Comparing Different Strategies of Decomposition to Solve the Short-Term Hydrothermal Scheduling Based on Lagrangian Relaxation

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***Abstract*-The Short-Term Hydrothermal Scheduling – STHS problem is addressed in this paper. In this problem, hydro and thermal plants must be coordinated in order to supply the demand at a minimum cost and comply with a set of constraints, concerning the system and the plants themselves, in the short-term planning horizon (from one day to one week). Given the nonlinearities associated with the problem, the large number of decision variables and constraints coupled in time periods, the STHS problem results in a complex mathematical programming problem. The Lagrangian Relaxation – LR method is the most widespread procedure for solving this problem. This work presents two strategies to solve STHS, which are based on different decomposition schemes of the primal problem. These strategies make it possible to define, for large-scale problems, the most appropriate decomposition scheme, which is particularly important for power systems with high proportion of hydraulic generation. The studies were accomplished using a reduced system composed of five reservoirs, 22 hydro and two thermal units of the Brazilian Hydrothermal Power System.**

Keywords- Hydrothermal Systems; Short-Term Scheduling; Lagrangian Relaxation

I. INTRODUCTION

The operation planning of hydrothermal systems is a very complex task. A possible approach to deal with this complexity involves the decomposition of the global task into long, medium and short-term scheduling problems [1, 2], which are modelled considering suitable aspects for their time horizons. In general, uncertainties are modelled in detail in the long and medium-term problems, as well as system and generation constraints are precisely detailed in the short-term problem. Coordination among the problems can be done in different ways. Examples of the medium-term and short-term model coordination can be seen in [3-5].

In this paper, the Short-Term Hydrothermal Scheduling – STHS is addressed. This problem has been intensely researched for the last decades, and numerous methods have been proposed to deal with it. Due to the significant number of coupling constraints and decision variables, decomposition techniques appear as a natural alternative for solving this problem; particularly, the Lagrangian Relaxation – LR, which is one of the most efficient strategies [6-8]. In [9] we used LR to solve the hydro unit commitment problem by using a reduced configuration of the Brazilian Hydrothermal System. However, in this previous work we took into consideration only the hydro system. This work is an extension of [9], so that thermal units and demand constraints are also modeled in the short-term horizon. In our model, we relate the amount of hydropower generation to nonlinear tailrace levels, and also, take into account hydraulic losses, turbine-generator efficiencies, as well, the multiple states related with forbidden zones. Our formulation also includes several constraints associated to the reservoirs operation, and it is suitable for a system with several plants in cascade.

By means of LR, the original STHS problem is split into a sequence of smaller and easy-to-solve subproblems, coordinated by a dual master program. The advantage of our approach is to obtain four separate subproblems: thermal, hydro, hydrothermal and hydraulic. The first two subproblems take into account the unit constraints (thermal and hydro unit commitment, respectively). The hydrothermal subproblem takes into account the demand and reserve requirements, as well as the transmission system limits. Finally, all of the reservoir constraints are modeled into the hydraulic subproblem. Each subproblem is solved with a specific mathematical programming technique.

In order to deal better with the combinatorial aspect introduced by the forbidden zones in the hydro unit commitment subproblem, we derive two different decomposition strategies. The first strategy relies on a complete enumeration of all possible 0-1 non-forbidden zones of the units, carried out for each hydro plant and time stage. In general, the hydro plants possess identical units, with a single forbidden zone. Therefore, a good strategy consists of carrying out an exhaustive enumeration of possible combinations. On the other hand, in those hydro plants with a large number of generating units and/or forbidden zones, an exhaustive enumeration becomes an inefficient procedure. Therefore, it is important to build another dual problem that can handle the combinatorial nature in a more efficient way.

This paper is organized as follows. The hydrothermal configuration and the STHS formulation are presented in Section 2. Section 3 details the proposed decomposition strategy and we report its computational results in Section 4. Finally, in Section 5, the main conclusions of this article are presented.

II. MODELING AND PROBLEM FORMULATION

The hydrothermal configuration used in this work is shown in Fig. 1.

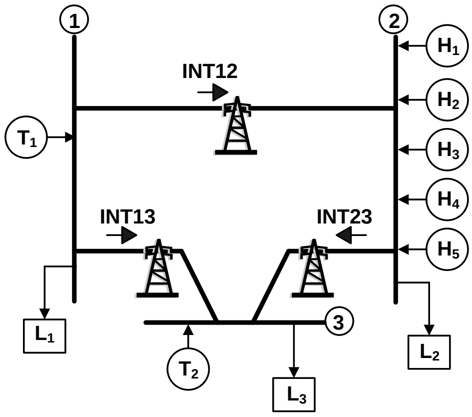


Fig. 1 Hydrothermal system

The system installed capacity is 8,474 MW. The main data referring the thermal plants are presented in Table 1.

TABLE 1 THERMAL PLANT DATA

Plant	pt^{min} [MW]	pt^{max} [MW]	Δ_i [MW/h]	RT_i [MW]	ct_{1i} [R$/MW^2]	ct_{2i} [R$/MW]
T_1	0	800	50	40	0.07	0.03
T_2	0	800	50	40	0.04	10

Table 1 shows generation limits, ramp data, plant spinning reserve and production costs coefficients, respectively. In this paper, minimum uptime and downtime constraints were not modeled [10], although the proposed decomposition strategy can treat this aspect in an efficient way.

The hydro configuration is illustrated in Fig. 2.

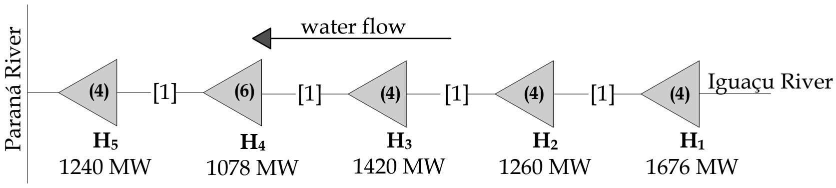


Fig. 2 Hydro system configuration

Fig. 2 shows the maximum installed capacity in each plant, water travel time, in hours, among reservoirs (in brackets), and still, generating units number of each plant (in parentheses). Table 2 shows other reservoirs operation data (volume, maximum turbined discharge and spillage, as well as the reserve requirement for each plant). The minimum turbined outflow and spillage values are zero.

