On Bessel Composite Function of the Third Order and First Kind Solution to Damped Wave Conduction and Relaxation Equation

Kal Renganathan Sharma

Lone Star College, Houston, TX, USA

jyoti_kalpika@yahoo.com

***Abstract*- Damped wave conduction and relaxation equation is solved in three dimensions in order to gauge the ampacity risks in PCB interconnections. V = u/r substitution is used and the temperature profile is obtained in three dimensions and in one dimension. In the creeping transfer limit, the spatio-temporal profile is given as a modified Bessel composite function in space and time of the third order. Three regimes of solution are identified; (i) lag regime; (ii) rising regime given by Bessel composite function in space and time and; (iii) rising regime given by modified Bessel composite function in space and time. In the general case, the order of the solution is found to be 7/2 and the order of the solution is two in the case of one dimension.**

Keywords- Damped Wave Conduction and Relaxation; Relativistic Transformation; Bessel Composite Function of Third Order; Ampacity Risks; Second Law of Thermodynamics

I. INTRODUCTION

Information handling systems (IHS) are used to process, compile, store and communicate data for business personnel or other similar purposes. Application specific requirements are used to configure IHS. It may use one or more computer systems, data storage systems and networking systems. Depending on the information handled, the IHS may differ from one application to another. They may differ as to what information is handled, how the information is processed, stored, communicated and how soon and efficiently the information may be processed, stored or communicated. IHS may be configured for general use or for specific use such as passenger reservation in the airlines, railways, credit card transactions, global communications, and enterprise data storage. A variety of hardware and software components may be configured to perform different tasks such as storage, communication, processing.

During the design and manufacture of HIS, one salient consideration is the detection of areas of the system or circuit that are prone to certain risks. Corrective steps have to be taken in order to minimize these risks. One example is the ampacity risks on circuit boards. The circuit or interconnection may go back to the drawing board stage for rerouting. "Ampacity' is defined [1] as the current in amperes that a conductor can carry continuously under the condition of use without exceeding the temperature rating or fuse point. The simulation, analysis, validation to laboratory data, direct current (DC) and short duration, alternating current (AC) transient pulse effects on the ampacity risks on PCB interconnectivity are needed. This would result in assessment of ampacity risks on PCB interconnections and improvement of the reliability of the operation of IHS. One critical aspect is the computation of heat conduction at short times where non-Fourier effects can be expected. The damped wave conduction equation may be applicable here. The damped wave conduction and relaxation equation was sought over Fourier's law of heat conduction for eight reasons by Sharma [2].

The damped wave conduction and relaxation equation was originally suggested by Maxwell [3], and postulated independently by Cattaneo [4, 5] and Vernotte [6]. The damped wave conduction and relaxation equation in one dimension across constant area may be written as follows;

$$q_x = -kA\frac{\partial T}{\partial x} - \tau_r A\frac{\partial q_x}{\partial t} \tag{1}$$

Where q_x is the heat transfer rate in x direction in (watts, w), A is the cross-sectional area across which the heat conduction occurs in (m^2), k is the thermal conductivity of the material in ($w.m^{-1}.K^{-1}$), τ_r is the relaxation time (s).

Reviews of the use of this equation have been presented by Joseph and Preziosi [7, 8] and Ozisik and Tzou [9]. Extensive theoretical treatments of the equation have been reported by Tzou [9] and Sharma [10]. Experimental measurement of relaxation times has been reported by Mitra et al. [11] recently for biological materials. Taitel [12] found an overshoot in his transient temperature solution for a finite slab subject to constant wall temperature boundary condition. Bai and Lavine [13] was concerned about Eq. (1) violating the second law of thermodynamics. Zanchini [14], Barletta and Zanchini [15] calculated an entropy production term and are concerned of a violation of Clausius' inequality. Al Nimir et al. [16, 17] discussed an "overshoot" and equilibrium entropy production. Haji Sheik et al. [18] pointed out some anomalies in Eq. (1). Tzou [9] has found Eq. (1) to be admissible within the framework of the second law of thermodynamics. Sharma [19-25] has presented closed form analytical solutions for different geometries within the bounds of the second law of thermodynamics; the damped wave equation was derived by accounting for acceleration of the molecules in the Stokes-Einstein formulation, demonstrating that the overshoot disappeared using the physically reasonable initial condition. Final condition in time was used and bounded solutions without violating the second law of thermodynamics were presented. Antaki [26] has discussed some analytical solutions for convective boundary condition.

