Flexural and Shear Resistance of High Strength Concrete Beams

Giuseppe Campione[1], Calogero Cucchiara[2], Alessia Monaco[3]

[1,2,3]Dipartimento di Ingegneria Civile, Ambientale e Aerospaziale e dei Materiali (DICAM), Università degli Studi di Palermo, Viale delle Scienze, 90128 Palermo (Italy)

[1]studioingcampione@libero.it

***Abstract*-In the present paper, an analytical model is proposed that is able to determine the shear resistance of high strength reinforced concrete beams with longitudinal bars, in the presence of transverse stirrups. The model is based on the evaluation of the resistance contribution due to beam and arch actions. For the resistance contribution of the main bars in tension the residual bond adherence of steel bars and the crack spacing of R.C. beams are considered. The compressive strength of the compressed arch is also verified by taking into account of the biaxial state of stresses.**

The model was verified on the basis of experimental data available in the literature and it is able to include the following variables in the resistance provision: - geometrical percentage of steel bars; - depth-to-shear span ratio; - resistance of materials; - crack spacing; - tensile stress in main bars; - residual bond resistance; - size effects. Finally, some of the more recent analytical expressions able to predict the shear and the flexural resistance of concrete beams are mentioned and a comparison is made with experimental data.

Keywords- Shear-Moment Interaction; High-Strength Concrete; Shear Resistance; Flexural Resistance

I. INTRODUCTION

Reinforced Concrete (RC) is being used extensively in the construction industry all over the world. The adoption of High Strength Concrete (HSC) has increased thanks to its obvious advantages: increased modulus of elasticity, chemical resistance, freeze thaw resistance, lower creep, lower drying shrinkage and lower permeability, to name a few. HSC is usually proportioned with a low water-to-cementitious material ratio and has high compressive strength in the range of 50 to 100 MPa. The supplementary cementitious materials may include blast furnace slag, fly ash, or silica fume, which are used either as cement replacement or as additives to the concrete mixture.

Complete knowledge of HSC properties and bond and anchorage characteristics is essential for evaluating structural response and behavior under monotonic and cyclic loads. In the presence of shear, beam flexural strength may be greatly reduced with respect to the pure flexure case, and failure may occur in a brittle way and without warning signs [1]. Due to the complexity of the phenomenon, much research has been addressed to evaluating the resisting mechanisms of only longitudinally reinforced beams, currently called concrete mechanisms [2-5]. The strength provided by any transverse reinforcement, which is constituted by stirrups in almost every study, is hence taken into account by adding the contribution of the truss mechanism to that of the concrete mechanisms. Some recent investigations [6-8] have shown that the expressions currently used for evaluating the shear strength can give unconservative predictions when applied to beams made up of HSC and so far the main building codes do not give specific design rules for such beams. A reliable expression for computing the shear strength of only longitudinally reinforced HSC beams has recently been proposed by the authors [3, 5]. A formula for the shear strength of HSC beams with stirrups is presented here. Design procedures proposed for regulatory standards should be safe, correct in concept and simple to understand, and should not necessarily add to either design or construction costs. These procedures are most effective if based on relatively simple conceptual models rather than complex empirical equations [9]. Hence, many equations have been proposed to estimate the ultimate and cracking shear strength of RC beams. Among them are Zsutty's equation [10], deduced by multiple regression analysis; Bazant's equation [5] (derived by Bazant and Kim [11]) which takes into account the size effect based on non-linear fracture mechanics; the CEB-FIP model code equation, introduced empirically; the formula of Japan Concrete Generally, in which the nominal shear strength of RC beams with stirrups is composed of two components: the resistance of stirrups resulting from the truss action and the concrete resistance. Shear design codes require a simple means of computing a realistic V_c term, the additional concrete contribution as a function of the shear stress level. This so-called concrete contribution is important in the design of beams where the factored shear force is near the value of the shear force required to produce diagonal tension cracking. This term is necessary for the economic design of beams and slabs with little or no shear reinforcement. In the previous study, Arslan's cracking shear strength equations [6] performed almost as well as the ACI 318 simplified equation [9] in terms of coefficient of variation and the cracking shear strength predicted by ACI 318 [9] tend to be more conservative than that obtained with Arslan's equations [6] in terms of mean value. In this study, Arslan's cracking shear strength equations [6] are used to take into account the concrete contribution while the contribution of stirrups is added to the concrete contribution to obtain the shear strength of RC beams. The proposed equation is justified comparing the results of existing shear tests on RC beams with stirrups and predicted values obtained with the current model.

