

Load Carrying Capacity of Glass Columns with Rectangular, T and X Transverse Cross-sections

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Abstract-In this study, the load carrying capacities of glass columns with rectangular, T and X cross-sections having different lengths were calculated. The proposed analytical procedure allows to calculate the load carrying capacity associated with strength of transverse cross-section including local and global flexural and/or torsional buckling phenomenon. Moreover, initial imperfections and time dependant effects, the latter associated with the viscoelastic behaviour of interlayer (PVB), were included in the model as suggested in the literature. Those effects were calculated by introducing the equivalent thickness of glass panels, which takes into account the shear modulus of PVB and the viscoelastic behaviour of a composite. Failure of the glass panels connected with structural silicone was not considered. Experimental research recently conducted by the authors was utilised for comparison with the analytical prediction. Cases of study were those of compressed glass panels constituted by single tempered glass jointed between PVB, and compressed members having T or X transverse cross-sections and constituted by single layered panels jointed between structural silicone. The analytical model had good prediction accuracy, and the experimental results were in agreement with the failure modes observed experimentally.

Keywords- Laminated Glass; Glass Members; Theoretical Model

I. INTRODUCTION

In the field of glass structures, it is well known that glass columns are made by assembling thin laminated glass panels. Every panel is made by coupling two or more glass foils, connected by means of an interlayer. In most cases, float or annealed glass is used, due to its better post-breakage behavior and to lower costs compared to fully-tempered glass, which is more expensive and in addition breaks into many small fragments. The interlayer has viscous-elastic behavior, and is normally formed by a polyvinylbutiral (PVB), polycarbonate (PC) or ethylene-vinyl-acetate (EVA) film. It is clear and already well known from the literature that the strength of the laminated glass columns is substantially affected by the buckling (flexural and torsional) effects due to the slenderness of the panels and the shape of the transverse cross-section [1-6]. An accurate investigation of second-order effects in glass panels has to take two main aspects into account: the effective connection level ensured by the interlayer, whose mechanical properties are affected by temperature and loading rate, and the initial imperfections due to the glass production process. Regarding the first aspect, different authors have investigated how the mechanical features of the viscous-elastic interlayer are affected by temperature and loading rate [7-11], highlighting the fact that low values of shear stiffness can be achieved in real applications. The aim of this work was to calculate the load carrying capacity of glass columns, including second order effects. The examined cases refer to columns constituted by two foils of laminated glass and assembled together with structural silicone to form glass columns with different shapes of the transverse cross-section (T and X shape). It was observed that buckling effect out of the plane strongly influences the compressive behavior of multilayer glass panels with rectangular cross-section, while torsional and local buckling effects strongly influence the compressive behavior of members with T or X transverse cross-sections.

II. CALCULUS OF LOAD CARRYING CAPACITY

Many studies have been carried out on the buckling phenomena in laminated glass panels in compression, highlighting the role of the time dependant effects and of the initial imperfection [2, 6]. For the strength verification of the transverse cross-section the interaction domain, moment-axial force (M_u - P_u) was determined under the hypothesis of linear elastic behavior, and the same values for Young modulus in both tension and compression were assumed. In this manner, the axial force-bending moment domain took a polygonal shape, and considering only the case of P positive (compression), it was simply characterized by three points:

$$\begin{aligned} M_u &= 0 \\ P_u &= A \cdot \sigma_c \end{aligned} \quad \text{pure compression,} \quad (1)$$

$$M_u = \left[\sigma_c - \left(\frac{\sigma_c + \sigma_t}{2} \right) \right] \cdot W \quad \text{maximum bending,} \quad (2)$$

$$P_u = A \cdot \left(\frac{\sigma_c + \sigma_t}{2} \right)$$

And pure bending $M_u = W \cdot \sigma_t$, (3)

where σ_c is the glass compressive strength (assumed positive), σ_t the tensile strength (negative), A is the transverse cross-section of the panel and W is the bending modulus calculated with the equivalent thickness. The expression utilised for equivalent thickness was the one suggested by CNR-DT (210-2012) [9] in the form:

$$t_{eq,w} = \sqrt[3]{2 \cdot t^3 + 6 \cdot \Gamma \cdot t \cdot (t + t_{int})^2}, \quad (4)$$

$$t_{eq,\sigma_1} = \sqrt{\frac{t_{eq,w}^3}{t + \Gamma \cdot (t + t_{int})}}, \quad (5)$$

where $t_{eq,w}$ is the equivalent thickness useful for the calculation of deformation and $t_{eq,1}$ is the equivalent thickness useful for the strength verification.