Analytical solutions are derived in this study for the damped wave conduction equation in three dimensions (3D). The solutions are in the form of Bessel composite function of the third order. The derived spatio-temporal temperature profiles can be used to gauge the ampacity risks in PCB interconnections.

II. THEORY

The 'defect' region is modeled as a spherical shell with radius R_0. The effects of the defects can be in any direction. The transient temperature field surrounding the surge needs to be represented using spherical coordinates. Spherical coordinates can be used when events in arbitrary direction becomes important. Spherical coordinates are used when the cross-sectional area along the path of heat conduction changes such as in conical objects. The governing equation for temperature, T (^{0}K) can be obtained as follows; an energy balance on the spherical shell at a distance r from the origin can be written. When combined with the damped wave diffusion and relaxation equation, it becomes

$$\tau_r \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{\partial T}{\partial r}\right)\right) \tag{2}$$

Let

$$u = \left(\frac{T - T_0}{T_s - T_0}\right); \tau = \frac{t}{\tau_r}; X = \frac{r}{\sqrt{\alpha \tau_r}} \tag{3}$$

Eq. (2) is made dimensionless by using the variables defined in Eq. (3), then Eq. (2) becomes

$$\frac{\partial^2 u}{\partial \tau^2} + \frac{\partial u}{\partial \tau} = \frac{1}{X^2}\frac{\partial}{\partial X}\left(X^2\left(\frac{\partial u}{\partial X}\right)\right) \tag{4}$$

The time and space conditions can be written as

$$\tau = 0, u = 0 \tag{5}$$

$$\tau = \infty, u = 1 \tag{6}$$

$$\tau > 0, X = X_{R0}, \quad u = 1 \tag{7}$$

$$X = \infty, \quad u = 0 \tag{8}$$

These boundary conditions represent the constant wall temperature surface boundary condition in an infinite medium. The infinite space assumption lends the solution with fewer terms and is physically reasonable. The two time conditions, i.e., one at initial time and the other after infinite time, has been assumed as a first approximation.

Substituting $V = u/X$ into Eq. (4), we can get

$$\frac{\partial^2 V}{\partial \tau^2} + \frac{\partial V}{\partial \tau} = \frac{2V}{X^2} + \frac{4}{X}\left(\frac{\partial V}{\partial X}\right) + \frac{\partial^2 V}{\partial X^2} \tag{9}$$

The damping term can be removed by the substitution V = Wexp(-nτ) and τ is defined in Eq. (3). As shown in the preceding sections for n = ½, Eq. (9) becomes;

$$\frac{\partial^2 W}{\partial \tau^2} - \frac{W}{4} = \frac{2W}{X^2} + \frac{4}{X}\left(\frac{\partial W}{\partial X}\right) + \frac{\partial^2 W}{\partial X^2} \tag{10}$$

Let $\eta = \tau^2 - X^2$

The term $2W/X^2$ can be neglected for a large X, because at large distances from the point of disturbance, the effect cannot be seen to an appreciable extent. W can be expected to be small for a large r as u = Wexp $(-\tau/2)/r$. For a large X, Eq. (10) can be modified as follows.

Now each of the terms in Eq. (10) in terms of X and τ is transformed in terms of η,

$$\frac{4}{X}\frac{\partial W}{\partial X} = -8\frac{\partial W}{\partial \eta} \tag{11}$$

$$4\eta\frac{\partial^2 W}{\partial \eta^2} + 12\frac{\partial W}{\partial \eta} - \frac{W}{4} = 0 \tag{12}$$

$$\eta^2\frac{\partial^2 W}{\partial \eta^2} + 3\eta\left(\frac{\partial W}{\partial \eta}\right) - \frac{\eta W}{16} = 0 \tag{13}$$

Comparing Eq. (13) with the generalized Bessel equation, the solution is;

a = 3; b = 0; c = 0; d = -1/16; s = ½

The order p of the solution is then p = 2 and $\frac{\sqrt{|d|}}{s} = 2$. The generalized Bessel equation and forms of Bessel solution are given in [10]. In the following equation (14), 2 is the order of the function, I_2 is the modified Bessel function of the first kind and second order and K_2 is the modified Bessel function of the second kind and second order.

