

The Boltzmann Entropy Decreasing for Angle Momentum in Formation of Concentrative Vortex

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Abstract- Based on the concept of Angular Momentum Geometric Space Configuration, this article discusses the sudden formation of the storms such as Typhoon, Tornado and Sandstorm, as well as their tremendous energy. The Boltzmann entropy for angular momentum of “gas of vortices” is calculated based on the ideal gas model in continuum mechanics using statistical method of non-localized particles in local region. It is shown that the formation of concentrative vortex is the spontaneous process of Boltzmann entropy decrease. The generalized thermal potential energy $T_{GA}S_{B,A}$ corresponding to the angular momentum geometric space configuration is used to reveal the excess energy in vortex formation. It is the result of the Boltzmann entropy decreasing for process with $\Delta T_{GA}S_{B,A}=0$.

Keywords- Concentrative Vortex; Gas Of Vortices; Boltzmann Entropy; Angular Momentum Geometric Space Configuration (AMGSC); Thermal Potential Energy; Entropy Elastic Force

I. THE ANGULAR MOMENTUM GEOMETRIC SPACE CONFIGURATION

Tornado, Typhoon and Sandstorm are all sudden-formation storms. Although different in causes, they all require stable meteorological condition. After its initiation, the storm contains tremendous and concentrated energy, and has completely different momentum and angular momentum distribution compared with the surrounding atmosphere. The causes of these phenomena are related with the angle momentum geometric space configuration (AMGSC) and its development process. Onsager defines 2-D vortex field as “gas of vortices”, which can be studied by statistical method as Coulomb interaction of vortices. However, it is still a concept based on position geometric space configuration (PGSC) [1]. The “gas of vortices” is the base of computer simulation method on large vortex. He also defines the decrease of phase space and increase of the energy as a negative temperature system to explain the origin of tremendous energy during the formation of the storm. Although there are still lots of arguments about whether the “gas of vortices” is a negative temperature system, only Montgomery and Matthaeus shows that the formation of Osceen vortex is the outcome of maximized entropy after long developing process [2-5]. The Clausius entropy and the Boltzmann entropy are different, depending on the position of the observer (in or out of the system). When staying in the system, decrease of phase space means decrease of vortices elements, and Boltzmann entropy should decrease. The Clausius entropy is calculated by heat transfer from system to environment. Therefore, the observer of the heat should be outside of the system.

The Clausius statement of the second law of thermodynamics is the criterion of spontaneous process direction. When we review the heat transformation process from high temperature object to low temperature one using the MGSC based Boltzmann entropy, we can conclude that decrease of Boltzmann entropy is a spontaneous process. This is equal to the conclusion that the Gibbs free energy decrease in an isolated system is the spontaneous process. (Appendix B).

The problem of “Maxwell demon” does not violate the second law of thermodynamics [6, 7]. The key point is whether MGSC (including AMGSC) exists, whether the particles in a statistic system are arranged following some rules in geometric space based on their momentum magnitude. The indistinguishability in a statistic system implies that the same particles with the same momentum can exchange to each other without generating new microscopic statistic state. Using no-wall space model of ideal gas in pressure field, it can be proven that under thermodynamic equilibrium condition, the molecular

momentum distributes by magnitude along radial direction. Total microscopic state number can be calculated by statistic method of no-localized particles in local region [8, 9]. The Boltzmann entropy is $S(N, \mathcal{V}, p(j), T(i), c_k) = k_B \sum_j n_i(\mathcal{V}, c_k)$.

The distribution of particle momentum magnitude arranged along radial direction has been proven in “hot cloud” of ideal alkali metal atom gas in Bose-Einstein condensation experiment [10, 11]. The $TS_{B(\mathcal{V})}$ of ideal gas in no-wall space model has been shown as a kind of thermal potential energy [9]. Since $S_{B(\mathcal{V})}$ is calculated for particles with same momentum in the certain geometric space, it should be a function of the geometric space. In analytical mechanics, all geometric space configurations should have corresponding potential energy. Any change of MGSC implies some heat effects. This potential energy should be a thermal potential energy. The angle momentum distribution in a concentrative vortex should also have corresponding geometric space configuration. The angle momentum geometric space configuration also has potential energy on the level of continuum mechanics. The huge energy in the storm is caused by the change of potential energy of AMGSC.