II. ANALYTICAL EXPRESSIONS FOR PREDICTION OF FLEXURAL AND SHEAR STRENGTH OF HSC BEAMS

A. Flexural Strength

All codes distinguish the case of over-reinforced and under-reinforced cross-section. Beams designed in practice are under-reinforced and their flexural strength is controlled by the yield force in the tensile steel. In these cases, most of the current codes utilize the stress-block approach for the determination of the ultimate flexural capacity of R.C. sections and neglect the presence of compressed bars.

From the translation and rotation equilibrium of internal forces shown in Fig. 1, we obtain, with reference to the symbols utilized in the same figure, the position of the neutral axis xc and the ultimate flexural strength Mu in the form:

$$x_c = \frac{A_s \cdot f_y}{\alpha \cdot f_c^{'} \cdot \beta \cdot b} \tag{1}$$

$$M_u = A_s \cdot f_y \cdot \left(d - 0.50 \cdot \beta \cdot x_c\right) \tag{2}$$

As being the area of longitudinal tensile bars, b the width of the beam, d the effective depth, α and β the stress block coefficients. Table 1 gives the most commonly utilized values for α and β, and for the ultimate compressive strain of concrete ε_{cu}.

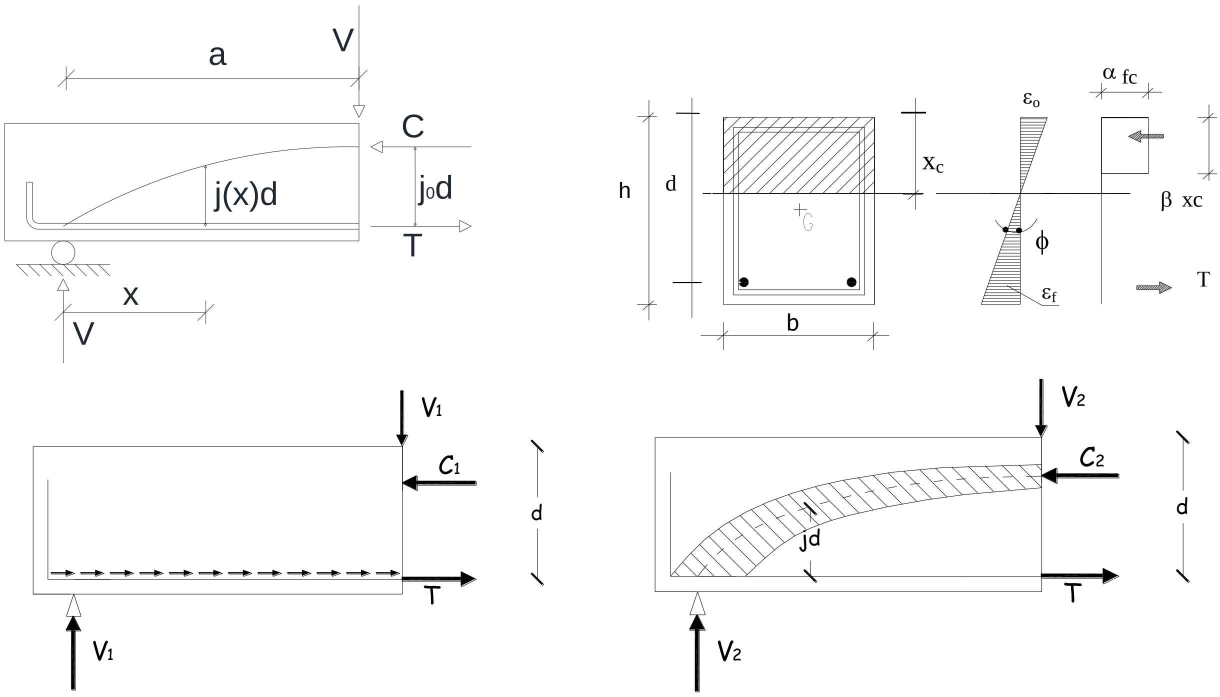


Fig. 1 Flexural-to-shear interaction model

TABLE 1 STRESS-BLOCK PARAMETERS FOR HSC

Code	α	β	εcu
A318-08 (2008)	0.85	$1.09 - 0.008 \cdot f_c^{'}$	0.0030
CAN3-A23-3-(2004)	$0.85 - 0.0015 \cdot f_c^{'}$	$0.97 - 0.0025 \cdot f_c^{'}$	0.0035
Eurocode 2 (2004)	0.85	$0.9 - \frac{f_c^{'}}{500}$	0.0035
Bae and Bayrak (2003)	$0.85 - 0.004 \cdot \left(f_c^{'} - 70\right)$	$0.85 - 0.004 \cdot \left(f_c^{'} - 30\right)$	$0.0025 \left(f_c^{'} > 55MPa\right)$ $0.0030 \left(f_c^{'} < 55MPa\right)$

