# Charged Fields in a Whole Abelian Model

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***Abstract*- Observing that nature also works as a group, a reinterpretation for field theory is given by taking the fields set $\{G_\mu^I\}$ as origin. Based on this assumption a whole abelian gauge theory is developed. It includes the usual abelian case and incorporates new structures as non-linearity and renormalizable mass without requiring spontaneous symmetry breaking. A next step for this systemic symmetry is to introduce an abelian internal structure. To propose an invariant action under the transformation law $G_\mu^{I\,\prime} = A_J^I G_\mu^J + k_I \partial_\mu \alpha$ where $A_J^I$ means a generic fields rotation matrix. This motivates to investigate on $A_J^I$ possibilities, understand on its possible physicities, as consider SO(N) symmetry and introduce charged fields through SO(2) symmetry. So given such systemic gauge symmetry based on a common gauge parameter, this work builds up a systemic abelian pattern of type $U(1)_{local} \times SO(2)_{global}$ for embracing $\gamma$, $Z^0$, $W^+$, $W^-$ or charged particles like that.**

***Keywords*- Whole Abelian Model; Unification; Charged Fields**

## I. INTRODUCTION

Unification has been the primary motivation for doing physics. Physics have been guided by this concept. Under such theme we have been opening space for doing research in physics. Great achievements have been done by this unification need mainly through electromagnetism and relativity. Under this compromise, nowadays physics is proposing the so-called Theory of Everything which is firstly proposed by Oscar Klein in 1939 [1-3].

Our viewpoint is that before adopting great lines of research through unification concept, one should focus on the meaning of parts. Instead of instantaneously looking to the Theory of Everything, the greatest moment of the reductionist view, there is another approach for making particles get together. Complexity and confinement are showing on possibilities for an antireductionist view being implemented. They are saying that physics is not necessarily ruled by isolated parts.

Thus, there are two ways to involve the parts. There are two distinct methods for particles comprehension: the reductionist and the antireductionist. In the first one, they are defined in terms of themselves, following the building block approach from molecules, protons to quarks. In the second one, inversely, following the totality arrangement, that is, instead of being constituted by quarks, parts that are derived as functions of a determined whole. It appears the principle of wholeness [4]. It says that given a group of particles there is something more than simple interactions to be considered. It rules that the phenomena happen in terms of systemic behaviours.

There is still something on the parts meaning to be understood. Following such second parts approach, there is a systemic physics to be derived from gauge symmetry. For this, one takes a fields set $\{G_\mu^I\}$ as origin. Then, taking such wholeness principle, our initial effort has being to develop the so-called whole abelian gauge theory [5, 6]. It proposes as starting point a fields set $\{G_\mu^I\}$ transforming as

$$G_\mu^I \to G_\mu^{I\,\prime} = G_\mu^I + (\Omega^{-1})_1^I \partial_\mu \alpha, \tag{1}$$

where $\alpha(x)$ means a systemic gauge parameter. Notice that every field transformation is specified by a weight $(\Omega^{-1})_1^I$ factor, where I varies from 1 to N.

Eq. (1) shows another physics approach. It expresses that there is also a nature manifestation which is to be understood in systemic terms. Differently from the reductionist meaning of unification, it does not depend on the ultimate elementary particles. It says that the origin is in the group. Our emphasis is that confinement and complexity support this reason. This is because while one does not allow the ultimate matter to be observed experimentally, the other shows a physics behaviour does not depend on individual properties.

Thus, given Eq. (1), firstly, it is basic to demonstrate how this systemic gauge model contains gauge invariance. For this, the simplest procedure is to derive it from the constructor basis $\{D_\mu, X_\mu^i\}$ written at Appendix A. Taking the $\Omega$ matrix rotation, which means that one diagonalizes the transverse sector, one gets

$$D_\mu = \Omega_I^1 G_\mu^I, \quad X_\mu^i = \Omega_I^i G_\mu^I \tag{2}$$

where the physical fields $\{G_\mu^I\}$ are a diagonalized spin 1 sector [5], [6]. Notice that the invertibility condition

$$\Omega_K^I (\Omega^{-1})_J^K = \delta_{IJ}, \tag{3}$$

is enough for proving on the model gauge invariance.

The corresponding field strengths are

$$G_{\mu\nu}^I = \partial_\mu G_\nu^I - \partial_\nu G_\mu^I,\ S_{\mu\nu}^I = \partial_\mu G_\nu^I + \partial_\nu G_\mu^I,\ z_{[\mu\nu]} = \gamma_{[IJ]} G_\mu^I G_\nu^J,\ z_{(\mu\nu)} = \gamma_{(IJ)} G_\mu^I G_\nu^J,\ \omega_{\mu\nu} = \tau_{(IJ)} G_\mu^I G_\nu^J. \quad (4)$$

Obviously, the $S_{\mu\nu}^I$ tensor is not gauge invariant, however the combination $a_I S_{\mu\nu}^I$ is. Similarly from Eq. (3), $z_{\mu\nu}$ tensors are gauge invariants.

Thus one derives the following gauge invariant Lagrangian written in terms of the physical fields $G_\mu^I$

$$L(G) = L_0 + L_I, \quad (5)$$

where $L_0 = L_K + L_m + L_{GF}$, $L_I = L_3 + L_4$.

The kinetic term can be expressed as the sum of its transversal and longitudinal parts, $L_K = L_K^A + L_K^S$. Then, the kinetic sector is given by

$$L_K^A = a_I G_{\mu\nu}^I G^{\mu\nu I},\ \ L_K^S = b_{(IJ)} S_{\mu\nu}^I S^{\mu\nu I} + c_{(IJ)} S_\alpha^{\alpha I} S_\beta^{\beta J}, \quad (6)$$

where $a_I$ can be arbitrary and $b_{(IJ)}$, $c_{(IJ)}$ must have, for attain gauge invariance, the following forms:

$$b_{(IJ)} = d_{(ij)} \Omega_I^i \Omega_J^j,\ \ c_{(IJ)} = e_{(ij)} \Omega_I^i \Omega_J^j, \quad (7)$$

where $d_{(ij)}$, $e_{(ij)}$ are free coefficients stipulated at constructor basis $\{D_\mu, X_\mu^i\}$ as in (A2). Taking $\Omega$ matrix invertibility condition notice as the term $b_{(IJ)} S_{\mu\nu}^I S^{\mu\nu I}$ is gauge invariant.

The gauge invariant mass term is

$$L_m = m_{(IJ)}^2 G_\mu^I G^{\mu I}, \quad (8)$$

where $m_{(IJ)}^2$ takes the form

$$m_{(IJ)}^2 = m_{(ij)}^2 \Omega_I^i \Omega_J^j, \quad (9)$$

which shows its gauge invariance from Eq. (3).