II. THE AMGSC OF “GAS OF VORTICES”

According to the vortex dynamics equations, the vorticity field is a function of space and time under the action of viscosity, non-inertial effect and oblique pressure. Therefore, the field of vortices is an AMGSC of macroscopic fluid element which isn't distributed completely randomly in geometric space. An actual vorticity field is formed by various vortices, so it is called “gas of vortices”. Moreover, the number of elements of “gas of vortices” is huge in storm scale and the degree of degeneracy is much bigger than the element number with same vortices level due to different directions for the same vorticity value. Its Boltzmann entropy should be calculated by statistics of non-localized particles in local region on analogous to ideal gas [8-9]. General Boltzmann entropy of the “gas of vortices” about angle momentum for stable vortices field on a similar plan of ideal gas can be obtained as the following formula:

$$S_{B,A}(\mathcal{V}, N, E(i), f(\bar{r}_j, t)) = \frac{k_B}{b} \sum_j n_i = \frac{k_B}{b} N(\mathcal{V}, j, i) \quad (1)$$

where b is the average number of molecules in macroscopic fluid element which defines the vorticity of a fluid element and the k_B is the Boltzmann constant for gas molecular; f is the forces on the vorticity gas elements about “gas of vortices”, which is the function of space ordinate r ; j is the position with the same interaction force; E is the total energy of “vortices of gas” in volume $\mathcal{V}$ of the concentrative vortex that contains N elements. (Refer to Appendix A).

The general temperature of the concentrative vortex can be defined as the average volume over all macroscopic kinetic energy in vortex

$$T_{G,A} = \frac{1}{\mathcal{V}} \iiint_{\mathcal{V}} \mathbf{V}_i \cdot (\mathbf{r} \times \boldsymbol{\omega}_i) d\mathcal{V}_j \quad (2)$$

According to the definition of potential energy in analytical mechanics, the $T_{G,A}S_B$ should be a potential energy. Since the Boltzmann entropy for angle momentum in “gas of vortices” is estimated by the geometric space zone with the same magnitude angle momentum, S_B is a function of space ordinates. By the following formula, the entropy force for average general temperature $T_{G,A}$ should not all be zero for all ordinates

$$F_{S,i} = -T_{G,A} \frac{\partial S_{B,A}}{\partial x_i} \quad (3)$$

It should be noted that the above formula defines a kind of entropy elastic force with negative sign. It is an action that leads

to a system with the minimum value of the free energy, i.e., a system in a steady equilibrium state.

The Gibbs free energy is defined for an isolated system, then $T_{G,A}S_{B,A}$ in the above formula (3) is always tenable and is not always zero. When the concentration vortex is considered as an adiabatic system with the same vorticity, there still exists difference of Boltzmann entropy between the system and surroundings (no vortices zone). Therefore, for the above AMGSC of vortices elements, the $T_{G,A}S_{B,A}$ should be a kind of potential energy, and can be written as

$$T_{G,A}S_{B,A} = \frac{k_B N}{b} \frac{1}{V} \iiint_V \mathbf{V}_i \cdot (\mathbf{r} \times \boldsymbol{\omega}_i) dV \quad (4)$$

It is not the potential energy of PGSC that results from interaction between all vortices over the total system discussed in many literatures, but it is the potential energy of macroscopic AMGSC due to restriction of corresponding entropy elasticity action [2~5]. It is an action that ought to induce annexation of small vortex to produce large concentration vortex. Since there is no corresponding term of entropy elastic force in vortex dynamic equations, therefore, only cascade energy dissipation process from large vortex to small eddy can be considered. The anti-cascade process from small vortices to large vortex could not be explained at all even though this phenomena is very popular in nature.

