Effect of Internal Heating on Double Diffusive Convection in a Couple Stress Fluid Saturated Anisotropic Porous Medium

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Abstract*-The effect of internal heat source on double diffusive convection in a couple stress fluid saturated horizontal anisotropic porous layer was studied analytically by performing linear and nonlinear stability analyses. For linear stability analysis, normal mode technique was used, whereas for nonlinear stability analysis a minimal representation of Fourier series upto two terms was used. Effects of anisotropic parameter, couple stress parameter, solute Rayleigh number, Vadász number, Lewis number, internal heat source parameter on stationary, oscillatory and finite amplitude convection were obtained and shown graphically. Also the graphs for heat and mass transport, streamlines, isotherms, isohalines for steady and unsteady cases were drawn. It was found that internal heat source parameter had destabilizing effect on all modes of convection. Heat transport increased and mass transport decreased with the increase in internal heat source parameter.

Keywords- Internal Heat Source; Double-diffusive Convection; Porous Media; Anisotropic; Couple-stress

NOMENCLATURE

LATIN SYMBOLS

a	Wave number
C	Couple stress parameter $C=\frac{\mu_c}{\mu d^2}$
d	Depth of the fluid layer
Da	Darcy number $Da=\frac{K_z}{d^2}$
g	Acceleration due to gravity
K_x	Permeability in x-direction
K_z	Permeability in z-direction
Le	Lewis number, $Le=\frac{\kappa_{Tz}}{\kappa_S}$
Nu	Nusselt number
p	Reduced pressure
Pr	Prandtl number, $Pr=\frac{\nu}{\kappa_{Tz}}$
Ra_T	Thermal Rayleigh number, $Ra_T=\frac{\beta_T g\Delta T d K_z}{\nu\kappa_{Tz}}$
Ra_S	Solute Rayleigh number, $Ra_S=\frac{\beta_S g\Delta S d K_z}{\nu\kappa_{Tz}}$
R_i	Internal heat source parameter, $R_i=\frac{Qd^2}{\kappa_{Tz}}$
S	Solute concentration
ΔS	Solute difference across the porous layer

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Sh Sherwood number
T Temperature
ΔT Temperature difference across the porous layer
t Time
Va Vadász number $Va = \frac{\phi Pr}{Da}$
(x,y,z) Space Co-ordinates

GREEK SYMBOLS

β_T Coefficient of thermal expansion
β_S Coefficient of solute expansion
γ Heat capacity ratio $\frac{(\rho c_p)_m}{(\rho c_p)_f}$
η Thermal anisotropy parameter,κ_{Tx}/κ_{Tz}
ξ Mechanical anisotropy parameter, K_x/K_z
κ_S Solutal diffusivity
κ_T Thermal diffusivity,$\kappa_{Tx}(\hat{\imath}\hat{\imath}+\hat{\jmath}\hat{\jmath})+\kappa_{Tz}(\hat{k}\ \hat{k})$
κ_{Tx} Effective thermal diffusivity in x-direction
κ_{Tz} Effective thermal diffusivity in z-direction
μ Dynamic viscosity of the fluid
μ_c Couple stress viscosity of the fluid
ϕ Porosity
ν Kinematic viscosity, $\left(\frac{\mu}{\rho_0}\right)$
ρ Fluid density
ψ Stream function

OTHER SYMBOLS

∇_1^2 $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$
∇^2 $\nabla_1^2+\frac{\partial^2}{\partial z^2}$

SUBSCRIPTS

b basic state
c critical
0 reference value

SUPERSCRIPTS

$'$ perturbed quantity
$*$ dimensionless quantity
F finite amplitude
osc oscillatory
st stationary

I. INTRODUCTION

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Double-diffusive convection in porous media provides many opportunities for researchers due to its analytical and practical accessibility. It arises in many physical situations such as solidification of binary mixture, migration of solutes in water-saturated soils, geophysical system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The reviews in this area have been well collected and presented by Nield and Bejan [1], Ingham and Pop [2,3], Vafai [4,5], Vadász [6]. Very first study on double-diffusive convection in porous media mainly concerns with linear stability analysis, and was performed by Nield [7]. Later, the study was continued by Taunton et al. [8], Rudraiah et al. [9], Poulikakos [10], Travison and Bejan [11], Rosenberg and Spera [12], Mamou [13], Kuznetsov and Nield [14-16], Nield and Kuznetsov [17,18]. Till now, there are numerous papers available on double-diffusive convection in porous media.

The importance of non-Newtonian fluids with suspended particle is of very significant, however it attracted less attention as compared to Newtonian fluids. Application of non-Newtonian fluids are found in extrusion of polymer fluids in industry, exotic suspension, fluid film lubrication, solidification of liquid crystals, cooling of metallic plate in bath, colloidal and suspension solutions. Non-Newtonian couple stress fluids carries a specific feature of polar effects. Couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules. Theory for couple stress fluid was proposed by Stokes [19], which is simpler polar fluid theory and shows all the important features and effects of couple stresses in fluids caused by the mechanical interactions that occur inside a deforming continuum. Stabilizing effect of couple stress parameter is reported in the works of Sharma and Thakur [20], who studied thermal instability in an electrically conducting couple stress fluid with magnetic field. Sunil et al. [21] studied the effect of suspended particles on double diffusive convection in a couple stress fluid saturated porous medium, Sharma and Sharma [22] investigated the effect of suspended particles on couple-stress fluid, heated from below, in the presence of rotation and magnetic field. Malashetty et al. [23] did an analytical study of linear and nonlinear double diffusive convection with Soret effect in couple stress liquids. Shivakumara [24] studied the effect of nonuniform temperature gradient on the onset of convection in a couple stress fluid saturated porous medium. Malashetty et al. [25] performed linear and weakly nonlinear analyses of double diffusive convection in an isotropic porous medium saturated with couple stress fluid. Malashetty and Kollur [26] studied the onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer. Shivkumara et al. [27] carried out linear and nonlinear stability analysis of double diffusive convection in a couple stress fluid saturated porous layer.

Most of the studies in relevant areas mainly dealt with isotropic porous media, however there are many physical situations where thermal and mechanical anisotropy exists in porous matrix, one of such examples is our geothermal environment. Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibres and is in fact encountered in numerous systems in industry and nature. Also artificial porous matrix anisotropy can be made deliberately according to applications. The first study in this area was given by Castinel and Combarnous [28], who obtained the criterion for the onset of convection in a layer with anisotropic permeability and impermeable upper and lower boundaries. Epherre [29] applied anisotropy in both permeability and thermal conductivity, Kvernvold and Tyvand [30] studied the steady finite amplitude convection and derived criterion for the onset of convection. Tyvand [31] studied the onset of thermohaline convection in anisotropic porous media, while anisotropy in only thermal conductivity has been studied by Storesletten [32]. It was found that the convection cells are rectangular with vertical lateral cell walls when longitudinal diffusivity is greater than transverse diffusivity and for reverse case the lateral cell walls are tilted as well as curved. Nield and Kuznetsov [33] studied the combined effects of horizontal and vertical heterogeneity and anisotropy on the onset of convection in a porous medium. Gaikwad et al. [34] performed linear and nonlinear stability analyses of double diffusive convection in anisotropic porous media including Soret effect and reported that the effect of mechanical anisotropic parameter is to destabilize and of thermal anisotropic parameter is to stabilize the system.

There are large number of practical situations in which convection is driven by internal heat source in the porous media. The wide applications of such convections occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions, and crystal growth. The study concerning internal heat source in porous media is provided by Tveitereid [35], who obtained the steady solution in the form of hexagons and two dimensional rolls for convection in a horizontal porous layer with internal heat source. Bejan [36] studied analytically the buoyancy induced convection with internal heat source, Parthiban and Patil [37] studied the effect of non-uniform boundaries temperatures on thermal instability in a porous medium with internal heat source and predicted that internal heat source parameter advances the onset of convection. Hill [38] performed linear and nonlinear stability analyses of double-diffusive convection in a porous layer with a concentration based internal heat source. Saravanan [39] investigated linear stability analysis for the onset of natural convection in a fluid saturated porous medium with uniform internal heat source and density maximum in an local thermal nonequilibrium model and predicted that internal heat source parameter advances the onset of convection, Cookey et al. [40] studied the onset of stationary convection in a low Prandtl number with internal heat source and found that effect of internal heat source parameter is destabilization. Borujerdi et al. [41] investigated the stability criterion for a horizontal porous layer with uniform heat source in both fluid and solid phases by considering thermal nonequilibrium model, Borujerdi et al. [42] performed linear stability analysis to predict the effect of Brinkman term on the onset criterion for the stability of natural convection in a horizontal porous layer with uniform heat generation. Capone et al. [43] carried

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out the linear and nonlinear stability analysis of double-diffusive penetrative convection in an anisotropic porous layer with a constant throughflow via internal heat source. Recently Bhadauria group [44-47] have studied the problem of thermal instability in porous media with internal heating, considering various physical models.

Double diffusive convection occurs in many systems in industry and nature, and in the present context, is of particular interest in the study of extraction of metals from ores where a mushy layer is formed during solidification of a metalic alloy. Further, the quality and structure of the resulting solid can be controlled by influencing the transport process externally, which can be done by thermal modulation, gravity modulation, rotation or by internal heating. However in the present study, internal heating of the system was used as an external means to influence the transport process, thereby controlling the quality and structure of the resulting solid. Further, many of the previous studies have modeled the mushy layer as isotropic porous medium, however in reality, the permeability of the porous medium is anisotropic as discussed above. In addtion, due to specific properties of couple stress fluids over Newtonian fluids, couple stress fluids are widely being used in modern industries. Therefore, in the present study the porous medium was assumed to be saturated with couple stress fluid as the melt in the mushy layer may be considered as couple stress fluid. With these motivations, a weak non−linear analysis of hydrodynamic stability was conducted to study the effect of internal heating on double diffusive convection in an anisotropic porous medium saturated with a couple stress fluid.