TABLE 2 HYDRO PLANT DATA

Plant	$v^{min/max}$ [hm^3]	Q_r^{max} [m^3/s]	s_r^{max} [m^3/s]	RH_r [MW]	Non-Forbidden Zones [MW]
H_1	1,974/5,779	1,376	2,752	77.82	[290-419]
H_2	2,562/2,950	1,268	2,536	60.18	[180-315]
H_3	2,662/6,775	1,576	3,152	68.56	[210-355]
H_4	1,014/1,124	1,784	3,568	53.52	[120-182] (4) [120-175] (2)
H_5	3,473/3,573	2,100	4,200	57.10	[205-310]

Table 2 also shows the non-forbidden zones for each unit. All plants have identical units, except H_4 that possesses two different groups of units[1]. As aforementioned, this work represents the hydro plant in a detailed way and, in this case, the output of the unit j depends on turbined outflow (q_j), the net head (h_j) and efficiency (η_j):

[1] One group possesses two identical units and the other possesses four identical units.

$$ph_j = 9{,}81 \times 10^{-3} \eta_j h_j q_j \tag{1}$$

The net head [11] is given by:

$$h_j = fbl - trl(Q) - k_j q_j^2 \tag{2}$$

In (2) *fbl* represents the forebay level of reservoir, which is considered as a fixed value. However, the tailrace level *trl*(.) varies significantly in the short-term horizon. For our system, *trl* (.) is represented by a fourth order polynomial[2], as shown in Table 3. Still regarding (2), $k_j q_j^2$ represents the penstock losses, where k_j is a constant in s^2/m^5.

TABLE 3 TAILRACE FUNCTION AND HYDRAULIC LOSSES CONSTANT

Plant	b_0(x10^3) [m]	b_1(x10^{-4}) [s/m^2]	b_2(x10^{-7}) [s^2/m^5]	b_3(x10^{-12}) [s^3/m^8]	b_4(x10^{-19}) [s^4/m^{11}]	k_j(x10^{-6}) [s^2/m^5]
H$_1$	0.602	11.060	4.209	-83.110	47.610	22.290
H$_2$	0.490	0.608	2.925	-23.200	4.565	18.300
H$_3$	0.394	21.110	-0.792	2.352	-271.4	10.776
H$_4$	0.322	22.810	-1.403	3.842	-536.3	36.156 71.270
H$_5$	0.258	6.208	-0.172	0.228	0.122	3.628

The unit efficiency, depending on hl_j and q_j, is usually represented by hill diagrams. We estimate the efficiency by interpolation; see [12, 13], using a polynomial function

$$\eta_j = \rho_{0j} + \rho_{1j} q_j + \rho_{2j} h_j + \rho_{3j} h_j q_j + \rho_{4j} q_j^2 + \rho_{5j} h_j^2 \tag{3}$$

where $\rho_{0j},\ldots,\rho_{5j}$ are the coefficients that should be estimated previously. Therefore, based in (1), (2) and (3), we have $ph_j = f\,(q^7, Q^{12})$.

Finalizing the data presentation, all the interchange limits are equal to 3000 MW. The demand data, initial and final reservoirs volumes, and incremental inflow are shown in the results section.

Based on the described data, the optimization model is formulated by:

$$\min\ f = \sum_{t=1}^{T=24} \sum_{i=1}^{n_t=2} (c_{1i}\, pt_{it}^2 + c_{2i}\, pt_{it}) \tag{4}$$

s.t.:

$$v_{r,t+1} - v_{rt} + c\left(Q_{rt} + s_{rt} - \sum_{m \in R_+^{(r)}} (Q_{m,t-\tau_{mr}} + s_{m,t-\tau_{mr}})\right) = c\,.\,y_{rt} \tag{5}$$

$$0 \le s_{rt} \le s_r^{max} \qquad 0 \le Q_{rt} \le Q_r^{max} \tag{6}$$

$$v_r^{min} \le v_{rt} \le v_r^{max} \qquad v_{r,T+1} \ge v_r^{LP} \tag{7}$$

$$Q_{rt} - \sum_{j=1}^{J_{rt}} q_{jrt} = 0 \tag{8}$$

$$\sum_{k=1}^{\Phi_{jr}} ph_{jkrt}^{\min} z_{jkrt} \le ph_{jrt}(q_{jrt}^7, Q_{rt}^{12}) \le \sum_{k=1}^{\Phi_{jr}} ph_{jkrt}^{\max} z_{jkrt} \tag{9}$$

$$0 \le q_{jrt} \le q_{jrt}^{max} \tag{10}$$

$$z_{jkrt} \in \{0,1\} \qquad \sum_{k=1}^{\Phi_{jr}} z_{jkrt} \le 1 \tag{11}$$

$$pt_{it}^{min} \le pt_{it} \le pt_{it}^{max} \tag{12}$$

$$\left| pt_{it} - pt_{i,t-1} \right| \le \Delta_i \tag{13}$$

$$\begin{aligned} & pt_{1t} - INT12_t - INT13_t = L_{1t} \\ & \sum_{r=1}^{R=5} \sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^7, Q_{rt}^{12}) + INT12_t - INT23_t = L_{2t} \\ & pt_{2t} + INT13_t + INT23_t = L_{3t} \end{aligned} \tag{14}$$

[2] $trl(Q) = b_0 + b_1 Q + b_2 Q^2 + b_3 Q^3 + b_4 Q^4$.

$$\begin{aligned} -INT12^{\max} \le INT12_t \le INT12^{\max} \\ -INT13^{\max} \le INT13_t \le INT13^{\max} \\ -INT23^{\max} \le INT23_t \le INT23^{\max} \end{aligned} \tag{15}$$

$$PH_r^{\max} - \sum_{r=1}^{R=5} \sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^7, Q_{rt}^{12}) \ge RH_{rt} \tag{16}$$

$$pt_{it} \ge PT_i^{\max} - RT_{it} \tag{17}$$

Where:

