Apparent Shear Stress in Symmetric-Straight Compound-Channel Flow

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***Abstract*—Momentum transfer results from apparent shear stress at the vertical intersection plane of compound channel section can be formed in roughness ratio, shape factors of width ratio and depth ratio between main channel and flood plain. It plays a quite important role for calculating flow rate and secondary currents to control both of the sediment and the contaminate transports. The analytical solutions on both of the apparent shear stress and the momentum transfer for symmetric-straight compound channel flows both in main channel and flood plain are derived and compared with the results of experimental and numerical outputs. Significant effects due to the ratios on roughness, width and depth are discussed.**

***Keywords*—Compound channel; Momentum transfer; Apparent shear stress; Secondary currents; Flow rate; Sediment transport; Contaminate transport;**

I. Introduction

In the natural flow channel, water level varies during flood period, the complicated behavior occurs at the interaction of the main-channel and flood-plain streams because of the difference in hydraulic resistances between the main-channel and flood-plain. This conditions considerable and significant velocity gradients near the channel edges, forms of various sizes of eddies in order to transverse momentum transfer and water masses exchange and increase energy losses, reduction of stream velocity in the main-channel and of its discharge capacity.

Normally river channels, berm, also flood-plane, always owns much larger hydraulic boundary roughness than the one of main channel, and the difference combining with different depth and width ratio leads to a slower movement of water in the berm section to retard the faster one, thus, creating a turbulent shear layer and extending laterally at the vertical intersection of the main-channel and the berm-section and to result in the unbalanced momentum transfer.

It is a very common case in the natural rivers in general practice by dividing the compound flow section into a number of supposedly homogeneous sub-areas with shear-free assumption in vertical boundaries at the edge of main channel to compute the flow rate for each subsection by generally using Manning's equation, and sum up to obtain the total quantity of discharge.

The flow characteristics of the compound channel could be expressed by four dimensionless parameters namely, D_r, B_r, n_r, and $H_r = \frac{(B/2)}{(D-d)}$, where the first two are used to show the geometrical properties of the whole channel; the third one for channel physical property and the last one is for the flow condition of main channel a two or three dimensional flow. Velocity profiles in centerline and the lateral direction will be derived, then the turbulent viscosity, for apparent shear stress and momentum transfer calculation.

II. ANALYTICAL RESULTS

The equations proposed in Luo〔1〕.are :

A. Primary velocity profile at the centerline U_{CL} (z)

$$\frac{U_{CL}}{U_{CLM}} = Exp\left[-0.71\left(\frac{z}{D}-0.65\right)^2\right] \qquad (1)$$

where U_{CLM} is the maximum velocity in the central line of compound channel.

B. Velocity profiles of primary flow to their corresponding horizontal-plane position U(y.z)

$$\frac{U}{U_{CL}} = 1.177(D_r)^{0.2004}(n_r)^{-0.2300}\left(\frac{y}{b/2}\right)^{0.2345}\left(\frac{z}{D}\right)^{-0.3556} \qquad (2)$$

C. Turbulent viscosity of compound section channels Momentum balance equation for uniform flow can be written as,

$$\rho g h s_0 + \frac{d}{dy}(h\tau_{xy}) - \tau_b = 0 \qquad (3)$$

Where τ_b=bottom shear stress $=\frac{1}{2}\rho c_f \overline{U}^2$,τ_{xy}= depth averaged apparent shear=ν_T (∂ U/∂ y) ; ν_T =eddy viscosity; and C_f = the resistance coefficient = $2gn^2h^{-1/3}$. Generally, the ν_T can be computed by depth averaged turbulent viscosity with paying attention that this model is rather depth-averaged turbulent shear stress than apparent one due to the secondary currents. The version of k-ε model with standard set of constants in current use has to be modified before application to computed channel flow. The dimensionless eddy viscosity, DEV, model will be applied to overcome the above weakness. The model is quite simple but the dimensionless eddy viscosity coefficient, **Cv**, is quite channel width dependent. By using the suggested DEV model form Fischer〔2〕,

