Analysis of Vibration Characteristics and Its Influencing Factors of Face-Gear Driving System

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Abstract- **Considering meshing gear backlashes, time-varying mesh stiffness, transmission errors, and ignoring influences of twist-oscillecting vibration on face-gear driving system and systematic axial errors, this paper builds the nonlinear dynamic bending-torsional coupling model of the face-gear drive. And nonlinear dynamic theory is used to analyze the vibration characteristic of face-gear driving system. Besides, some analyses are conducted on influences of different parameters such as modulus, pressure angle, tooth difference, rotational speed and torque on the time domain and frequency domain of face-gear displacement, velocity, acceleration, angular velocity and angular acceleration, in an attempt to find out the influence degree of these parameters on the vibration characteristics of face-gear driving system, making preparations for high quality face-gear machining.**

Keywords- Face-Gear Driving System, Dynamic Model, Vibration Characteristic, Dynamic Analysis

In the early 1990s, with the development of correlative technology, high-quality face-gear drive is applied in the split - junction flow transmission of the main decelerator of armed helicopters, which could reduce about 40% of the total mass for the main decelerator of armed helicopters, and the bearing capacity could increase about 35%. So the unique split-flow features of face-gear drive draws the attention of people [1-13]. In the face-gear drive, the stress features of cylindrical gear meshing with a bevel gear are different from general cylindrical gear drive or bevel gear drive. The three-dimensional space model of the orthogonal face-gear driving system, which is a dynamic model with multi-freedom bending-torsional-axial shifting-pendular coupling vibration, was built by Jin Guanghu et al. Some digital simulations, in which the dynamic responses of face-gear driving system were obtained, were conducted on the dynamic response of face-gear driving system by Yang Zhen et al. by use of vibration theory and ANSYS software based on the Wilson-θ method. Besides, finite element modal analyses were conducted on 3D solid model, and influences of tooth number difference, contact position and pressure angle on mode were analyzed.

The nonlinear dynamic bending-torsional coupling model of the face-gear drive was built, through which we analyzed the influences of different parameters such as modulus, pressure angle, tooth difference, rotational speed and torque on the time domain and frequency domain of the parameters such as displacement, velocity, acceleration, angular velocity and angular acceleration of the face-gear and cylindrical gear, trying to find out the influence degree of these parameters on the vibration characteristics of face-gear driving system, and making preparations for high quality face-gear machining.

I. VIBRATION ANALYSIS OF FACE-GEAR DRIVING SYSTEM

A. The vibration model of face-gear driving system

For the cylindrical spur gear meshing with a face-gear, on the basis of the principle of action & reaction, the face-gear does not have radial force because the cylindrical spur gear has no axial force. And the normal force between the gear pairs is

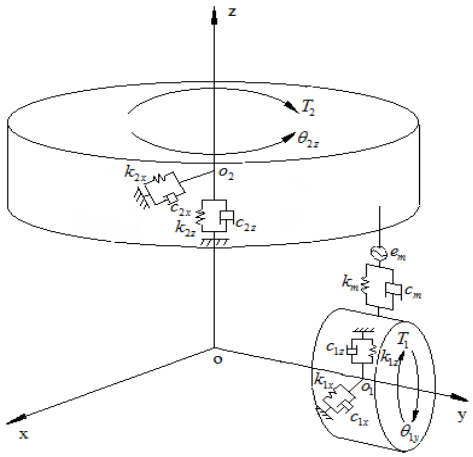

Fig. 1 the nonlinear dynamic model for orthogonal face-gear driving system

tangent with the base circle of the cylindrical spur gear. Besides, the influence of twist-oscillecting vibration on face-gear driving system is lesser, and face-gear driving system is insensitive to axial tolerances. For the convenience of problem analysis, twist-oscillecting vibration and axial errors are both neglected here.

The dynamic model of face-gear driving system under elastic supporting condition was built based on the theory of face-gear drive, as illustrated in Fig. 1. A global coordinate system was built with the intersection of two gears' perpendicular axes as coordinate origin, the cylindrical gear's axis as X-axis, and the face-gear's axis as Y-axis. In Fig. 1, k is stiffness coefficient, c is damping coefficient, T1 and T2 are the torques brought to bear on the cylindrical gear and face-gear respectively. The cylindrical gear and face-gear are regarded as mass blocks with rotation inertia, and supporting shafts are equivalent as elastic supports at the heart chakras of two gears by simulating elastic supports with springs and dampers.