T total stages, [h];

t stage index, so that t=1, T;

R number of reservoirs in the system;

r index of the reservoirs in the system, so that r=1, R;

v_{rt} volume of reservoir r at the beginning of stage t [hm^3];

c conversion factor of water discharge units [m^3/s] into stored water units [hm^3]. For unit consistency, it should be noted that time periods of 1 hour are considered;

s_{rt} spillage of reservoir r during stage t [m^3/s];

$\Re_+^{(r)}$ set of reservoirs immediately upstream of reservoir r;

τ_{mr} time of water travel between reservoir m and r [h];

y_{rt} incremental inflow of reservoir r during stage t [m^3/s];

Q_{rt} plant turbined outflow in reservoir r during stage t [m^3/s];

J_{rt} total of hydro units available of reservoir r, during stage t;

q_{jrt} turbined outflow of unit j of reservoir r during stage t [m^3/s];

$v_r^{min,max}$ minimum (maximum) volume of reservoir r [hm^3];

v_r^{LP} volume of reservoir r at the end of the study horizon [hm^3];

Φ_{jr} total non-forbidden zones of unit j and reservoir r;

k index of the operating zones of the units, so that k=1, Φ_{jr};

$ph_{jrt}(.)$ power output of unit j, reservoir r and stage t [MW];

$ph_{jkrt}^{min,max}$ minimum (maximum) power of unit j, reservoir r and stage t, operating in zone k [MW];

z_{jkrt} binary variable that indicates if unit j of reservoir r is operating (z_{jkrt} = 1) or not (z_{jkrt} = 0) in zone k during stage t;

pt_{it} power output of thermal unit i, during stage t [MW];

c_{1i}, c_{2i} operating costs of quadratic function of thermal production pt_{it};

$pt_{it}^{min,max}$ minimum (maximum) power of unit i and stage t [MW];

Δi ramp rate maximum of the thermal unit i [MW];

RH_{rt} hydro plant reserve during stage t [MW];

RT_{it} thermal unit reserve during stage t [MW];

$INT12_t$ power interchange among the bus 1 and 2, during stage t [MW];

$INT13_t$ power interchange among the bus 1 and 3, during stage t [MW];

$INT23_t$ power interchange among the bus 2 and 3, during stage t [MW];

B index of bus in the system, b=1, 3;

Lb_t hourly power demand of bus b [MW].

The objective function (1) seeks to minimize thermal generation cost over the planning horizon. In this work, the horizon is one day and the time step is one hour. The coordination with the medium term model is obtained by means of stored volumes targets (7).

The problem constraints can be classified in different subsets, each one characterized by a specific type coupling, as detailed ahead. This classification is fundamental for comprehension of the decomposition strategies proposed in the work.

A. *Reservoirs Constraints* (CHH)

This subset is given by the constraints (5) – (7), which correspond to the reservoirs operation limits (stream-flow balance, spillage, turbined outflow and storage). Notice that these constraints are coupled in time and space. In a compact notation, we can define CHH=C_{HH} (Q, s, v), where Q, s and v are vectors associated with the variables showed in (5) – (7).

B. Hydro Unit Constraints (CHU)

This subset includes the constraints that involve the penstock water balance in each reservoir (8), non-forbidden zones (9), unit turbined outflow limits (10) and integer variables (11). Except (8), the constraints only possess variables referring to the hydro units and are characterized as nonlinear mixed-integer constraints. In compact notation we can define CHU=C_{HU} (z, q, Q), where z and q are vectors associated with the variables previously showed.

C. Thermal Plant Constraints (CT)

This subset includes the power limit constraints (12) and ramp constraints (13). In compact formulation, we define CT=C_T (pt), where pt is the vector with all thermal generation variables.

D. Hydrothermal Constraints (CHT)

This subset includes the demand supply requirements per bus (14), the power interchange limits (15), and hydro (16) and thermal (17) reserve constraints. We define CHT=C_{HT} (pt, q, Q, INT), where, INT represents the vector with all power interchange variables.

E. Compact Notation

The optimization problem, in compact notation, has the following structure:

$$\begin{aligned} &\min\ f = \sum_{t=1}^{T=24} \sum_{i=1}^{n_t=2} (c_{1i}\, pt_{it}^2 + c_{2i}\, pt_{it}) \\ \text{s.t.:}\quad & C_{\mathrm{HH}}(Q,s,v) \cap C_{\mathrm{HU}}(z,q,Q) \cap C_{\mathrm{T}}(pt) \cap C_{\mathrm{HT}}(pt,q,Q,INT) \end{aligned} \tag{18}$$

III. DUAL DECOMPOSITIONS

The classical application form of LR relaxes coupling constraints, such as demand and reserve requirements. The reason is that these constraints couple all generators at every time stage. Nevertheless, even using this type of decomposition, the hydro subproblem is still very complex because it is time and space linked. An efficient approach to deal with this difficulty consists in combining LR with Variable Splitting – LRVS method [6-8], where the decomposition is achieved by duplicating some variables.

A. First Decomposition – Dual Problem I

In STHS Problem (18), the subset C_{HT} (.) is coupled with C_T (.), C_{HU} (.) and C_{HH} (.), due to the demand supply requirements. Initially, the objective is to uncouple hydro and thermal plant variables. In this sense, we include in (18) the artificial variables, pta and PHa, in the following way:

$$\begin{aligned} &\min\ f = \sum_{t=1}^{T=24} \sum_{i=1}^{n_t=2} (c_{1i}\, pt_{it}^2 + c_{2i}\, pt_{it}) \\ \text{s.t.:}\quad & \mathrm{C}_{\mathrm{HH}}(Q,s,v) \cap \mathrm{C}_{\mathrm{HU}}(z,q,Q) \cap \mathrm{C}_{\mathrm{T}}(pt) \cap \mathrm{C}_{\mathrm{HT}}(pta,PHa,INT) \\ & pta_{it} = pt_{it};\ PHa_{rt} = \sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^{7}, Q_{rt}^{12}) \end{aligned} \tag{19}$$