II. GOVERNING EQUATIONS

An infinite horizontal anisotropic porous layer saturated by couple stress fluid, confined between the planes $z = 0$ and $z = d$ with internal heat source, heated and salted from below, was considered. Cartesian frame of reference was chosen as; origin in the lower boundary and the z-axis in vertically upward direction. The gravity force acted in vertically downward direction, and only free-free boundaries were considered. It was assumed that the mechanical properties and thermal properties in x and y directions were the same. Uniform adverse temperature gradient $\Delta T/d$ and concentation gradient $\Delta S/d$ were maintained across the surfaces. Further, the density variation was considered under Boussinesq approximation. The governing equations under the above considerations are given by

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\frac{\rho_0}{\phi}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \rho \mathbf{g}\hat{k} - \frac{1}{K}\left(\mu - \mu_c \nabla^2\right)\mathbf{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = \nabla.(\kappa_T \nabla.T) + Q(T - T_0), \tag{3}$$

$$\phi \frac{\partial S}{\partial t} + (\mathbf{q}.\nabla)S = \kappa_S \nabla^2 S, \tag{4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)] \tag{5}$$

The thermal and solutal boundary conditions are

$$T = T_0 + \Delta T \text{ at } z = 0 \text{ and } T = T_0 \text{ at } z = d, \tag{6}$$

$$S = S_0 + \Delta S \text{ at } z = 0 \text{ and } S = S_0 \text{ at } z = d. \tag{7}$$

The basic state of liquid is quiescent and given by

$$q_b = 0,\ p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z), \tag{8}$$

which satisfy the following equations

$$\kappa_T \frac{d^2(T_b - T_0)}{dz^2} + Q(T_b - T_0) = 0, \tag{9}$$

$$\frac{d^2 S_b}{dz^2} = 0, \tag{10}$$

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$$\frac{\partial p_b}{\partial z} = -\rho_b g, \tag{11}$$

$$\rho_b = \rho_0\left[1-\beta_T\left(T-T_0\right)+\beta_S\left(S-S_0\right)\right]. \tag{12}$$

The solution of Eq. (9), subject to the boundary conditions (6) , is given by

$$T_b = T_0 + \Delta T\,\frac{\sin\sqrt{\dfrac{Qd^2}{\kappa_{Tz}}}\left(1-\dfrac{z}{d}\right)}{\sin\sqrt{\dfrac{Qd^2}{\kappa_{Tz}}}}. \tag{13}$$

Also Eq. (10) has been solved subjected to the boundary conditions (7) , and

$$S_b = S_0 + \Delta S\left(1-\frac{z}{d}\right). \tag{14}$$

Infinitesimal perturbation was applied to the basic state of the system and then the pressure term waseliminated by taking curl twice of Eq. (2). The resulting equations were nondimensionlized using the following transformations,

$$\begin{aligned}&(x,y,z) = \left(x^*,y^*,z^*\right)d,\ t = t^*\left(\gamma d^2/\kappa_{Tz}\right),\\ &(u,v,w) = \left(u^*,v^*,w^*\right)\left(\frac{\kappa_{Tz}}{d}\right),\ T = (\Delta T)T^*,\ S = (\Delta S)S^*\end{aligned} \tag{15}$$

The nondimensionlized equations (after dropping the asterisks for simplicity) and by setting $\gamma = 1$ are

$$\left[\frac{1}{Va}\frac{\partial}{\partial t}\nabla^2 + \left(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\left(1-C\nabla^2\right)\right]w - Ra_T\nabla_1^2 T + Ra_S\nabla_1^2 S = 0, \tag{16}$$

$$\left[\frac{\partial}{\partial t} - \left(\eta\nabla_1^2 + \frac{\partial^2}{\partial z^2}\right) - R_i\right]T + w\frac{\partial T_b}{\partial z} = 0, \tag{17}$$

$$\left[\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right]S - w = 0 \tag{18}$$

where $Va = \frac{\phi Pr}{Da}$ is Vadász number ($Pr = \frac{\nu}{\kappa_{Tz}}$ is Prandtl number and $Da = \frac{K_z}{d^2}$ is Darcy number), $Ra_T = \frac{\beta_T g \Delta T d K_z}{\nu\kappa_{Tz}}$ is thermal Rayleigh number, $Ra_T = \frac{\beta_S g \Delta S d K_z}{\nu\kappa_{Tz}}$is solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is internal heat source parameter, $C = \frac{\mu_c}{\mu d^2}$ is couple stress parameter, $Le = \frac{\kappa_{Tz}}{\kappa_S}$ is Lewis number, $\xi = \frac{K_x}{K_z}$ is mechanical anisotropy parameter, $\eta = \frac{\kappa_{Tx}}{\kappa_{Tz}}$ is thermal anisotropy parameter.

The stress free, isothermal, isohalines boundary conditions are given by

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \text{ at } z = 0,1. \tag{19}$$

III. LINEAR STABILITY ANALYSIS

For linear stability analysis, normal mode technique was used to solve the eigenvalue problem defined by Eqs. (16)-(18), subject to the boundary conditions given by Eq. (9), using time periodic disturbance in horizontal plane. For fundamental mode, there is

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$$\begin{pmatrix} w \\ T \\ S \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \exp[i(lx+my)+\sigma t]\sin(\pi z) \tag{20}$$

where l, m are horizontal wave number and $\sigma = \sigma_r + i\sigma_i$ is growth rate, and in general a complex quantity. Substituting Eq. (20) in Eqs. (16)-(18), the nontrivial solution in the form of thermal Rayleigh number can be obtained as

$$Ra_T = \frac{\left(R_i - \left(\sigma + \delta_2^{\ 2}\right)\right)}{2a^2F}\left[\frac{\sigma\delta^2}{Va} + \delta_1^{\ 2}\left(1 + C\delta^2\right) + \frac{a^2 Ra_S}{\left(\sigma + \dfrac{\delta^2}{Le}\right)}\right] \tag{21}$$

where $a^2 = l^2 + m^2, \delta^2 = \pi^2 + a^2, \delta_1^{\ 2} = \dfrac{\pi^2}{\xi} + a^2, \delta_2^{\ 2} = \pi^2 + \eta a^2, F = \int_0^1 \dfrac{dT_b}{dz}\sin^2(\pi z)dz$.

For neutral stability state $\sigma_r = 0$, whereas for $\sigma_r < 0$, the system is always stable and for $\sigma_r > 0$, the system is always unstable.

A. Stationary State

The expression of thermal Rayleigh number for the onset of stationary convection at the marginally stable steady state, for which the exchange of stabilities are valid correspond to the $\sigma = 0$ i.e. ($\sigma_r = 0$ and $\sigma_i = 0$) becomes

$$Ra_T^{st} = \frac{\left[R_i - \left(\pi^2 + \eta a^2\right)\right]}{2a^2F}\left[\left(\frac{\pi^2}{\xi} + a^2\right)\left(1 + C\left(\pi^2 + a^2\right)\right) + \frac{a^2 Ra_S Le}{\left(\pi^2 + a^2\right)}\right]. \tag{22}$$

Onset of double-diffusive convection with couple stress fluid saturated isotropic porous layer in presence of internal heat source for stationary mode is obtained by putting $\xi = 1$ and $\eta = 1$ in Eq. (22)

$$Ra_T^{st} = \frac{\left[R_i - \left(\pi^2 + a^2\right)\right]}{2a^2F}\left[\left(\pi^2 + a^2\right)\left(1 + C\left(\pi^2 + a^2\right)\right) + \frac{a^2 Ra_S Le}{\left(\pi^2 + a^2\right)}\right]. \tag{23}$$

In case of single component fluid with internal heat source $Ra_S = 0$, Eq. (22) gets the form

$$Ra_T = \frac{\left[R_i - \left(\pi^2 + \eta a^2\right)\right]}{2a^2F}\left(\frac{\pi^2}{\xi} + a^2\right)\left[1 + C\left(\pi^2 + a^2\right)\right] \tag{24}$$

In absence of internal heat source one has to put $F = -\dfrac{1}{2}$ and $R_i = 0$ in Eq. (22) to obtain

$$Ra_T^{st} = \frac{\left(\pi^2 + \eta a^2\right)}{a^2}\left[\left(\frac{\pi^2}{\xi} + a^2\right)\left(1 + C\left(\pi^2 + a^2\right)\right) + \frac{a^2 Ra_S Le}{\left(\pi^2 + a^2\right)}\right]. \tag{25}$$

The above expression of thermal Rayleigh number is the result of stationary mode for onset of double-diffusive convection in a couple stress saturated anisotropic porous layer given by Malashetty and Premila Kollur [26]. In the case when medium is isotropic i.e. ($\xi = 1$ and $\eta = 1$)

$$Ra_T^{st} = \frac{1}{a^2}\left(\pi^2 + a^2\right)^2\left[1 + C\left(\pi^2 + a^2\right)\right] + Ra_S Le. \tag{26}$$

This is the result for stationary mode for the onset of double-diffusive convection with couple stress fluid saturated isotropic porous media given by Malashetty et al [25]. For $C = 0$ above Eq. (26) reduces to

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$$Ra_T^{st} = \frac{1}{a^2}\left(\pi^2 + a^2\right)^2 + Ra_S Le. \tag{27}$$

The above expression of thermal Rayleigh number for double-diffusive convection in porous media is the result for stationary mode given by Nield [7]. For single component couple stress fluid saturated porous medium i.e. $Ra_S = 0$, Eq. (25) becomes

$$Ra_T^{st} = \frac{1}{a^2}\left(\frac{\pi^2}{\xi} + a^2\right)\left(\pi^2 + \eta a^2\right)\left[1 + C\left(\pi^2 + a^2\right)\right]. \tag{28}$$

When $C = 0$, the above Eq. (28)

$$Ra_T^{st} = \frac{1}{a^2}\left(\frac{\pi^2}{\xi} + a^2\right)\left(\pi^2 + \eta a^2\right) \tag{29}$$

is the result for single component fluid in anisotropic porous media given by Storeslette [32]. For isotropic case, the above Eq. (29) takes the form

$$Ra_T^{st} = \frac{\left(\pi^2 + a^2\right)^2}{a^2}, \tag{30}$$

which is the classical result obtained by Horton and Rogers [48] and Lapwood [49] for single component fluid in porous layer.