B. Oscillatory differential equation of face-gear driving system

The Relative Displacement caused by Vibration and Errors between engaging points of the face-gear and cylindrical gear in normal direction is given by:

$$X_n = \left(X_1 - X_2 + r_{b1}\theta_{1y} - r_m\theta_{2z}\right)c_1 + \left(Z_1 - Z_2\right)c_2 - e_n(t) \tag{1}$$

Where $c_1 = \cos\alpha_n, c_2 = \sin\alpha_n$, r_{b1} is base circle radius of the cylindrical gear, r_m is the face-gear's nominal radius, α_n is the pressure angle on the gear meshing point, and $e_n(t)$ is general transmission errors of gear pairs.

$$e_n(t) = e_0 + A_e \sin\left(\omega_h t + \varphi_e\right) \tag{2}$$

Where e_0 is static constant transmission error in normal direction, A_e is the amplitude of the variable for static transmission error in normal direction, ω_h is meshing frequency of gear pairs, φ_e is initial phase.

Then the dynamic load in the direction of meshing line and component force on each coordinate axis of gear pairs are shown as follows.

$$\begin{cases} F_n = k(t)f(X_n) + c_m \dot{X}_n \\ F_x = F_n c_1 \\ F_z = F_n c_2 \end{cases} \tag{3}$$

In which $k(t)$ is the time-varying mesh stiffness of gear pairs; c_m is meshing damping: $f(X_n)$ is gap function, where $k(t)$、$f(X_n)$ can be expressed as follow:

$$k(t) = k_m + A_k \cos\left(\omega_h t + \varphi_k\right) \tag{4}$$

$$f(X_n) = \begin{cases} X_n(t) - b, X_n(t) > b \\ 0, -b \le X_n(t) \le b \\ X_n(t) + b, X_n < -b \end{cases} \tag{5}$$

Where k_m is the average value of time-varying mesh stiffness, A_k is the fluctuating amplitude of meshing stiffness, φ_k is the initial phase of time-varying mesh stiffness, b is half of the average meshing backlash in normal direction. Then, the vibration equations of face-gear driving system can be shown as follow:

$$\begin{cases} m_1 \ddot{X} + c_{1x}\dot{X}_1 + k_{1x}X_1 = -F_x \\ m_1 \ddot{Z}_1 + c_{1z}\dot{Z}_1 + k_{1z}Z_1 = -F_z \\ I_{1y}\ddot{\theta}_{1y} = T_1 - F_n r_{b1} \\ m_2 \ddot{X}_2 + c_{2x}\dot{X}_2 + k_{2x}X_2 = F_x \\ m_2 \ddot{Z}_2 + c_{2z}\dot{Z}_2 + k_{2z}Z_2 = F_z \\ I_{2z}\ddot{\theta}_{2z} = -T_2 + F_n r_m \end{cases} \tag{6}$$

In which m_1 and m_2 are the lumped mass of cylindrical gear and face-gear respectively, I_{1y} and I_{2z} are rotary inertia of cylindrical gear and face-gear respectively, k_{ij} and c_{ij} ($i=1,2; j=x,z$) are the support stiffness and damping in x axis orientation and in z axis orientation respectively, T_1 is the driving torque of drive wheel (cylindrical gear), T_2 is the resistance torque of driven wheel (face-gear).

The expression (6) is a six degree of freedom nonlinear second-order differential equation system with positive Semi-definiteness and variable parameters. In order to eliminate systematic rigid displacement, we take the normal relative displacements X_n of meshing points as a new freedom degree, and combine twisting vibration equations of the tow gears in equation system (6), the result is shown as follow.

$$-m_e c_1 \ddot{X}_1 - m_e c_2 \ddot{Z}_1 + m_e c_1 \ddot{X}_2 + m_e c_2 \ddot{Z}_2 + m_e \ddot{X}_n + c_1 k_h(t) f(X_n) + c_1 c_m \dot{X}_n = c_1 F + m_e \ddot{e}_n(t) \tag{7}$$

Where the equivalent mass of gear pair is $m_e = \dfrac{I_{1y} I_{2z}}{I_{1y} r_m^2 + I_{2z} r_{b1}^2}$, the load of gears are $F = \dfrac{T_1}{r_{b1}} = \dfrac{T_2}{r_m}$.

Simplify the equation system (7) to a non-dimensional equation system as follows.

$$\begin{cases} \ddot{x}_1 + 2\xi_{1x} \dot{x}_1 + 2c_1\xi_{1m} \dot{x}_n + k_{1x} x_1 + c_1 k_{1m} f(x_n) = 0 \\ \ddot{z}_1 + 2\xi_{1z} \dot{z}_1 + 2c_2\xi_{1m} \dot{x}_n + k_{1z} z_1 + c_2 k_{1m} f(x_n) = 0 \\ \ddot{x}_2 + 2\xi_{2x} \dot{x}_2 - 2c_1\xi_{2m} \dot{x}_n + k_{2x} x_2 - c_1 k_{2m} f(x_n) = 0 \\ \ddot{z}_2 + 2\xi_{2z} \dot{z}_2 - 2c_2\xi_{2m} \dot{x}_n + k_{2z} z_2 - c_2 k_{2m} f(x_n) = 0 \\ \ddot{x}_n - c_1 \ddot{x}_1 - c_2 \ddot{z}_1 + c_1 \ddot{x}_2 + c_2 \ddot{z}_2 + c_1 k(\tau) f(x_n) + 2c_1\xi_m \dot{x}_n = f + f_e \end{cases} \tag{8}$$

