Exact Solution for Overhanging Slabs of Bridges under Arbitrary Concentrated Load Position

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***Abstract*-Overhanging slabs of bridges are the most sensitive part of the bridge deck, because they have to tolerate high moments and shear forces acting on the cantilever part. So, the behaviors of bridges with overhanging slabs under concentrated loads are complicated. Several load mechanisms can develop depending on the loading and the geometry of the structure. The overhanging slabs of spine beam bridges can be treated as infinite or semi-infinite cantilevers. In this paper, cantilever slabs of varying or constant thickness, with or without edge reinforcement are analyzed under the action of an arbitrary placed point load. The structural actions of spine beam bridges are composed of interacting between the longitudinal behavior and transverse behavior. Therefore, solutions were obtained by means of Fourier Integrals for the deflection, rotation, moment and shear values along the longitudinal and transverse sections of the cantilever. The numerical calculation was done in MATLAB program. Also, a Finite Element analysis was performed using ANSYS to exhibit the accuracy of the numerical solution.**

Keywords- Exact Solution; Overhanging Slabs of Bridges; Arbitrary Concentrated Load

I. INTRODUCTION

It is customary these days to resort to available general purpose structural analysis programs based on the finite element method for the solution to plate problems. While acknowledging the obvious use and versatility of such programs, an attempt was made in the present work to extend the known analytical solutions for plates under concentrated loads to a reasonable level of complication such as is to be found in typical concrete bridge cantilever slabs, most of which are of tapering thickness and edge reinforced. In this respect, the earlier works of Jaramillo (1950) [1], Reismann and Cheng [2] and Sawko and Mils (1971) [3] deserve especial mention. Jaramillo's solution (1950) is an exact solution to constant thickness slab without edge beam in terms of improper integrals for the deflections and moments due to a transverse concentrated load. His formulation and the computation in his solution are difficult to understood and have several limitations in its use. Based on a similar approach, Lee (1965) [4] studied the effective width considering force boundary conditions between slab and beam. He considered resultant axial forces and bending moment developed in the slab and beam due to interactive forces. Reismann and Cheng's solution (1970) [2] is for the analysis of a cantilever plate strip of finite width and infinite length with constant thickness slab and an edge beam. Their analysis is close to Jaramillo's solution but the computation is too difficult for general use. In particular, the paper of Sawko and Mills [3] has served as a starting point for the present investigation. The importance of assessing the transverse moments at the root of the cantilever so as to determine both the maximum depth of the cantilever deck and the quantity of transverse prestressing steel has recently been evaluated in the paper by Thoman et al. [5]. Lu [6] performed a series of nine tests on reduced scale cantilevers. The tested cantilevers had a relatively small thickness of h = 50 mm to 60 mm. Nevertheless, the behavior of cantilevers under concentrated loads was well represented. The predominant failure mode was shear. The flexural transverse reinforcement varied from 0.15% to 1.0%. The cantilevers were tested under one or two concentrated loads introduced by means of square loading pads with a side length of 76 mm. One of them was tested under one concentrated load applied near the free edge. The bending reinforcement ratio was low (0.15%). This cantilever underwent significant ductile deformation before failing in shear. The failure seems to have propagated from the shear cracks in longitudinal direction. Another Cantilever, with an edge beam, was tested under a single concentrated load near the free edge. The failure mode was punching shear, but the punching shear crack did not cross the edge beam. The edge beam had a width of 60 mm and an overall depth of 150 mm. This cantilever illustrated well the behavior of cantilevers without edge beam subjected to concentrated loads. Therefore, the distribution of the transverse bending moment and shear force along the overhanging slab due to a concentrated point load has not been tackled. In this study, exact solution for varying thickness overhanging slab in two case of linear and parabolic with an edge beam was presented. Also, some experimental work was done related to loading on the cantilevers. Some of these works are mentioned here.

II. THEORETICAL BACKGROUND

The plate was considered infinitely long in the longitudinal direction and of constant width in the transverse direction (Fig. 1). It was clamped along one of its parallel edges, but as a special case a support rotation can be incorporated. The thickness of the plate could be made to vary linearly in the transverse direction, and optionally an edge beam existed to reinforce the free edge.

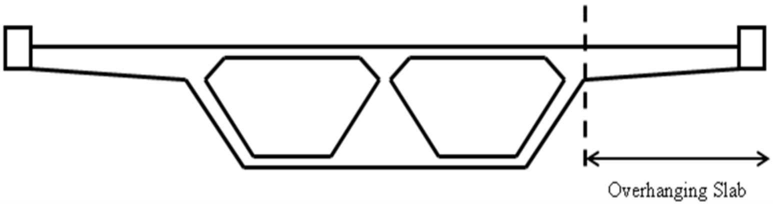


Fig. 1 Overhanging slabs of bridges

A concentrated load acted at an arbitrary point on the slab or the edge beam (Fig. 2). Assuming that the two opposite edges of the cantilever slab at infinity were simply supported, Levy's method was applicable. An exact solution to the problem for a cantilever of uniform thickness and an almost exact solution for the linearly varying thickness case can be obtained in terms of improper Fourier Integrals.