B. Oscillatory State

To obtain the expression of thermal Rayleigh number for oscillatory convection at the marginal state, we have $\sigma = i\sigma_i$ (since the real part of σ for marginal oscillatory state is zero i.e $\sigma_r = 0$). After some simplification, we have

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2. \tag{31}$$

Since Ra_T cannot be imaginary as thermal Rayleigh number is a physical quantity, for marginal oscillatory state $\sigma_i \neq 0$, therefore we must have $\Delta_2 = 0$. This gives the expression for the frequency of oscillation

$$\sigma_i^2 = \frac{a^2 Ra_S\left(\left(R_i - \delta_2^{\,2}\right) + \frac{\delta^2}{Le}\right)}{\left[\left(R_i - \delta_2^{\,2}\right)\frac{\delta^2}{Va} - \delta_1^{\,2}\left(1 + C\delta^2\right)\right]} - \left(\frac{\delta^2}{Le}\right)^2. \tag{32}$$

Corresponding to the above value of σ_i, the expression of thermal Rayleigh number for oscillatory convection becomes

$$Ra_T^{osc} = \frac{1}{2a^2 F}\left[\left(R_i - \delta_2^{\,2}\right)\left(\delta_1^{\,2}\left(1 + C\delta^2\right) + \frac{a^2 Ra_S\left(\frac{\delta^2}{Le}\right)}{\left(\frac{\delta^2}{Le} + \sigma^2\right)}\right) + \sigma^2\left(\frac{\delta^2}{Va} - \frac{a^2 Ra_S}{\left(\frac{\delta^2}{Le} + \sigma^2\right)}\right)\right] \tag{33}$$

From the close observation of Eq. (23), it was found that for the physical significance of thermal Rayleigh number, there must be $R_i < \delta_2^{\,2}$ as F is negative. The effect of various parameters on neutral stability of the system are given in the Figs. 1-2.

IV. WEAK NONLINEAR ANALYSIS

Although the linear stability analysis result is significant, some important physical quantities like the value of convection amplitude, heat transfer, mass transfer, cannot be calculated using linear stability analysis, thus nonlinear stability analysis of the system is needed. Nonlinear stability analysis provides useful information which helps to describe the physical mechanism of convective flow with minimum amount of mathematics.

For this, only two-dimensional rolls were considered. The calculation started from Eqs. (2)-(5) by introducing stream function

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in the form $u = \frac{\partial \psi}{\partial z}$ and $w = -\frac{\partial \psi}{\partial x}$ then, taking curl to eliminate pressure term from Eq. (2) and then nondimensionlizing the resulting equations by using transformations given by Eq. (15). Finally, it was set $\gamma = 1$ and the following equations were obtained

$$\frac{1}{Va}\frac{\partial}{\partial t}\nabla_1^2\psi + \left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\left(1 - C\nabla_1^2\right)\psi + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{34}$$

$$\frac{\partial T}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + R_i\right)T - \frac{\partial(\psi, T)}{\partial(x, z)} - \frac{dT_b}{dz}\frac{\partial\psi}{\partial x} = 0, \tag{35}$$

$$\frac{\partial S}{\partial t} - \frac{1}{Le}\nabla_1^2 S - \frac{\partial(\psi, S)}{\partial(x, z)} + \frac{\partial\psi}{\partial x} = 0. \tag{36}$$

A local nonlinear stability analysis shall be performed by using a severely truncated representation of Fourier series for stream function, temperature and concentration, Veronis [50], Rudraiah et al. [51]. This study will help in understanding the physics of the problem with minimum mathematical expressions. Further the results can be used as starting point to generalize it for full nonlinear problem. Also, it is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T, and ψ and S respectively. As a result a component of the form $\sin(2\pi z)$ will be generated. Therefore, the minimal expression which describes the finite amplitude convection are of the forms

$$\psi = A(t)\sin(ax)\sin(\pi z), \tag{37}$$

$$T = B_1(t)\cos(ax)\sin(\pi z) + B_2(t)\sin(2\pi z), \tag{38}$$

$$S = E_1(t)\cos(ax)\sin(\pi z) + E_2(t)\sin(2\pi z), \tag{39}$$

where the amplitudes $A(t)$, $B_1(t)$, $B_2(t)$, $E_1(t)$ and $E_2(t)$ are functions of time and are to be determined. Substituting above expressions in Eqs. (34)-(36) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$\frac{dA(t)}{dt} = -\frac{Va}{\delta^2}\left[\delta_1^2\left(1 + C\delta^2\right)A(t) + aRa_T B_1(t) - aRa_S E_1(t)\right], \tag{40}$$

$$\frac{dB_1(t)}{dt} = -\left[\pi a A(t)B_2(t) + \left(\delta_2^2 - R_i\right)B_1(t) - 2aFA(t)\right], \tag{41}$$

$$\frac{dB_2(t)}{dt} = -\left[\left(4\pi^2 - R_i\right)B_2(t) - \frac{\pi a}{2}A(t)B_1(t)\right], \tag{42}$$

$$\frac{dE_1(t)}{dt} = -\left[aA(t) + \frac{\delta^2}{Le}E_1(t) + \pi a A(t)E_2(t)\right], \tag{43}$$

$$\frac{dE_2(t)}{dt} = -\left[\frac{4\pi^2}{Le}E_2(t) - \frac{\pi a}{2}A(t)E_1(t)\right]. \tag{44}$$

The above system of autonomous nonlinear differential equation is not suitable for analytical study. Numerical method was used to solve the above set of nonlinear differential equations to find the amplitudes. After determining the amplitudes, they were computed to plot the graphs for heat transfer, mass transfer, streamlines, isotherms and isohalines in unsteady case.

A. Steady Finite Amplitude Convection

For steady state finite amplitude convection we have to set left hand side of the Eqs. (40)-(44) to zero. This will give set of equations in amplitudes of convection, as

$$\delta_1^2\left(1 + C\delta^2\right)A + aRa_T B_1 - aRa_S E_1 = 0, \tag{45}$$

$$\pi a A B_2 + \left(\delta_2^2 - R_i\right)B_1 - 2aFA = 0, \tag{46}$$

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$$\left(4\pi^2 - R_i\right)B_2 - \frac{\pi a}{2} AB_1 = 0, \tag{47}$$

$$aA + \frac{\delta^2}{Le} E_1 + \pi a A E_2 = 0, \tag{48}$$

$$\frac{4\pi^2}{Le} E_2 - \frac{\pi a}{2} AE_1 = 0. \tag{49}$$

From the nontrivial solution of the above Eqs. (45)-(49), a quadratic equation in $\left(\frac{A^2}{8}\right)$ is given by

$$a_1\left(\frac{A^2}{8}\right)^2 + b_1\left(\frac{A^2}{8}\right) + c_1 = 0 \tag{50}$$

where

$$a_1 = 4\pi^2 a^4 {\delta_1}^2 Le^2 (1 + C\delta^2),$$
$$b_1 = a^2 {\delta_1}^2 (1 + C\delta^2)\left(4\pi^2\delta^2 + Le^2(4\pi^2 - R_i)({\delta_2}^2 - R_i)\right)$$
$$+2a^4 Le\left((4\pi^2 - R_i)LeRa_T F + 2\pi^2 Ra_S\right),$$
$$c_1 = (4\pi^2 - R_i)\left[\delta^2 {\delta_1}^2 (1 + C\delta^2)({\delta_2}^2 - R_i) + a^2\left(2\delta^2 Ra_T F + Ra_S Le({\delta_2}^2 - R_i)\right)\right].$$

The required root of the above equation is

$$\left(\frac{A^2}{8}\right) = \frac{-b_1 + \sqrt{{b_1}^2 - 4a_1c_1}}{2a_1}. \tag{51}$$

For the expression of thermal Rayleigh number to characterize the onset of finite amplitude steady convection, the radical of above equation must vanish, this condition gives

$$Ra_T^F = \frac{-f + \sqrt{f^2 - 4ph}}{2p}, \tag{52}$$

where

$$p = 4a^4 Le^4\left(4\pi^2 - R_i\right)^2 F^2,$$
$$f = 4a^2 Le^2\left(4\pi^2 - R_i\right)F\left(4a^2 Le\pi^2 Ra_S - \left(1 + C\delta^2\right){\delta_1}^2\left(4\pi^2\delta^2 - Le^2\left(4\pi^2 - R_i\right)\left({\delta_2}^2 - R_i\right)\right)\right),$$
$$h = \left(4a^2 Le\pi^2 Ra_S + {\delta_1}^2\left(1 + C\delta^2\right)\left(4\pi^2\delta^2 - Le^2\left(4\pi^2 - R_i\right)\left({\delta_2}^2 - R_i\right)\right)\right)^2.$$

B. Heat And Mass Transports

The quantification of heat and mass transport is very important for the study of convection in porous media. This is because the onset of convection, as Rayleigh number is increased is more readily detected by its effect on the heat and mass transport. However, in the basic state, heat and mass transfer is given by conduction alone.