The interaction sector is given by

$$L_3 = a_{I[JK]} G_{\mu\nu}^I G^{\mu J} G^{\nu K} + b_{I(JK)} S_{\mu\nu}^I G_\mu^J G^{\nu K} + c_{I(JK)} S_\mu^{\mu I} G_\nu^J G^{\nu K}, \quad (10)$$

where

$$a_{I[JK]} = \tfrac{1}{2}\{a_{[jk]} \Omega_I^1 \Omega_J^j \Omega_K^k + a_{i[jk]} \Omega_I^i \Omega_J^j \Omega_K^k\},\ \ a_{I(JK)} = \tfrac{1}{2} a_{i(jk)} \Omega_i^i \Omega_J^j \Omega_k^k,\ \ c_{I(JK)} = \tfrac{1}{2} b_{i(jk)} \Omega_I^i \Omega_J^j \Omega_K^k, \quad (11)$$

and

$$L_4 = a_{IJKL} G_\mu^I G_\nu^J G^{\mu K} G^{\nu L}, \quad (12)$$

$a_{IJKL}$ must have the following form, to satisfy gauge invariance:

$$a_{IJKL} = a_{ijkl} \Omega_I^i \Omega_J^j \Omega_K^k \Omega_L^l, \quad (13)$$

where the free coefficients $a_{ijkl}$ have the property: $a_{ijkl} = a_{jilk} = a_{klij}$, and, consequently, $a_{IJKL} = a_{JILK} = a_{KLIJ}$.

A second way to write the physical Lagrangian L(G) is directly in terms of their field strengths. It gives

$$\begin{aligned} L(G,S,z,\omega) = {} & a_I G_{\mu\nu}^I G^{\mu\nu I} + b_{(IJ)} S_{\mu\nu}^I S^{\mu\nu I} + c_{(IJ)} S_\alpha^{\alpha I} S_\beta^{\beta J} + d_I G_{\mu\nu}^I z^{[\mu\nu]} + e_I S_{\mu\nu}^I z^{(\mu\nu)} \\ & + f_I S_\alpha^{\alpha I} z_{(\beta}^{\ \beta)} + g_I S_\alpha^{\alpha I} \omega_{(\beta}^{\ \beta)} + z_{[\mu\nu]} z^{[\mu\nu]} + z_{(\mu\nu)} z^{(\mu\nu)} + 2 z_{(\mu}^{\ \mu)} \omega_{(\nu}^{\ \nu)} + 4 \omega_{(\mu}^{\ \mu)} \omega_{(\nu}^{\ \nu)}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} a_{I[JK]} &= d_I \gamma_{[JK]},\ \ b_{I(JK)} = e_I \gamma_{(JK)},\ \ c_{I(JK)} = f_I \gamma_{(JK)} + g_I \tau_{(JK)}, \\ a_{IJKL} &= \gamma_{[IJ]} \gamma_{[KL]} + \gamma_{(IJ)} \gamma_{(KL)} + 2\gamma_{(IK)} \tau_{(JL)} + 4\tau_{(IK)} \tau_{(JL)}. \end{aligned} \quad (15)$$

L(G) introduces new aspects. It produces a non-linear abelian gauge model [5, 6] and a gauge invariant mass term preserving renormalizability [7, 8]. Consequently, it develops a model alternative to Born-Infeld [9] and to spontaneous symmetry breaking [10-13].

## II. ABELIAN INTERNAL STRUCTURE

L(G) has developed a non-linear whole abelian model. It works as the first bridge between usual abelian and non-abelian models. The second one should provide them with an abelian internal structure. It is built up by the following gauge transformation

$$G_{\mu}^{I\,\prime} = A_{IJ}\,G_{\mu}^{J} + k_I\partial_\mu\alpha, \tag{16}$$

where $A_{IJ}$ means a generic rotation matrix between flavours fields and $k_I \equiv (\Omega^{-1})_1^I$ correspond to their local systemic transformation. $A_{IJ}$ represents a global symmetry which is expected to be added to Eq. (1). It can depend on the gauge parameter or not. Its importance is on incorporating internal properties on this systemic model. Provide new associative relationships. Our effort here is to unify $A_{IJ}$ and $k_I$ under the same gauge parameter.

## III. INTERNAL SYMMETRIES

In order to study such abelian internal symmetry given by matrix $A_{IJ}$, let us firstly consider the most general global symmetry

$$\delta G_{\mu}^{I} = A_{IJ}\,G_{\mu}^{J}, \tag{17}$$

where $A_{IJ}$ means a generic matrix. Its corresponding Ward identity is

$$\int d^4x\; A_{IJ}\,G_{\mu}^{J}\,\frac{\delta S}{\delta G_{\mu}^{I}} = 0 = \int d^4x\;\partial_\mu J^{\mu}. \tag{18}$$

Calculating it explicitly, one gets the following relationship from L(G)

$$\begin{gathered}\int d^4x\;\{\partial_\mu\big(\omega_{IJ}^{(1)}G^{\mu I}\,\partial_\nu G^{\nu J} + \omega_{IJ}^{(2)}G_{\nu}^{I}\partial^\mu G^{\nu J} + \omega_{IJK}^{(1)}G^{\mu I}\,G_{\nu}^{J}\,G^{\nu K}\big) + \omega_{IJ}^{(3)}\big(\partial_\mu G^{\mu I}\big)(\partial_\nu G^{\nu J})\\ +\omega_{IJ}^{(4)}\big(\partial_\mu G_{\nu}^{I}\big)(\partial^\mu G^{\nu J}) + \omega_{IJ}^{(5)}G_{\mu}^{I}G^{\mu J} + \omega_{IJK}^{(2)}G_{\mu}^{I}G^{\mu J}\,\partial_\nu G^{\nu K} + \omega_{IJK}^{(3)}G_{\mu}^{I}G_{\nu}^{J}\,\partial^\mu G^{\nu K}\\ +\omega_{IJKL}\,G_{\mu}^{I}G^{\mu J}\,G_{\nu}^{K}\,G^{\nu L}\} = 0,\end{gathered} \tag{19}$$

where

$$\begin{gathered}\omega_{IJ}^{(1)} = 2{A^K}_I(b_{(JK)} + c_{(JK)}),\quad \omega_{IJ}^{(2)} = 2{A^K}_I a_{(JK)},\quad \omega_{IJ}^{(3)} = -2{A^K}_J(b_{(IK)} + c_{(IK)}),\\ \omega_{IJ}^{(4)} = -2{A^K}_J a_{(IK)},\quad \omega_{IJ}^{(5)} = -2A_{JI}m_J^2,\quad \omega_{IJK}^{(1)} = {A^L}_I b_{L(JK)} + {A^L}_J a_{LIK},\\ \omega_{IJK}^{(2)} = -2{A^L}_I b_{K(JL)} - {A^L}_K b_{L(IJ)},\quad \omega_{IJK}^{(3)} = -{A^L}_I a_{KLJ} - {A^L}_J a_{KIL} - {A^L}_K a_{LJI},\\ \omega_{IJKL} = -2{A^M}_I(a_{(JM)(KL)} + a_{(KL)(JM)} + b_{(J_{(K}M)_{L)}} + b_{(K_{(J}L)_{M)}}).\end{gathered} \tag{20}$$

Consequently, for this unknown symmetry to be implemented, the last six parameters that are not connected with a total derivative must vanish. A necessary condition is $\det A \neq 0$. However, the implementation condition will also depend on relationships between the global parameters. This means that this implementation of this global symmetry must be studied specifically for every number of flavours being introduced. A further investigation is to write down $A_{IJ}$ in terms of a generators set as $A_{IJ} = \alpha_M(Q_M)_{IJ}G_{\mu}^{J}$.