And

$$\Gamma = \frac{1}{1 + 9.6 \cdot 1.03 \cdot \left(\frac{b}{L} \right)^2 \cdot \frac{E \cdot \frac{1}{2} \cdot t \cdot (t + t_{int})^2 \cdot t_{int}}{G_{int} \cdot (t + t_{int})^2 \cdot [\min(L; b)]^2}}, \quad (6)$$

where E is the elastic modulus of glass (assumed 70000 MPa) and G_{int} is the shear modulus of PVB interlayer.

A. Second Order Flexural Effects

To study the second-order flexural effects in a glass column subject to pure compression and affected by initial geometrical imperfections (out of straightness) w_o , the first-order moment was calculated as:

$$M^I = P \cdot w_o. \quad (7)$$

Belis et al. [3] proposed to assume w_o in the range between $L/500$ and $L/350$, suggesting the value $L/400$ for straightforward calculations.

The second order moment is

$$M^{II} = P \cdot w(z), \quad (8)$$

where $w(z)$ is the lateral displacement of the beam due to the second-order effect.

The total moment is

$$M = P \cdot [w_o + w(z)]. \quad (9)$$

The differential equilibrium equation gives:

$$w''(x) + \left(\frac{P}{EI} \right) w(x) = - \frac{P \cdot w_o}{EI}. \quad (10)$$

By introducing

$$K^2 = \frac{P}{EI}, \quad (11)$$

Eq. (10) becomes

$$w''(x) + K^2 w(x) = - \frac{P \cdot w_o}{EI}. \quad (12)$$

The solution to Eq. (8) is

$$w(z) = C_1 \cos(Kz) + C_2 \sin(Kz) - w_0. \quad (13)$$

The constants C_1 and C_2 are obtained by imposing the boundary conditions, which for a beam simply pinned are:

$$\begin{aligned} w(0) &= 0, \\ w'(L/2) &= 0. \end{aligned} \quad (14)$$

Finally the solution is

$$w(z) = w_0 [\cos(Kz) + \tan(KL/2) \sin(Kz) - 1]. \quad (15)$$

The maximum deflection of the beam is

$$w_{\max} = w_0 \left[\frac{1 - \cos(KL/2)}{\cos(KL/2)} \right]. \quad (16)$$

And therefore the moment is

$$M_{\text{tot}} = \frac{P \cdot w_0}{\cos(KL/2)}. \quad (17)$$

By introducing the axial force P acting on the column in Eq. (17), the moment variation was obtained. The coupled P, M values describes the non-linear path loading. Each pair of M - P values can be placed in the M_u - P_u interaction domain, making it possible to calculate the failure load, which corresponds to the ordinate of the intersection point between the load pathway and the strength domain.

By applying this procedure, it is possible to derive the strength domain including second-order effects. Fig. 1 shows the strength domain of a single compressed panels in the length of $L = 300$ mm, width of $b = 300$ mm and thickness of $t = 9.52$ mm ($4+4+1.52$ mm). The initial out of straightness of $1/400 L$ was considered. Duration of the external load was assumed 3 sec. Corresponding values of G_{int} were calculated as in Bennison [13-14]. In the same graph elastic domain, elastic and inelastic paths loads were also given.