$$W = c_1\frac{I_2\left(\frac{\sqrt{\tau^2 - X^2}}{2}\right)}{\left(\tau^2 - X^2\right)} + c_2\frac{K_2\left(\frac{\sqrt{\tau^2 - X^2}}{2}\right)}{\left(\tau^2 - X^2\right)} \tag{14}$$

Where c_2 can be seen to be zero as W is finite and not infinitely large at $\eta = 0$. At $\eta = 0$, the velocity of heat times, the time ordinate t becomes the space ordinate r. This can be seen at the point where the disturbance would have travelled at the speed of heat.

$$V = c_1 e^{-\frac{\tau}{2}}\frac{I_2\left(\frac{\sqrt{\tau^2 - X^2}}{2}\right)}{\left(\tau^2 - X^2\right)} \tag{15}$$

Where c_1 can be eliminated between the above equation and the equation from the boundary condition. The boundary condition is given by Eq. (7).

$$\frac{1}{X_{R0}} = c_1 e^{-\frac{\tau}{2}}\frac{I_2\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)}{\left(\tau^2 - X_{R0}^2\right)} \tag{16}$$

Thus for $\tau > X$

$$V = \frac{1}{X_{R0}} \left(\frac{\tau^2 - X_{R0}^2}{\tau^2 - X^2} \right) \frac{I_2\left(\frac{\sqrt{\tau^2 - X^2}}{2} \right)}{I_2\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2} \right)} \tag{17}$$

For X > τ,

$$u = \frac{X}{X_{R0}} \left(\frac{\tau^2 - X_{R0}^2}{X^2 - \tau^2} \right) \frac{J_2\left(\frac{\sqrt{X^2 - \tau^2}}{2} \right)}{I_2\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2} \right)} \tag{18}$$

On examining Eq. (18), it can be seen that the Bessel function of the second order and first kind will go to zero at some value of η. The first root of the Bessel function occurs when

$$½(X^2 - \tau^2)^{1/2} = 5.1356 \tag{19}$$

How 5.1366 was obtained is discussed in details by graph in Ref. [10]. It has something to do with the zeros of the function considered. In this case, it's the first root of the Bessel function.

$$\text{Or} \quad X^2 - \tau^2 = 105.498 \tag{20}$$

When an exterior point in the infinite sphere is considered, a lag time can be calculated prior to when there is no heat transfer to that point. After the lag time, there exist two regimes. One is described by Eq. (18) and the third regime is described by Eq. (17). Thus,

τ_{lag} = sqrt(X_p^2 - 105.498)

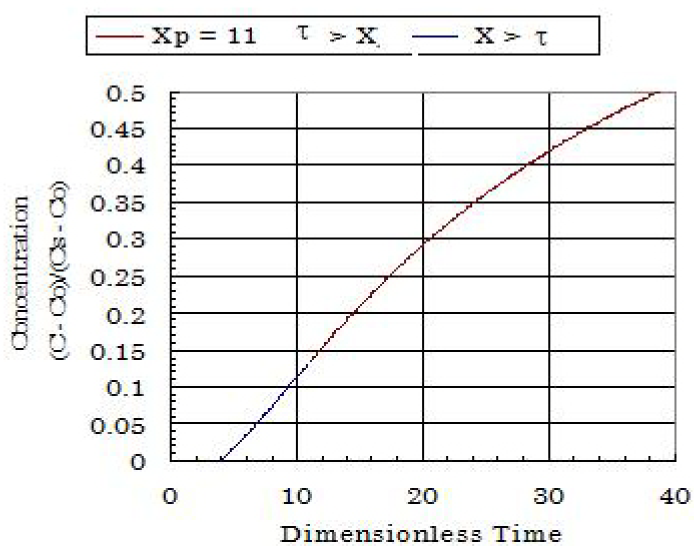

Fig. 1 Three regimes of dimensionless temperature at an exterior point from the defect

All the three dimensions of the spherical coordinates are considered. The V = u/r substitution is used and the spatio-temporal temperature in the infinite sphere is derived as follows.

The governing equation for the temperature is obtained when the energy balance equation and the constitutive damped wave diffusion and relaxation equation are combined. The equation is made dimensionless by using the substitutions in Eq. (3).