where $x_i = \dfrac{X_i}{b} : z_i = \dfrac{Z_i}{b} : \omega_n = \sqrt{\dfrac{k_m}{m_e}} : \omega_{ij} = \sqrt{\dfrac{k_{ij}}{m_i}} : \tau = \omega_n t : \Omega_h = \dfrac{\omega_h}{\omega_n} : \xi_{ij} = \dfrac{c_{ij}}{2m_i\omega_n} : \xi_{im} = \dfrac{c_m}{2m_i\omega_n} : \xi_m = \dfrac{c_m}{2m_e\omega_n} : k_{ij} = \dfrac{\omega_{ij}^2}{\omega_n^2} :$

$k_{im} = \dfrac{m_e}{m_i} k(\tau) : k(\tau) = \dfrac{k_h(t)}{k_m} = \dfrac{k_m}{m_e b \omega_n^2} + a_k \dfrac{k_m}{m_e b \omega_n^2} \cos(\Omega_h \tau + \varphi_k) : a_k = \dfrac{A_k}{k_m} : f(x_n) = \dfrac{f(X_n)}{b} = \begin{cases} x_n - 1, x_n > 1 \\ 0, -1 \le x_n \le 1 \\ x_n + 1, x_n < -1 \end{cases} : f = \dfrac{c_1 F}{b m_e \omega_n^2}$

$: f_e = \dfrac{A_e \Omega_h^2}{b} \cos(\Omega_h \tau + \varphi_e) : \quad i = 1,2; j = x, z$.

II. DYNAMIC ANALYSIS OF FACE-GEAR DRIVING SYSTEM

A. *Model building of face-gear driving*

Face-gear driving Model is built by using three-dimensional modeling software. The parameters are: modulus is 7, pressure angle is 20°, face-gear tooth number is 60 (tooth number of the gear cutter is 20) , the tooth number of cylindrical gear is 17. The model building of face-gear driving is shown in Fig. 2. The model is saved as a file with x_t format and imported into ADAMS, modifying the material of gears as steel, adding a revolute pair between face-gear and the ground at the face-gear center, as well as the cylindrical gear, adding a contact pair between the two gears, and adding a driving on the revolute pair of cylindrical gear, which is shown in Fig. 3.

B. *Dynamic analysis of face-gear drive*

(1) Analysis on face-gear displacement

Simulation step is set to 500 and simulation time is set to 5s. The driving speed of cylindrical gear is 1000r/min. A dynamic simulation is performed without loads on two gears, and then parameters setting and simulations are conducted. The parameters setting about the simulations of the model are as follows. In CONTACT—l, damping is 10.0, dynamic coefficient is 0.1, material stiffness is 1.0xl05N/m, and the Force Exponent for calculating the contribution value of material stiffness in the instancy is 2.2, static coefficient is 0.3. The simulation result is illustrated in Fig. 4.