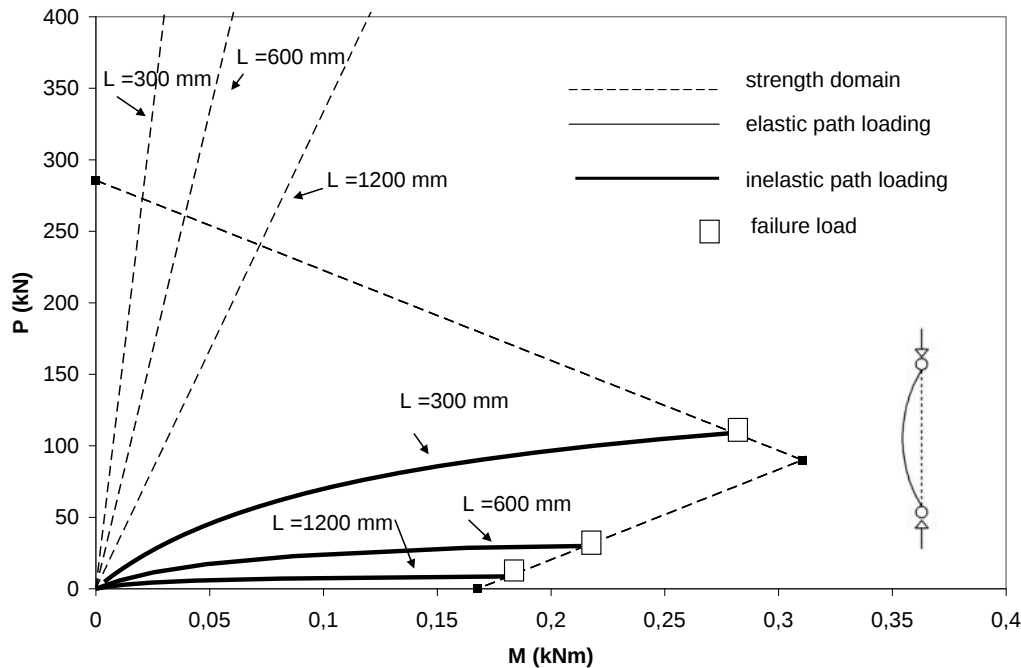


Fig. 1 Strength domain of multilayered glass panels of 300x9.52 mm with second-order effects and for short time load duration
a) $L = 300$ mm; b) $L = 600$ mm; c) $L = 1200$ mm.

In Fig. 1, continuous line indicates inelastic path-load, straight dashed line indicates linear elastic path-load and dashed line indicates strength domain. To derive the strength domain, compressive strength was assumed conventionally as $\sigma_c = 100$ MPa, although the value of 1000 MPa was also possible. Choosing the value of 100 MPa for the maximum compressive strength instead of 1000 MPa was only for graphical purpose in representation of the strength domain, yet it did not influence the value of load carrying capacity that is related to the tensile strength of glass. The tensile strength assumed was the value determined experimentally in the study of Campione et al. [12] and equals to $\sigma_t = 37$ MPa. The analysis of graphs in Fig.1 highlights a significant reduction in the loading carrying capacity with the increase in the slenderness of the panel.

B. Second-Order Torsional and Local Effects

In the case of T and X cross-sections, a further risk of local and torsional buckling arises. For flexural buckling out of the plane, the same procedure of the previous section was utilized by adopting the strength modulus defined in Table 1. In the case of elastic torsional buckling, the members exhibit a deformation shape similar to that shown in Fig. 2.

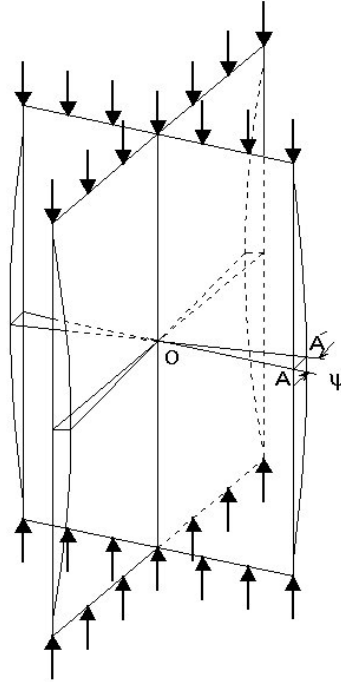


Fig. 2 Buckled mode of failure of cruciform cross-section

In this case, the expression of resistance N_{OZ} of a simply supported column with length of L is as following.

$$N_{OZ} = \left(G \cdot J + \frac{\pi \cdot E \cdot I_w}{L^2} \right) \cdot \frac{1}{r_o^2}, \quad (18)$$

with G the shear modulus of glass expressed as

$$G = \frac{E}{2 \cdot (1 + \nu)}, \quad (19)$$

Where J is the torsion section constant, I_w is the warping section constant and r_o is the polar radius of gyration and is expressed as:

$$r_o^2 = (I_x + I_y) \cdot \frac{1}{A}. \quad (20)$$

I_x and I_y are the principal axis moments of inertia and A the area of the cross-section.

Table 1 present the expressions of J , I_w , I_x , I_y , r_o and A .