The Nusselt number and Sherwood number are defined by

$$Nu = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial T}{\partial z} dx}{\int_0^{\frac{2\pi}{a}} \frac{dT_b}{dz} dx}\right]_{z=0}, \tag{53}$$

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$$Sh = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial S}{\partial z} dx}{\int_0^{\frac{2\pi}{a}} \frac{dS_b}{dz} dx} \right]_{z=0} . \tag{54}$$

Substituting the value of T, T_b, S and S_b in Eqs. (53)-(54), there is

$$Nu = 1 - \frac{2\pi B_2}{\sqrt{R_i}\cot\left(\sqrt{R_i}\right)}, \tag{55}$$

$$Sh = 1 - 2\pi E_2. \tag{56}$$

Substituting the value of B_2 and E_2 in Eqs. (55)-(56), the expressions for Nu and Sh were obtained in terms of A

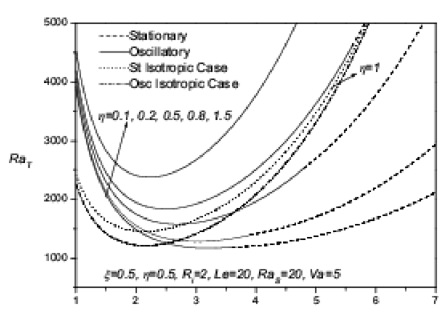
$$Nu = 1 - \frac{4F\left(\frac{A^2}{8}\right)}{\sqrt{R_i}\cot\left(\sqrt{R_i}\right)\left[\frac{\left(4\pi^2 - R_i\right)\left(\delta_2{}^2 - R_i\right)}{4a^2\pi^2} + \left(\frac{A^2}{8}\right)\right]}, \tag{57}$$

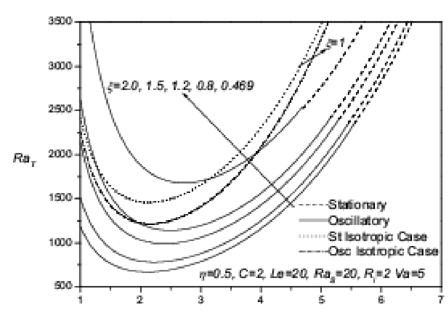
$$Sh = 1 + \frac{2\left(\frac{A^2}{8}\right)}{\left[\frac{\delta^2}{a^2 Le^2} + \left(\frac{A^2}{8}\right)\right]}. \tag{58}$$

V. RESULTS AND DISCUSSION

In this section, the results obtained from the graphs of various parameters on the onset of convection, finite amplitude convection, heat transfer and mass transfer are discussed.

From the expressions of thermal Rayleigh number of stationary and oscillatory state convection, the graphs of neutral stability curves for various parameters are shown in Figs. 1(a-g). The values of parameters were fixed at $\eta = 0.5$, $\xi = 0.5$, $C = 2$, $Ra_S = 20$, $Le = 20$, $Va = 5$, $R_i = 2$ except for the varying one. The critical thermal Rayleigh number was defined as at which onset of convection takes place. It can be found from Fig. 1(a) that the critical thermal Rayleigh number increases with the increment in thermal anisotropic parameter η, resulting in delay in the onset of convection. Similar effects were produced by couple stress parameter as in Fig. 1(c), and solute Rayleigh number in Fig. 1(d) on the onset of convection. However, from Fig. 1(b), it was found that with increasing value of mechanical anisotropic parameter ξ, the critical thermal Rayleigh number decreased, resulting in advancing of the onset of convection. Similar effects were found for Lewis number Le as in Fig. 1(e) and for internal heat source parameter R_i as in Fig. 1(f). The effect of Vadász number Va on oscillatory neutral curve is shown in Fig. 1(g), which reveals that the effect of Vadász number Va is to advance the onset of convection with increment in Va .





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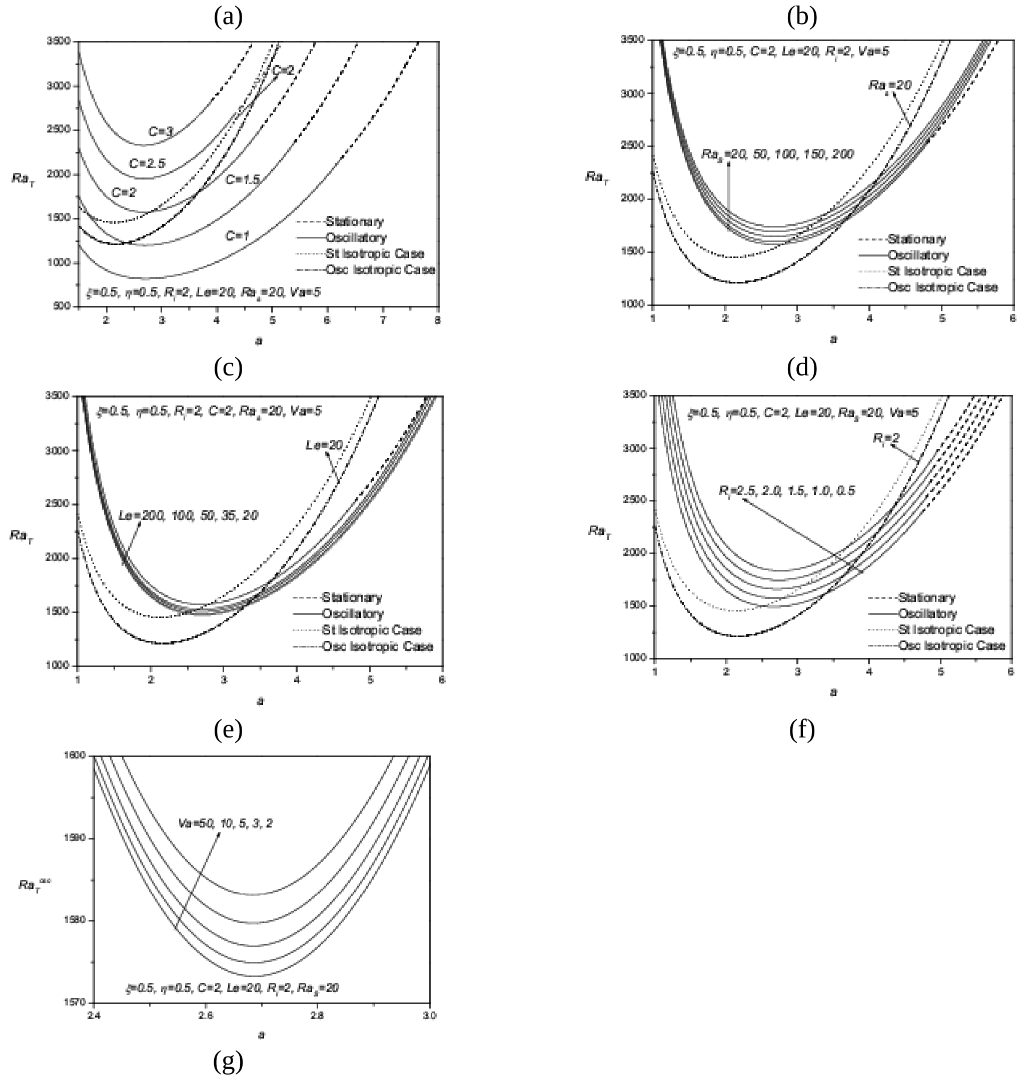

Fig. 1 Neutral stability curves for different values of (a) η , (b) ξ , (c) C , (d) Ra_S , (e) Le ,(f) R_i , (g) Va

Figs. 2(a-f) show the variation of critical thermal Rayleigh number Ra_{Tc} for both stationary and oscillatory convection with solute Rayleigh number Ra_S for different values of varying parameters. It is clear from Fig. 2(a), that for the fixed value of solute Rayleigh number Ra_S , the effect of the increase in thermal anisotropic parameter η was to increase the value of critical thermal Rayleigh number Ra_{Tc} for both stationary and oscillatory convection. Fig. 2(b) shows that the effect of mechanical anisotropic parameter ξ was to decrease the Ra_{Tc} with corresponding increment in ξ . Further, it was found that the effect of mechanical anisotropic parameter ξ was to advance the onset of convection as compared to isotropic case. The variation of couple stress parameter C is shown by Fig. 2(c), it is clear that when couple stress parameter increased, the critical thermal Rayleigh number Ra_{Tc} also increased for both stationary and oscillatory convection. From Fig. 2(d), an interesting effect of Lewis number Le was observed for stationary and oscillatory convection, to increase in Le the critical thermal Rayleigh number Ra_{Tc} increased for stationary convection, whereas it decreased for oscillatory convection, moreover the effect of Lewis number Le for oscillatory convection was found very frail. Fig. 2(e) depicts that the effect of internal heat parameter R_i was to decrease the value of critical thermal number Ra_{Tc} with an increment in the value of R_i for both stationary and oscillatory

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cases. Therefore, the effect of internal heat parameter R_i was to destabilize the system. In Fig. 2(f) the effect of Va is shown on oscillatory convection, critical thermal Rayleigh number decreased with increasing value of Va and hence advanced the onset of convection.