III. THE SPONTANEITY OF CONCENTRATIVE VORTEX FORMATION

The Clausius statement of the second law of thermodynamics is a direction criterion of spontaneous process. The decrease of the Boltzmann entropy in the above spontaneous process is contradictory to the Clausius entropy increase due to the choice of Q and TS_B , which depends on how to choose the thermodynamic system (See Appendix B). In classical analytic solutions of concentrative vortex, all vortices elements can only contain one or two common vorticity values Ω . It implies that the number of initial angle momentum state for N vortices elements decreases to one or two. For example, for Osceen vortex, it is equal to one. The random initial angular momentum for all directions of all N elements become one Ω of one direction. The Boltzmann S_B change of “gas of vortices” for the change of AMGSC will be.

$$\Delta S_B = k_B (N-1)/b < 0 \quad (5)$$

The formation process of all concentrative vortexes is spontaneous. The “gas of vortices” develops into the Osceen vortex by use of Lagrange multipliers method to seek the maximum of Clausius entropy under the condition of the infinite time owing to the calculation of heat for a reversible process. Then it is an equivalence problem of minimizing Boltzmann entropy instantly. Lagrange problem of the condition limit-value does not specify the limit of maximum or minimum value. In addition, the Boltzmann entropy within a system is a state function and all energy is time symmetrical, there is no time limit for physical process as long as the initial and terminal states are the same. Therefore, the instant developing will not influence the universality of minimum problem of Boltzmann entropy. In fact, the storm formation is an instant event which is difficult to forecast these storms accurately.

IV. SOURCE OF STORM ENERGY

The formation process of concentrative vortex is sudden paroxysm, and only a little heat exchange by heat-conduction takes place. In addition, the temperature gradient direction inside the vortex is perpendicular to the streamline, which suggests that there is no convection heat transfer. Therefore, this can be considered as an adiabatic process. In the vortex dynamics equation, there is no relation correlating with adiabatic process of “gas of vortices”. There is no corresponding term of entropy elastic force in the vortices dynamics. So, an addition equation is needed.

$$\Delta(T_{G,A} S_{B,A}) = 0 \quad (6)$$

Based on the above equation, we can get excess kinetic energy

$$T_{G,A}(\Omega) = NT_{G,A}(\omega_i)_0 \quad (7)$$

It is shown that the formation of a concentrative vortex produces the huge kinetic energy from the redistribution process of AMGSC in the atmosphere. It indicates that storms such as Typhoon, Tornado and Sandstorm with huge energy can start from peaceful weather. There is also contribution of potential energy of PGSC, which has been discussed previously in many papers. This result tallies well with the negative temperature system introduced by Onsager that has decreasing phase space and increasing energy. The thermo potential energy of AMGSC or MGSC is a real energy which is not well understood before.

V. CONCLUSION

1. The formation of concentrative vortex is an ordering process with a spontaneous decrease of generalized Boltzmann entropy for macroscopic AMGSC under the action of entropy elasticity.
2. The entropy elastic force is an action of process to minimize free energy and decrease Boltzmann entropy. For different problems, they could have different forms. The Boltzmann entropy change has no time limit, therefore, the paroxysm of concentrative vortex should be related with entropy elastic force.
3. The generalized potential energy change caused by the AMGSC change is the real source of excess kinetic energy in the formation process of a concentration vortex.

Appendix .A The Statistics Of System Of Non-Localized Particles In Local Region

The ideal gas in the statistical thermodynamics can be considered as system of indistinguishable non-localized particles. The total microscopic states of this system that has conservation conditions of total energy and total particles number by the quantum statistic can be written as

$$W = \sum_{N,E} \prod_i \frac{g_i^{n_i}}{n_i!} \quad (A1)$$

Where n_i is the particle numbers on energy level ϵ_i and g_i is the degeneracy about ϵ_i . Due to the indetermination of energy level number and partition number of the particle on every energy levels, the partition function must be used which is constructed through the macroscopic thermodynamic state quantity under the most probable distribution condition.

The indistinguishableness of iso-particles implies that any exchange among particles will not produce new microscopic state. The momentum of ideal gas particle in no-wall space model arranges along the radial direction, therefore the indistinguishable can be only kept for particle exchanging among particles with the same momentum and energy level that must lie to the sphere shell from r to $r+dr$. Therefore the indistinguishableness of non-localized particles is real only in local region. It is a statistic problem of a non-localized particle in local region.