$$v_T = C_v hu \qquad (4)$$

and regressing the data sets from Qgink〔3〕, we have:

$$C_v = 0.2(b/B) - 0.15 \quad \text{for } b/B \le 2.0$$

$$= 0.25; \quad \text{for } b/B \ge 2.0 \qquad (5)$$

Eqs.(4) and (5) are used to compare the one of Cv with Hanxiang〔4〕 with analytical solution for v_{xy} at steady uniform flow,

$$v_t(y) = \frac{C_h gH}{C_c}\overline{U}(y) \qquad (6)$$

in which $C_H = 4.8$; $M = 6 + 0.6C_c$; Combining Eqs. (4), (5) and (6), the new form of the eddy viscosity is,

$$v_t = \left(\frac{n^* h^* \sqrt{g}}{R^{*1/6}}\right)\left[0.2\left(\frac{b}{B}\right) - 0.15\right] \cdot U(y \cdot z_c) \qquad (7)$$

The Eq. (7) can be used in each subsection with the corresponding n*, h* and R* which takes into account the intersection surface of the virtual vertical division. In 2-DH model, U(y , z_c) and U(y , z) and U(y^*_a , z) for 2-DV model, and the variation of turbulent viscosity between main channel and flood-plain is rather obvious and that means the wet perimeter should not be neglected.

D. Apparent Shear stress

The apparent shear stress resulting from the lateral momentum transfer will form.

$$U(y,z) = 1.177(D_r)^{0.2004}(n_r)^{-0.2300}\left(\frac{Y}{b/2}\right)^{0.2545}\left(\frac{z}{D}\right)^{-0.3536} U_{CL}(y,z_c) \qquad (8)$$

And $U_{C.L.}(y,z_c) = Exp\left[-0.71\left(\frac{z_c}{D} - 0.65\right)^2\right] \cdot U_{C.L.M.}$

$$\frac{\partial U(y,z)}{\partial y} = 0.3(D_r)^{0.2004}(n_r)^{-0.2300}\left(\frac{b}{2}\right)^{-0.2545}(y)^{-0.7455}(z/D)^{-0.3556} U_{C.L.}(y,z_x) \qquad (9)$$

In the berm-section, near to the edge of the main-channel, say $y=y_0$, and $z=d_f$, then

$$\frac{\tau_{f\cdot p}}{\rho} = \left(\frac{n_f d_f \sqrt{g}}{R_f^{1/6}}\right)\left[\frac{\partial U(y_0 z)}{\partial y}\right]\left[0.2\left(\frac{b}{B}\right) - 0.15\right]U(y_0 z) \qquad (10\text{-}1)$$

where $R_f = \frac{\left[\frac{1}{2}d_f(b-B)\right]}{\left[2d_f + \frac{1}{2}(b-B)\right]}$;

d_f: measures from bed of flood plain to the position of $z=z_c$. In the main-channel, near to the intersection dividing surface, also, $y=y_0$ and $z=z_c$, then

$$\frac{\tau_{m.c.}}{\rho} = \left(\frac{n_{m.c.} z_c \sqrt{g}}{2R_{m.c.}^{1/6}}\right)\left[\frac{\partial U(y_0 z_c)}{\partial y}\right]\left[0.2\left(\frac{b}{B}\right) - 0.15\right]U(y_0 z_c) \qquad (10\text{-}2)$$

where $\frac{(Bz_c)}{(2z_c + B)} = R_{m.c.}$

z_c: measured from bed of main channel to the position of $z = z_c$.

At the vertical interfaces, the forces acting on each sub-section are indicated and can be defined as Figs. 1 and 2:

$$\frac{\tau_*}{\rho} = \left|\frac{\tau_{m.c.}}{\rho} - \frac{\tau_{f.p.}}{\rho}\right| \qquad (11)$$