Another case is to consider a set of symmetries as

$$\int d^4x\; A_{IJK}\,G_{\mu}^{J}\,\frac{\delta S}{\delta G_{\mu}^{K}} = 0 = \int d^4x\;\partial_\mu J^{\mu I}. \tag{21}$$

It yields,

$$\begin{gathered}\int d^4x\;\{\partial_\mu\big(r_{IJK}^{(1)}G^{\mu J}\,\partial_\nu G^{\nu K} + r_{IJK}^{(2)}G_{\nu}^{J}\,\partial^\mu G^{\nu K} + r_{IJKL}^{(1)}\,G^{\mu J}\,G_{\nu}^{K}\,G^{\nu L}\big) + r_{IJK}^{(3)}(\partial_\mu G^{\mu J})(\partial_\nu G^{\nu K})\\ +r_{IJK}^{(4)}\big(\partial_\mu G_{\nu}^{J}\big)(\partial^\mu G^{\nu K}) + r_{IJK}^{(5)}G_{\mu}^{J}\,G^{\mu K} + r_{IJKL}^{(2)}\,G_{\mu}^{J}\,G^{\mu K}\,\partial_\nu G^{\nu L} + r_{IJKL}^{(3)}\,G_{\mu}^{J}\,G_{\nu}^{K}\,\partial^\mu G^{\nu L}\\ +r_{IJKLM}\,G_{\mu}^{J}\,G^{\mu K}\,G_{\nu}^{L}\,G^{\nu M}\} = 0,\end{gathered} \tag{22}$$

where

$$r_{IJK}^{(1)} = 2A_I{}^L{}_J(b_{(KL)} + c_{(KL)}),\ \ r_{IJK}^{(2)} = 2A_I{}^L{}_J a_{(KL)},\ \ r_{IJK}^{(3)} = -2A_I{}^L{}_K(b_{(JL)} + c_{(JL)}),$$
$$r_{IJK}^{(4)} = -2A_I{}^L{}_K a_{(JL)},\ \ r_{IJK}^{(5)} = -2A_I{}^L{}_J d_{KL},\ \ r_{IJKL}^{(1)} = A_I{}^M{}_J b_{M(KL)} + A_I{}^M{}_K a_{MJL},$$
$$r_{IJKL}^{(2)} = -2A_I{}^M{}_J b_{L(KM)} - A_I{}^M{}_L b_{M(JK)},\ \ r_{IJKL}^{(3)} = -A_I{}^M{}_J(a_{LMK} + a_{LKM}) - A_I{}^M{}_L a_{MKJ},$$
$$r_{IJKLM} = -2A_I{}^N{}_J\left(a_{(KN)(LM)} + a_{(LM)(KN)} + b_{(K_{(L}N)_M)} + b_{(L_{(K}M)_N)}\right). \tag{23}$$

## IV. U(1)×SO(N) SYMMETRY

Given Eq. (16) internal symmetry, the next step to be considered is the case where the rotation depends on the gauge symmetry, $A_{IJ} = R_J^I(\alpha)$. For this, we are going to incorporate SO(N) symmetry to the original Lagrangian at Eq. (5).

A new physical aspect for this systemic Lagrangian research is to introduce charged vector fields on the original fields set $\{G_\mu^I\}$. Expanding the Eq. (6) for LK, in terms of $G_\mu^I$, and rearranging appropriately, we obtain

$$L_K^A = 2a_I\left(\partial_\mu G_\nu^I\right)(\partial^\mu G^{\nu I}) - 2a_I(\partial_\mu G_\nu^I)(\partial^\nu G^{\mu I}),$$
$$L_K^S = \alpha_{(IJ)}\left(\partial_\mu G_\nu^I\right)(\partial^\mu G^{\nu J}) + \beta_{(IJ)}(\partial_\mu G_\nu^I)(\partial^\nu G^{\mu J}), \tag{24}$$

where $\alpha_{(IJ)} = 2b_{(IJ)},\ \beta_{(JK)} = 2b_{(IJ)} + 4c_{(IJ)}$.

The trilinear term of the interaction sector, given by Eq. (10), can be written as

$$L_3 = \alpha_{IJK}\left(\partial_\mu G_\nu^I\right)G^{\mu J}G^{\nu K} + \beta_{I(JK)}(\partial_\mu G^{\mu I})G_\nu^J G^{\nu K}, \tag{25}$$

with $\alpha_{IJK} = 2a_{I[JK]} + 2b_{I(JK)},\ \beta_{I(JK)} = 2c_{I(JK)}$, and for the quadrilinear term of the interaction sector we must refer to Eqs. (12) and (13).

Then, these equations mean a systemic model where $G_\mu^I$ are real fields. A further development is to introduce on possible internal symmetries. They will configure the presence of charged fields through this whole concept.

The global SO(N) symmetry corresponding to L(G) is given by

$$G_\mu^I \xrightarrow{SO(N)} G_\mu^{I\prime} = R_J^I G_\mu^J, \tag{26}$$

where $R_J^I$ is the I-th row and J-th column of a well defined matrix R, representing SO(N) group on its vectorial representation. It characterizes the SO(N) transformation given by the parameter $\alpha_a$, $R = e^{i\alpha_a t_a}$, which makes $L(G) \xrightarrow{SO(N)} L(G')$. Inserting Eq. (26) in Eq. (24),

$$L_K = \left(\partial_\mu G_\nu\right)^t R^t AR(\partial^\mu G^\nu) + (\partial_\mu G_\nu)^t R^t BR(\partial^\nu G^\nu), \tag{27}$$

one has the conditions

$$R^t AR = A,\ \ R^t BR = B,\ \ R^t MR = M, \tag{28}$$

which trivial solution is with matrices A, B, M be diagonal. See Appendix A.

Eq. (26) also imposes constraints on the coupling constants written at Eqs. (25) and (12). It implies that they must be invariant tensor under SO(N) group. It gives,

$$a'_{PQR} = R_P^I R_Q^J R_R^K a_{IJK} = a_{PQR},\ \ a'_{PQRS} = R_P^I R_Q^J R_R^K R_S^L a_{IJKL} = a_{PQRS}, \tag{29}$$

where the generic expressions written above are representing the coupling constants $a_{IJK}$, $b_{I(JK)}$, $a_{(IJ)(KL)}$, $b_{(I_{(J}K)_L)}$.