Observe that the variables pta and PHa are used in C_{HT} (.), replacing pt and $\sum_{j=1}^{J_{rt}} ph_{jrt}(.)$, respectively. Now, we can note that the variables of C_{HT} (.) are not coupled with C_T (.), C_{HU} (.) and C_{HH} (.). The subset C_T (.) also is not coupled with other subsets. However, the variable Q still couples the subsets C_{HH} (.) and C_{HU} (.). In the sense, to break this coupling, the artificial variables Qa are replaced in the stream-flow balance equations in C_{HH}. Thus, the Problem (19) is rewritten as follows:

$$\begin{aligned} &\min\ f = \sum_{t=1}^{T=24} \sum_{i=1}^{n_t=2} (c_{1i}\, pt_{it}^2 + c_{2i}\, pt_{it}) \\ \text{s.t.:}\quad & \mathrm{C}_{\mathrm{HH}}(Qa,s,v) \cap \mathrm{C}_{\mathrm{HU}}(z,q,Q) \cap \mathrm{C}_{\mathrm{T}}(pt) \cap \mathrm{C}_{\mathrm{HT}}(pta,PHa,INT) \\ & pta_{it} = pt_{it};\ PHa_{rt} = \sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^{7}, Q_{rt}^{12}) \\ & Qa_{rt} = Q_{rt} \end{aligned} \tag{20}$$

As a result, at this moment, the coupling is given only by the artificial constraints. To break this coupling, it is necessary to relax these constraints using Lagrange multipliers λ_{pt}, λ_{PH} e λ_Q:

$$\max \theta_{\mathrm{I}} = \min f + \sum_{t=1}^{T=24}\left\{\sum_{i=1}^{n_t=2}\lambda_{pt_{it}}(pta_{it}-pt_{it})+\sum_{r=1}^{R=5}\left[\lambda_{PH_{rt}}(PHa_{rt}-\sum_{j=1}^{J_{rt}}ph_{jrt}(q_{jrt}^{7},Q_{rt}^{12}))+\lambda_{Q_{rt}}(Qa_{rt}-Q_{rt})\right]\right\}$$

s.t.: $$\mathrm{C_{HH}}(Qa,s,v)\cap \mathrm{C_{HU}}(z,q,Q)\cap \mathrm{C_T}(pt)\cap \mathrm{C_{HT}}(pta,PHa,INT) \tag{21}$$

The dual problem above could be separated in smaller subproblems, with distinct mathematical characteristics, as will be showed in the sequence.

1) Thermal Subproblem:

The first subproblem is denominated as Thermal Subproblem (θ_T), which contains only thermal constraints, as seen previously in the set C_T (*pt*).

$$\theta_T = \min \sum_{t=1}^{T=24}\left[\sum_{i=1}^{n_t=2} ct_{1i}\, pt_{it}^2 + (ct_{2i}-\lambda_{pt_{it}})pt_{it}\right]$$

s.t.: $$\mathrm{C_T}(pt) \tag{22}$$

Subproblem (22) is a Quadratic Programming – QP problem, coupled along time steps, but not along plants. Each QP is solved by the PLCBAS software [14].

2) Hydrothermal Subproblem:

This subproblem includes constraints with thermal and hydro plant variables:

$$\theta_{HT} = \min \sum_{t=1}^{T=24}\left[\sum_{i=1}^{n_t=2}\lambda_{pt_{it}}\, pta_{it} + \sum_{r=1}^{R=5}\lambda_{PH_{rt}}\, PHa_{rt}\right]$$

s.t.: $$\mathrm{C_{HT}}(pta,PHa,INT) \tag{23}$$

Subproblem (23) is a standard Linear Programming – LP problem, coupled along plants, but not along time steps, which can be solved by any LP commercial software. The software used for the solution is the CPLEX 7.0 [15].

3) Hydraulic Subproblem:

This subproblem given is:

$$\theta_H = \min \sum_{t=1}^{T=24}\left[\sum_{r=1}^{R=5}\lambda_{Q_{rt}}\, Qa_{rt}\right]$$

s.t.: $$\mathrm{C_{HH}}(Qa,s,v) \tag{24}$$

Subproblem (24) is also a LP problem, coupled in time and space, which can be efficiently solved by CPLEX.

4) Hydro Unit Commitment Subproblem:

The last subproblem from this first decomposition has the following formulation:

$$\theta_{HUC} = \min \sum_{t=1}^{T=24}\left\{\sum_{r=1}^{R=5}\left[-\lambda_{PH_{rt}}\sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^{7},Q_{rt}^{12}) - \lambda_{Q_{rt}}Q_{rt}\right]\right\}$$

s.t.: $$\mathrm{C_{HU}}(z,q,Q) \tag{25}$$

Subproblem (25) is nonlinear and mixed-integer optimization problem uncoupled in time and hydro plants. Therefore, (25) can be solved by $R\times T$ subproblems separated.

The computational burden to solve each subproblem of (25) depends on the size of the state space, which is given by the total of generating units combinations that can be enumerated in each plant and time stage.

Plants H_1, H_2, H_3 and H_5 possess identical units, with a single non-forbidden zone. In this case, the total of combinations, in each time stage, is J_{rt}+1. Plant H_4 possesses two different groups (Group I, with two identical units and Group II, with four identical units), each unit possessing one non-forbidden zone. Thus, H_4 has 15 unit combinations.

Each combination is a configuration where the corresponding binary variables are fixed to one of the feasible values. Once the binary values are fixed, the problem becomes a nonlinear program, whose size is dependent on J_{rt}. The software used for Nonlinear Programming – NLP solution was developed by [9, 13], which used Sequential Quadratic Programming methodology concepts [16, 17].

In this first decomposition – Dual I, we solve (25) by enumeration of non-forbidden zones combination at plant *r* and stage *t*.

5) Dual Problem I Schematic Representation:

Fig. 3 illustrates the associated Dual Problem I subproblems.