III. COMPARISONS

The apparent shear stresses was calculated in

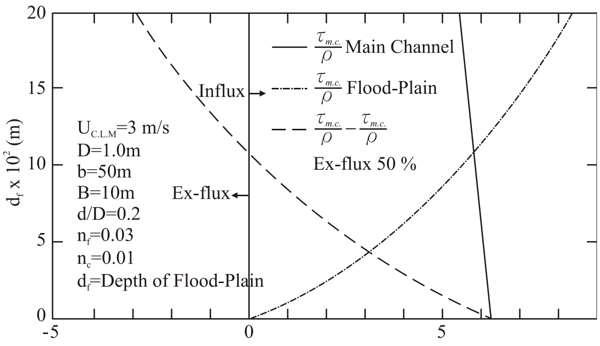


Fig. 1 The apparent shear stresses at the edge of intersection above bankful of main – channel , d / D = 0.2

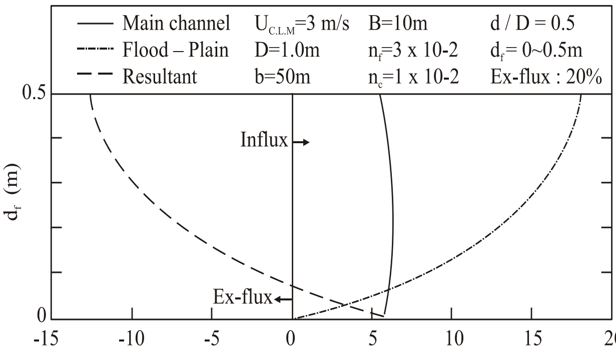


Fig. 2 The apparent shear stresses at the edge of intersection above bankful of main – Channel, d / D = 0.5

Figs.1. and 2. The dimensionless apparent shear stress formed $\bar{\tau}_* / \bar{\tau}_{\infty.f.p}$ were also construed by defining $\tau_{\infty.f.p.}$ following the relationship of Rajaratnam and Ahmadi〔5〕 with slight modification.

$$\bar{\tau}_* / \bar{\tau}_{\infty.f.p} = 0.15(D/d-1)^2 \qquad (12)$$

Fig. 3 is constructed and compared with the experimental data from Tamai〔6〕, Prison〔7〕, Knight〔8〕, Tominaga〔9〕, and Shiono and Knight〔10〕, with different roughness values. The data from Shiono, and Knight〔10〕are the experiments in trapezoidal cross-section with shallow bankful depth and short inclined side wall on the bed with which we also could find the effects of cross-section shape, whether wide-shallow or narrow-deep, on the apparent shear stresses translation. The apparent shear stresses were obtained by subtracting the bed wall shear stress from the value of τ_a which was defined as apparent shear stress. The acceptable agreement in Fig. 4 is presented.

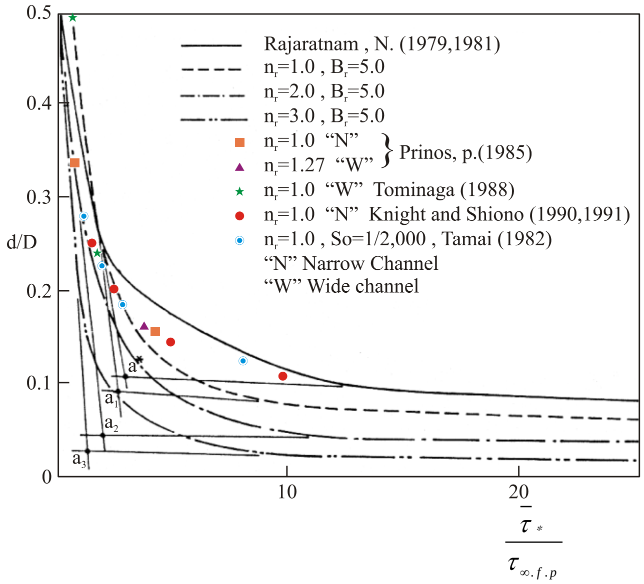
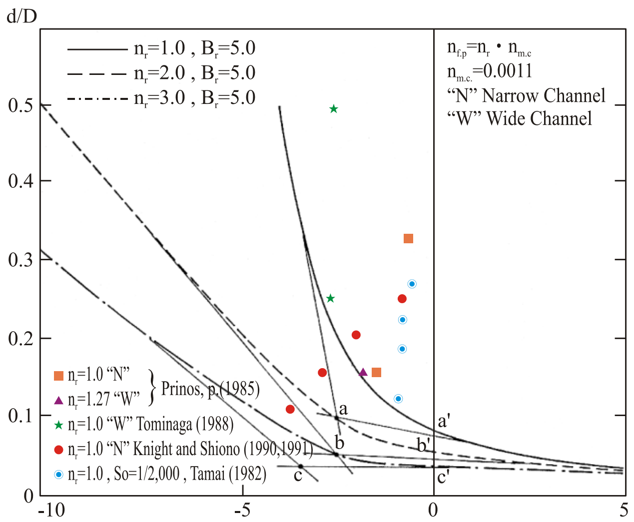