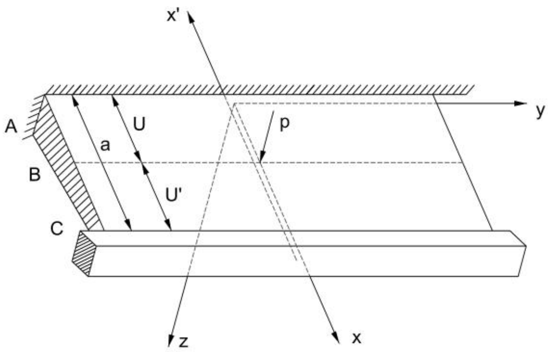


Fig. 2 Edge reinforced, infinitely long cantilever plate under arbitrarily placed point load

The plate was divided into two strips, AB and BC, one from $x=0$ to $x=u$, and the other from $x=u$ to $x=a$ (or $x'=0$ to $x'=u'$), respectively, as shown in Fig. 2. Therefore, the plate governing equation became homogeneous, and the concentrated load was considered as part of the boundary conditions. A support rotation was given to the cantilever root assuming that at the support there were continuous springs all having a given rotational stiffness k, and a beam was monolithically attached to the opposite parallel free edge. The thickness of the plate was assumed to vary linearly in the transverse direction, resulting in a flexural rigidity D which is a function of x.

According to the small deflection plate theory for varying thickness in the transverse direction, the deflections of the middle plane of the plate were characterized by the following plate governing equation:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{dD}{dx}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)+\frac{d^2D}{dx^2}\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right)=q \tag{1}$$

The bending moments, twisting moments and shear forces were related to the deflections by means of the following known equations:

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right) \tag{2}$$

$$M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right)$$

$$M_{xy}=-M_{yx}=D(1-\upsilon)\frac{\partial^2 w}{\partial x\partial y} \tag{3}$$

$$Q_x = -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) - \frac{dD}{dx}\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y = -D\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}\right) - \frac{dD}{dx}(1-\upsilon)\frac{\partial^2 w}{\partial x \partial y} \tag{4}$$

Formulation of the Problem

The plate flexural rigidity variation was taken as exponential in order to render Eq. (1) amenable to solution. The following expression closely approximates a uniform taper, i.e. the thickness variation is not sudden:

$$D_x = D_1 e^{cx} \tag{5}$$

where D_1 is the flexural rigidity of the plate along the cantilever root, $D_2 = D_1 e^{ca}$ is the flexural rigidity of the plate along the boundary between the beam and the plate, and $D_x = D_1 e^{cx}$ is the flexural rigidity of the plate at any arbitrary point. Also, $c = \frac{Ln\left(D_2/D_1\right)}{a}$.

Substituting Eq. (5) into Eq. (1) for an edge loaded plate gives

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + 2c\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) + c^2\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right) = 0 \tag{6}$$

In the case of infinite strips with arbitrary boundary conditions on the two parallel edges, the method of M. Levy can be used by assuming that the two edges at infinity are simply supported. Expressing the deflection w in terms of an Improper Fourier Integral in the y direction,

$$w(x,y) = \int_0^\infty F(x,\alpha)\cos\alpha y\, d\alpha \tag{7}$$

Substituting Eq. (7) into Eq. (6) gives

$$\int_0^\infty \left[\frac{d^4F}{dx^4} - 2\alpha^2\frac{d^2F}{dx^2} + \alpha^4 F + 2c\left(\frac{d^3F}{dx^3} - \alpha^2\frac{dF}{dx}\right) + c^2\left(\frac{d^2F}{dx^2} - \upsilon\alpha^2 F\right)\right]\cos\alpha y\, d\alpha = 0 \tag{8}$$

Simplification leads to the following result:

$$\frac{d^4F}{dx^4} - 2\alpha^2\frac{d^2F}{dx^2} + \alpha^4 F + 2c\left(\frac{d^3F}{dx^3} - \alpha^2\frac{dF}{dx}\right) + c^2\left(\frac{d^2F}{dx^2} - \upsilon\alpha^2 F\right) = 0 \tag{9}$$

The above equation is a linear homogeneous differential equation of fourth order with constant coefficients. Its solution may be written in the form of hyperbolic functions

$$F(x,\alpha) = e^{-cx/2}\left(C_1 shr_1 x + C_2 chr_1 x + C_3 shr_2 x + C_4 chr_2 x\right) \tag{10}$$

where $r_1 = \frac{1}{2}\left(c^2 + 4\alpha^2 + 4c\alpha\sqrt{\upsilon}\right)^{1/2}$,

$$r_2 = \frac{1}{2}\left(c^2 + 4\alpha^2 - 4c\alpha\sqrt{\upsilon}\right)^{1/2},$$

and C_1, C_2, C_3, C_4 are the constants to be determined from the boundary conditions.

Substituting Eq. (10) into Eq. (7) gives:

$$w(x,y) = \int_0^\infty e^{-cx/2}\left(C_1 shr_1 x + C_2 chr_1 x + C_3 shr_2 x + C_4 chr_2 x\right)\cos\alpha y\, d\alpha \tag{11}$$

and

$$w'(x,y)=\int_0^\infty e^{cx'/2}\left(C_1'shr_2x'+C_2'chr_2x'+C_3'shr_1'x+C_4'chr_1x'\right)\cos\alpha y d\alpha \tag{12}$$

for the deflection equations for the strips AB and BC respectively.