If in Eqs. (1) and (2), we introduce the mechanical ratio of the main steel ω_l defined as $\omega_l = \frac{A_s}{b \cdot d} \cdot \frac{f_y}{f_c^{'}}$, with f'$_c$ the compressive cylindrical strength of the concrete proves to be the following:

$$\frac{x_c}{d} = \frac{A_s}{b \cdot d} \cdot \frac{f_y}{f_c^{'}} \cdot \frac{1}{\alpha \cdot \beta} = \rho_l \cdot \frac{f_y}{f_c^{'}} \cdot \frac{1}{\alpha \cdot \beta} = \frac{\omega_l}{\alpha \cdot \beta} \tag{3}$$

with $\rho_l = \frac{A_s}{b \cdot d}$

$$\frac{M_u}{b \cdot d^2} \frac{1}{f_c^{'}} = \omega_l \cdot \left(1 - 0.5 \cdot \frac{\omega_l}{\alpha}\right) \tag{4}$$

The arm of the internal forces can be expressed as:

$$j_o \cdot d = \left(1 - 0.5 \cdot \frac{x_c}{d}\right) \cdot d \tag{5}$$

The application of Eq. (3) is subject to the condition that:

$$\frac{x_c}{d} \le \frac{x_{c\lim}}{d} = \frac{\varepsilon_u}{\varepsilon_y + \varepsilon_u} \tag{6}$$

Eq. (6) is verified when the steel bars are considered to have yielded and the concrete to be crushed.

Substituting Eq. (6) in Eq. (3) gives the limit geometrical ratio of the longitudinal steel:

$$\rho_{\lim} = \alpha \cdot \beta \cdot \frac{f_c^{'}}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_y + \varepsilon_u} \tag{7}$$

B. Shear Strength

For the concrete shear resistance contribution ACI 318 [9] suggests:

$$v_{uc} = \left(0.157 \cdot \sqrt{f_c^{'}} + 17.2 \cdot \rho \cdot \frac{d}{a}\right) < 0.30 \cdot \sqrt{f_c^{'}} \qquad \text{in S.I. units} \tag{8}$$

For the stirrup contribution ACI 318 [9] considers stirrups to have yielded and the main crack to be inclined 45°, giving:

$$v_{us} = \frac{A_{sw}}{b \cdot s} \cdot f_{yw} = \rho_{sw} \cdot f_{yw} \tag{9}$$

Therefore the shear strength is expressed as:

$$v_u = v_{uc} + v_{us} \tag{10}$$

The CAN3 (2004) shear provisions [12] are based on the shear resisting mechanism consisting in a free body diagram of the end portion of a beam. This portion cuts the flexural compression region and the longitudinal reinforcement and stirrups following the diagonal shear crack.

If the dowel action is neglected, the shear strength equation is:

$$v_{uc} = \phi_c \cdot \beta \cdot \sqrt{f_c^{'}} + \phi_s \cdot \rho_{sw} \cdot f_{yw} \cdot \cot\theta \le 0.25 \cdot \phi_c \cdot f_c^{'} \qquad \text{in S.I. units} \tag{11}$$

where ϕ_c, ϕ_s are material reduction factors for concrete and steel stirrups.

β represents the ability of the member to resist aggregate interlock stresses and θ is the angle of principal compressive stresses and indicates how many stirrup legs will cross the crack.

As stressed in [13] for β we can assume (in an approximate way) the expression:

$$\beta = \frac{0.4}{1 + 1500 \cdot \frac{f_y}{2 \cdot E_s}} \tag{12}$$

and for θ the expression

$$\theta = 29^0 + 7000 \cdot \frac{f_y}{2 \cdot E_s} \tag{13}$$

According to Eurocode 2 [14] adopting the variable truss angle method the shear strength can be assumed as:

$$v_{uc} = \min\left\{\left(0.9 \cdot \left[0.6 \cdot \left(1 - \frac{f_c^{'}}{250}\right)\right] \cdot \frac{f_c'}{\gamma_c} \cdot \frac{1}{\cot\theta + \tan\theta}\right) \;;\; 0.90 \cdot \rho_{sw} \cdot \frac{f_{yw}}{\gamma_s} \cdot \cot g\theta\right\} \tag{14}$$

$$\rho_{sw} \cdot \frac{f_{yw}}{\gamma_s} \cdot \le 0.5 \cdot \left[0.6 \cdot \left(1 - \frac{f_c^{'}}{250}\right)\right] \cdot \frac{f_c'}{\gamma_c} \tag{15}$$

$$\text{with } 0.4 \le \cot g\theta \le 2.5 \tag{16}$$

Some other simple and interesting expressions are those in [6-8].