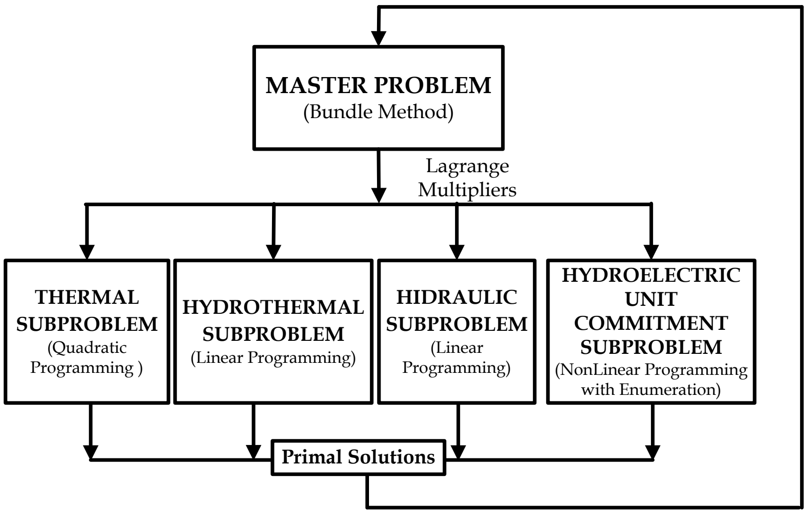


Fig. 3 Dual problem I schematic diagram

For update the Lagrange multipliers, this work uses the Bundle Method [18], through N1CV2 [19] academic use free software.

B. Second Decomposition –Dual Problem II

Generally, hydro plants possess identical units and each unit has a single non-forbidden zone. In this case, the total combinations are given by J_{rt}+1 and an exhaustive enumeration can be a good strategy, according to previous section. Sometimes, however, there are power plants with many different types of units, and several non-forbidden zones. For these configurations, an enumeration procedure may become too expensive. We now introduce an alternative decomposition scheme.

First of all, in the second decomposition strategy, the set C_{HUC} (.) is divided into two smaller subsets: $C_{AH1}(q, Q)$ and $C_{AH2}(q, Q, z)$. C_{AH1} (.) contains penstock balance constraints and C_{AH2} (.) contains non-forbidden zones and integer constraints:

$$\min\ f = \sum_{t=1}^{T=24} \sum_{i=1}^{n_t=2} (c_{1i}\, pt_{it}^2 + c_{2i}\, pt_{it})$$

$$\text{s.t.:} \quad C_{HH}(Q,s,v) \cap C_{AH1}(q,Q) \cap C_{AH2}(q,Q,z) \cap C_T(pt) \cap C_{HT}(pt,q,Q,INT) \tag{26}$$

The strategy follows the same decomposition showed in Dual Problem I; however, the duplication of hydro generation variables is included in following way:

$$min\ f = \sum_{t=1}^{T=24} \sum_{i=1}^{n_t=2} (c_{1i}\, pt_{it}^2 + c_{2i}\, pt_{it})$$

$$\text{s.t.:} \quad C_{HH}(Qa,s,v) \cap C_{AH1}(q,Q) \cap C_{AH2}(pha,z) \cap C_T(pt) \cap C_{HT}(pta,PHa,INT)$$

$$pta_{it} = pt_{it};\ PHa_{rt} = \sum_{j=1}^{J_{rt}} ph_{jrt}(q_{jrt}^7, Q_{rt}^{12}); \tag{27}$$

$$Qa_{rt} = Q_{rt};\ pha_{jrt} = ph_{jrt}(q_{jrt}^7, Q_{rt}^{12})$$

As seen above, we use *pha* to replace the hydro production function in the non-forbidden zones constraints which belongs to the C_{AH2} (.) set, therefore decoupling integer variables of nonlinear constraints.

Relaxing the artificial constraints of (27) and using Lagrange multipliers λ_{pt}, λ_{PH}, λ_Q e λ_{ph}, the following dual problem is obtained:

$$\max \theta_{\mathrm{II}} = \min\ f + \sum_{t=1}^{T=24} \left\{ \sum_{i=1}^{n_t=2} \lambda_{pt_{it}} (pta_{it} - pt_{it}) + \sum_{r=1}^{R=5} \left[\lambda_{PH_{rt}} (PHa_{rt} - \sum_{j=1}^{J_{rt}} ph_{jrt}(q^{7}_{jrt}, Q^{12}_{rt})) + \lambda_{Q_{rt}} (Qa_{rt} - Q_{rt}) \right] \right\} +$$
$$+ \sum_{t=1}^{T=24} \sum_{r=1}^{R=5} \sum_{j=1}^{J_{rt}} \lambda_{ph_{jrt}} (pha_{jrt} - ph_{jrt}(q^{7}_{jrt}, Q^{12}_{rt}))$$

$$\text{s.t.:} \quad \mathrm{C_{HH}}(Qa,s,v) \cap \mathrm{C_{AH1}}(q,Q) \cap \mathrm{C_{AH2}}(pha,z) \cap \mathrm{C_T}(pt) \cap \mathrm{C_{HT}}(pta,PHa,INT) \tag{28}$$

The Dual Problem II, θ_{II}, can also be decomposed in smaller subproblems. In this sense, the same subproblems of the Dual Problem I, θ_{I}, Thermal (θ_{T}), Hydrothermal (θ_{HT}) and Hydraulic (θ_{H}) are developed. However, two new subproblems (Continuous and Integer) appear in this new decomposition.

1) Continuous Subproblem:

$$\theta_{AH_1} = \min \sum_{t=1}^{T=24} \sum_{r=1}^{R=5} \left[\sum_{j=1}^{J_{rt}} (-\lambda_{PH_{rt}} - \lambda_{ph_{jrt}}) ph_{jrt}(q_{jrt}, Q_{rt}) - \lambda_{Q_{rt}} Q_{rt} \right]$$

$$\text{s.t.:} \quad \mathrm{C_{AH1}}(q,Q) \tag{29}$$

This subproblem is very similar to the NLP problems associated to (25). However, the subproblem size is not variable. In (29) exist $R \times T$ nonlinear subproblems with an equality linear constraint and (J_{rt}+1) variables.