Substitution of the plate boundary conditions proceeds as follows:

Along the clamped edge, $x=0$,

$$(w)_{x=0}=0 \tag{13}$$

$$(M_x)_{x=0}=k(\theta_x)_{x=0} \tag{14}$$

and along the boundary between the beam and the plate, $x'=0$,

$$-EI\left(\frac{\partial^4 w}{\partial y^4}\right)_{x'=0}=-(V_{x'})_{x'=0} \tag{15}$$

$$GJ\left(\frac{\partial^3 w'}{\partial x'\partial y^2}\right)_{x'=0}=-D_2\left(\frac{\partial^2 w'}{\partial x'^2}+\upsilon\frac{\partial^2 w'}{\partial y^2}\right)_{x'=0} \tag{16}$$

where $V_{x'}=Q_{x'}-\dfrac{\partial M_{x'y}}{\partial y}$.

Fig. 3 depicts the boundary conditions given in Eqs. (15) and (16).

In Fig. 3(a), the plate shear at any point of the edge equals the beam shear at the corresponding point, therefore $-EI\dfrac{\partial^4 w}{\partial y^4}=-V_{x'}=-\left(Q_{x'}-\dfrac{\partial M_{x'y}}{\partial y}\right)$.

In Fig. 3(b), the plate bending moment at any point of the edge equals the twisting moment for the beam at the corresponding point, therefore $M_{x'}=M_t$, where M_t is found from Fig. 3 (c) to be given by:

$$T-M_t dy=T+\frac{\partial T}{\partial y}dy \text{ and } T=GJ\frac{\partial^2 w}{\partial x\partial y}.$$

As $M_{x'}$ can be written as:

$$M_{x'}=-D_2\left(\frac{\partial^2 w'}{\partial x'^2}+\upsilon\frac{\partial^2 w'}{\partial y^2}\right),$$

the equation $M_{x'}=M_t$ becomes:

$$-D_2\left(\frac{\partial^2 w'}{\partial x'^2}+\upsilon\frac{\partial^2 w'}{\partial y^2}\right)=-GJ\frac{\partial^3 w}{\partial x\partial y^2}=GJ\frac{\partial^3 w'}{\partial x'\partial y^2}.$$

Since continuity conditions must be satisfied for the two boundaries,

$$(w)_{x=u}=(w')_{x'=u'} \tag{17}$$

$$\left(\frac{\partial w}{\partial x}\right)_{x=u}=-\left(\frac{\partial w'}{\partial x'}\right)_{x'=u'} \tag{18}$$

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_{x=u}=\left(\frac{\partial^2 w'}{\partial x'^2}\right)_{x'=u'} \tag{19}$$

$$(V_x)_{x=u}+(V_{x'})_{x'=u'}=p(y) \tag{20}$$

where $p(y)=\dfrac{P}{\pi}\displaystyle\int_0^\infty\cos\alpha y d\alpha$ is the Fourier Integral representation for $p(y)$.

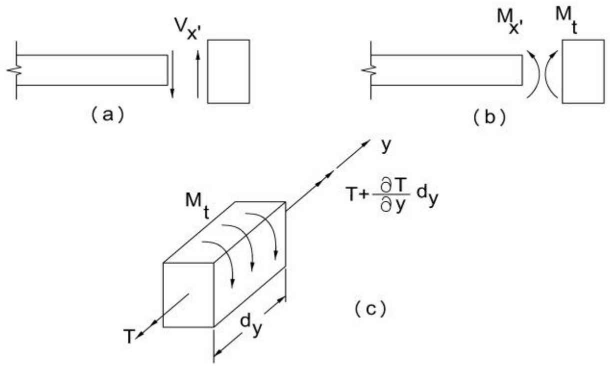


Fig. 3 Edge beam plate boundary conditions a) Shear condition b) Moment condition c) Torsion applied to beam

Although the integral in the above expression is not convergent, it is possible to give the integral a meaning in the sense of distributions, and according to the theory of distributions, all subsequent computations are still valid.

Solving Eqs. (13)-(20), the unknown coefficients $C_1, C_2, C_3, C_4,\ C_1', C_2', C_3', C_4'$ are determined. The rotation, moment and shear equations for the two strips of the plate are:

$$\theta_x(C_1,C_2,C_3,C_4,c,r_1,r_2,x,y)=\int_0^{\infty} e^{-cx/2}\Big[C_1\left(-\frac{c}{2}shr_1x+r_1chr_1x\right)+C_2\left(-\frac{c}{2}chr_1x+r_1shr_1x\right) C_3\left(-\frac{c}{2}shr_2x+r_2chr_2x\right)+C_4\left(-\frac{c}{2}chr_2x+r_2shr_2x\right)\Big]\cos\alpha y d\alpha \tag{21}$$

$$\theta_{x'}=-\theta_x\left(C_1',C_2',C_3',C_4',-c,r_2,r_1,x',y\right) \tag{22}$$