Considering an infinitesimal rotation, we obtain the following relationships

$$(t_a)_{pi}a_{iqr} + (t_a)_{qi}a_{pir} + (t_a)_{ri}a_{pqi} = 0,\ \ (t_a)^i{}_p a_{iqrs} + (t_a)^i{}_q a_{pirs} + (t_a)^i{}_r a_{pqis} + (t_a)^i{}_s a_{pqri} = 0, \tag{30}$$

were Eqs. (30) are relating the constraints between the coupling constants and the associated SO(N)generators.

The corresponding Ward identities for this SO(N) global symmetry, $\delta G_\mu^I = \alpha_a(t_a)_{IJ}G_\mu^J$, are derived from the generating functional invariance, $\delta Z[J^I] = 0$. It gives the generic expression $J^I(t_a)_{IJ}\frac{\delta Z}{\delta G_\mu^J} = 0$. The first relationship defined from it is $(t_a)_{IJ} < G_\mu^J >$, which is trivially zero. For a two point Green's functions, it results in

$$(t_a)_{IM} < G_\mu^M \, G_\nu^J > + (t_a)_{JM} < G_\mu^M \, G_\nu^I > = 0 \tag{31}$$

where $< G_\mu^I \, G_\nu^J > = \frac{\delta^2 Z}{\delta_\mu^I \delta_\nu^J} \equiv \mathcal{G}_{IJ}^{(2)}$. Eq. (31) can be integrated in the group sense and expressed as

$$\mathcal{G}_{IJ}^{(2)\prime} = R_{IM}(\omega) R_{JN}(\omega) \mathcal{G}_{MN}^{(2)}. \tag{32}$$

Eq. (32) is showing that fields rotations correspond to the same Green's functions rotations. This result can be generalized for a n-point Green's function $\mathcal{G}_{IJ\ldots R}^{(n)}$ [14]

$$\mathcal{G}_{IJ\ldots R}^{(n)}{}' = R_{IA} R_{JB} \ldots R_{RS} \mathcal{G}_{AB\ldots S}^{(n)}. \tag{33}$$

## V. FOUR FIELDS

A further step concerning L(G) is to introduce charged vector fields. Given the fields set $\{G_\mu^0, G_\mu^1, G_\mu^2, G_\mu^3\}$ associate to the last two ones a global SO(2) symmetry.

From Eq. (6), choosing $a_2 = a_3 = a$, we have

$$L_K^A = a_0 G_{\mu\nu}^0 G^{\mu\nu\,0} + a_1 G_{\mu\nu}^1 G^{\mu\nu\,1} + a\{G_{\mu\nu}^2 G^{\mu\nu\,2} + G_{\mu\nu}^3 G^{\mu\nu\,3}\} \tag{34}$$

which produces the global SO(2) symmetry

$$G_\mu^{2'} = \cos(q\alpha)\, G_\mu^2 + \sin(q\alpha)\, G_\mu^3, \quad G_\mu^{3'} = -\sin(q\alpha)\, G_\mu^2 + \cos(q\alpha)\, G_\mu^3, \tag{35}$$

whose complexification,

$$G_\mu^2 = \tfrac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-), \quad G_\mu^3 = \tfrac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-), \tag{36}$$

results in the transverse kinetic Lagrangian

$$L_K^A = a_0 G_{\mu\nu}^0 G^{\mu\nu\,0} + a_1 G_{\mu\nu}^1 G^{\mu\nu\,1} + 2a W_{\mu\nu}^+ W^{\mu\nu\,-}, \quad W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm, \tag{37}$$

with a global symmetry $W_\mu^{\pm'} = e^{\pm iq\alpha} W_\mu^\pm$, $\pm q$ being the charge of $W_\mu^\pm$ field in response to U(1) global symmetry. At this way a global internal symmetry depending on the gauge parameter is introduced. It is proportional to $\begin{pmatrix} \cos q\alpha & \sin q\alpha \\ -\sin q\alpha & \cos q\alpha \end{pmatrix} = e^{iq\alpha \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$

Similarly, under this SO(2) symmetry the longitudinal part of the kinetic sector takes the form

$$\begin{gathered} L_K^S = b_{(00)} S_{\mu\nu}^0 S^{\mu\nu\,0} + b_{(11)} S_{\mu\nu}^1 S^{\mu\nu\,1} + c_{(00)} S_\mu^{\mu\,0} S_\nu^{\nu 0} + c_{(11)} S_\mu^{\mu\,1} S_\nu^{\nu 1} \\ + b_{(22)}\{S_{\mu\nu}^2 S^{\mu\nu\,2} + S_{\mu\nu}^3 S^{\mu\nu\,3}\} + c_{(22)}\{S_\mu^{\mu 2} S_\nu^{\nu 2} + S_\mu^{\mu 3} S_\nu^{\nu 3}\}. \end{gathered} \tag{38}$$

Thus, Eq. (35) endows L(G) with an internal abelian structure provided by the SO(2) symmetry. It generates the expression

$$G_\mu^I \to G_\mu^{I'} = R_K^I(\alpha) G_\mu^K + k_I \partial_\mu \alpha(x), \tag{39}$$

with a rotation matrix $R_I^K(\alpha)$ containing the following systemic transformation

$$\begin{pmatrix} G_\mu^{0\prime} \\ G_\mu^{1\prime} \\ G_\mu^{2\prime} \\ G_\mu^{3\prime} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos q\alpha & \sin q\alpha \\ 0 & 0 & -\sin q\alpha & \cos q\alpha \end{pmatrix} \begin{pmatrix} G_\mu^0 \\ G_\mu^1 \\ G_\mu^2 \\ G_\mu^3 \end{pmatrix} + \begin{pmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{pmatrix} \partial_\mu \alpha, \tag{40}$$

what becomes possible to implement a charged whole model with four fields transforming under a common gauge parameter.