In the case of open section, the warping section constant is negligible and therefore Eq. (18) with Eq. (19) and Eq. (20) in terms of normal stress becomes:

$$\sigma_{OZ} = \frac{N_{OZ}}{A} = \frac{1}{2} \cdot \frac{E}{(1 + \nu)} \cdot \frac{J}{(I_x + I_y)}. \quad (21)$$

TABLE 1 GEOMETRICAL CHARACTERISTICS OF T AND X CROSS-SECTIONS

Cross-section	I_x	I_y	J	I_w	A	r_o^2
R	$\frac{1}{12} \cdot b \cdot t^3$	$\frac{1}{12} \cdot b^3 \cdot t$	$\frac{1}{3} \cdot b \cdot t^3$	$\frac{1}{48} \cdot b^3 \cdot t^3$	$b \cdot t$	$\frac{1}{12} \cdot (b^2 + t^2)$

X	$\frac{4 \cdot b^2 + t^2}{6} \cdot b \cdot t$	$\frac{4 \cdot b^2 + t^2}{6} \cdot b \cdot t$	$\frac{4}{3} \cdot b \cdot t^3$	$\frac{1}{9} \cdot b^3 \cdot t^3$	$4 \cdot b \cdot t$	$\frac{4 \cdot b^2 + t^2}{12}$
T	$\frac{b \cdot t}{2} \cdot \left[(2 \cdot t^2 + b^2) + 2 \cdot (t + b)^2 \right]$	$\frac{1}{12} \cdot b \cdot t \cdot (8 \cdot b^2 + t^2)$	$b \cdot t^3$	$\frac{1}{4} \cdot b^3 \cdot t^3$	$3 \cdot b \cdot t$	$\frac{I_x + I_y}{A}$

R=rectangular-shaped; X=X-shaped; T= T-shaped; $W=\min(I_x, I_y)/y_g$ with y_g position of center of cross-section

In the case of members with X cross-sections, Eq. (21) gives the load carrying capacity N_{oz} . Consequently, the stress results:

$$\sigma_{oz} = \frac{N_{oz}}{A} \quad (22)$$

And therefore, substituting in Eq. (19) and expressions of inertia given in Table 1 results:

$$\sigma_{oz} = \frac{8 \cdot E}{(1 + \nu)} \cdot \frac{1}{1 + 4 \cdot \left(\frac{b}{t}\right)^2} \quad \text{for X cross-section} \quad (23)$$

While the Eulerian load results:

$$P_{cr} = \frac{\pi^2 \cdot E \cdot J}{L^2} \quad (24)$$

Therefore substituting in Eq. (24) the expressions of inertia given in Table 1 it results:

$$\sigma_{cr} = \frac{5}{3} \cdot E \cdot \frac{1 + 4 \cdot \left(\frac{b}{t}\right)^2}{\left(\frac{L}{b}\right)^2} \cdot \left(\frac{t}{b}\right)^2 \quad \text{for X cross-section} \quad (25)$$

Against buckling phenomenon a further control against local effects consists in calculating the critical load, expressed as suggested in [16-17] as follows:

$$N_c = A \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot \left[\frac{6 \cdot (1 - \nu)}{\pi^2} + \left(\frac{b}{L}\right)^2 \right] \quad (26)$$

With t assumed as the effective thickness of steel panels or the equivalent thickness for glass panels (see Eq. (2)) to take into account of the time dependant effects. Eq. (21) refers to perfect panels therefore a further penalization should be introduced if initial imperfection has to be introduced (see [16-17]).

Eq. (25) in terms of normal stress results:

$$\sigma_c = \frac{1}{48 \cdot b \cdot t \cdot (1 - \nu^2)} \cdot \frac{\pi^2 \cdot E}{\left(\frac{L}{b}\right)^2} \cdot \left(\frac{t}{b}\right)^2 \cdot \left[\frac{6 \cdot (1 - \nu)}{\pi^2} + \left(\frac{b}{L}\right)^2 \right] \quad (27)$$

For a twisted cruciform section the maximum shear stress due to the torsional buckling load, can be calculated as suggested in [17] as:

$$\tau = G \cdot \frac{\pi \cdot t}{L} \cdot \phi_m \quad (28)$$

As demonstrated in [16-17] it results:

$$\frac{\phi_m}{\phi_{om}} = \frac{N/N_{oz}}{1 - N/N_{oz}} \quad (29)$$

To take into account of initial imperfection the initial crookedness ϕ_{om} can be assumed as

$$\phi_{om} = \frac{w_o}{2 \cdot b} \quad (30)$$

With $w_o = L/400$ the initial imperfection due to the out of straightness.