$$M_x(C_1,C_2,C_3,C_4,c,r_1,r_2,x,y)=-D_x\int_0^{\infty} e^{-cx/2}\Big[C_1\left(\frac{c^2}{4}shr_1x-cr_1chr_1x+r_1^2shr_1x\right. \left.\upsilon\alpha^2shr_1x\right)+C_2\left(\frac{c^2}{4}chr_1x-cr_1shr_1x+r_1^2chr_1x-\upsilon\alpha^2chr_1x\right)+C_3\left(\frac{c^2}{4}shr_2x-cr_2chr_2x\right. \left.+r_2^2shr_2x-\upsilon\alpha^2shr_2x\right)+C_4\left(\frac{c^2}{4}chr_2x-cr_2shr_2x+r_2^2chr_2x-\upsilon\alpha^2chr_2x\right)\Big]\cos\alpha y d\alpha \tag{23}$$

$$M_{x'}=M_x\left(C_1',C_2',C_3',C_4',-c,r_2,r_1,x',y\right) \tag{24}$$

$$V_x\left(C_1,C_2,C_3,C_4,c,r_1,r_2,x,y\right)=-D_x\int_0^{\infty}e^{cx/2}\left\{C_1\left[\left(shr_1x\right)\left(\frac{c^3}{8}+\alpha^2c-\frac{1}{2}cr_1^{\ 2}-\frac{3}{2}\upsilon\alpha^2c\right)+\left(chr_1x\right)\right.\right.$$
$$\left.\left(-\frac{1}{4}c^2r_1-2\alpha^2r_1+\upsilon\alpha^2r_1+r_1^{\ 3}\right)\right]+C_2\left[\left(chr_1x\right)\left(\frac{c^3}{8}+\alpha^2c-\frac{1}{2}cr_1^{\ 2}-\frac{3}{2}\upsilon\alpha^2c\right)+\left(shr_2x\right)\left(-\frac{1}{4}c^2r_1\right.\right.$$
$$\left.\left.-2\alpha^2r_1+\upsilon\alpha^2r_1+r_1^{\ 3}\right)\right]+C_3\left[\left(shr_2x\right)\left(\frac{c^3}{8}+\alpha^2c-\frac{1}{2}cr_2^{\ 2}-\frac{3}{2}\upsilon\alpha^2c\right)+\left(chr_2x\right)\left(-\frac{1}{4}c^2r_2-2\alpha^2r_2+\right.\right. \tag{25}$$
$$\left.\left.\upsilon\alpha^2r_2+r_2^{\ 3}\right)\right]+C_4\left[\left(chr_2x\right)\left(\frac{c^3}{8}+\alpha^2c-\frac{1}{2}cr_2^{\ 2}-\frac{3}{2}\upsilon\alpha^2c\right)+\left(shr_2x\right)\left(-\frac{1}{4}c^2r_2-2\alpha^2r_2+\upsilon\alpha^2r_2+r_2^{\ 3}\right)\right]\Bigg\}$$
$$\cos\alpha y d\alpha$$

$$V_{x'}=-V_x\left(C_1^{'},C_2^{'},C_3^{'},C_4^{'},-c,r_2,r_1,x',y\right) \tag{26}$$

III. NUMERICAL SOLUTION

The unknown coefficients $C_1,C_2,C_3,C_4,C_1^{'},C_2^{'},C_3^{'},C_4^{'}$ in the deflection, rotation, moment and shear equations are functions of α and can be calculated from the boundary condition equations, i.e. Eqs. (13)-(20), for a specific α value during the numerical integration process for each interval, where α changes value from zero to infinity. A numerical integration procedure was followed for the evaluation of these integrals in view of the fact that they are not expressible in terms of elementary functions. Since the calculation of the unknown coefficients requires the solution of a set of simultaneous algebraic equations, an elimination method was also used. A MATLAB code was prepared to provide efficient solutions for any given plate geometry and load position. In order to compare results for non-prismatic slabs, cantilever plate examples analyzed by Sawko and Mills [3], as shown in Fig. 4, and Thoman, Redfield and Hollenbook [5], as shown in Fig. 5, were solved and the results are given in Table 1. For the plate in Fig. 4, the poission ratio is 0.25 and results for $\frac{M_x}{P}$ are compared. For the cantilever plate in Fig. 5, the modulus of elasticity is 27580 MPa, Poisson ratio is 0.15, and P is 397.2 kN.

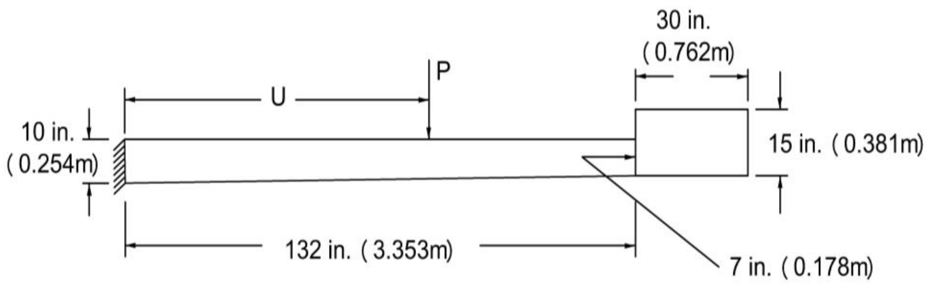


Fig. 4 Cross section of the cantilever plate analyzed by Sawko and Mills [3]