Next, we rewrite L (G) in terms of $\{G_\mu^0, G_\mu^1, W_\mu^+, W_\mu^-\}$, after determining the necessary conditions for attain this global SO(2) invariance.

$$\begin{gathered} L_K = 2a_0\{(\partial_\mu G_\nu^0)(\partial^\mu G^{\nu 0}) - (\partial_\mu G_\nu^0)(\partial^\nu G^{\mu 0})\} + 2a_1\{(\partial_\mu G_\nu^1)(\partial^\mu G^{\nu 1}) - (\partial_\mu G_\nu^1)(\partial^\nu G^{\mu 1})\} \\ + 4a\{(\partial_\mu W_\nu^+)(\partial^\mu W^{\nu -}) - (\partial_\mu W_\nu^+)(\partial^\nu W^{\mu -})\} + \alpha_{(00)}(\partial_\mu G_\nu^0)(\partial^\mu G^{\nu 0}) + \beta_{(00)}(\partial_\mu G_\nu^0)(\partial^\nu G^{\mu 0}) \\ + \alpha_{(11)}(\partial_\mu G_\nu^1)(\partial^\mu G^{\nu 1}) + \beta_{(11)}(\partial_\mu G_\nu^1)(\partial^\nu G^{\mu 1}) + 2\alpha_{(01)}(\partial_\mu G_\nu^0)(\partial^\mu G^{\nu 1}) \\ + 2\beta_{(01)}(\partial_\mu G_\nu^0)(\partial^\nu G^{\mu 1}) + 2\alpha_{(22)}(\partial_\mu W_\nu^+)(\partial^\mu W^{\nu -}) + 2\beta_{(22)}(\partial_\mu W_\nu^+)(\partial^\nu W^{\mu -}), \end{gathered} \tag{41}$$

where $\alpha_{(IJ)} = 2b_{(IJ)}$, $\beta_{(IJ)} = 2b_{(IJ)} + 4c_{(IJ)}$.

The mass term is

$$L_m = m^2_{(00)} G^0_\mu G^{\mu 0} + m^2_{(11)} G^1_\mu G^{\mu 1} + 2m^2_{(22)} W^+_\mu W^{\mu -}. \tag{42}$$

The trilinear term of the interaction sector is given by the following equation

$$\begin{aligned} L_3 &= \alpha_{000}\left(\partial_\mu G^0_\nu\right)G^{\mu 0}G^{\nu 0} + \alpha_{001}\left(\partial_\mu G^0_\nu\right)G^{\mu 0}G^{\nu 1} + \alpha_{010}\left(\partial_\mu G^0_\nu\right)G^{\mu 1}G^{\nu 0} + \alpha_{100}\left(\partial_\mu G^1_\nu\right)G^{\mu 0}G^{\nu 0} \\ &+\alpha_{011}\left(\partial_\mu G^0_\nu\right)G^{\mu 1}G^{\nu 1} + \alpha_{101}\left(\partial_\mu G^1_\nu\right)G^{\mu 0}G^{\nu 1} + \alpha_{110}\left(\partial_\mu G^1_\nu\right)G^{\mu 1}G^{\nu 0} + \alpha_{111}\left(\partial_\mu G^1_\nu\right)G^{\mu 1}G^{\nu 1} \\ &+\beta_{0(00)}\left(\partial_\mu G^{\mu 0}\right)G^0_\nu G^{\nu 0} + 2\beta_{0(01)}\left(\partial_\mu G^{\mu 0}\right)G^0_\nu G^{\nu 1} + \beta_{1(00)}\left(\partial_\mu G^{\mu 1}\right)G^0_\nu G^{\nu 0} + \beta_{0(11)}\left(\partial_\mu G^{\mu 0}\right)G^1_\nu G^{\nu 1} \\ &+2\beta_{1(01)}\left(\partial_\mu G^{\mu 1}\right)G^0_\nu G^{\nu 1} + \beta_{1(11)}\left(\partial_\mu G^{\mu 1}\right)G^1_\nu G^{\nu 1} + 2\Re\{(\mathrm{i}\alpha_{023} + \alpha_{022})\left(\partial_\mu G^0_\nu\right)W^{\mu -}W^{\nu +}\} \\ &+2\Re\left\{(i\alpha_{203} + \alpha_{202})G^{\mu 0}\left(\partial_\mu W^-_\nu\right)W^{\nu +}\right\} + 2\Re\{(i\alpha_{230} + \alpha_{220})G^{\nu 0}\left(\partial_\mu W^-_\nu\right)W^{\mu +}\} \\ &+2\Re\{\{(i\alpha_{123} + \alpha_{122})\left(\partial_\mu G^1_\nu\right)W^{\mu -}W^{\nu +}\} + 2\Re\{(i\alpha_{213} + \alpha_{212})G^{\mu 1}\left(\partial_\mu W^-_\nu\right)W^{\nu +}\} \\ &+2\Re\{(i\alpha_{231} + \alpha_{221})G^{\nu 1}\left(\partial_\mu W^-_\nu\right)W^{\mu +}\} + 2\beta_{0(22)}\left(\partial_\mu G^{\mu 0}\right)W^+_\nu W^{\nu -} + 2\beta_{1(22)}(\partial_\mu G^{\mu 1})W^+_\nu W^{\nu -} \\ &+4\Re\left\{\left(i\beta_{2(03)} + \beta_{2(02)}\right)G^0_\nu\left(\partial_\mu W^{\mu -}\right)W^{\nu +}\right\} + 4\Re\left\{\left(i\beta_{2(13)} + \beta_{2(12)}\right)G^1_\nu\left(\partial_\mu W^{\mu -}\right)W^{\nu + \nu}\right\}, \end{aligned} \tag{43}$$

where $\alpha_{IJK} = 2a_{I[JK]} + 2b_{I(JK)}$, $\beta_{I(JK)} = 2c_{I(JK)}$, and $\Re$ means the real part of the corresponding complex expression.

The quadrilinear term of the interaction sector is

$$\begin{aligned} L_4 &= a_{0000}\left(G^0_\mu G^{\mu 0}\right)^2 + a_{1111}\left(G^1_\mu G^{\mu 1}\right)^2 + 4a_{0001}\left(G^0_\mu G^{\mu 0}\right)\left(G^0_\nu G^{\nu 1}\right) + 4a_{0111}\left(G^1_\mu G^{\mu 1}\right)\left(G^0_\nu G^{\nu 1}\right) \\ &+2(a_{0011} + a_{0110})\left(G^0_\mu G^{\mu 1}\right)^2 + 2a_{0101}\left(G^0_\mu G^{\mu 0}\right)\left(G^1_\nu G^{\nu 1}\right) \\ &+8\Re\{[(a_{0122} + a_{2012}) + i(a_{0123} - a_{2013})]\left(G^0_\mu G^1_\nu\right)W^{\mu -}W^{\nu +}\} \\ &+4a_{0202}\left(G^0_\mu G^{\mu 0}\right)W^+_\nu W^{\nu -} + 4a_{1212}\left(G^1_\mu G^{\mu 1}\right)W^+_\nu W^{\nu -} + 8a_{0212}\left(G^0_\mu G^{\mu 1}\right)W^+_\nu W^{\nu -} \\ &+4(a_{0022} + a_{0220})\left(G^0_\mu G^0_\nu\right)W^{\mu +}W^{\nu -} + 4(a_{1122} + a_{1221})\left(G^1_\mu G^1_\nu\right)W^{\mu +}W^{\nu -} \\ &+2(a_{2222} - a_{2323})\left(W^+_\mu W^-_\nu\right)\left(W^{\mu +}W^{\nu -}\right) + 2(a_{2222} + a_{2323})\left(W^+_\mu W^{\mu -}\right)^2. \end{aligned} \tag{44}$$

Thus Eq. (40) introduces a systemic abelian model containing the $\{\gamma \equiv G^0, Z^0 \equiv G^1, W^+, W^-\}$ particles. It shows that based on only one gauge parameter it is possible to build up a whole Lagrangian with renormalizable mass and charged fields.