Fig. 3 Relationship of the apparent shear stress ratio with roughness ratio



$\tau_{xy} \times 10^4 (kg/m.\sec^2)$

Fig. 4 The relationship between dimensional apparent shear stresses and depth – ratio for different dimensionless roughness

Chin-lien Yen *et al*〔11〕also supposed when $H_r = \dfrac{(B/2)}{(D-d)}$ is very large the flow is nearly two dimensional situation (2-D) in the transverse direction, while it is extremely small, the flow becomes nearly 2-D in the vertical direction. Because a decrease in $H_r = \dfrac{(B/2)}{(D-d)}$ from extremely large values means that flows translate from two-to three dimension. From Fig. 3 and 4, we can obtain the same trends.

It is quite obviously seen that Figs. 3 and 4 give us the analytical solutions owning wider applicable utilization.

IV. APPLICATION

By using the 2-Dh depth-averaged model with the *standard* k-ε equations expressed in the vertical direction by x-y momentum equations, with substituting the zero continuity equation in (x,y) and flow level fluctuation also zero by the equation of the kinematic boundary condition,

$$\frac{\bar{\tau}_{wx}}{B\rho} + \frac{1}{\rho}\frac{\partial(\bar{\tau}_{xx})}{\partial x} + \frac{1}{\rho}\frac{\partial(\bar{\tau}_{xy})}{\partial y} = 0 \qquad \text{.........(13)}$$

$$\frac{\bar{\tau}_{wy}}{B\rho} + \frac{1}{\rho}\frac{\partial(\bar{\tau}_{xy})}{\partial x} + \frac{1}{\rho}\frac{\partial(\bar{\tau}_{yy})}{\partial y} = 0 \qquad \text{.........(14)}$$

$$\frac{\tau_{wx}}{\rho} = \frac{gU^2}{C_c^2}; \quad \frac{\tau_{wy}}{\rho} = 0; \qquad \text{.........(15)}$$

$$\frac{\partial(-\overline{u^2})}{\partial x} - \frac{1}{\rho}\frac{\partial(\tau_{xy})}{\partial y} - \frac{\tau_{xw}}{\rho B} = \frac{\partial\left(-\bar{\nu}_{ty}\dfrac{\partial U}{\partial y}\right)}{\partial y} - \frac{gU^2}{BC_c^2} \qquad (16)$$

$$\frac{\partial(-\overline{v^2})}{\partial y} = \frac{\partial\left(-\bar{\nu}_{ty}\dfrac{\partial U}{\partial y}\right)}{\partial x} = 0 \qquad \text{.........(17)}$$

-15 -10 -5 0 5 10 15 20

and ν_t is in Eq. (7) :

(a) In main-channel of depth direction:

$$\frac{\partial U(y.z_c)}{\partial z_c} = 1.177(D_r)^{0.2004}.\left(\frac{2y}{b}\right)^{0.2545}.(D)^{0.3556}.(n_r)^{-0.2300}.U_{C.L.M}.f(z_c) \qquad (18)$$

$$f(z_c) = -\left(\frac{z_c^{-1.3556}}{4}\right) + \frac{0.59}{D}z_c^{-0.3556} - \frac{1.17}{D^2}z^{0.6444} \qquad (19)$$

$$\nu_{t_{m.c}} = \left[\frac{n_{m.c}z_c\cdot\sqrt{g}}{2\left(\dfrac{z_cB}{B+2z_c}\right)^{\frac{1}{6}}}\right](0.2B_r - 0.15).U(y.z_c) \qquad (20)$$

with Eqs (1) and (2), and substituting the quantuty:

$$\left(\frac{B\ +\ 2z_c}{Bz_c}\right)^{\frac{1}{6}}\ =\ 1\ +\ \frac{5}{24}\left(\frac{2z_c}{B}\right)^{\frac{1}{6}}$$
……………(21)

the turbulent shear stress and the corresponding value of momentum can be solved.