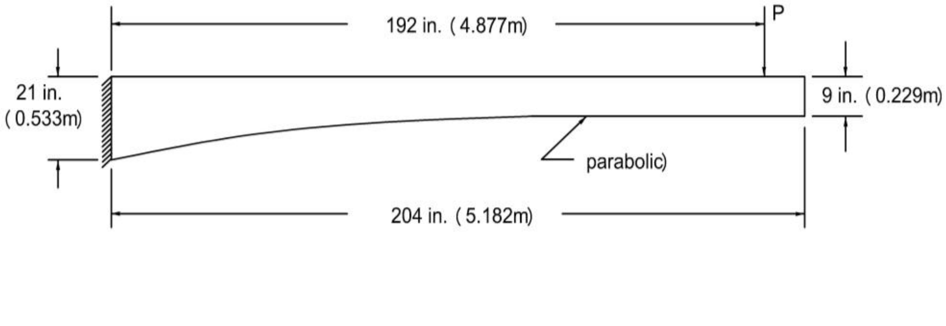


Fig. 5 Cross section of the cantilever plate analyzed by Thoman, Redfield and Hollenbeck [5]

IV. FINITE ELEMENT WORK

These examples wew analyzed using the finite element method and compared with the exact approaches. The finite element analysis was carried out using ANSYS software. The quadrilateral elements by eight nodes having three degrees of freedom at each node (translations in the nodal x, y, and z directions) were used. The model test results for two different cases of linear and parabolic thicknesses were used as a basis for comparison with the theoretical results. The finite element solution to the problem was verified with the exact solution for Poisson's ratio of 0.25 for the linear case and 0.15 for the parabolic case. The modulus of elasticity was 4000 ksi (27580 MPa) in both cases.

V. RESULTS AND DISCUSSION

Only limited results are available in the literature for nonprismatic edge-reinforced cantilever bridge slabs. Good agreement was observed between the analytical results of Sawko and Mills [3] and the proposed method, as shown in Table 1. Agreement between the finite element results of Thoman et al. [4] and the proposed method, as shown in the Table 1, while adequate for the moment values, was less than acceptable for the tip deflection. The discrepancy may be explained by the manner in which the exact solution program the tapering thickness of a cantilever slab. The program uses only the tip and root thickness and passes a smooth curve that approaches a linear taper. However, the slab modeled by Thoman et al. [4] varies parabolically in thickness from 228.6 mm at the tip to 533.4 mm at the root. Figs. 6 to 9 compare the results of moment distribution along the overhang length at different concentrated load positions between Finite Element, exact solution, and two more methods. In addition, contours of moment distribution as well as vertical displacement are shown in Figs. 10 to 12.