Reference [8] shows that the nominal shear stress at diagonal tension cracking is the product of the ratio of the neutral axis depth to the effective depth of the beam and the splitting tensile strength of the concrete. The problem of the size effect on the shear strength of beams comes down to the problem of the size effect on the splitting tensile strength of the concrete. The contribution of stirrups which are assumed to have yielded is considered in the splitting region and the expression of the shear strength proves to be:

$$v_{uc} = \xi \cdot \left(f_c^{'}\right)^{0.66} \cdot \frac{x_c}{d} + \left(0.5 + 0.25 \cdot \frac{a}{d}\right) \cdot \rho_{sw} \cdot f_{yw} \tag{17}$$

with $\xi = 0.3 \cdot \left(1.2 - 0.2 \cdot \frac{a}{d} \cdot d\right)$ the size effect term and

$$\left(\frac{x_c}{d}\right)^2 + 600 \cdot \frac{\rho + \rho^{'}}{f_c^{'}} \cdot \frac{x_c}{d} - 600 \cdot \frac{\rho + \frac{d}{h} \cdot \rho^{'}}{f_c^{'}} = 0 \tag{18}$$

Reference [6] is modifying the original approach of [4], calculates the shear strength of HSC slender beams considering the stirrups to have yielded and the main crack inclined 45°, giving:

$$v_{uc} = 0.12 \cdot \sqrt{f_c^{'}} + 0.02 \cdot \left(f_c^{'}\right)^{0.65} + \rho_{sw} \cdot f_{yw} \tag{19}$$

Finally, according to [7] the shear strength can be calculated as:

$$v_{uc} = \frac{1 + \sqrt{\frac{5.08}{d_a}}}{\sqrt{1 + \frac{d}{25 \cdot d_a}}} \cdot \left[\rho^{0.4} \cdot f_c^{'0.39} + 0.5 \cdot \rho^{0.83} \cdot f_y^{0.89} \cdot \left(\frac{a}{d}\right)^{-1.2 - 0.45\frac{a}{d}}\right] + 0.36 \cdot \rho^{0.2} \cdot \sqrt{f_c^{'}} \cdot \left(\rho_{sw} \cdot f_{yw}\right)^{0.6} \tag{20}$$

with d_a the maximum aggregate size and a the shear span.

C. Proposed Model

According to [11], to calculate the bearing capacity in shear of reinforced concrete beams, it is possible to sum the resistance contributions due to the beam and arch actions. The arch and beam contributions are identified as shown in Fig. 1, by imposing the equilibrium conditions of a portion of the beam comprised between the support and the loaded section (shear span a). With reference to the symbols utilized in Fig. 1 the bending moment M and the shear force V at any given cross-section can be related to the axial force T in the longitudinal bar and to the internal arm jd by means of:

$$M = V \cdot x = T \cdot jd \tag{21}$$

Moreover, the shear force V is related to the variation in M by the relationship:

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} = j_o d \cdot \frac{\mathrm{d}T}{\mathrm{d}x} + T \cdot \frac{\mathrm{d}(jd)}{\mathrm{d}x} \tag{22}$$

obtaining by means of Eq. (22) the two fundamental resistance contributions well known in the literature as beam effect (jd constant) and arch effect (jd variable).

From Eq. (22) it emerges that the resistance contribution due to the beam effect is defined by:

$$V_1 = j_0 d \cdot \frac{\mathrm{d}T(x)}{\mathrm{d}x} \tag{23}$$

in which j_0 takes on a constant value and the variation in the moment M is due to T.

Referring to Fig. 2, which highlights the contribution due to the beam effect, denoted as V_1, it has to be observed that dT/dx can be expressed in terms of residual splitting bond stresses qres transmitted by the longitudinal bar.