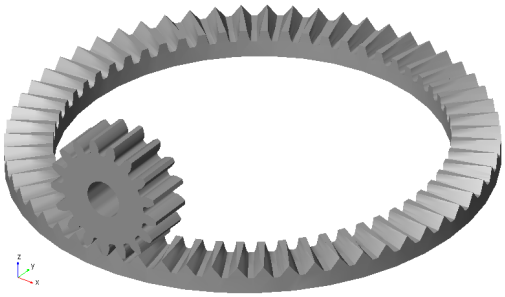

Fig. 2 Face-gear driving model

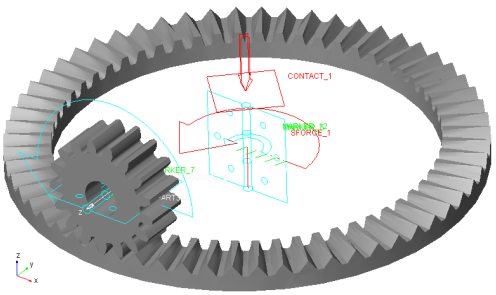

Fig. 3 Face-gear driving model with constraints

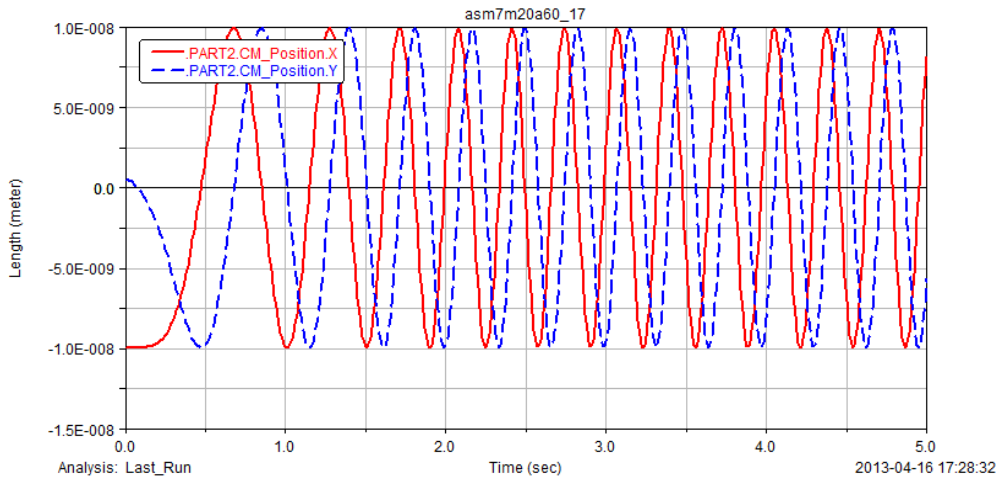

Fig. 4 displacement diagram of face-gear in x and y axis orientation

From Fig. 4, it is shown that face-gear displacements in x and y axis orientation vary by sine curve or cosine curve after speed stabilizing. The amplitude of displacements is about 9.93E-009m,and the phase difference of face-gear displacements in x and y axis orientation is about 1/4 period.

(2) Analyses on face-gear rotation speed

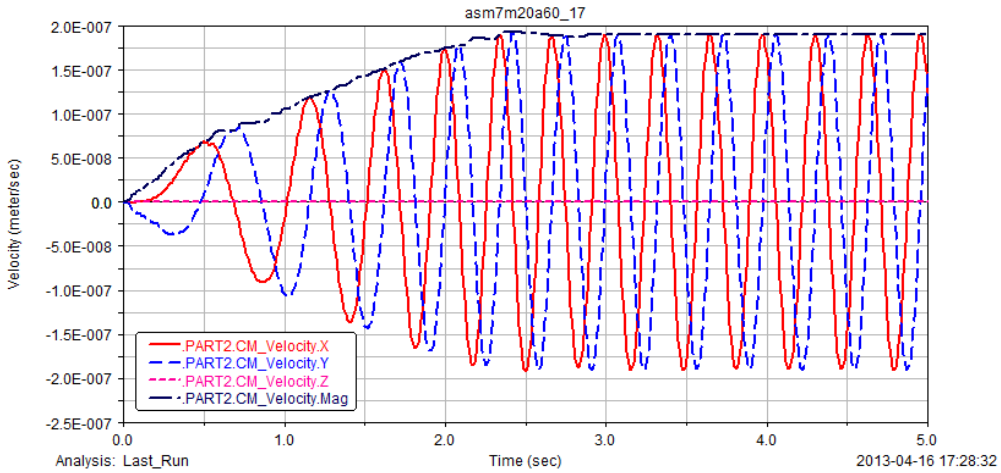

Fig. 5 The rotation speed diagram of the face-gear

From what is shown in Fig. 5, it can be known that face-gear rotation speed reaches stability when the time is about 3s. The velocity curves vary by sine curve or cosine curve in x and y axis orientation, their amplitude is about 1.91E-007m, and their phase difference in x and y axis orientation is about 1/4 period. The speed in z axis orientation is always 0.

(3) Analyses on face-gear acceleration

Fig. 6 shows that face-gear acceleration reaches stability when the time is about 3s. The acceleration curves vary by sine curve or cosine curve in x and y axis orientation, their amplitude is about 4.66E-006m, absolute value is about 4.64E-006m/s2, and their phase difference in x and y axis orientation is about 1/4 period. The speed in z axis orientation is always 0. Besides,

Fig. 7 reveals that the acceleration reaches the maximum value when frequency is about 0.