2) Integer Subproblem:

This subproblem possesses only non-forbidden zones constraints.

$$\theta_{AH_2} = \min \sum_{t=1}^{T=24} \sum_{r=1}^{R=5} \sum_{j=1}^{J_{rt}} \lambda_{ph_{jrt}}\, pha_{jrt}$$

$$\text{s.t.:} \quad \mathrm{C_{AH2}}(pha,z) \tag{30}$$

In (30), for each reservoir and stage t, there is J_{rt} mixed-integer linear problems, which can be solved independently. Each subproblem, each one possessing ($1+\Phi_{jr}$) variables and box constraints in pha_{jrt}, has analytical solution:

- If $\lambda ph_{jrt} > 0$, $pha_{jrt} = 0$;
- If $\lambda ph_{jrt} < 0$, $pha_{jrt} = pha_{jr}^{max}$[3];
- If $\lambda ph_{jrt} = 0$, pha_{jrt} can have any feasible value.

3) Dual Problem II Schematic Representation:

Fig. 4 illustrates the Dual Problem II subproblems.

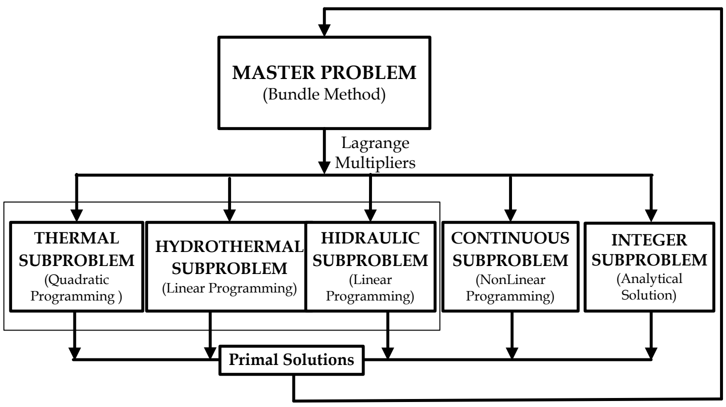


Fig. 4 Dual problem II schematic diagram

To obtain feasible primal solutions it is important to emphasize that, following the dual problems solution, it is necessary to make some adjustments. This LR algorithm phase is known as Primal Recovery – PR and, frequently is based on heuristics. This phase is not implemented in this work. Details on PR can be seen in [7, 8].

[3] Maximum value among the maximum limits of Φ_{jr} non-forbidden zones.

IV. COMPUTATIONAL TESTS

This section aims to analyze the results obtained on dual problems I and II. The number of stages is 24 which refer to an hourly discretization in a one-day horizon. The computational tests were performed in a computer AMD Athlon XP 2400+ with 512 Mbytes of RAM memory with Windows 2000 Professional operating system. The programming language used was FORTRAN, using Compaq Visual FORTRAN Professional Edition 6.1.0 as compiler.

The main data problem can be seen in the Section 2. The incremental inflows were considered null. Table 4 illustrates initial and final reservoirs storage.

Fig. 5 illustrates demand curve for the horizon.

TABLE 4 RESERVOIRS INITIAL AND FINAL STORAGES

Plant	Initial Storage [hm^3]	%Storage max	Final Storage [hm^3]	%Storage max
H_1	4,637.50	70	4,557.50	67.90
H_2	2,794.80	60	2,774.80	54.85
H_3	5,335.45	65	5,305.45	64.27
H_4	1,069.00	50	1,069.00	50.00
H_5	3,523.00	50	3,523.00	50.00

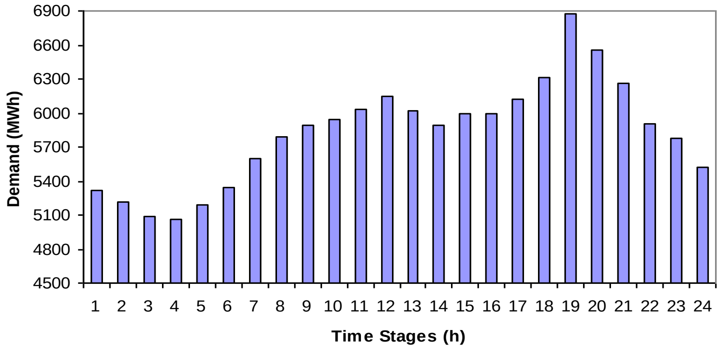


Fig. 5 Hydrothermal system demand

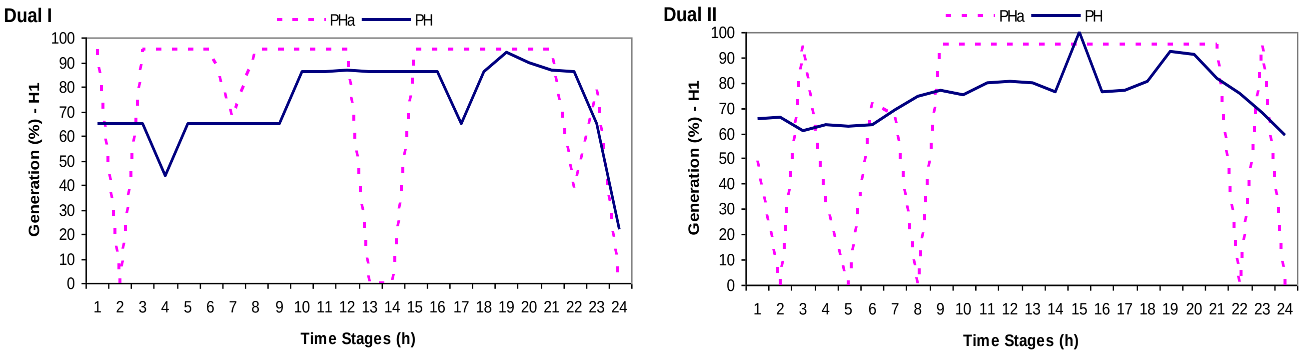
Table 5 illustrates computational results obtained from the dual problems. Notice that in the Dual I strategy, the dual function reaches a value higher than the Dual II strategy.