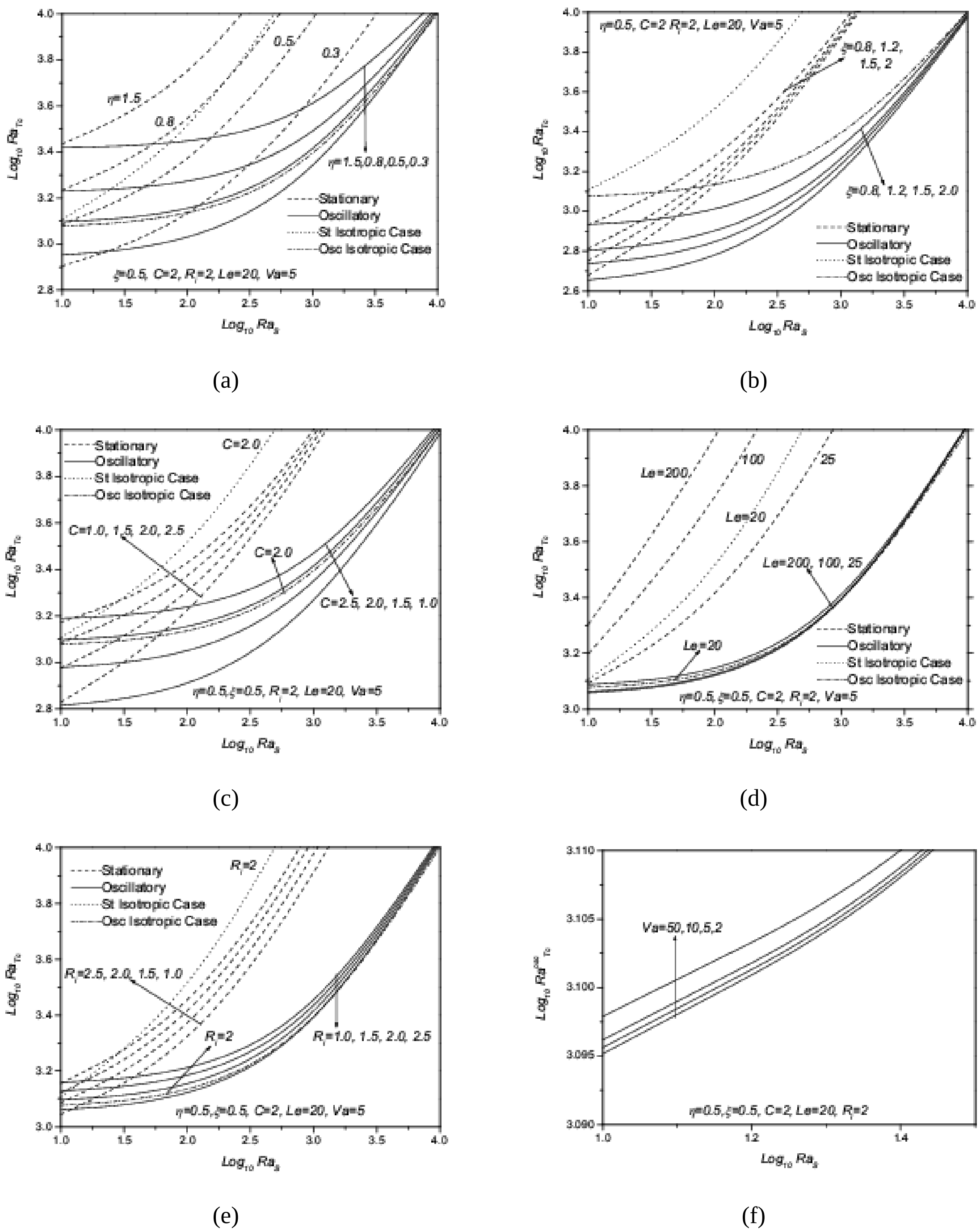

Fig. 2 Variations of $Ra_{T,c}$ with Ra_S for different values of (a) η, (b) ξ, (c) C, (d) Le, (e) R_i, (f) Va

Figs. 3(a-f) depict the variations of critical finite amplitude thermal Rayleigh number Ra_{Tc}^{F} with respect to solute Rayleigh number Ra_S. Figs. 3(a, c) show that increments in thermal anisotropic parameter η and couple stress parameter C

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increasedthe values of critical finite amplitude thermal Rayleigh number Ra_{Tc}^{F}, thus delaying the finite amplitude convection. However, from Figs. 3(b, e) it was found that on increasing the values of mechanical anisotropic parameter ξ and internal heat source parameter R_i, the values of critical finite amplitude thermal Rayleigh number Ra_{Tc}^{F} decreased, thus advancing the finite amplitude convection. Fig. 3(d) depicts the effect of Lewis number Le on the value of critical finite amplitude thermal Rayleigh number Ra_{Tc}^{F}, and shows that an increment in the value of Le increased the value of Ra_{Tc}^{F} at small values of Ra_S, however the trend was reversed at higher values of Ra_S.

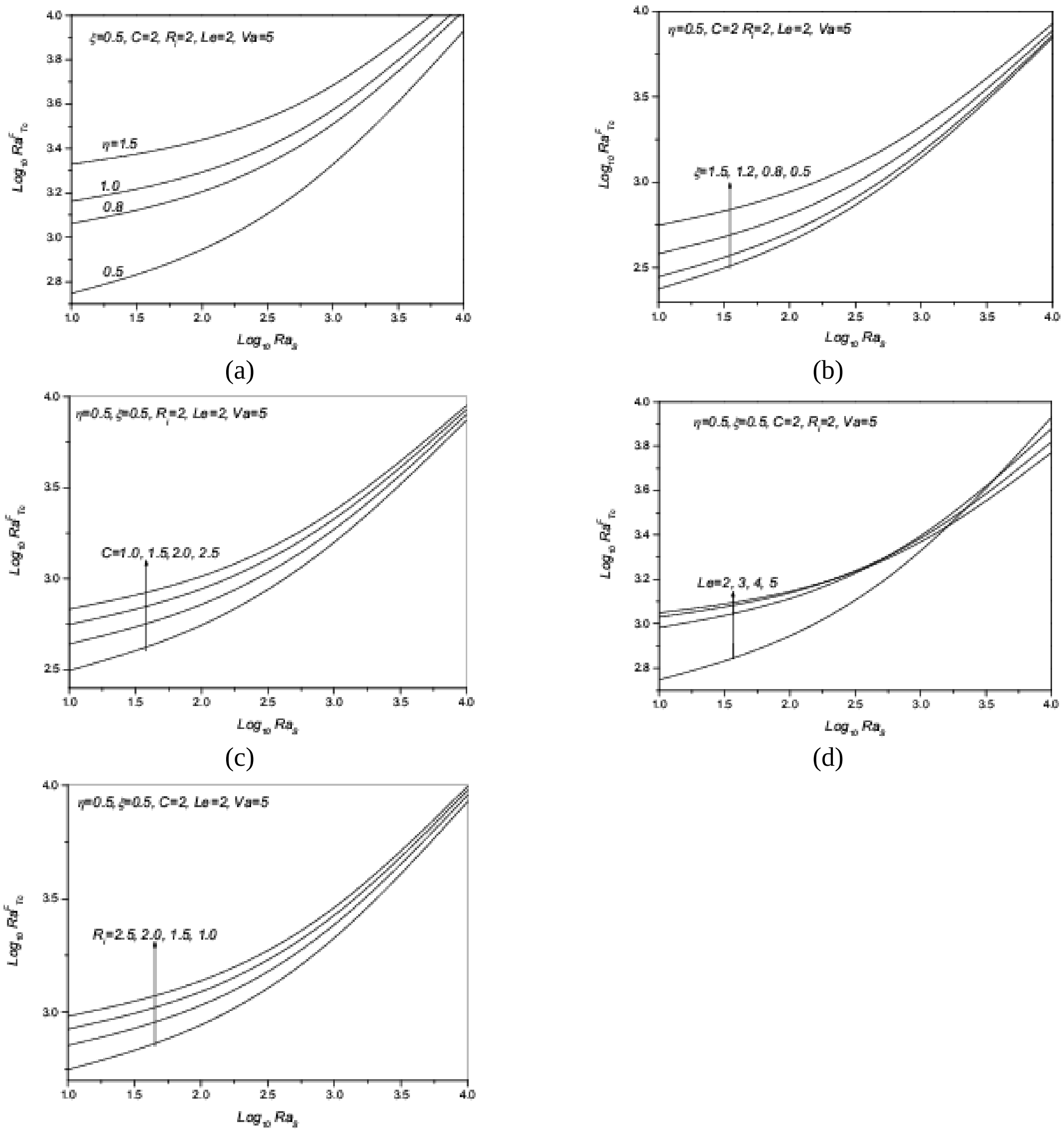

Fig. 3 Variations of $Ra_{T,c}^{F}$ with Ra_S for different values of (a) η, (b) ξ, (c) C, (d) Le, (e) R_i

In Figs. 4(a-f) and 5(a-f), the variation of Nusselt number Nu and Sherwood number Sh with respect to Ra_T/Ra_{Tc}^{st} for different values of parameters were depicted. The values of Nu and Sh started with 1, thus showing conduction state initially. As the value of Ra_T increased, the values of Nu and Sh increased thus increasing the heat and mass transfers across the

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porous layer. However, on further increasing Ra_T the values of Nu and Sh became constants and approached fixed values. In Figs. 4(a) and 5(a), it can be found that, on increasing the value of thermal anisotropic parameter η, the values of Nu and Sh increased, thus increasing the heat and mass transfers across the porous layer. Similar effects were also found for mechanical anisotropic parameter ξ as in Figs. 4(b) and 5(b), for Lewis number Le as in Figs. 4(d) and 5(d), and for solute Rayleigh number Ra_S as in Figs. 4(e) and 5(e). However as seen in Figs. 4(c) and 5(c), the effect of increasing couple stress parameter C was to decrease the values of Nu and Sh, thus reducing the heat and mass transfer. Also from Figs. 4(f) and 5(f) respectively, it was found that on increasing R_i, the value of Nu increased while that of Sh decreased, thus the effect of an increment in R_i was to increase the heat transfer but to decrease the mass transfer across the porous layer. The results corresponding to the isotropic case are also presented in the Figs. 4-5 and shown by dotted lines.