(b) In flood-plain of depth direction:

$$\frac{\partial U(y.d_f)}{\partial d_f}\ =\ 1.177(D_r)^{0.2004}.\left(\frac{2y}{b}\right)^{0.2545}.(D)^{0.3556}.(n_r)^{-0.2300}.U_{C.L.M}.f(d_f)$$
(22)

where H*=D-df = the bankful depth of the main channel.

$$f(d_f)\ =\ \left(\frac{0.41}{d_f^{0.3556}}\right)-\left(\frac{0.905}{D^2}\right)\left(\frac{H_*^2}{d_f^{0.3556}}\right)-\left(\frac{1.81}{D^2}\right)\left(H_* d_f^{0.6444}\right)+$$

$$+\left(\frac{d_f^2}{D}\right)\left(\frac{0.905}{d_f^{0.3556}}\right)+\left(\frac{H_*}{D}\right)\left(\frac{1.18}{d_f^{0.3556}}\right)+\left(\frac{1.18}{D}\right)d_f^{0.6444}$$
(23)

$$\nu_{t_{fp}}\ =\ \left[\frac{n_{fp}d_f.\sqrt{g}}{\left(d_f^{1/6}\right)}\right](0.2B_r\ -\ 0.15).U_{C.L.M}.Exp\left[-0.71\left(\frac{z_c}{D}-0.65\right)^2\right]F(y.d_f)$$
…………(24)

$$F(y,d_f)\ =\ 1.177(D_r)^{0.2004}.\left(\frac{2y}{b}\right)^{0.2545}.\left(\frac{d_f}{D}\right)^{-0.3556}.(n_r)^{-0.2300}$$
…………..(25)

The lateral gradient of the velocity in the transverse direction in main channel and the flood-plain is:

$$\frac{\partial U(y.z_c)}{\partial y}=0.357(D_r)^{0.2004}.(y)^{-0.7455}.(b)^{-0.2545}.\left(\frac{z_c}{D}\right)^{-0.3556}.(n_r)^{-0.2300}.U_{C.L.}$$
(26)

For the 2-DH with depth-averaged models, the depth-averaged turbulent viscosity, $\nu_{ty,}$

$$\overline{\nu}_{ty}\ =\int_0^{z_a}\frac{\nu_t dz}{(z_a)}$$
……………………………………(27)

The kinetic turbulent energy, k, solved from Eqs. (16) and (17), the energt dissipation rate:

$$\overline{\varepsilon}\ =\ C_\mu.\frac{\overline{k}^2}{\overline{\nu}_{ty}}$$
……………………………………(28)

The lateral momentum transfer:

$$\frac{\tau_{xy}}{\rho}\ =\ \nu_{ty}\frac{\partial U}{\partial y}$$
……………………………………(29)

By using Eqs. (1), (2), (20), (21), (24)(25), and (26), then the analttical shear stress profiles are plotted with:

(1) In flood-plain:

$$\left(\frac{\tau}{\rho}\right)_{f.p.eff}\ =\ \left(\frac{\tau}{\rho}\right)_{f.p}\ -\ \left(\frac{\tau}{\rho}\right)_{m.c.IN}\ =\ \left(-\overline{u'v'}\right)_{f.p.eff}$$
..(30)

(2) In main channel for z≧H*:

$$\left(\frac{\tau}{\rho}\right)_{m.c.eff}\ =\ \left(\frac{\tau}{\rho}\right)_{m.c}\ -\ \left(\frac{\tau}{\rho}\right)_{f.p.IN}\ =\ \left(-\overline{u'v'}\right)_{m.c.eff}$$
(31)