TABLE 5 DUAL PROBLEMS RESULTS

	Dual I	Dual II
Initial multipliers	-0.1	-0.1
Dual function (1st iteration)	-29,814.95	-29,814.95
Dual function (convergence)	346,298.98	334,859.01
Number of iterations	338	187
Computational time (seconds)	185.7	70.89

Regarding the computational performance, the difference between the two strategies is evident. Considering the computational burden the Dual II is approximately 2.5 times faster than Dual I.

Fig. 6 depicts the dispatches of H_1 and H_4 plants, in percentile terms with relation to the maximum nominal values.



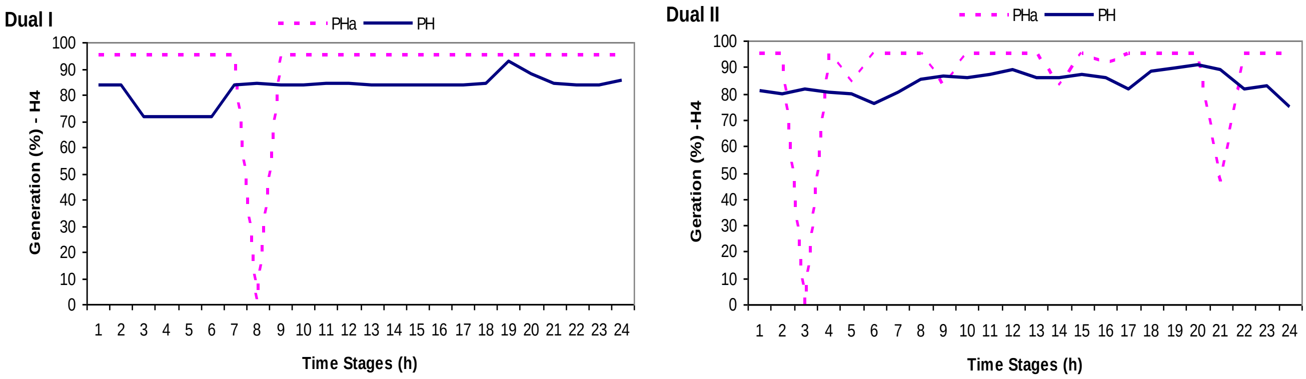


Fig. 6 Dual I and Dual II hydro generation

Stippled line represents *PHa* generation for Hydrothermal Subproblem. The bang-bang behavior can be observed due to the linear modelling of this problem. The full line represents the generation from the nonlinear problems (*PH*): Hydro Unit Commitment Subproblem (Dual I) and Continuous Subproblem (Dual II).

Fig. 7 illustrates T_1 generation. The *pta* variable results from the Hydrothermal Subproblem, while *pt* comes from the Thermal Subproblem. Again the oscillations (*pta*) can be observed in linear problems while *pt*, of quadratic nature, such behavior is avoided by the ramp constraints. The *pt* behavior is similar in the two decomposition strategies.

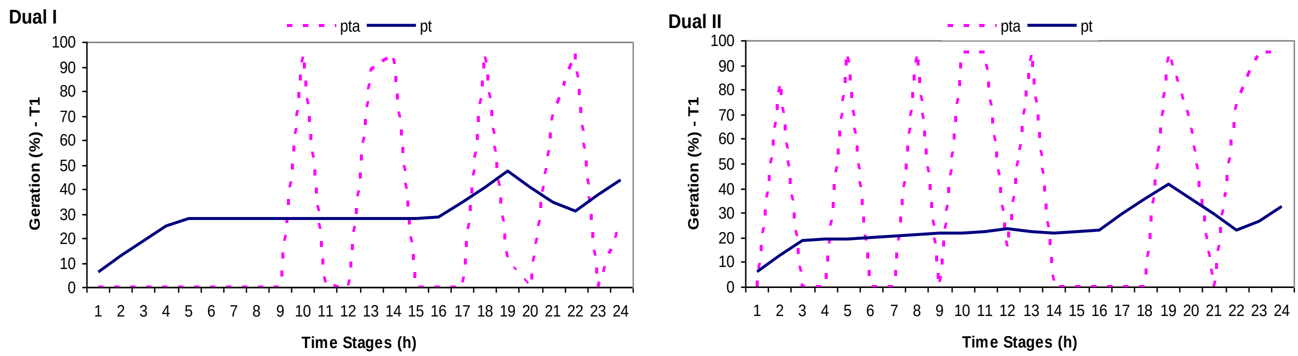


Fig. 7 Dual I and dual II thermal generation

The number of units allocated in H_1 and H_4 is illustrated in Fig. 8.