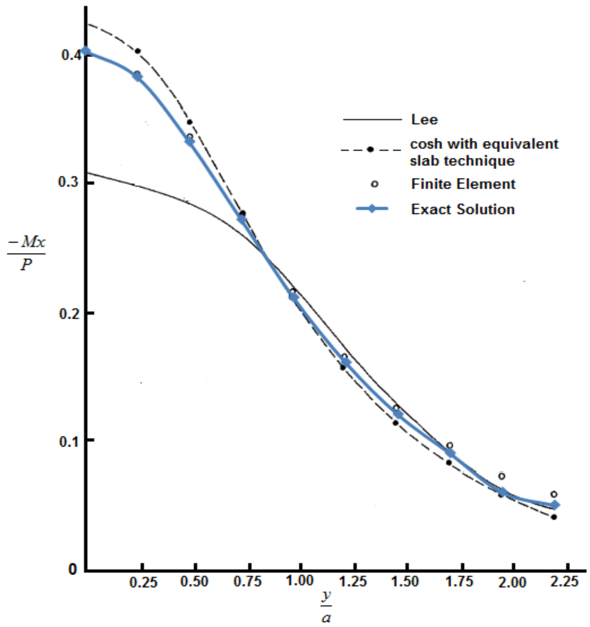


Fig. 6 Distribution of moment along the overhang length for $\frac{u}{a} = 1.00$

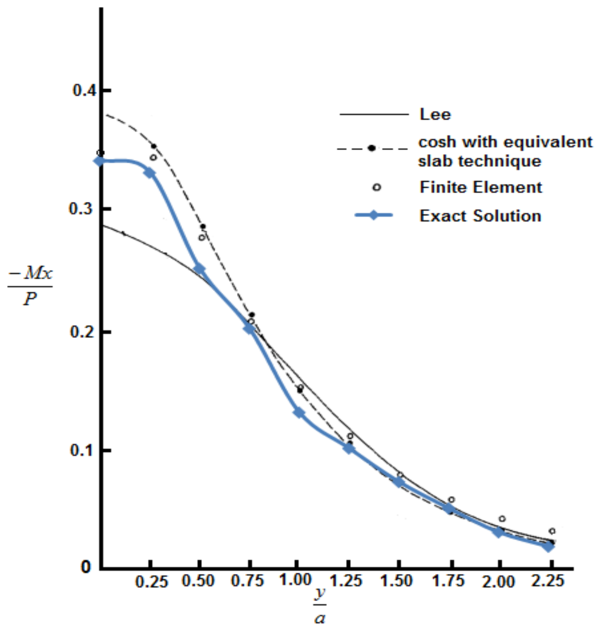


Fig. 7 Distribution of moment along the overhang length for $\frac{u}{a} = 0.75$

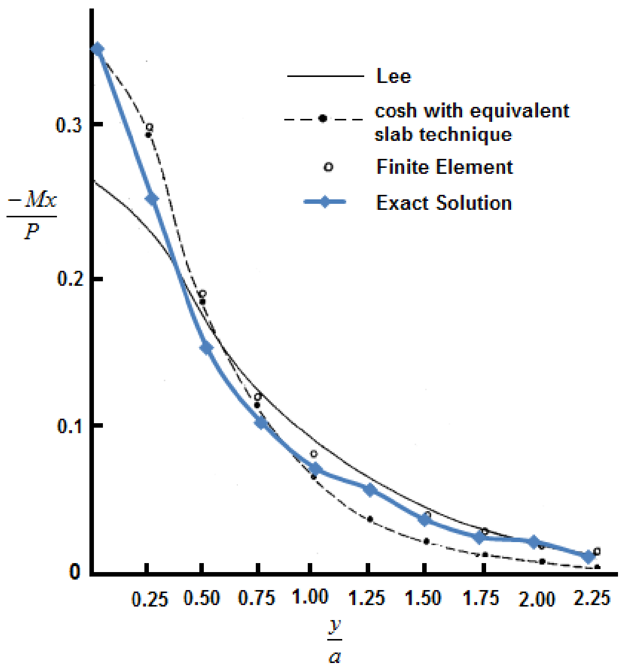


Fig. 8 Distribution of moment along the overhang length for $\frac{u}{a} = 0.5$

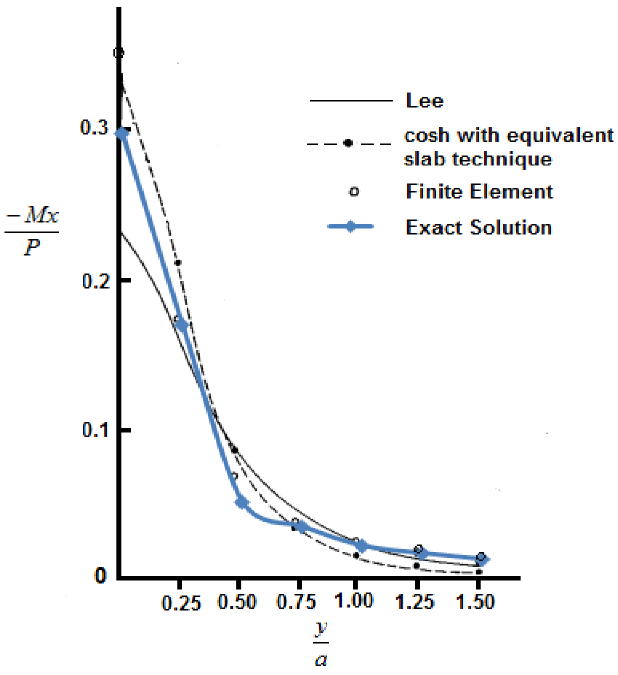
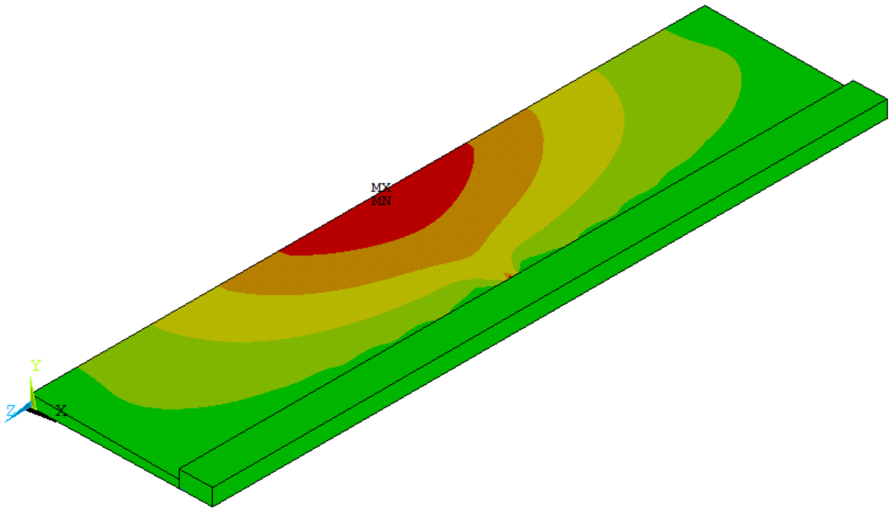
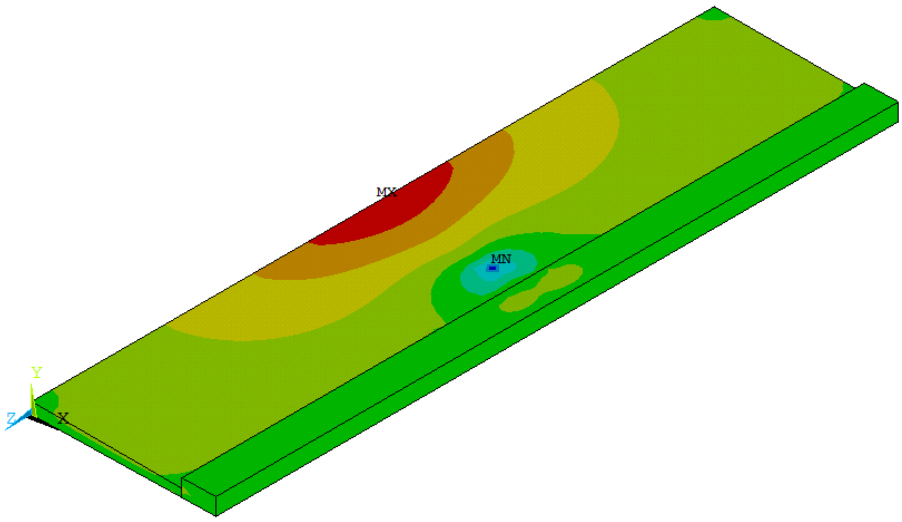