## VI. NOETHER IDENTITIES

Considering this systemic interpretation for the gauge parameter three Noether's identities are consequently derived. They are

$$\partial_\mu J^\mu = 0, \quad J^\mu = W^+_\nu \frac{\partial L}{\partial \partial_\mu W^+_\nu} - W^-_\nu \frac{\partial L}{\partial \partial_\mu W^-_\nu}, \tag{45}$$

$$\partial_\mu \left[k_0 \frac{\partial L}{\partial \partial_\mu G^0_\nu} + k_1 \frac{\partial L}{\partial \partial_\mu G^1_\nu} + k_+ \frac{\partial L}{\partial \partial_\mu W^+_\nu} + k_- \frac{\partial L}{\partial \partial_\mu W^-_\nu}\right] + J^\nu = 0, \tag{46}$$

$$k_0 \frac{\partial L}{\partial \partial_\mu G^0_\nu} + k_1 \frac{\partial L}{\partial \partial_\mu G^1_\nu} + k_+ \frac{\partial L}{\partial \partial_\mu W^+_\nu} + k_- \frac{\partial L}{\partial \partial_\mu W^-_\nu} = 0, \tag{47}$$

where the above equations yield the following Gauss's law

$$\partial_\mu Z^{[\mu\nu]} = J^\nu, \tag{48}$$

where $Z_{\mu\nu}$ means an antisymmetric field strength composed as

$$Z_{[\mu\nu]} = aG^0_{\mu\nu} + bG^1_{\mu\nu} + cW^+_{\mu\nu} + dW^-_{\mu\nu} + z_{[\mu\nu]}.$$

## VII. CONCLUSION

A systemic unification is introduced. While the usual unification plays properties as space and time, electric and magnetic fields, this study focuses on the fields interdependence meaning. Its wholeness principle provides interconnected fields. It considers that interdependent fields associated in a same whole works as a new approach for the unification principle.

Under this view, Eq. (16) is introduced as a closer frontier between the abelian and the non-abelian structures. It incorporates two symmetry types: the local gauge symmetry and the global symmetry (which can have gauge origin or not). Both are expressing ways for collecting $\{G_\mu^I\}$ fields set. Consequently, through Eq. (39) it is introduced a gauge rotation and translation. It allows charged fields to be introduced under SO(2) symmetry.

Thus based on such whole unification viewpoint, this work associates the intermediate $\{\gamma, Z^0, W^+, W^-\}$ in a same whole expressed by Eq. (40). It proposes an abelian scenario for a whole family of self-interacting vector bosons. Also, Eqs. (41-44) are expressing a renormalizable abelian model that introduces mass and interactions without requiring spontaneous symmetry breaking and Yang-Mills approach.

We are under a new context, which says that instead of individualized particles as in the Standard Model, these fields should act as parts in an interconnected whole. Considering that LHC energy will compose a high number of $\gamma, Z^0, W^\pm$ particles, a new fact is that, perhaps it will require a model able to express the association between these concentrated number of particles. Perhaps points out that the Standard Model reductionist view is more propitious when a small number of particles is considered. However, in cases with a high concentration we probably should take the antireductionist approach.

Thus considering this whole unification view, it is possible to open up a new investigation for the phenomena involving $\gamma, Z^0, W^\pm$ particles. It is through the interconnected fields meaning. Differently from Standard Model it does not require spontaneous symmetry breaking and SU(2)⊗U(1) symmetry for introducing mass and electric charge respectively. Based just on only one gauge parameter it is possible to build up a four-field interconnected whole model. It is a study under development.

## Appendix A SO(N) symmetry on constructor basis $\{D, X^i\}$

Given such whole symmetry and its collection of fields the appropriated platform for studying the associated systemic symmetry is the so-called constructor basis $\{D, X^i\}$. It is defined through fields reparametrizations where one field is a genuine gauge field $D_\mu$ transforming as $D_\mu{}' = D_\mu + \partial_\mu \alpha$ and the others $(N-1)$ fields are Proca fields transforming as $X_\mu^{i\,\prime} = X_\mu^i$.

The corresponding gauge invariant Lagrangian is

$$L(D, X^i) = Z_{[\mu\nu]} Z^{[\mu\nu]} + Z_{(\mu\nu)} Z^{(\mu\nu)} - \frac{1}{2} m_{ij}^2 X_\mu^i X^{\mu j} + L_{GF}, \tag{A1}$$

where

$$Z_{\mu\nu} \equiv dD_{\mu\nu} + \alpha_i X_{\mu\nu}^i + \gamma_{[ij]} X_\mu^i X_\nu^j + \beta_i \Sigma_{\mu\nu}^i + \rho_i g_{\mu\nu} \Sigma_\alpha^{i\alpha} + \gamma_{(ij)} X_\mu^i X_\nu^j + \tau_{ij} g_{\mu\nu} X_\alpha^i X^{\alpha j},$$

$$D_{\mu\nu} \equiv \partial_\mu D_\nu - \partial_\nu D_\mu, \quad X_{\mu\nu}^i \equiv \partial_\mu X_\nu^i - \partial_\nu X_\mu^i, \quad \Sigma_{\mu\nu}^i \equiv \partial_\mu X_\nu^i + \partial_\nu X_\mu^i. \tag{A2}$$

The associated gauge fixing term is

$$L_{GF} = \frac{1}{2\xi} (\partial_\mu D^\mu + \sigma_i \partial_\mu X^{\nu j})^2.$$

We rewrite the Lagrangian as

$$L = L_0 + L_1, \quad L_0 = L_K + L_{GF} + L_m, \quad L_I = L_I^3 + L_I^4,$$

where

$$L_K = a\partial_\mu D_\nu (\partial^\mu D^\nu - \partial^\nu D^\mu) + c_i \partial_\mu D_\nu (\partial^\mu X^{\nu i} - \partial^\nu X^{\mu i}) + e_{ij} (\partial_\mu X_\nu^i)(\partial^\mu X^{\nu j})$$
$$+ f_{ij} (\partial_\mu X_\nu^i)(\partial^\nu X^{\mu j}) + s_{ij} (\partial_\mu X^{\mu i})(\partial_\nu X^{\nu j}),$$

$$L_m = -\frac{1}{2} m_{ij}^2 X_\mu^i X^{\mu j}, \tag{A3}$$

$$L_I^3 = a_{[jk]} (\partial_\mu D_\nu) X^{\mu j} X^{\nu k} + a_{ijk} (\partial_\mu X_\nu^i) X^{\mu j} X^{\nu k} + b_{i(jk)} (\partial_\mu X^{\mu i}) X_\nu^j X^{\nu j},$$