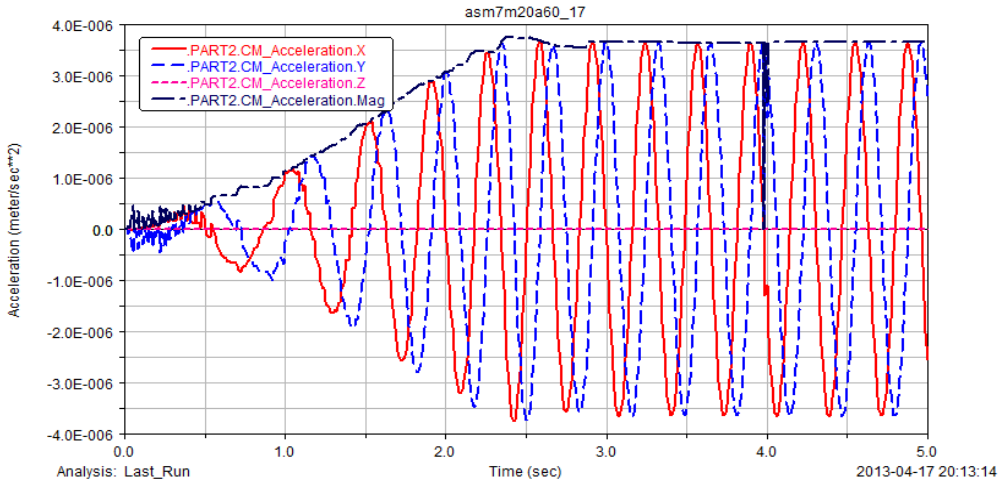

Fig. 6 The acceleration diagram of the face-gear

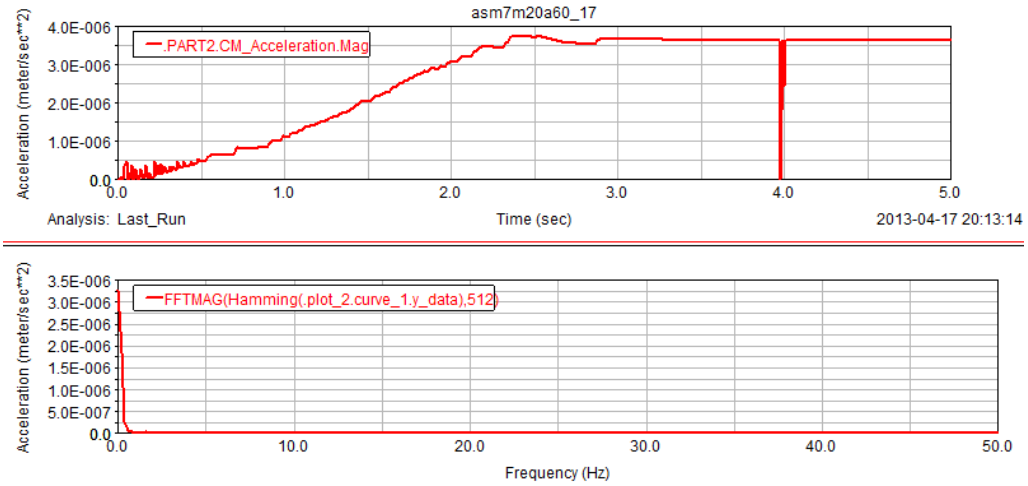

Fig. 7 The time-frequency figure of face-gear absolute acceleration

(4) Analysis of angle acceleration of face-gear

As shown in Fig. 8, angular velocity of face-gear reaches stable when the time is about 3s. The angular velocity in x and y axis orientation is always 0, and absolute angular velocity is equal to the z-axis angular velocity, 1100.08deg/sec.