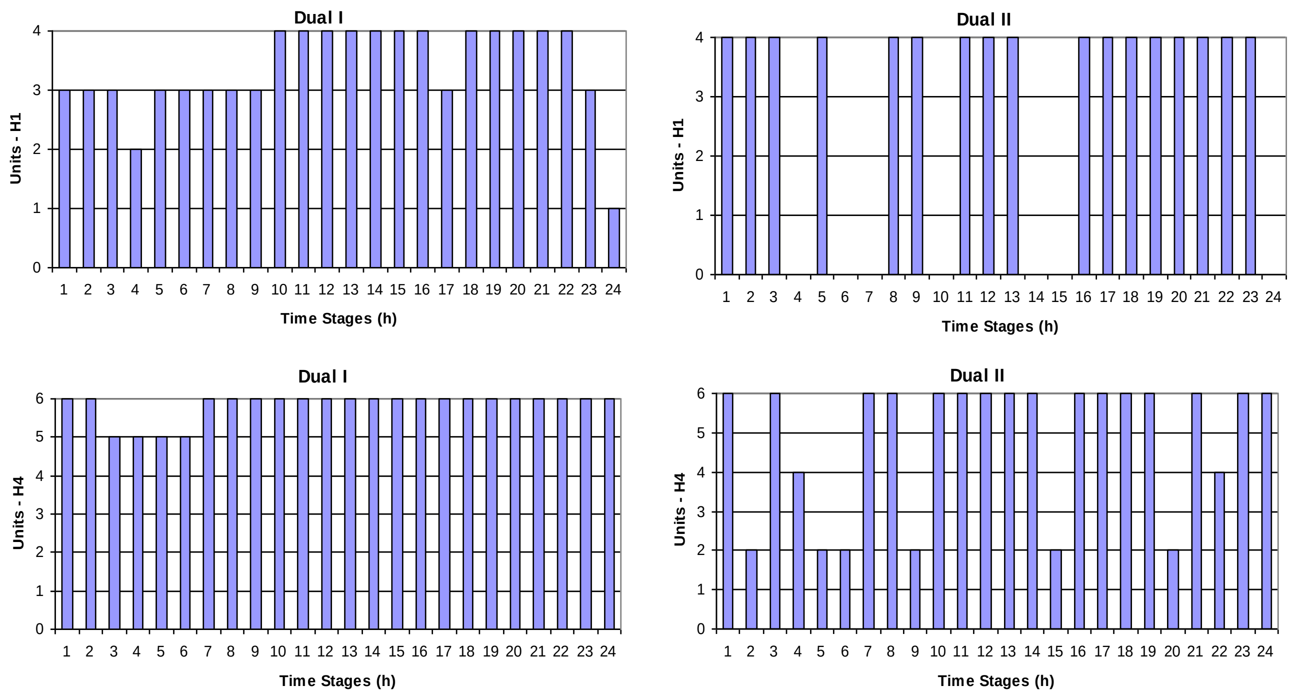


Fig. 8 Dual I and dual II hydro unit allocated

The Dual Problem I strategy presents more realistic solutions because it avoids units turns on/off which does not occur in the Dual II. Moreover, sometimes, in the Dual II, the problem solution determines the operation into forbidden zones, as it can be observed in Fig. 9.

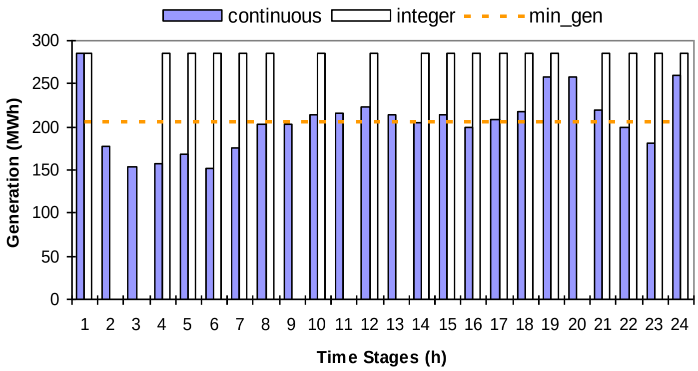


Fig. 9 Integer and continuous subproblems (Unit 1 of H_5)

Fig. 10 illustrates Lagrange multipliers associated to the T_1 thermal plant and H_1 hydro plant. As can be observed, Dual Problem II suggests being a good initial point for Dual I, in some time stages.

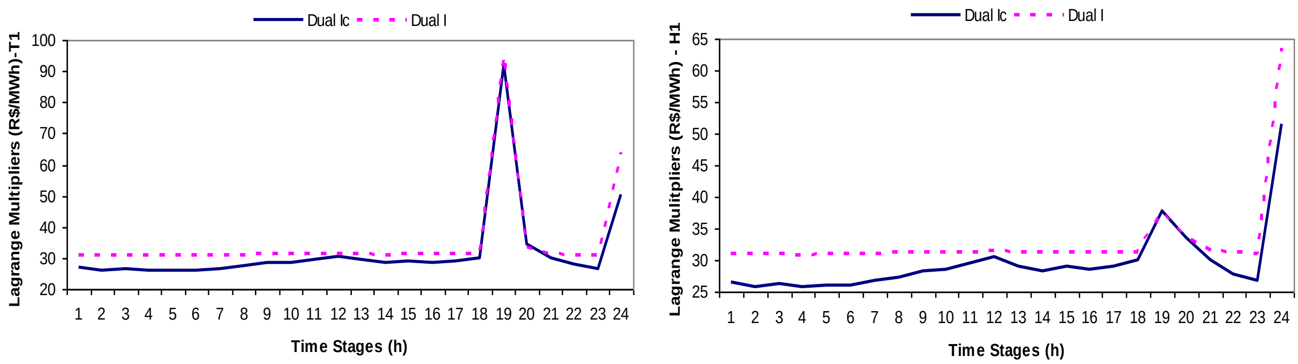


Fig. 10 Lagrange multipliers (Dual I x Dual II)

We conclude from our experiments, in terms of primal and dual solutions, that the Dual I decomposition is the best option. However, given the required enumerative process its computational performance is worse. Thus, the Dual I is not recommended to be used for hydro plants with complex configurations. In such cases, like Brazilian system, a combined strategy could be more suitable.

V. CONCLUSIONS

This work presents a solution strategy for the STHS problem, which uses LR and artificial variables technique, decomposing the original problem into smaller subproblems with distinct mathematical characteristics (Thermal, Hydrothermal, Hydraulic and Hydro Unit Commitment). In order to deal better with the combinatorial aspect introduced by the forbidden zones in the Hydro Unit Commitment subproblem, we proposed two different decomposition strategies. The first strategy relies on a complete enumeration of all possible 0-1 non-forbidden zones of the units, carried out for each hydro plant and time stage. And, the second strategy relies only on the enumeration of each hydro unit and time stage, because the Hydro Unit Commitment subproblem is separated into two other subproblems (Continuous and Integer).

The strategies based on LR were evaluated with respect to primal and dual solutions. We conclude from our experiments, that the Dual I decomposition is the best option in terms of primal and dual solutions, but its computational performance is worse. Thus, for hydro plants with complex configurations, like Brazilian system, a combined strategy could be more suitable.

The proposed strategies also make possible the inclusion of other constraints without modifying the solution structure. Therefore, the minimum up and downtime constraints for thermal and hydro units and the consideration of the electrical network would be modelled on the STHS problem. These constraints would be included at the proposed subsets of subproblems.

The obtained results point out that the proposed strategy is promising for the application in bigger problems like the whole Brazilian power systems.

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