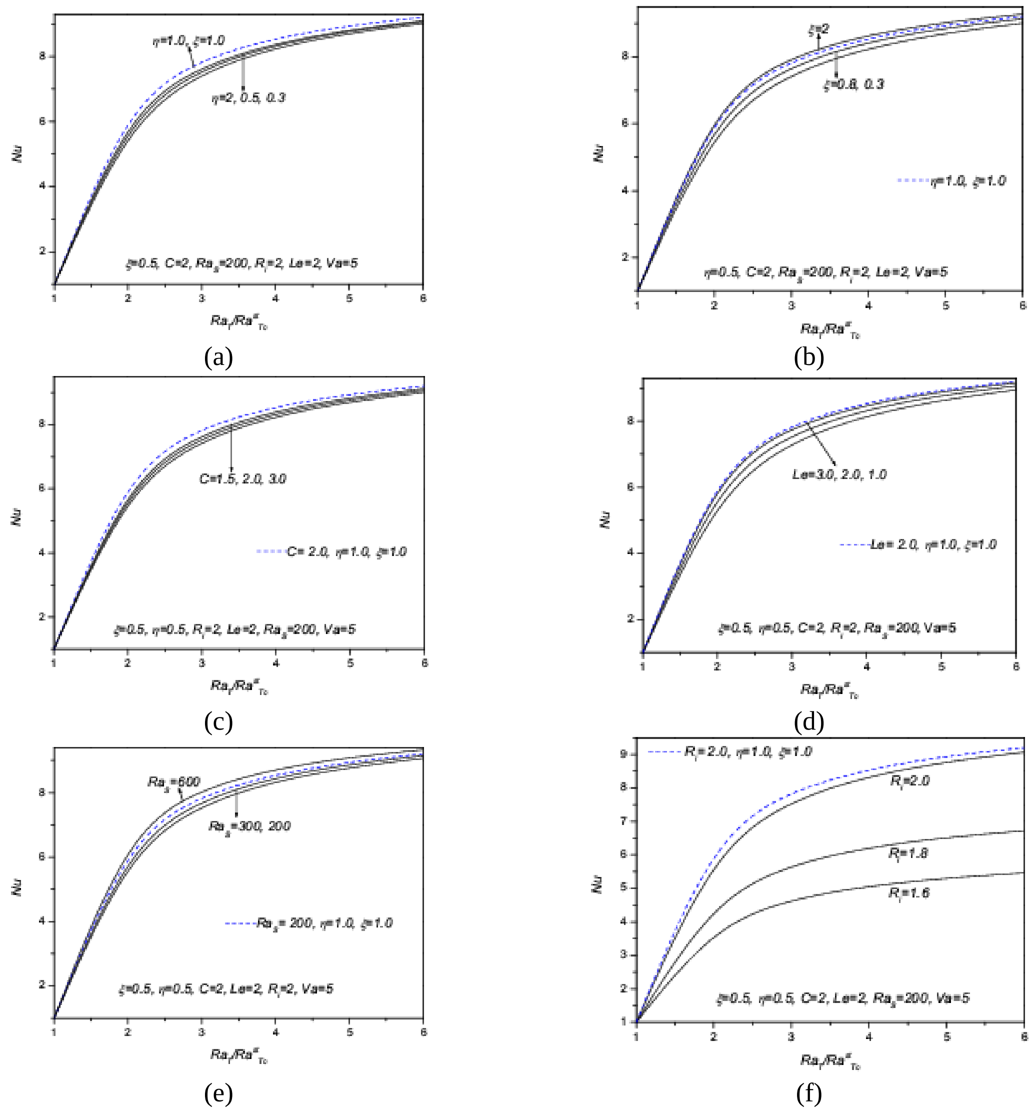

Fig. 4 Variations of Nusselt number with $Ra_T/Ra_{T,c}^{St}$ for different values of (a) η, (b) ξ, (c) C, (d) Le, (e) R_i, (f) Va

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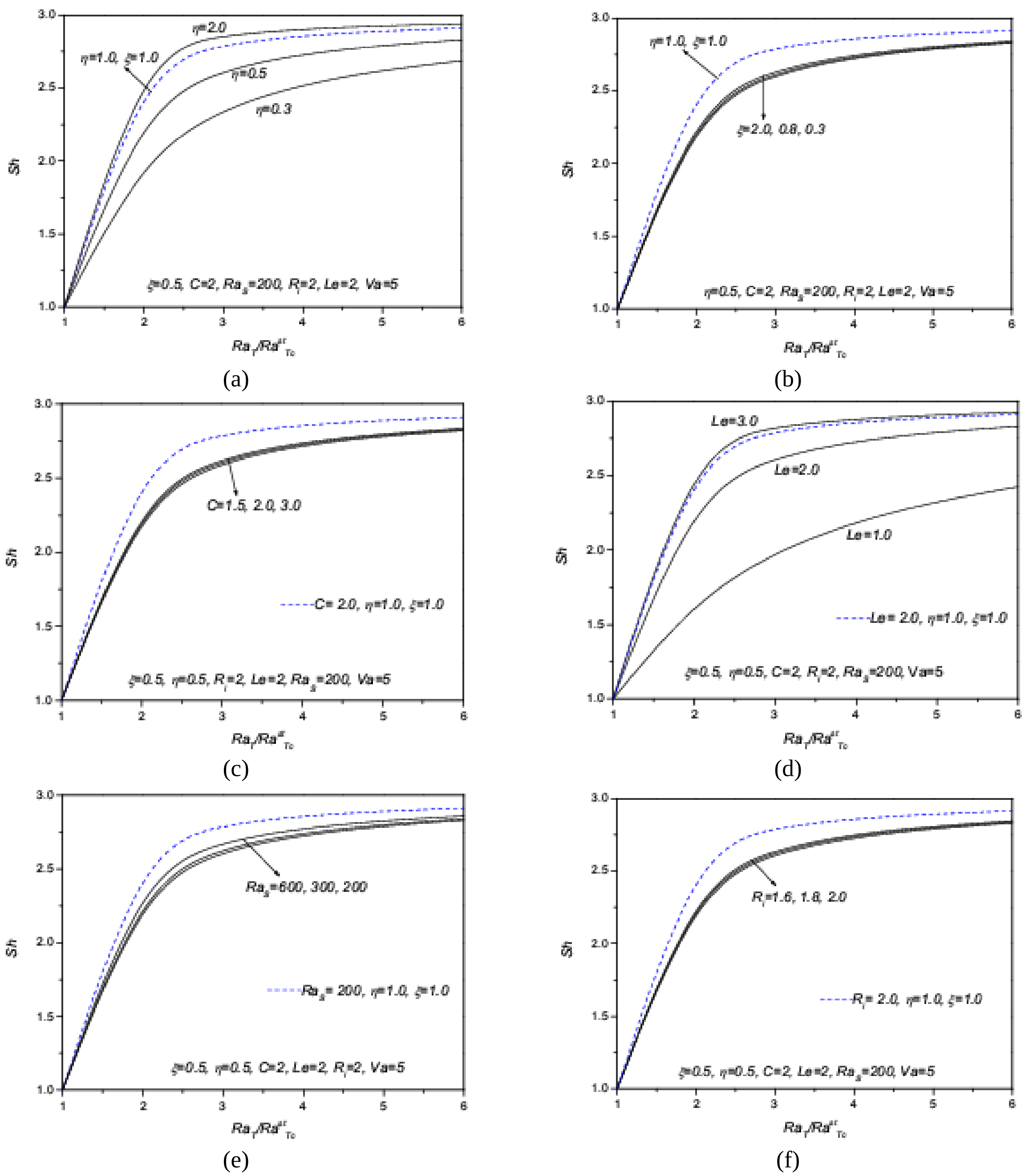

Fig. 5 Variations of Sherwood number with $Ra_T/Ra_{T,c}^{St}$ for different values of (a) η,(b) ξ, (c) C, (d) Le, (e) R_i, (f) Va

The variation of streamlines, isotherms, isohalines with critical thermal Rayleigh number Ra_{Tc}^{st} are shown in Figs. 6-8. The values of parameters were kept at fixed values $\eta = 0.5$, $\xi = 0.5$, $C = 2$, $Le = 20$, $Ra_S = 20$, $R_i = 2$, $Va = 5$. Figs. 6(a), 7(a) and 8(a), respectively correspond to streamlines, isotherms, and isohalines calculated at $Ra_T = Ra_{Tc}^{st}$ whereas Figs. 6(b), 7(b) and 8(b) correspond to streamlines, isotherms, and isohalines calculated at $Ra_T = 3 \times Ra_{Tc}^{st}$. From Figs. 6(a, b), it was observed that the magnitude of contours of streamlines increased rapidly with increasing value of critical thermal Rayleigh number Ra_{Tc}^{st}. From Figs. 7(a, b), it is clear that at $Ra_T = Ra_{Tc}^{st}$ heat transfer was almost by conduction only, whereas at

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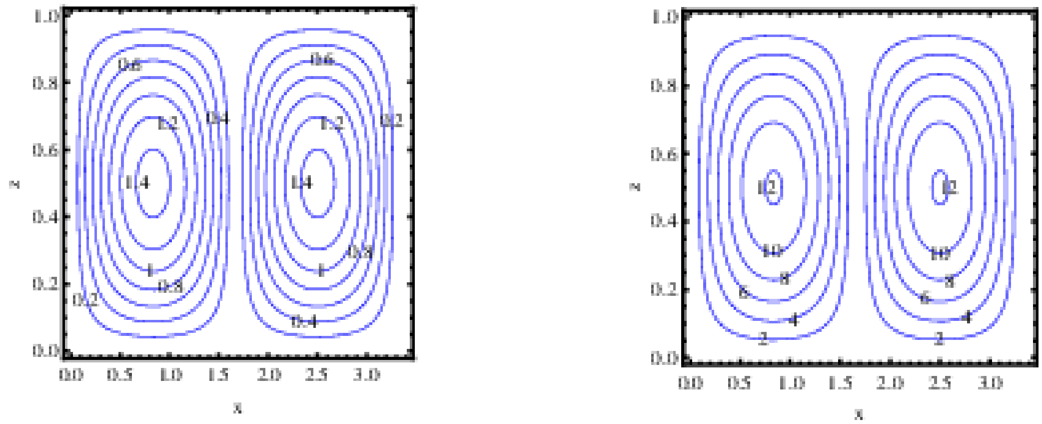
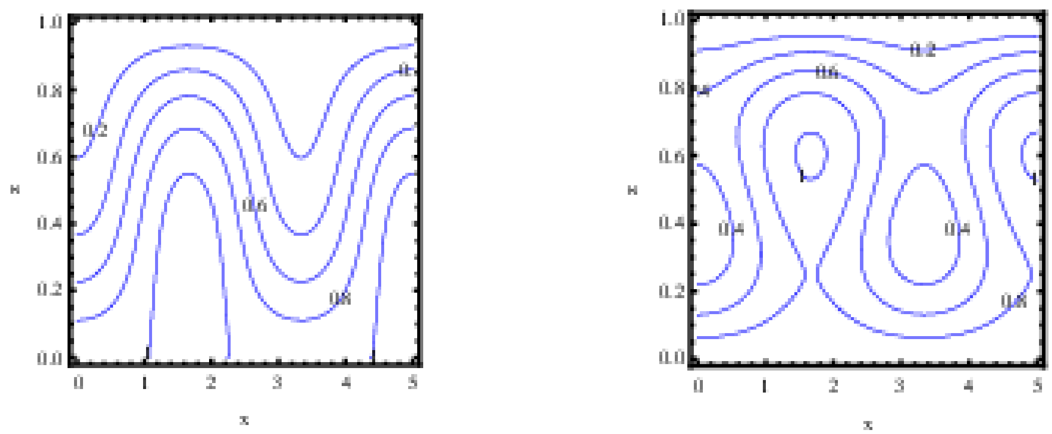
$Ra_T = 3\times Ra_{Tc}^{st}$ heat transfer came from the account of convection. From Figs. 8(a,b), it was found that at $Ra_T = Ra_{Tc}^{st}$ mass transfer was by conduction whereas at $Ra_T = 3\times Ra_{Tc}^{st}$ mass transfer was by convection.