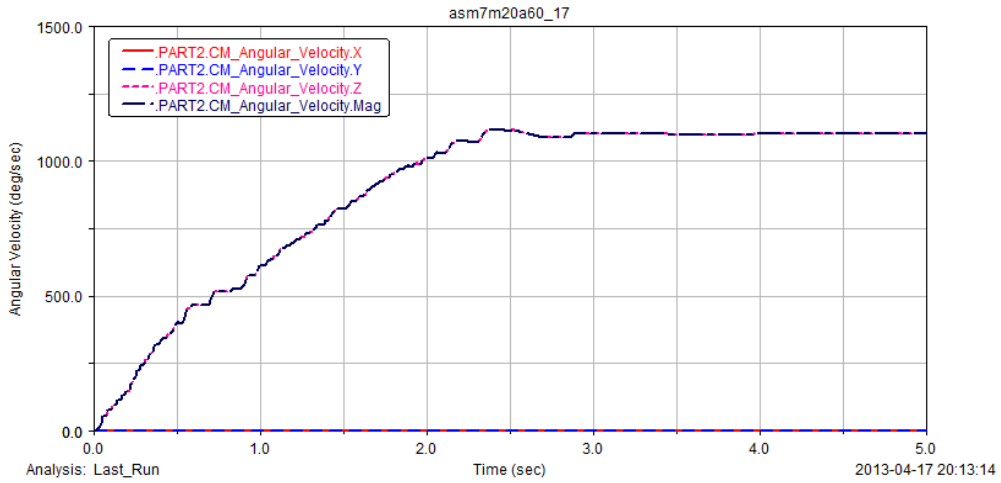

Fig. 8 The angle velocity diagram of the face-gear

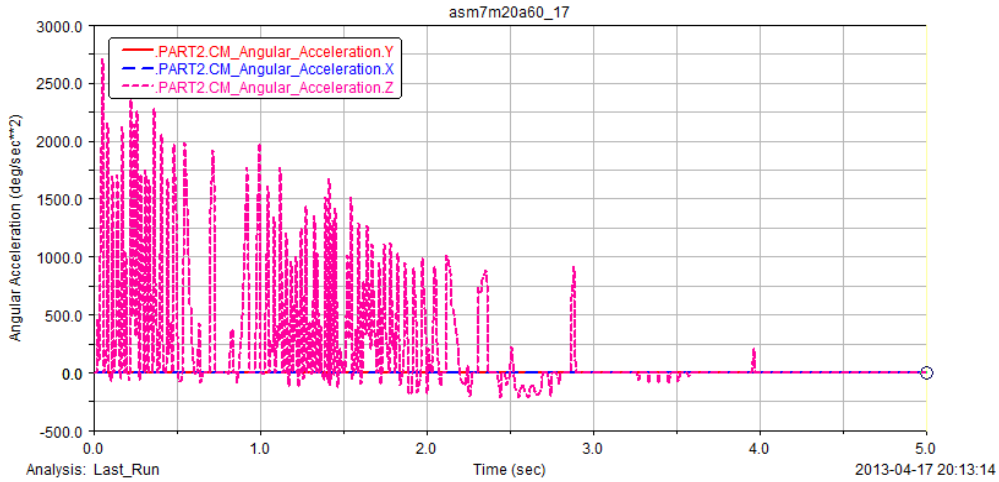

Fig. 9 The angular acceleration diagram of the face-gear

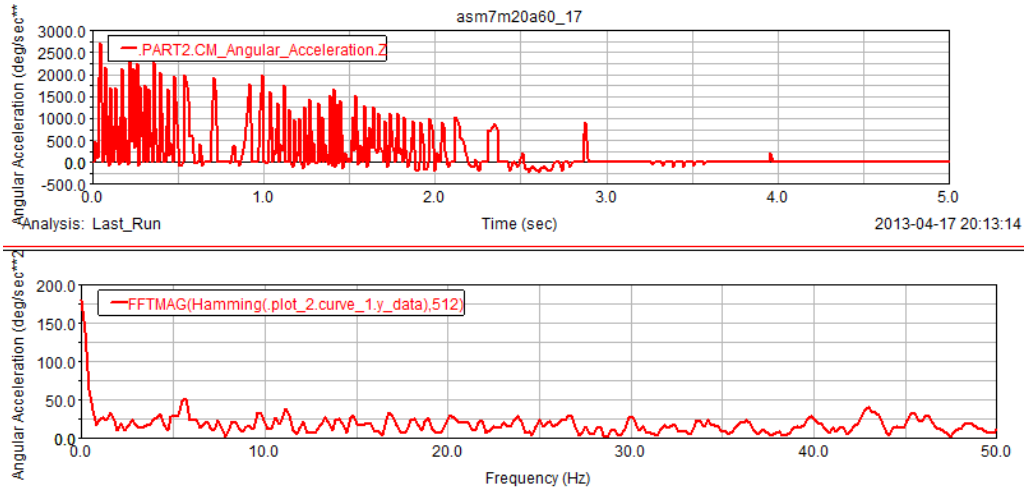

Fig. 10 the time-frequency figure of face-gear angle acceleration

As shown in Fig. 9, the face-gear angle acceleration in x and y axes orientation is 0. However, the z-axis angular acceleration is not 0 before 3s. After 3s, the angular acceleration becomes stable with a value of 0. Fig. 10 is the time-frequency Fig. of face-gear angle acceleration in z axis orientation, showing that angle velocity reaches the maximum when frequency is about 0, and the angle velocity in other time segment undulates in a small range. As shown in Fig. 11, the coordinate value of the geometric center point of the cylindrical gear is as follows: x=-1.02E-008mm, y=-0.2275mm, z=4.24E-008mm, the general coordinate location is 0.2275mm, which correspond with the setting of JOINT1. The velocity and acceleration of the cylindrical gear in three directions are both 0 without velocity fluctuation, which corresponds with the ideal condition.

III. ANALYSIS OF THE MAIN FACTORS WHICH INFLUENCE FACE-GEAR VIBRATION CHARACTERISTIC

In face-gear drive, there are many factors influencing face-gear vibration characteristic. And different factors have different influence degree. The research and analysis on some main factors are of great significance to improve efficiency of face-gear drive. By analyzing the absolute acceleration of face-gear, we analyze the influence of major parameters to the face-gear vibration characteristic under the different external conditions.

A. The influence of modulus on the face-gear vibration characteristic

The face-gears are built with modulus 3, 4, 5, 6, 7, 8 while other parameters remain unchanged. Their analysis results in Fig. 11 show that face-gear absolute acceleration decreases in proper order. The face-gear absolute acceleration value is larger when the modulus value is 3, 4 and 8. It shows that when the tooth number is a certain constant and in a proper range, a larger modulus will lead to a smaller the face-gear vibration; in addition, appropriate modulus plays an obvious role in decreasing face-gear vibration. So choosing appropriate modulus plays a non-ignorable role in decreasing the vibration of face-gear driving system and improving the reliability of face-gear driving system before manufacturing face-gear. In fact, there are many various factors, such as the load of the face-gear bearing, the space limitation of face-gear drive, acceptable manufacturing cost of face-gear, which should be taken into account to choose a suitable modulus.