Then the governing equation in three dimensions in spherical coordinates can be written as;

$$\frac{\partial^2 u}{\partial \tau^2}+\frac{\partial u}{\partial \tau}=\frac{2}{X}\left(\frac{\partial u}{\partial X}\right)+\frac{\partial^2 u}{\partial X^2}+\frac{1}{X^2}\left(\frac{\partial^2 u}{\partial \theta^2}\right)+\frac{1}{X^2}\sin^2\theta\frac{\partial^2 u}{\partial \phi^2}+\frac{\cot\theta}{X^2}\left(\frac{\partial u}{\partial \theta}\right) \tag{21}$$

Substituting V = u/X into Eq. (21), we can get

$$\frac{\partial^2 V}{\partial \tau^2}+\frac{\partial V}{\partial \tau}=\frac{2V}{X^2}+\frac{4}{X}\left(\frac{\partial V}{\partial X}\right)+\frac{1}{X}\frac{\partial^2 V}{\partial X^2}+\frac{1}{X^2}\left(\frac{\partial^2 V}{\partial \theta^2}\right)+\frac{1}{X^2}Sin^2\theta\frac{\partial^2 V}{\partial \phi^2}+\frac{1}{X^2}\left(\frac{\partial^2 V}{\partial \theta^2}\right)+\frac{\cot\theta}{X^2}\left(\frac{\partial V}{\partial \theta}\right) \tag{22}$$

The damping term can be removed by a V = wexp(-nτ) substitution. As shown in the preceding sections for n = ½, Eq. (22) becomes

$$\frac{\partial^2 W}{\partial \tau^2}-\frac{W}{4}=\frac{2W}{X^2}+\frac{4}{X}\left(\frac{\partial W}{\partial X}\right)+\frac{\partial^2 W}{\partial X^2}+\frac{1}{X^2}\left(\frac{\partial^2 W}{\partial \theta^2}\right)+\frac{1}{X^2}\sin^2\theta\frac{\partial^2 W}{\partial \phi^2}+\frac{1}{X^2}\left(\frac{\partial^2 W}{\partial \theta^2}\right)+\frac{\cot\theta}{X^2}\left(\frac{\partial W}{\partial \theta}\right) \tag{23}$$

For small θ by Taylor approximation as shown in [10],

$$\frac{\partial^2 W}{\partial \tau^2}-\frac{W}{4}=\frac{2W}{X^2}+\frac{4}{X}\left(\frac{\partial W}{\partial X}\right)+\frac{\partial^2 W}{\partial X^2}+\frac{1}{X^2}\left(\frac{\partial^2 W}{\partial \theta^2}\right)+\frac{1}{X^2}\sin^2\theta\frac{\partial^2 W}{\partial \phi^2}+\frac{1}{\theta X^2}\left(\frac{\partial W}{\partial \theta}\right) \tag{24}$$

Let ξ = θX,

$$\frac{1}{X^2}\left(\frac{\partial^2 W}{\partial \theta^2}\right)=\frac{\partial^2 W}{\partial \xi^2} \tag{25}$$

ψ = ϕXSinθ,

$$\text{Then, }\frac{1}{X^2}Sin^2\theta\left(\frac{\partial^2 W}{\partial \phi^2}\right)=\frac{\partial^2 W}{\partial \psi^2} \tag{26}$$

Eq. (24) then becomes for a large X,

$$\frac{\partial^2 W}{\partial \tau^2}-\frac{W}{4}=\frac{4}{X}\left(\frac{\partial W}{\partial X}\right)+\frac{\partial^2 W}{\partial X^2}+\left(\frac{\partial^2 W}{\partial \xi^2}\right)+\frac{\partial^2 W}{\partial \psi^2}+\frac{1}{\xi}\left(\frac{\partial W}{\partial \xi}\right) \tag{27}$$

Consider the transformation, $\eta = (\tau^2 - X^2 - \xi^2 - \psi^2)$

As shown in Eq. (11), the derivatives in Eq. (27) in four independent variables become converted into one independent variable (η), and Eq. (27) becomes

$$4\eta\frac{\partial^2 W}{\partial \eta^2}+18\left(\frac{\partial W}{\partial \eta}\right)-\frac{W}{4}=0 \tag{28}$$

$$\text{Or,}\quad \eta^2 \frac{\partial^2 W}{\partial \eta^2} + \frac{9}{2}\eta\left(\frac{\partial W}{\partial \eta}\right) - \frac{W\eta}{16} = 0 \tag{29}$$