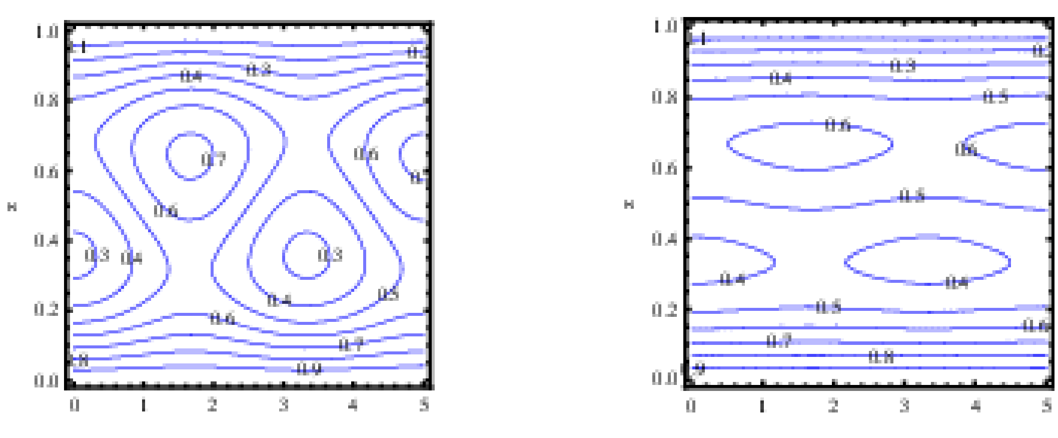
(a) (b)

Fig. 6 Variations of Stream lines with Ra_T (a) $Ra_T = Ra_{T,c}^{St}$, (b) $Ra_T = 3\times Ra_{T,c}^{St}$



(a) (b)

Fig. 7 Variations of Isotherms with Ra_T (a) $Ra_T = Ra_{T,c}^{St}$, (b) $Ra_T = 3\times Ra_{T,c}^{St}$



(a) (b)

Fig. 8 Variations of Isohalines with Ra_T (a) $Ra_T = Ra_{T,c}^{St}$, (b) $Ra_T = 3\times Ra_{T,c}^{St}$

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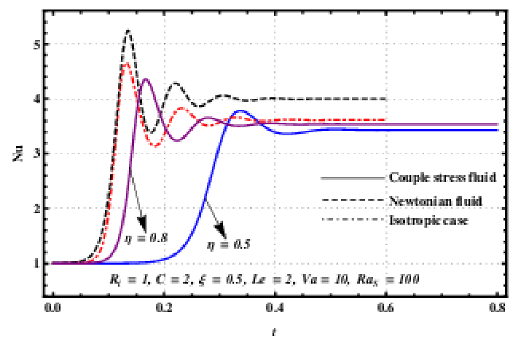
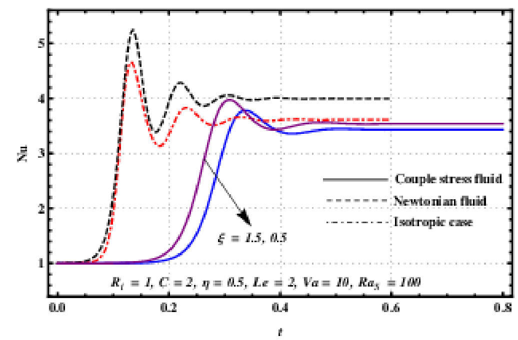
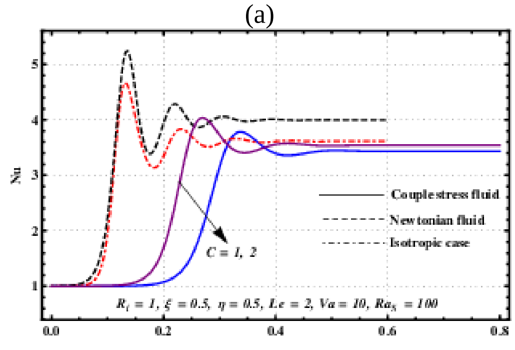
The effects of parameters on Nusselt number Nu and Sherwood number Sh for unsteady case are depicted in Figs. 9(a-g) and Figs. 10(a-g) respectively with qualitative comparison to Newtonian fluid. Runge-Kutta method was used to numerically solve the autonomous system of differential equations at fixed values of parameters $\eta = 0.5$, $\xi = 0.5$, $C = 2$, $Le = 2$, $Ra_S = 100$, $R_i = 1$, $Va = 10$ and $Ra_T = 5 \times Ra_{Tc}^F$. From these figures, it is clear that initially when time $t = 0$, the values of Nusselt and Sherwood numbers were 1, as the heat and mass transfers were taking place only due to conduction. However, as time passed, both Nu and Sh oscillated with time, and reached a steady state on further elapses of time. Figs. 9(a-g) show the behaviours of heat transfer with different values of parameters. From Fig. 9(a), it is clear that heat transfer increased with increasing value of thermal anisotropic parameter. Furthermore, it was noticed that heat transfer in the case of Newtonian fluid was more than that in the case of couple-stress fluid. Similar effects were found for mechanical anisotropic parameter, Lewis number, solute Rayleigh number and internal heat source parameter(see Figs. 9(b, d, e, f)). It was observed from Fig. 9(c) that heat transfer decreased with increasing value of couple stress parameter, and similar effect was also found for Vadász number(see Fig. 9(g)). Figs. 10(a-g) show the variations of mass transfer with respect to time for different values of parameters. Fig. 10(a) reveals that mass transfer increased with increasing value of thermal anisotropic parameter. Also, more mass transfer took place in the case of Newtonian as compared to the case of couple-stress fluid. Similar effect was also found for mechanical anisotropic parameter, Lewis number, and solute Rayleigh number (see Figs. 10(b, d, e)). From Fig. 10(c), it was concluded that with increasing value of couple stress parameter, mass transfer decreases, and similar effect was also found for internal heat source and Vadász(see Figs. 10(f, g)).



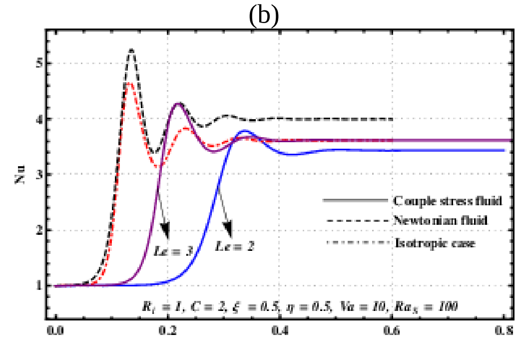
(a)



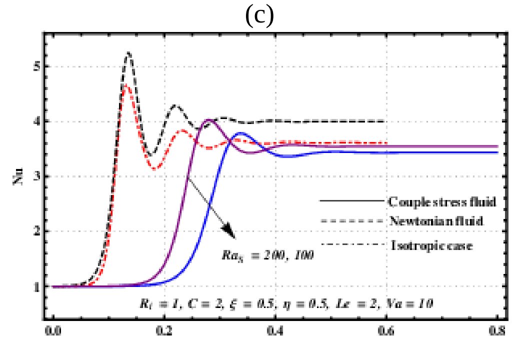
(b)



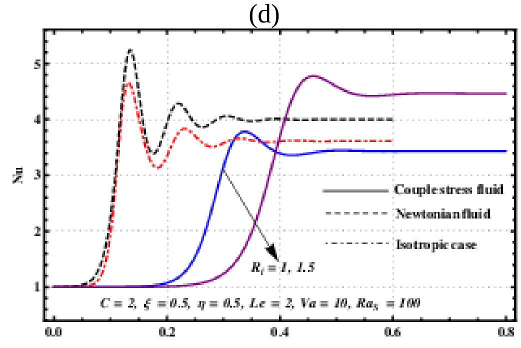
(c)



(d)



(e)



(f)

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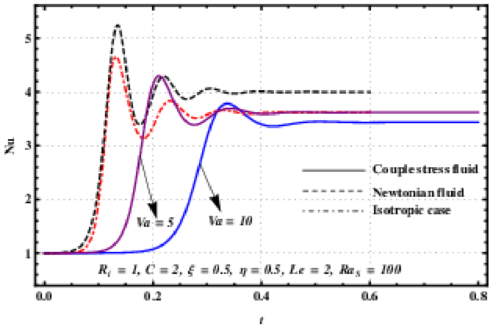
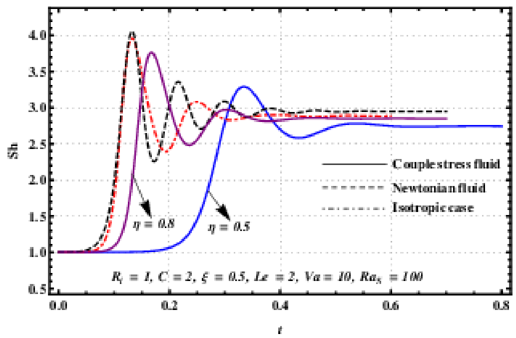
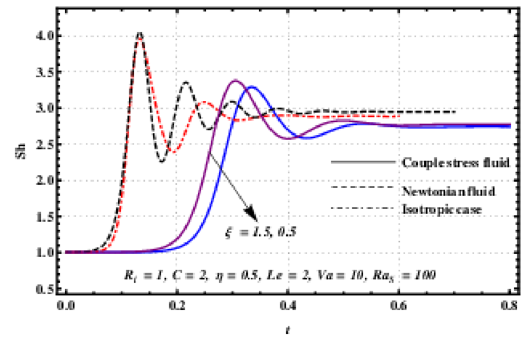


(g)