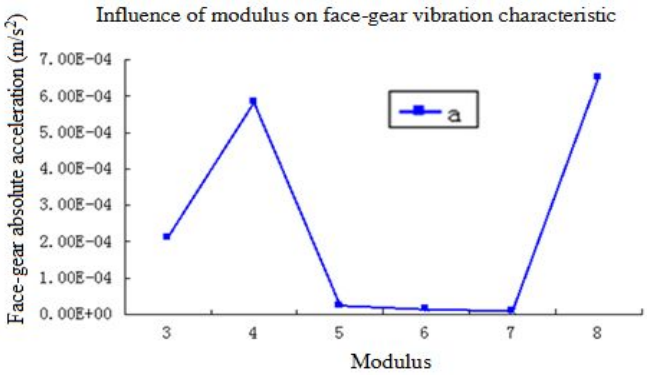

Fig 11 Influence of modulus on face-gear vibration characteristic

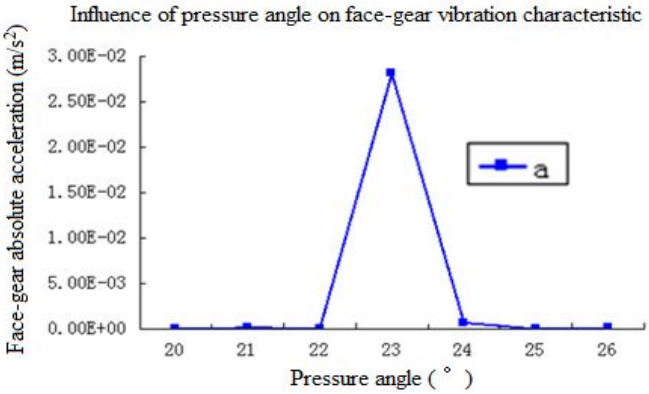

Fig. 12 Influence of pressure angle on face-gear vibration characteristic

B. Influence of pressure angle to face-gear vibration characteristic

Face-gears are built at the pressure angles of 20°, 21°, 22°, 23°, 24°, 25°, 26°without changing other parameters. After accomplishing and assembling face-gear model, it is simulated in ADAMS to detect and analyze absolute acceleration of the face-gear in each group of face-gear driving system. Analysis results shown in Fig. 12 reveal that face-gear absolute acceleration is much smaller when the pressure angle is 20°and 25°; and face-gear absolute acceleration value of 23°is obviously larger than that of other degrees. Therefore, pressure angle 20°or 25°is more appropriate for face-gear. But the selection of pressure angle should also consider other requirements of face-gear drive.

C. Influence of tooth difference on face-gear vibration characteristic

Face-gears are built with the tooth number of cylindrical gear 17, 18, 19 under the condition of keeping other parameters unchanged. Analysis results are clearly shown in Fig. 13: face-gear absolute acceleration changes obviously and regularly, and decreasing linearly with the increase of tooth number approximately. Face-gear absolute acceleration value reaches the minimum when the tooth number of cylindrical gear is 17. That means the face-gear vibration amplitude is the minimum. That is to say, the tooth difference of 3 may be appropriate to decrease vibration of face-gear driving system when selecting a cylindrical gear to assemble a face-gear.