Comparing Eq. (29) with the generalized Bessel equation given in [10], the solution is

a = 9/2; b = 0; c = 0; d = -1/16; s = ½

Then the order p of the solution is p = 7/2

$$W = c_1 \frac{I_{7/2}\left(\frac{\sqrt{\tau^2 - X^2 - \xi^2 - \psi^2}}{2}\right)}{\left(\tau^2 - X^2 - \xi^2 - \psi^2\right)} + c_2 \frac{I_{-7/2}\left(\frac{\sqrt{\tau^2 - X^2 - \xi^2 - \psi^2}}{2}\right)}{\left(\tau^2 - X^2 - \xi^2 - \psi^2\right)} \tag{30}$$

Where c_2 can be seen to be zero as W is finite and not infinitely large at $\eta = 0$. An approximate solution can be obtained by eliminating c_1 between the above equation and the equation from the boundary condition. The equation from the boundary condition can be written as;

$$1 = X_{R0} e^{-\frac{\tau}{2}} c_1 \frac{I_{7/2}\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)}{\left(\tau^2 - X_{R0}^2\right)} \tag{31}$$

Dividing Eq. (30) by Eq. (31)

$$u = \frac{X}{X_{R0}}\left(\frac{\tau^2 - X_{R0}^2}{\tau^2 - X^2 - \xi^2 - \psi^2}\right) \frac{I_{7/2}\left(\frac{\sqrt{\tau^2 - X^2 - \xi^2 - \psi^2}}{2}\right)}{I_{7/2}\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)} \tag{32}$$

For small X,

$$u = \frac{X}{X_{R0}}\left(\frac{\tau^2 - X_{R0}^2}{\tau^2 - X^2 - \xi^2 - \psi^2}\right) \frac{J_{7/2}\left(\frac{\sqrt{X^2 + \xi^2 + \psi^2 - \tau^2}}{2}\right)}{I_{7/2}\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)} \tag{33}$$

In the creeping heat transfer limit, Eq. (27) can be approximated as;

$$\frac{\partial^2 W}{\partial \tau^2} - \frac{W}{4} = \frac{4}{X}\left(\frac{\partial W}{\partial X}\right) + \frac{\partial^2 W}{\partial X^2} + \left(\frac{\partial^2 W}{\partial \xi^2}\right) + \frac{\partial^2 W}{\partial \psi^2} \tag{34}$$

Creeping limit is suitable for problems in spherical geometry. The characteristics of transient temperature close to the sphere R_0 can be expected to be completely different from that far from the sphere. Creeping flow assumptions have been made successfully in fluid mechanics in the flow with sphere problems. By analogy between momentum transfer and heat transfer, Eq. (27) is approximated as Eq. (34).

After the transformation, the PDE with 4 variables is converted to a Bessel equation with one variable:

$$\eta^2 \frac{\partial^2 W}{\partial \eta^2} + 4\eta\left(\frac{\partial W}{\partial \eta}\right) - \frac{W\eta}{16} = 0 \qquad (35)$$

The order of the Bessel solution for Eq. (35) can be calculated by comparing Eq. (35) with the generalized Bessel equation and the solution is

when a = 4, b = 0, c = 0, d = -1/16, s = ½ , the order p of the solution is 3

$$W = c_1 \frac{I_3\left(\frac{\sqrt{\tau^2 - X^2 - \xi^2 - \psi^2}}{2}\right)}{\left(\tau^2 - X^2 - \xi^2 - \psi^2\right)} + c_2 \frac{K_3\left(\frac{\sqrt{\tau^2 - X^2 - \xi^2 - \psi^2}}{2}\right)}{\left(\tau^2 - X^2 - \xi^2 - \psi^2\right)} \qquad (36)$$

c_2 can be seen to be zero as W is finite and not infinitely large at η = 0. The boundary condition is given in Eq. [7]. An approximate solution can be obtained by eliminating c_1 between the above equation and the equation from the boundary condition. The equation from the boundary condition can be written as;

$$1 = X_{R0} e^{-\frac{\tau}{2}} c_1 \frac{I_3\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)}{\left(\tau^2 - X_{R0}^2\right)} \qquad (37)$$