$$L_I^K = a_{ijkl} X_\mu^i X_\nu^j X^{\mu k} X^{\nu l},$$

with the relationships

$$a = 2d^2, \;\; c_i = 4d\alpha_i, \;\; e_{ij} = 2(\alpha_i \alpha_j + \beta_i \beta_j), \;\; f_{ij} = 2(\beta_i \beta_j - \alpha_i \alpha_j), \;\; s_{ij} = 8(\beta_i \rho_j + 2\rho_i \rho_j), \tag{A4}$$

$$a_{[ij]} = 4d\gamma_{[ij]},\ \ a_{ijk} = 4\left(\alpha_i\gamma_{[jk]} + \beta_i\gamma_{(jk)}\right),\ \ b_{i(jk)} = 4\left(\rho_i\gamma_{(jk)} + 4\rho_i\tau_{(jk)} + \beta_i\tau_{(jk)}\right),$$

$$a_{ijkl} = \gamma_{[ij]}\gamma_{[kl]} + \gamma_{(ij)}\gamma_{(kl)} + \gamma_{(ik)}\tau_{(jl)} + \tau_{(ik)}\gamma_{(jl)} + 4\tau_{(ik)}\tau_{(jl)},$$

where the coefficients $\mathrm{a_{ijkl}}$ displays the symmetry: $\mathrm{a_{ijkl}} = \mathrm{a_{jilk}} = \mathrm{a_{klij}}$.

Notice that although being a non-linear abelian Lagrangian, Eq. (A1) is gauge invariant. Under this constructor basis $\{\mathrm{D_\mu}, \mathrm{X^i_\mu}\}$ is immediate to understand on mass presence without Higgs mechanism.

Denoting $\mathrm{V_\mu} = (\mathrm{D_\mu}, \mathrm{X^i_\mu})^{\mathrm{t}}$, one gets the matricial form for the kinetic sector $\mathrm{L_0}$. After some algebraic manipulation and making use of the transversal and longitudinal projectors, $\theta^{\mu\nu}$ and $\omega^{\mu\nu}$, we obtain, $\mathrm{L_0} = -\frac{1}{2}\mathrm{V^t_\mu}\mathcal{O}^{\mu\nu}\mathrm{V_\nu}$, where $\mathcal{O}^{\mu\nu} = \mathcal{O}_{K_T}\partial^\alpha\partial_\alpha\theta^{\mu\nu} + \left(\mathcal{O}_{K_L} + \mathcal{O}_{GF}\right)\partial^\alpha\partial_\alpha\omega^{\mu\nu} + M^2\eta^{\mu\nu}$, with

$$\mathcal{O}_{K_T} = \begin{bmatrix} 2a & c_1 & \cdots & c_{N-1} \\ c_1 & 2e_{11} & \cdots & 2e_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N-1} & 2e_{N-1,1} & \cdots & 2e_{N-1,N-1} \end{bmatrix},\quad \mathcal{O}_{K_L} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 2t_{11} & \cdots & 2t_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 2t_{N-1,1} & \cdots & 2t_{N-1,N-1} \end{bmatrix},$$
$$\mathcal{O}_{GF} = \frac{1}{\xi}\begin{bmatrix} 1 & \sigma_1 & \cdots & \sigma_{N-1} \\ \sigma_1 & {\sigma_1}^2 & \cdots & \sigma_1\sigma_{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N-1} & \sigma_1\sigma_{N-1} & \cdots & {\sigma_{N-1}}^2 \end{bmatrix},\quad M^2 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & m^2_{11} & \cdots & m^2_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & m^2_{N-1,1} & \cdots & m^2_{N-1,N-1} \end{bmatrix}. \tag{A5}$$

where $\mathrm{t_{ij}} = \mathrm{e_{ij}} + \mathrm{f_{ij}} + \mathrm{s_{ij}}$.

Then, similarly to Eq. (28), the SO(N) symmetry conditions are given by

$$R^t\mathcal{O}_{K_T}R = \mathcal{O}_{K_T},\quad R^tM^2R = M^2,\quad R^t\left(\mathcal{O}_{K_L} + \mathcal{O}_{GF}\right)R = \mathcal{O}_{K_L} + \mathcal{O}_{GF}. \tag{A6}$$

Next, we study the case involving four fields $\mathrm{D_\mu}, \mathrm{X^1_\mu}, \mathrm{X^2_\mu}, \mathrm{X^3_\mu}$, where we consider the SO(2) symmetry represented by the matrix

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & -\sin\alpha & \cos\alpha \end{bmatrix}. \tag{A7}$$

Then, from (A5) and (A6) we obtain:

$$\mathcal{O}_{K_T} = \begin{bmatrix} 2a & c_1 & 0 & 0 \\ c_1 & 2e_{11} & 0 & 0 \\ 0 & 0 & 2e_{22} & 0 \\ 0 & 0 & 0 & 2e_{22} \end{bmatrix},\quad \mathcal{O}_{K_L} + \mathcal{O}_{GF} = \begin{bmatrix} 1/\xi & \sigma_1/\xi & 0 & 0 \\ \sigma_1/\xi & 2t_{11} + {\sigma_1}^2/\xi & 0 & 0 \\ 0 & 0 & 2t_{22} & 2t_{23} \\ 0 & 0 & -2t_{23} & 2t_{22} \end{bmatrix}$$
$$M^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & m^2_{11} & 0 & 0 \\ 0 & 0 & m^2_{22} & m^2_{23} \\ 0 & 0 & -m^2_{23} & m^2_{22} \end{bmatrix}, \tag{A8}$$

which rewrites under SO(2) the kinetic sector as

$$\begin{aligned} L_0(D, X^i) = {} & a\partial_\mu D_\nu(\partial^\mu D^\nu - \partial^\nu D^\mu) + c_1\partial_\mu D_\nu(\partial^\mu X^{\nu 1} - \partial^\nu X^{\mu 1}) + e_{11}(\partial_\mu X^1_\nu)(\partial^\mu X^{\nu 1}) \\ & + e_{22}\left[(\partial_\mu X^2_\nu)(\partial^\mu X^{\nu 2}) + (\partial_\mu X^3_\nu)(\partial^\mu X^{\nu 3})\right] + (f_{11} + s_{11})(\partial_\mu X^{\mu 1})^2 \\ & + (f_{22} + s_{22})\left[(\partial_\mu X^{\mu 2})^2 + (\partial_\mu X^{\mu 3})^2\right] + \frac{1}{2\xi}(\partial_\mu D^\mu + \sigma_1\partial_\mu X^{\mu 1})^2 - \frac{1}{2}m^2_{11}X^1_\mu X^{\mu 1} \\ & - \frac{1}{2}m^2_{22}(X^2_\mu X^{\mu 2} + X^3_\mu X^{\mu 3}), \end{aligned} \tag{A9}$$