N_{oz} represents the asymptotic value of critical load for member with initial imperfections.

Finally, load carrying capacity is the minimum between Eq. (18), Eq. (23) and Eq. (25) coupled with the moment axial forces domain.

III. COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

The experimental research utilised for comparison with analytical model are those of Aiello et al. [11] and of Campione et al. [12]. In both researches, specimens utilised are made of laminated glass, which thickness of 9.52 mm (4+1.52+4). This choice is connected to the use of a structural glass available commercially. It was constituted by two single foils of float glass had thickness t of 4 mm connected by a polyvinylbutiral (PVB) film having a thickness t_{int} equal to 1.52 mm. Compressive tests were carried out on single panels and columns. Two different cross-section shapes characterized the latter; based on the number of assembled panels, the column had a T-shaped or X-shaped cross-section. The columns were assembled with a main panel having a width of 300 mm and one (T-shaped section) or two (X-shaped section) orthogonally disposed panels with side $b=150$ mm. Assemblage was achieved by structural silicone glazing. Single panels had width 300 mm, thickness 9.52 mm and height L equal to 300, 400, 500 and 600 mm, while for columns it was equal to 600, 800 and 1000 mm. Table 2 and Table 3 give geometrical characteristics of specimens tested with load carrying capacity determined experimentally and theoretically.

For local effects it was assumed $\nu=0.2$ and $E=70000$ MPa. Table 2 and Table 3 give data of specimens tested with experimental load carrying capacity values and analytical predictions. Analytical values calculated with the procedure proposed gives in the majority of cases examined in which torsional buckling was observed good prediction of experimental results.

TABLE 2 DATA OF AIELLO ET AL. [11] AND CAMPIONE ET AL. [12] FOR PANELS WITH RECTANGULAR CROSS-SECTION

Data from	b(mm)	L(mm)	$P_{exp.}$ (kN)	P_{teor} (kN) (Eq.17)
[11]	300	400	56.00	59.00
[11]	300	400	60.40	
[11]	300	500	47.00	42.84
[11]	300	500	49.60	
[11]	300	600	32.70	31.42
[11]	300	600	33.10	
[12]	300	300	116.00	109.48
[12]	300	300	128.00	
[12]	300	500	40.00	42.84
[12]	300	500	48.00	
[12]	300	600	29.00	28.56
[12]	300	600	30.00	
[12]	300	800	16.00	19.04
[12]	300	800	22.00	

TABLE 3 DATA OF CAMPIONE ET AL. [12] FOR COLUMNS WITH T AND X CROSS-SECTION

T cross-section				
Data from	b(mm)	L(mm)	P _{exp.} (kN)	P _{teor} (kN) (Eq.18)
[12]	300+150	600	231	204
[12]	300+150	600	200	
[12]	300+150	800	224	194
[12]	300+150	800	200	
[12]	300+150	1000	214	190
[12]	300+150	1000	193	
X cross-section				

[12]	300+150+150	600	269	272
[12]	300+150+150	600	288	
[12]	300+150+150	800	208	255
[12]	300+150+150	800	227	
[12]	300+150+150	1000	296	248
[12]	300+150+150	1000	272	

IV. CONCLUSIONS

In this paper, an analytical procedure able to calculate the load carrying capacity associated with compressive strength cross-section (including local and global flexural and/or torsional buckling phenomenon) of shapes rectangular, X and T is proposed. Initial imperfections and time dependant effects, the latter associated with the viscoelastic behaviour of PVB, are included in the model highlighting their influences on the reduction of load carrying capacity.

Experimental results recently obtained by the authors referred to compressive tests on glass members with different lengths and cross-sections (rectangular panels, T and X shape) are assumed for comparison with the analytical results obtained with the proposed model.

For the range of variables analyzed the following conclusions can be drawn:

- the buckling strength of single panels depends very much on the slenderness and time dependant effect that reduce drastically the load carrying capacity mainly governed by the tensile strength of glass;
- T and X shapes of the transverse cross-section are affected mainly by local buckling and torsional buckling effects, which significantly reduce the bearing capacity of columns associated with compressive strength cross-section.

Finally, analytical prediction was satisfactory when compared with experimental results and the proposed method is a useful instruments for a preliminary design of glass columns. Failure of glass panels connected with structural silicone often observed experimentally are not considered in the proposed model and further studies will be addressed to analyse these effects.

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