant case is that of a plate with a length as large as 40. For this case the density field is presented in Fig. 3 at $t = 300$.

In contrast to the previous variants, the shock interaction region is clearly visible and contains the triple shock configuration typical of the continuum flow regime. The reflected shock enters the developed wake region and interacts with the latter at $x > 200$. From analyzing the main laws governing flow regimes with a shock attached to the lattice, we conclude that for $S = 0.14$ the kinetic effects influence the development of the main flow at $L < 30-40$. At larger L the flow develops in the continuum regime, except of the wall layer on the plate (Fig. 4).

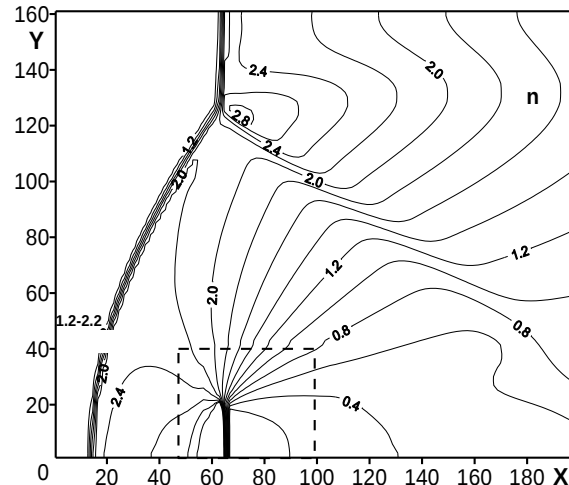


Fig. 4 Steady-State Density Distribution for $L = 40$, $S = 0.14$, $M_0 = 2.5$

As the distance between the plates decreases (S increases), there comes a moment when the supersonic flow is no longer able to pass through the lattice and, after having been reflected from it, forms a plane shock wave traveling upstream and leaving behind it a homogeneous region of hot compressed gas. The lattice is in essentially subsonic flow. The gas in the gap between the plates accelerates to supersonic velocities, flows out into the space behind the lattice, and mixes with the similar jets issuing from the neighboring gaps. In this regime, a collective interaction of the lattice elements manifests itself strongly.

This case is shown in Fig. 5 where the density distribution for $S = 0.28$, $L = 10$, $M_0 = 2.5$ is plotted at $t = 100$.

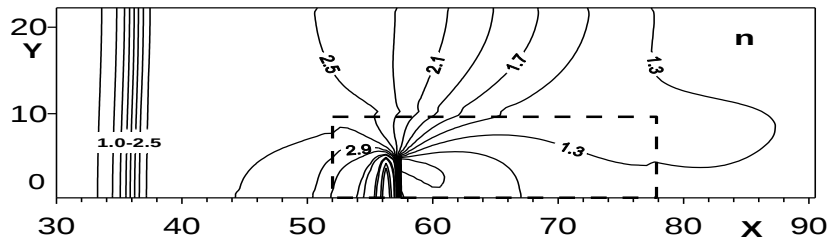


Fig. 5 Density Field in Time-Dependent Regime for $L = 10$, $S = 0.28$, $M_0 = 2.5$ at $t = 100$

The reflected shock wave velocity is $D = -0.22$, the front being at $x = 33$, and gas parameters behind the shock are: $n = 2.7$, $T = 3.15$, $u = 1.1$ and $M = 0.47$. The gas density on the plate is about 6.1 and is determined by the gas attachment to the cold surface rather than by the flow deceleration. In the shadow region $n = 1$ and $T = 1.2$. In the region above the upper edge a two-dimensional expansion wave is formed. Across this wave the density and temperature are reduced by factors of 3 and 1.5, respectively, while the gas accelerates to $M = 1.5$. Downstream of the lattice total density flow rate in each section $x = \text{const}$ is equal to 0.9 of that of the undisturbed inflow. In the time-dependent regimes only part of the gas passes through the lattice, and the rest is absorbed by the moving shock.

V. RAREFIED GAS FLOW AROUND AN INCLINED PLATE

Consider the gas flow around a single flat plate by the supersonic uniform flow at different angles of attack. The size of the plate is equal to 10 molecular mean free paths of the incoming flow, Mach number is fixed and is equal to 2.83, and the angle of attack varies from 0 up to 90 degrees.

Solution of the Boltzmann equation was carried out on the grid with $dx = 1$, $dy = 1$ in a rectangle containing the plate. The number of computational nodes in the coordinates x, y is equal to 10 and 120. The number of nodes in the velocity space is about 2000. In the outer region of the rectangle the Navier-Stokes equations were solved on the same grid with the number of

nodes 350×350 . The matching of the solutions was done on each temporal layer $dt = 0.1$. The stationary distribution was achieved at $t = 300-400$. The most interesting case presents the flow at 60 degrees angle of attack, for which the detailed picture of density field is given in Fig. 6. Here contours of density are drawn with the step 0.5. The plate is located at $x = 150$ and y between 90 and 100.

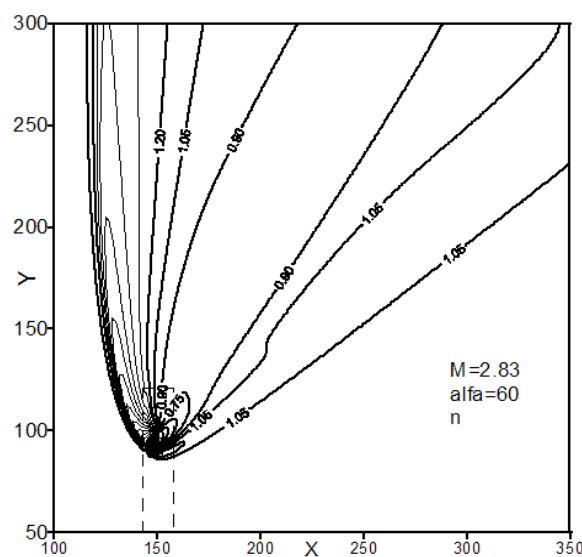


Fig. 6 Density Field around a Single Plate at Angle of Attack

The applied method gives a possibility to determine the behavior of dynamic and thermal loads on the plate depending on the value of angle of attack. Zero angle of attack corresponds to cross-flow. It was found that the variation of the drag coefficient is very close to the cosine of the angle of attack. The friction coefficient and the heat flux change linearly. For the transverse flow the heat flux is about two times higher than for the flow in longitudinal direction. The chosen geometry of computation domains and the considered set of parameters allow one to study the process of gas flow around the plates for different physical conditions.

VI. CONCLUSIONS

We described the methodology of matching the solutions of the Boltzmann and Navier-Stokes equations for computing gas flows in which relatively small highly not-equilibrium regions are incorporated in essentially weakly perturbed flow. The efficiency of the approach was demonstrated on some examples of computation of 2D plane subsonic and supersonic flows. With some minimal modifications, the proposed approach can be used to study a number of classical problems, for example the structure of inclined shock wave, the interaction of shock waves among themselves and with streamlined surfaces, boundary and shear layers, and so on. It can be extended on 3D flows and applied for modeling more complex geometries of technical systems as well.

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