and the interaction sector as

$$\begin{aligned} L^3_I = {} & a_{[23]}\left(\partial_\mu D_\nu\right)\{X^{\mu 2}X^{\nu 3} - X^{\mu 3}X^{\nu 2}\} + a_{111}\left(\partial_\mu X^1_\nu\right)X^{\mu 1}X^{\nu 1} + a_{122}\left(\partial_\mu X^1_\nu\right)\{X^{\mu 2}X^{\nu 2} + X^{\mu 3}X^{\nu 3}\} \\ & + a_{123}\left(\partial_\mu X^1_\nu\right)\{X^{\mu 2}X^{\nu 3} - X^{\mu 3}X^{\nu 2}\} + a_{212}X^{\mu 1}\{\left(\partial_\mu X^2_\nu\right)X^{\nu 2} + \left(\partial_\mu X^3_\nu\right)X^{\nu 3}\} \\ & + a_{213}X^{\mu 1}\{\left(\partial_\mu X^2_\nu\right)X^{\nu 3} - \left(\partial_\mu X^3_\nu\right)X^{\nu 2}\} + a_{221}X^{\nu 1}\{\left(\partial_\mu X^2_\nu\right)X^{\mu 2} + \left(\partial_\mu X^3_\nu\right)X^{\mu 3}\} \\ & + a_{231}X^{\nu 1}\{\left(\partial_\mu X^2_\nu\right)X^{\mu 3} - \left(\partial_\mu X^3_\nu\right)X^{\mu 2}\} + b_{1(11)}\left(\partial_\mu X^{\mu 1}\right)X^1_\nu X^{\nu 1} + b_{1(22)}\left(\partial_\mu X^{\mu 1}\right)\{X^2_\nu X^{\nu 2} + X^3_\nu X^{\nu 3}\} \\ & + 2b_{2(12)}X^1_\nu\{\left(\partial_\mu X^{\mu 2}\right)X^{\nu 2} + \left(\partial_\mu X^{\mu 3}\right)X^{\nu 3}\} + 2b_{2(13)}X^1_\nu\{\left(\partial_\mu X^{\mu 2}\right)X^{\nu 3} - \left(\partial_\mu X^{\mu 3}\right)X^{\nu 2}\}, \end{aligned} \tag{A10}$$

$$L_I^4 = a_{1111}(X_\mu^1 X^{\mu 1})^2 + 2(a_{1122} + a_{1221})\left\{\left(X_\mu^1 X^{\mu 2}\right)^2 + \left(X_\mu^1 X^{\mu 3}\right)^2\right\} + 2a_{1212} X_\mu^1 X^{\mu 1}\{X_\nu^2 X^{\nu 2} + X_\nu^3 X^{\nu 3}\}$$
$$+a_{2222}\left\{\left(X_\mu^2 X^{\mu 2}\right)^2 + \left(X_\mu^3 X^{\mu 3}\right)^2 + 2\left(X_\mu^2 X^{\mu 3}\right)^2\right\} + 2a_{2323}\left\{\left(X_\mu^2 X^{\mu 2}\right)(X_\nu^3 X^{\nu 3}) - \left(X_\mu^2 X^{\mu 3}\right)^2\right\}. \quad \text{(A11)}$$

Appendix B Charged fields from SO(2)

Given the SO(2) symmetry stipulated at Eq. (A8) we are going to rewrite the fields set $\{D_\mu, X_\mu^1, X_\mu^2, X_\mu^3\}$ in terms of charged fields. Redefining the fields,

$$X_\mu^2 = \frac{1}{\sqrt{2}}\left(W_\mu^+ + W_\mu^-\right), \quad X_\mu^3 = \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-). \quad \text{(B1)}$$

Thus through the constructor basis, and the SO(2) generator $t_A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and the vertices coefficients are related through Eq. (A2), one derives the following Lagrangian invariant under U(1) and SO(2)

$$L = L_0 + L_1, \quad \text{(B2)}$$

with

$$L_0 = L_K^{D,X^1} + L_K^{W^+,W^-} + L_m + L_{GF}, \quad L_I = L_3 + L_4, \quad \text{(B3)}$$

where

$$L_K^{D,X^1} = a\partial_\mu D_\nu(\partial^\mu D^\nu - \partial^\nu D^\mu) + c_1\partial_\mu D_\nu(\partial^\mu X^{\nu 1} - \partial^\nu X^{\mu 1})$$
$$+e_{11}\left(\partial_\mu X_\nu^1\right)(\partial^\mu X^{\nu 1}) + (f_{11} + s_{11})(\partial_\mu X^{\mu 1})^2, \quad \text{(B4)}$$

$$L_K^{W^+,W^-} = 2e_{22}(\partial_\mu W_\nu^+)(\partial^\mu W^{\nu -}) + 2(f_{22} + s_{22})(\partial_\mu W^{\mu +})(\partial_\nu W^{\nu -}), \quad \text{(B5)}$$

$$L_m = -\frac{1}{2}m_{11}^2 X_\mu^1 X^{\mu 1} - m_{22}^2 W_\mu^+ W^{\mu -}, \quad \text{(B6)}$$

$$L_{GF} = \frac{1}{2\xi}(\partial_\mu D^\mu + \sigma_1\partial_\mu X^{\mu 1})^2, \quad \text{(B7)}$$

$$L_3 = a_{111}\left(\partial_\mu X_\nu^1\right)X^{\mu 1}X^{\nu 1} + b_{1(11)}\left(\partial_\mu X^{\mu 1}\right)X_\nu^1 X^{\nu 1} + 2\Re\left\{\left[ia_{[23]}\partial_\mu D_\nu + (a_{122} + ia_{123})\partial_\mu X_\nu^1\right]W^{\mu -}W^{\nu +}\right\}$$
$$+2b_{1(22)}\left(\partial_\mu X^{\mu 1}\right)W_\nu^+ W^{\nu -} + 2\Re\left\{(a_{212} + ia_{213})X^{\mu 1}(\partial_\mu W_\nu^-)W^{\nu +}\right\}$$
$$+2\Re\left\{(a_{221} + ia_{231})X^{\nu 1}(\partial_\mu W_\nu^-)W^{\mu +}\right\} + 2\Re\left\{(b_{2(12)} + ib_{2(13)})X_\nu^1(\partial_\mu W^{\mu -})W^{\nu +}\right\}, \quad \text{(B8)}$$

$$L_4 = a_{1111}(X_\mu^1 X^{\mu 1})^2 + 4(a_{1122} + a_{1221})X_\mu^1 X_\nu^1 W^{\mu +}W^{\nu -} + 4a_{1212}X_\mu^1 X^{\mu 1}W_\nu^+ W^{\nu -}$$
$$+2(a_{2222} + a_{2323})\left(W_\mu^+ W^{\mu -}\right)^2 + 2(a_{2222} - a_{2323})W_\mu^+ W^{\mu +}W_\nu^- W^{\nu -}. \quad \text{(B9)}$$

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