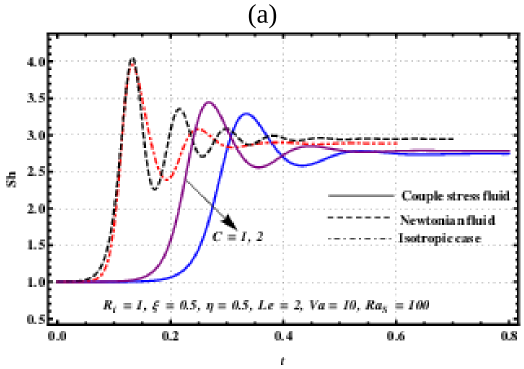
Fig. 9 Variations of Nusselt number with time for different values of (a) η, (b) ξ, (c) C, (d) Ra_S, (e) Le, (f) R_i, (g) Va



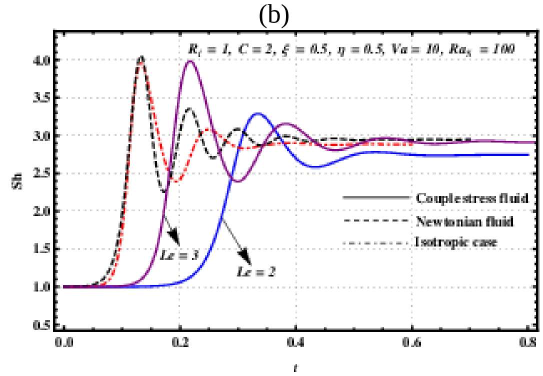
(a)



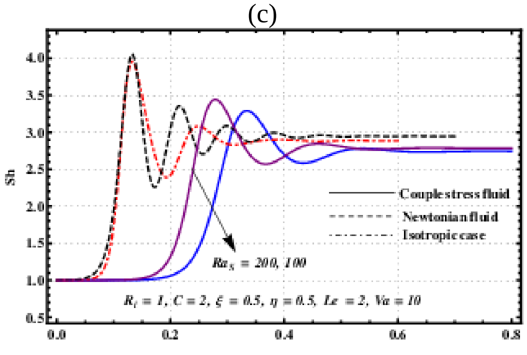
(b)



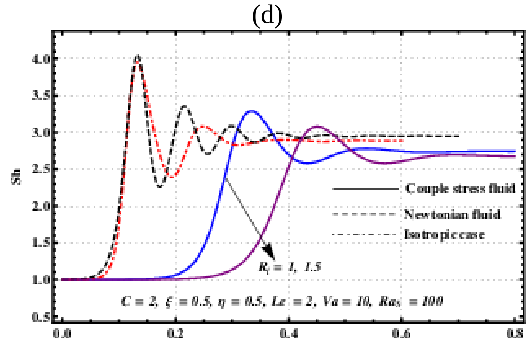
(c)



(d)



(e)



(f)

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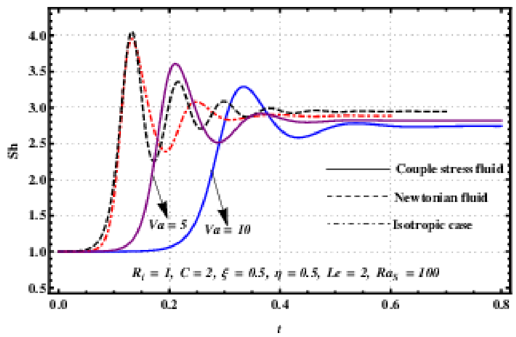
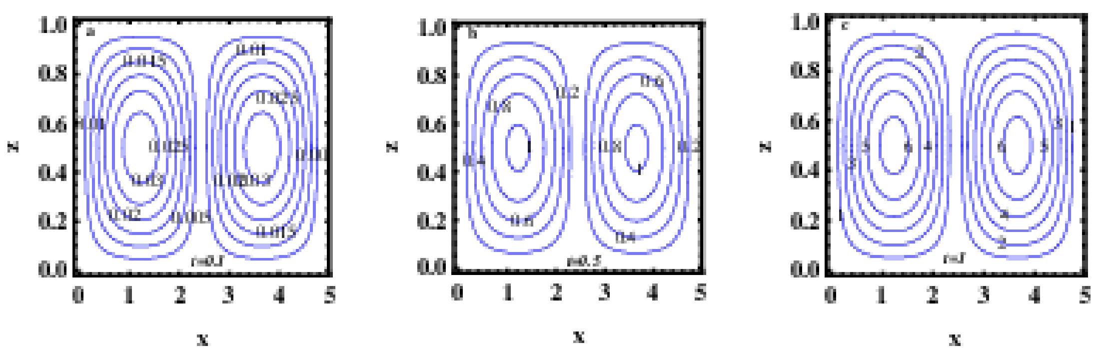


(g)

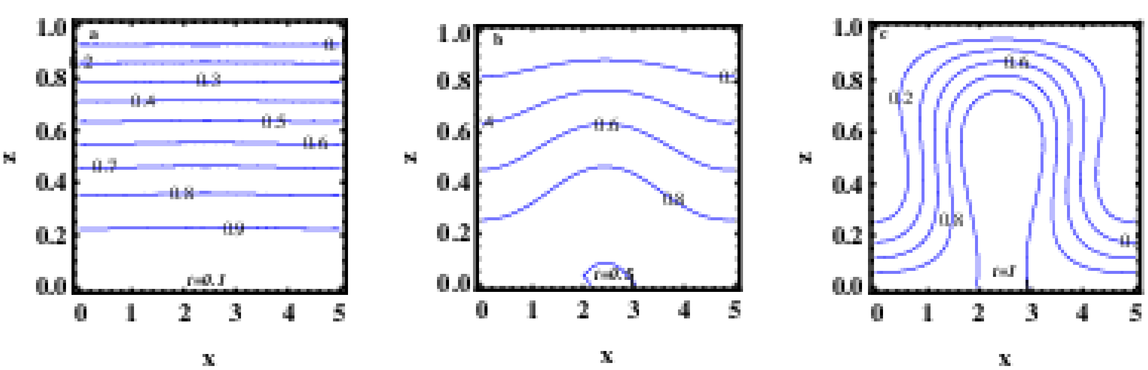
Fig. 10 Variations of Sherwood number with time for different values of (a) η, (b) ξ, (c) C, (d) Ra_S, (e) Le, (f) R_i, (g) Va

In Figs. 11-13, variations of streamlines, isotherms and isohalines respectively are shown with time at fixed values of $\eta = 0.5$, $\xi = 0.5$, $C = 2$, $Le = 20$, $Ra_S = 20$, $R_i = 2$, $Va = 5$ and $Ra_T = 5 \times Ra_{Tc}^F$. The streamlines, isotherms, and isohalines were plotted for the time $t = 0.1$, $t = 0.5$ and $t = 1$. It was found that the magnitude of streamlines increased with time. Further, the graphs show that initially heat and mass transfers were by conduction alone. However, as time increases, the graphs of isotherms and isohalines oscillated and formed contours showing that convection has taken place.



(a) (b) (c)

Fig. 11 Variations of Stream lines with time (a) t=0.1, (b) t=0.5, (c) t=1.0



(a) (b) (c)

Fig. 12 Variations of Isotherms lines with time (a) t=0.1, (b) t=0.5, (c) t=1.0

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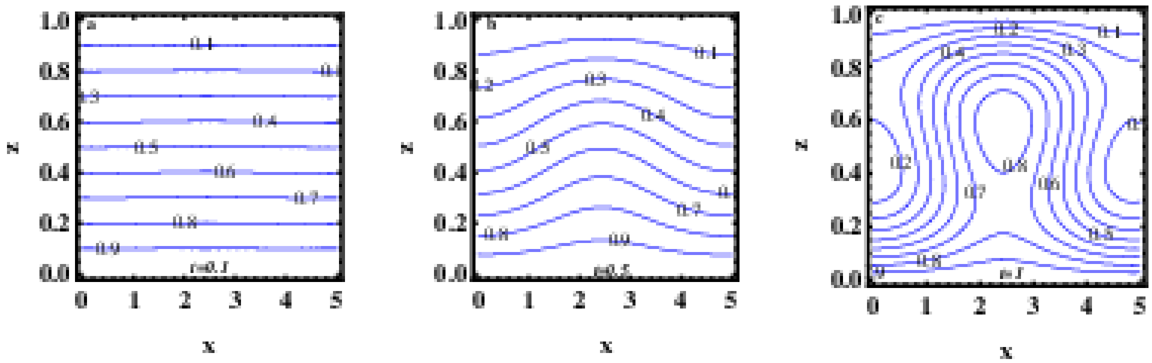

(a) (b) (c)

Fig. 13 Variations of Isohalines lines with time (a) t=0.1, (b) t=0.5, (c) t=1.0

VI. CONCLUSIONS

An analytical study of linear and nonlinear double-diffusive convection saturated by couple stress anisotropic porous media with internal heat source is presented in this paper. A horizontal anisotropic porous layer which is heated and salted from below was assumed. The effect of parameters on neutral stability curve, finite amplitude convection, heat and mass transfers, steamlines, isotherms and isohalines were discussed. Following conclusions were drawn:

(1) Stabilizing effect was found for thermal anisotropic parameter η, couple stress parameter C and solute Rayleigh number Ra_S on the onset of stationary, oscillatory and finite amplitude convection.

(2) Destabilizing effect was found for mechanical anisotropic parameter ξ, internal heat source parameter R_i on the onset of stationary, oscillatory and finite amplitude convection.

(3) The effect of Lewis number Le was to stabilize the stationary convection, and destabilize the oscillatory convection, however dual effect was found for finite amplitude convection.

(4) Vadász number advanced the oscillatory convection.

(5) Heat transfer increased with increasing values of thermal anisotropic parameter η, mechanical anisotropic parameter ξ, Lewis number Le, solute Rayleigh number Ra_S and internal heat source parameter R_i and decreased with increasing value of couple stress parameter C.

(6) Mass transfer increased with increasing values of thermal anisotropic parameter η, mechanical anisotropic parameter ξ, Lewis number Le and solute Rayleigh number Ra_S, and decreased with increasing values of couple stress parameter C and internal heat source parameter R_i.

(7) Magnitude of streamlines increased while isotherms and isohalines took more contour forms with increasing value of Rayleigh number. Same behaviour was also observed with respect to time.

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