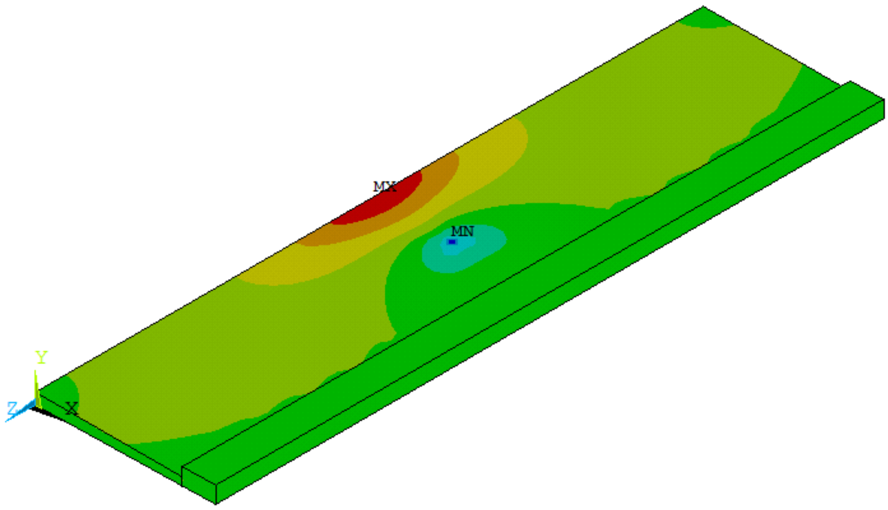
Fig. 9 Distribution of moment along the overhang length for $\frac{u}{a} = 0.25$



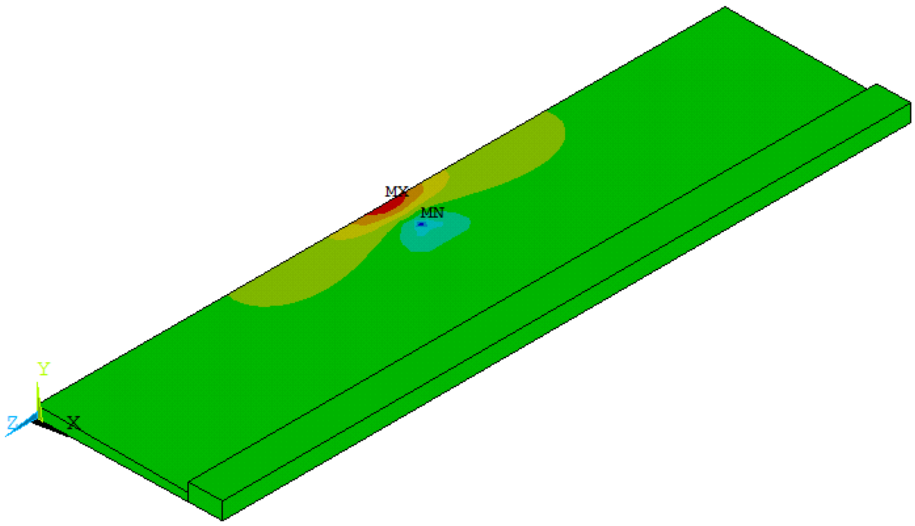
(a) u/a =1



(b) u/a = 0.75



(c) u/a = 0.5



(d) u/a = 0.25

Fig. 10 Contours of moment distribution along the overhang length with linear thickness