The force equilibrium determinates the distribution of particle number density along the radial direction in no-wall space, and then regulates the combination of quantum energy levels that corresponds to a certain pressure. The total energy determinates a certain temperature that implies uniqueness partition number of particles on every energy level for total particle

number N . Therefore the quantum statistics need no partition function under the condition of thermo and force equilibrium.

The microscopic states number is $g_i^{n_i}/n_i!$ for n_i particles with g_i degree of degeneracy in local region with ϵ_i energy. Therefore total microscopic state number for N particles over all energy levels in certain volume V is

$$W = \prod_{i(N,E,p(j),T)} \frac{g_i^{n_i}}{n_i!} = \prod_{i(N,E,p(j),T)} e^{n_i} \quad (A2)$$

Above formula uses Stirling approximation. The Boltzmann entropy of ideal gas can be gotten as

$$S_B(E, N, p(j), c_k) = k_B \ln W = k_B \sum_{j(p)} n_i(E, p, c_k) \quad (A3)$$

The Boltzmann entropy of ideal gas for certain particle number N is a function about pressure, temperature and component that is not a constant. It is universal for ideal gas in box with fix wall with uniformity of MGSC. The n_i particles for every energy level that equably distribute in whole box will overlap together. The uniformity configuration in box is also non-localized particle system in local region because any two particles exchange on the same energy level will not produce new microscopic state but particle exchange on different energy levels will make total microscopic state numbers change.

The Boltzmann entropy is determined by the microscopic state number under the most probable distribution as observed inside system. The Clausius entropy is calculated through the heat observed outside system. Therefore the Boltzmann entropy should be the entropy in free energy. The relation between the heat and TS_B in free energy is similar to the one between the work and potential energy of PGSC that have same value but different physical meanings.

Appendix .B *The Boltzmann entropy decreasing criterion of spontaneous process about thermodynamic second law*

Let two containers have the same volume V and hold the same molecules numbers N with different temperatures $T_A > T_B$. Heat transfers across one contacted wall surface but all other surfaces keep adiabatic. Ultimately, two containers come to thermo equilibrium and have the same pressure. The container A discharges heat ($Q_A < 0$) and container B absorbs heat ($Q_B > 0$). Therefore the Clausius entropy can be calculated by the heat of reversal isochoric process as a spontaneous irreversible process

$$\Delta S_C = \int \frac{dQ_A}{T_A} + \int \frac{dQ_B}{T_B} > 0 \quad (B1)$$

The Boltzmann entropy change in this process needs formula (A3) to calculate. The initial state of Boltzmann entropy of two containers will be

$$\begin{aligned} S_0 &= S_A + S_B = k_B (\ln W_A + \ln W_B) \\ &= k_B \ln \prod e^{n_{i,A}} \prod e^{n_{i,B}} = k_B (\sum n_{i,A}(\Psi, N, p_1, T_{A1}) + \sum n_{i,B}(\Psi, N, p_2, T_{B1})) \end{aligned} \quad (B2)$$

When two containers get to thermo equilibrium with temperature T and pressure p , the $2n_i$ particles for every energy level ϵ_i will be equably distributed in whole $2V$ container with $2N$ particles, and then overlap together. Any V volume in the system should have N particles.

$$S_B(2N, 2V, p, T) = S_B(N, V, p, T) \quad (B3)$$

Above result can be only gotten from the view point of MGSC. The Boltzmann entropy change in this spontaneous isochoric process should be

$$\Delta S_B = S_B(N, V, p, T) - S_0 < 0 \quad (B4)$$

Using a common expression, the two macroscopic states change to one macroscopic state.

It is true that the Clausius entropy increase is equivalence to Boltzmann entropy decrease in a spontaneous process because they are two kinds of physical quantity with different concepts. It can be induced by a simple syllogistic method

$$\Delta S_C = \int \frac{dQ}{T} = \int \frac{TdS_C}{T} = \int \frac{-TdS_B}{T} = -\Delta S_B \quad (B5)$$

The Boltzmann entropy decreasing is a spontaneous process criterion and the Clausius entropy increasing is an irreversible process criterion.

ACKNOWLEDGEMENT

The authors would like to thank the National Natural Science Foundation of China under Grant No. 50276036 and 50476079 for support of this research

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