Dividing Eq. (36) by Eq. (37),

$$u = \frac{X}{X_{R0}}\left(\frac{\tau^2 - X_{R0}^2}{\tau^2 - X^2 - \xi^2 - \psi^2}\right) \frac{I_3\left(\frac{\sqrt{\tau^2 - X^2 - \xi^2 - \psi^2}}{2}\right)}{I_3\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)} \qquad (38)$$

For small X,

$$u = \frac{X}{X_{R0}}\left(\frac{\tau^2 - X_{R0}^2}{\tau^2 - X^2 - \xi^2 - \psi^2}\right) \frac{J_3\left(\frac{\sqrt{X^2 + \xi^2 + \psi^2 - \tau^2}}{2}\right)}{I_3\left(\frac{\sqrt{\tau^2 - X_{R0}^2}}{2}\right)} \qquad (39)$$

For small X,

$$u = \frac{X}{X_{R0}}\left(\frac{\tau^2}{\tau^2 - X^2 - \xi^2 - \psi^2}\right) \frac{J_3\left(\frac{\sqrt{X^2 + \xi^2 + \psi^2 - \tau^2}}{2}\right)}{I_3\left(\frac{\tau}{2}\right)} \qquad (40)$$

The solution is in terms of a Bessel composite function of the third order and first kind for a small X and a modified Bessel composite function of the third order and first kind for a large X. The first root of the Bessel function of the third order is calculated by using 17 terms of the series expansion of the Bessel function in a Pentium IV microprocessor using a Micorsoft Spreadsheet up to four decimal places. The root is found to be 6.3802.

$$½(X^2 + \xi^2 + \psi^2 - \tau^2)^{1/2} = 6.3802 \tag{41}$$

$$\text{Or} \quad X^2 + \xi^2 + \psi^2 - \tau^2 = 162.828 \tag{42}$$

When an exterior point in the infinite sphere is considered, a lag time can be calculated prior to when there is no heat transfer to that point. After the lag time, there exist two regimes. One is described by Eq. (40) and the third regime is described by Eq. (39). Thus,

$$\tau_{lag} = \sqrt{X_p^2 + \xi_p^2 + \psi_p^2 - 162.828} \tag{43}$$

III. CONCLUSIONS

Analytical solutions are presented to evaluate the ampacity risks in PCB interconnections. Earlier studies have used numerical solutions. In [1], conjugate gradient numerical analysis method is used for the non-Fourier heat conduction equation. In this study, analytical solutions are presented for the damped wave conduction and relaxation non Fourier model. Method of relativistic transformation is used. Solutions in one dimension and in three dimensions are given. The orders of the Bessel composite functions are: (i) 7/2 for the 1 dimensional case; (ii) 2 for the three dimensional case and (iii) 3 for the case with the 'creeping limit' assumption.Three regimes can be seen in each solution: (i) inertia lag regime; (ii) rising regime characterized by Bessel composite function and; (iii) rising regime characterized by modified Bessel composite function in space and time. The final condition is made to obtain physically realistic solutions. These solutions are within the framework permitted by the second law of thermodynamics. Other reports with violating the second law and overshoot occurrence can be seen due to the use of unrealistic initial accumulation condition [10]. The occurrence of solution with spatio-temporal symmetry can have implications in the special theory of relativity proposed by Sir Albert Einstein. Model can be used to prevent potential damage to predetermined excitation.

REFERENCES

[1] R. J. Murugan and S. Krishnan, "System and method to analyze and determine ampacity risks on PCB interconnections," US Patent 7 596 482 2009, Dell Products, Round Rock, TX.

[2] K. R. Sharma, "On the solution of damped wave conduction and relaxation equation in a semi-infinite medium subject to constant wall flux," Int. Journal of Heat and Mass Transfer, vol. 51, pp. 6024-6031, 2008.

[3] C. Cattaneo, "A form of heat conduction which eliminates the paradox of instantaneous propagation," Comptes Rendus, vol. 247, pp. 431-433, 1958.

[4] C. Cattaneo, Sulla Coduzione del Calone, Atti. Sem. Fis. Univ. Moderna, 3, 83, 1948.