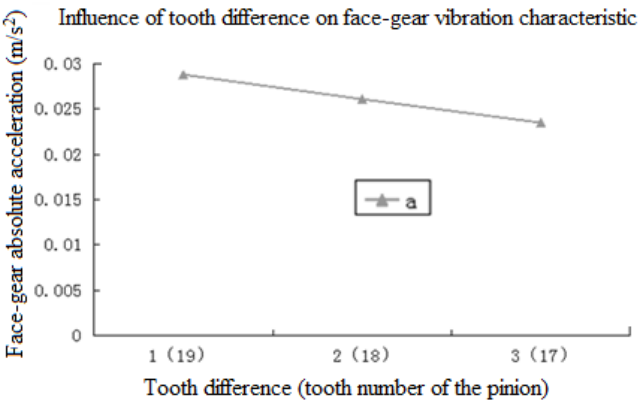

Fig 13 Influence of tooth difference on face-gear vibration characteristic

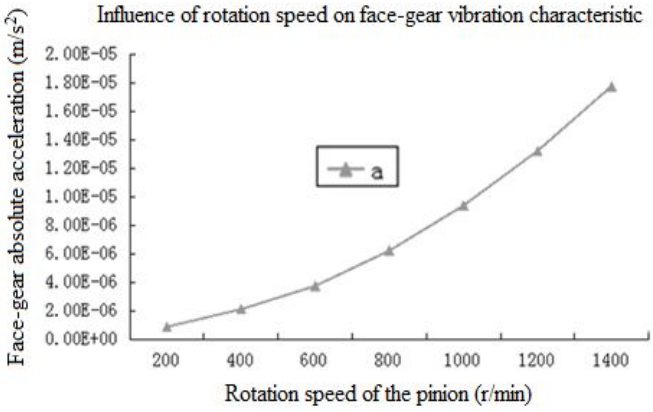

Fig 14 Influence of rotation speed on face-gear vibration characteristic

D. Influence of rotation speed on face-gear vibration characteristic

Analysis results are shown clearly in Fig. 14. The influence of rotation speed on face-gear driving system is complicated. Normally it shows that face-gear absolute acceleration value increases in a smooth curve with the increase of the rotation speed of the input cylindrical gear shaft. That is to say, the vibration response increases. The influence of rotation speed on face-gear vibration characteristic is obvious and regular. The higher rotation speed is, the stronger the face-gear vibration will be. In fact, rotation speed selecting should be combined with practical status, which is significant to controlling the vibration of face-gear driving system.

E. Influence of torque on face-gear vibration characteristic

Analysis results are shown clearly in Fig. 15. The load of face-gear usually is supplied by the form of torque in face-gear driving system. The magnitude of torque plays an important role in the vibration of face-gear driving system. In Fig. 15, the increase of face-gear absolute acceleration would lead to a larger vibration of face-gear driving system, with the increase of the torque supplied on face-gear. That would seriously influence the service life of face-gear. Therefore, in the practical application, the suitable torque supplied on face-gear should be selected in combination with other vibrating factors and actual situation, in order to decrease the vibration of face-gear driving system and prolong the service life of face-gear.

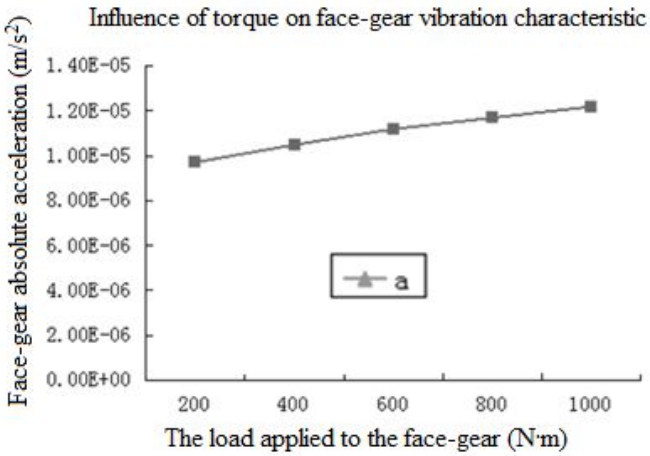

Fig 15 Influence of torque on face-gear vibration characteristic

There are many factors which influence vibration characteristic of face-gear driving system. Different factors lead to various influence result, and even the same factor may cause different effect under different external environmental conditions. Therefore, in the process of decreasing the vibration of face-gear driving system, the practical situation and the most economic cost should be fully considered to control the vibration of face-gear driving system within the acceptable extent.

IV. CONCLUSIONS

By utilizing ADAMS to dynamically simulate the transmission of face-gear, this paper analyzes the influence of factors, such as tooth difference, pressure angle, modulus, rotation speed and load on the face-gear vibration characteristic, from which some parameters beneficial to face-gear drive are obtained.

Considering the backlash of gear pairs meshing, the time-varying mesh stiffness and transmission error stimulation, and neglecting the influence of twist-oscillecting vibration on the system and systemic axial errors, the oscillatory differential equation of face-gear driving system is solved. The changing amplitude of the displacement, velocity, acceleration, angular velocity and angular acceleration of face-gear is quite big; especially the variation of face-gear angular acceleration is the most obvious in face-gear driving system. When the modulus is a certain constant, the larger the modulus is, the smaller the vibration of face-gear driving system will be. Furthermore, the vibration of face-gear driving system reaches the maximum at the pressure angle of 23°,which is obviously superior than that at 20°or 25°. In practical application, pressure angle 20°or 25°is selected. Besides, it can decrease the systematic vibration properly to select a cylindrical gear with tooth number less than the gear cutter by 1~3 to assemble the face-gear. The tooth difference of 3 is optimal for decreasing systematic vibration. In addition, with the increase of rotation speed, the systematic vibration increases and the impact load born by the system also increases. The systematic vibration increases too. Finally, the face-gear absolute acceleration will increase with the increase of the load torque of face-gear. That is to say, the vibration of face-gear driving system will increase.

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