(3) In main channel for z < H*:

$$\left(\frac{\tau}{\rho}\right)'_{m.c.}\ =\ \left(\frac{\tau}{\rho}\right)_{m.c}\ -\ \left(\frac{\tau}{\rho}\right)^*_{f.p.eff}$$
(32)

The analytical shear stress profiles based on the above values are compared with the numerical results from Kawahara and Tamai〔12〕based on the algebraic stress model with k-ε equations. The quite good trends are expressed on Fig. 5 with:

$$u_*^2\ =\ C_f.\overline{U}^2$$
…………………………………..(33)

$$\overline{U}=\frac{\displaystyle\int_{\left(\frac{b}{2}-\frac{B}{2}\right)}^{\frac{b}{2}}\int_0^{D}U dz dy+\int_0^{\left(\frac{b}{2}-\frac{B}{2}\right)}\int_{H_*}^{D}U dz dy}{\left[\dfrac{BD}{2}+\dfrac{(b-B)(D-H_*)}{2}\right]}$$
…………(34)

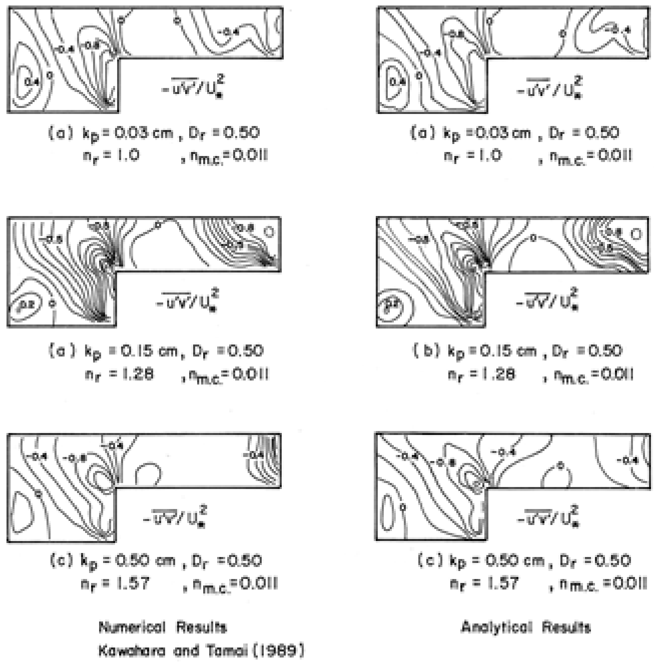


Fig. 5 Comparisons of effect of the flood-plain roughness on shear stress

V. DISCUSSION AND CONCLUSIONS

A force balance may be rewrirren for the flood-plain sub-section and it is then possible to eliminate the apparent shear force term between the main-channel and flood-plain equation to give the relationship:

$$A_f(\phi_f\ -\ 1)\ =A_c(1\ -\ \phi_c)$$
…………………..(35)

Ø is the index to characterize the degree of interaction between the main-channel and the flood-plain sub-section and indicating the degree of direction of the momentum transfer across the specific interface. If the index changes, the fluid weight component varies to balance forces. It demonstrates that if Øc greater than unity, then Øf less than 1, the net flow of momentum in the flood-plain will transfer into the main-channel to keep the force balance.

The obvious conclusions are presented as follows:

1. The dimensionless apparent shear stress with different depth-ratio and roughness-ratio were given with good trend by comparing the results of Rajaratnam, *et a*〔13〕 and other results of experiments.
2. The dimensional apparent shear stress with depth-ratio were compared with the ones from Tamai〔6〕, Prinos〔7〕, Knighy〔8〕,and Tominaga〔9〕 experimental, the acceptable results and expressing the dimensional apparent shear stress from Eqs.(9) and (10) has wider application than the results of Tulay Ozbek, et al〔14〕, Christopher I. Thornton〔15〕and Lai Sai Hin, et al〔16〕.
3. The analytical shear stress for a given roughness ratio expressed good trends with the numerical results of Kawahara and Tamai〔12〕.

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