[5] P. Vernotte, "Les paradoxes de la theorie continue de l'equation de la chaleur," C. R. Hebd. Seanc. Acad. Sci., vol. 246(22), pp. 3154-3155, 1958.

[6] D. D. Joseph and L. Preziosi, "Heat waves," Reviews of Modern Physics, vol. 61, pp. 41-73, 1989.

[7] D. D. Joseph and L. Preziosi, "Addendum to heat waves," Reviews of Modern Physics, vol. 62, 375-391, 1990.

[8] M. N. Ozisik and D. Y. Tzou, "On the wave theory of heat conduction," ASME J of Heat Transfer, vol. 116, pp. 526- 535, 1994.

[9] D. Y. Tzou, Macro to microscale heat transfer: the lagging behavior, CRC Press, New York, 1996.

[10] K. R. Sharma, Damped wave transport and relaxation, Elsevier, Amsterdam, 2005.

[11] K. Mitra, S. Kumar, A. Vedavarz and M. K. Moallemi, "Experimental evidence of hyperbolic heat conduction in processed meat," Journal of Heat Transfer, vol. 117, 568-573, 1995.

[12] Y. Taitel, "On the parabolic, hyperbolic and discrete formulation of heat conduction equation," International Journal of Heat and Mass Transfer, vol. 15(2), pp. 369-371, 1972.

[13] C. Bai and A. S. Lavine, "On hyperbolic heat conduction and second law of thermodynamics," Journal of Heat Transfer, vol. 117 (2), pp. 256-263, 1995.

[14] E. Zanchini, "Hyperbolic heat conduction theories and non-decreasing entropy," Phy. Review B-Condensed Matter and Material Physics, vol. 60(2), pp. 991-997, 1999.

[15] A. Barletta and E. Zanchini, "Thermal-wave heat conduction in a solid cylinder which undergoes a change of boundary temperature," Heat and Mass Transfer/ Warema-und Stoffuebertragung, vol. 32(4), pp. 285-291, 1997.

[16] M. A. Al-Nimr and O. M. Haddad, "The dual-phase-lag heat conduction model in thin slab under

a) Fluctuating Thermal Disturbance," Heat Transfer Engineering, vol. 24(5), pp. 47-54, 2003.

[17] M. A. Al-Nimr and M. Naji, "The hyperbolic heat conduction equation in an anisotropic material," Int. J of Thermophysics, vol. 21, pp. 281-287, 2000.

[18] A. M. Haji-Sheik, W. J. Minkowycz, and E. M. Sparrow, J. Heat Transfer, vol. 124(2), pp. 307-319, 2002.

[19] K. R. Sharma, "Is overshoot a mathematical artefact," JP Journal of Heat and Mass Transfer, vol. 7(1), pp. 35-64, 2013.

[20] K. R. Sharma, "On analytical solutions during damped wave conduction and relaxation in a finite slab subject to the convective boundary condition," International Journal of Thermophysics, vol. 31(2), pp. 430-443, 2010.

[21] K. R. Sharma, "Damped wave conduction and relaxation in cylindrical and spherical coordinates," Journal of Thermophysics and Heat Transfer, vol. 21(4), pp. 688-693, 2007.

[22] K. R. Sharma, “Manifestation of acceleration during transient heat conduction,” Journal of Thermophysics and Heat Transfer, vol. 20(4), pp. 799-808, 2006.
[23] K. R. Sharma, “A fourth mode of heat transfer called damped wave conduction,” 42nd Annual Convention of Chemists Meeting, Santiniketan, India, February, 2006.
[24] K. R. Sharma and R. Turton, “Mesoscopic approach to correlate surface heat transfer coefficients with pressure fluctuations in gas-solid fluidized beds,” Powder Technology, vol. 99(2), pp. 109-118, 1998.
[25] K. R. Sharma, “On the derivation of an expression for relaxation time from stokes-einstein relation,” 233rd
a) ACS National Meeting, Chicago, IL, March 2007, American Chemical Society, Washington, DC.
[26] P. J. Antaki, “Solution for non-fourier dual phase lag heat conduction in a semi-infinite slab with surface heat flux,” International Journal of Heat and Mass Transfer, vol. 41(14), pp. 2253-2258, 1998.