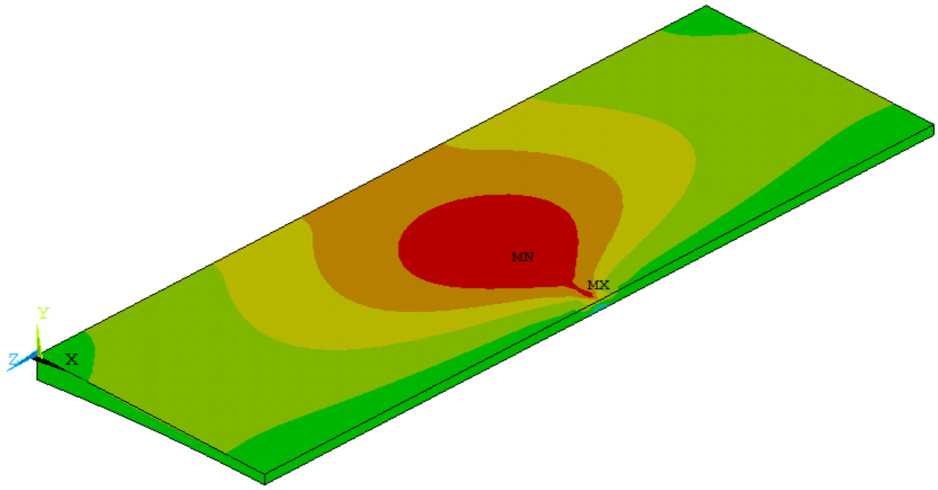


Fig. 11 Contours of moment distribution along the overhang length with parabolic thickness

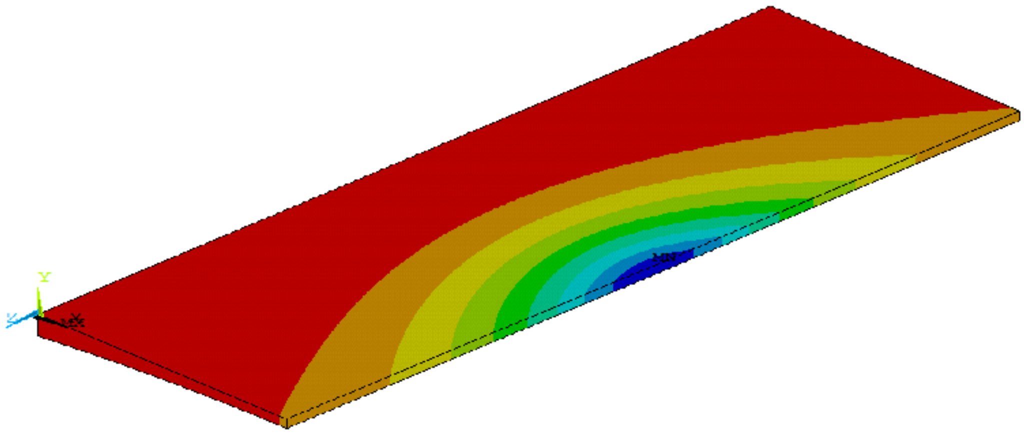


Fig. 12 Contours of vertical displacement along the overhang length with parabolic thickness

For parabolic case the results of the proposed exact solution were compared with those of finite element solution and Thoman as shown in Table 1.

TABLE 1 MOMENT DISTRIBUTION ALONG THE PARABOLIC OVERHANG

	$x/a = 0.00$			$x/a = 1.00$		
y/a	$-M_x$ (Kips.ft/ft) Thoman et al.	$-M_x$ (Kips.ft/ft) Exact Solution	$-M_x$ (Kips.ft/ft) Finite Element	w in Thoman et al.	w in Exact Solution	$-M_x$ (Kips.ft/ft) Finite Element
0.00	54	56.3	56.35	0.63	0.54	0.52
0.25	48	50.4	50.48	Not available	0.44	0.43
0.50	34	36.9	37	.	0.31	0.30
0.75	21	23.5	23.58	.	0.20	0.19
1.00	12	13.7	13.73	.	0.12	0.10
1.25	6.5	7.5	7.6	.	0.07	0.069
1.50	3.5	4.0	4.4	Not available	0.04	0.02

1.75		2.1	2.6		0.03	0.026
2.00		1.0	1.5		0.01	0.096

VI. CONCLUSIONS

In this research, an analytical solution for overhanging bridge slabs under arbitrary concentrated load position was developed in a form especially convenient for application due to its simplicity. This analytical solution is applicable for cantilever slabs of varying or constant thickness, with or without edge reinforcement. At the end, the results of the analytical solution were compared and verified with the numerical results using ANSYS software, equivalent slab technique and experimental findings by Thoman. Good agreement was observed among the three different approaches.

Notation

a Cantilever width

Plate flexural rigidity $\left(Et^3 / 12\left(1-\upsilon^2\right) \right)$

E Young's modulus

EI Beam flexural rigidity

GJ Beam torsional rigidity

k Spring rotational stiffness

k_1 Stiffness ratio of beam flexural rigidity to plate flexural rigidity $\left(EI / D \right)$

k_2 Stiffness ratio of beam torsional rigidity to plate flexural rigidity $\left(GJ / D \right)$

k_3 Stiffness ratio of spring rotational stiffness to plate flexural rigidity $\left(k / D \right)$

M_x, M_y Bending moments per unit length

M_{xy} Twisting moment per unit length

P Arbitrarily placed concentrated load

q Load intensity

Q_x, Q_y Shear forces per unit length

t Plate thickness

u Distance of the point load from the clamped edge

u' Distance of the point load from the free edge

V_x Reaction function of the plate per unit length

w Deflection of the plate

x Coordinate along the plate width from the clamped edge

x' Coordinate along the plate width from the free edge

y Coordinate along the clamped edge

z Coordinate perpendicular to the $xy-$plane

υ Poisson's ratio

θ Rotation

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