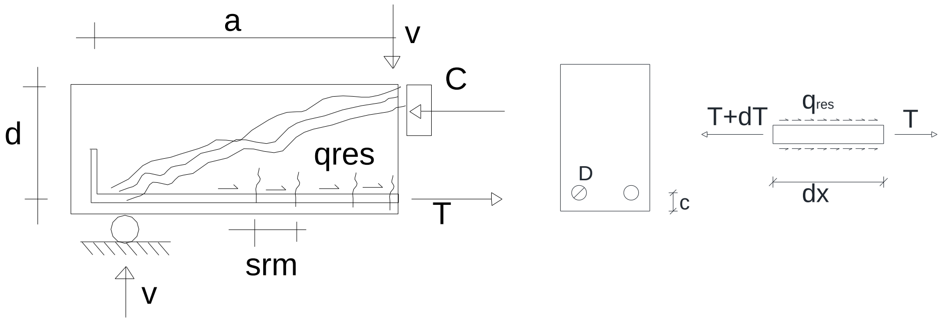


Fig. 2 Equilibrium of a longitudinal bar between two successive cracks

Considering a beam portion of length dx, the equilibrium of the internal forces (T and T+dT) gives the following:

$$\frac{\mathrm{d}\,\mathrm{T}}{\mathrm{d}\,x} = \pi \cdot \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{q}_{\mathrm{resi}} \cdot \mathrm{D}_{\mathrm{i}} \tag{24}$$

Di being the diameter of the i-th bar belonging to the main reinforcements of the area As.

Therefore utilizing Eq. (23) and Eq. (24) we obtain

$$V_1 = j_o \cdot d \cdot \frac{dT}{dx} = j_o \cdot d \cdot \pi \cdot \sum_{\mathrm{i}=1}^{\mathrm{n}} q_{resi} \cdot \mathrm{D}_{\mathrm{i}} \tag{25}$$

On the basis of Eq. (24) and Eq. (25) we then obtain the following:

$$v_1 = \frac{j_o \cdot \pi \cdot \sum q_{resi} \cdot D_i}{b} \tag{26}$$

Bond splitting failure with constant bond resistance is assumed here. In the case of ordinary concrete it was shown [15] that the bond stress q_b can be assumed to be proportional to the square root of the cylindrical compressive resistance f'_c as is done in [9]. Another code [14] also suggests assuming the bond stress to be proportional to the power 1/3.

In the case of HSC it was demonstrated by Harajili et al. [16] that the bond splitting resistance between reinforcing bars and concrete can be assumed as:

$$q_{resi} \cong 0.33 \cdot \sqrt{f_c^{'}} \cdot \left(\frac{\delta}{D_i}\right)^{0.66} \quad \text{(in MPa)} \tag{27}$$

δ being the cover of the longitudinal bars. In the following section for simplicity's sake we assume $\frac{\delta}{D_i} = 1$ and therefore $q_{resi} \cong 0.33 \cdot \sqrt{f_c^{'}}$.

We can introduce the equivalent diameter of the longitudinal bars defined as:

$$D_{eq} = \sum D_i \tag{28}$$

This gives

$$D_{eq} = 2 \cdot \sqrt{\frac{b \cdot d \cdot \rho}{\pi}} \tag{29}$$

Substituting Eq. (29) into Eq. (26) and taking Eq. (27) into account gives:

$$v_1 = 0.886 \cdot j_o \cdot \sqrt{\rho} \cdot \sqrt{\frac{d}{b}} \cdot \sqrt{f_c^{'}} \tag{30}$$

Similar expressions are also obtained in [2, 8, 17].

Reference [17] suggests calculating the contribution in shear strength due to bond splitting failure as:

$$v_c = 2 \cdot \rho^{0.33} \cdot f_t \tag{31}$$

ft being the tensile strength of the concrete. If ft is assumed as in the ACI [9] code in the form:

$$f_t = 0.3 \cdot \sqrt{f_{c'}^{'}} \qquad \text{(MPa)} \tag{32}$$

Substituting Eq. (32) into Eq. (31) gives:

$$v_c = 0.6 \cdot \rho^{0.33} \cdot \sqrt{f_c^{'}} \tag{33}$$

From Eq. (33) it emerges that the shear strength depends on the geometrical ratio of the main steel and on the square root of the compressive strength of concrete, as is also observed utilizing Eq. (30).

Analogously, [2] assuming a splitting bond failure occurring when the circumferential tensile stress reaches the tensile strength of the concrete, obtains:

$$v_c = 0.2 \cdot \sqrt{f_c^{'}} \tag{34}$$

Reference [9] utilizes Eq. (34) but assumes 1/7 instead of 0.2.

From the comparison between Eq. (34), Eq. (33) and Eq. (30) shown graphically in Fig. 3, it emerges that for low percentages of steel the shear strength is overestimated while for ρ>0.025 in the limit $0.2 \cdot \sqrt{f_c^{'}}$ appears appropriate.

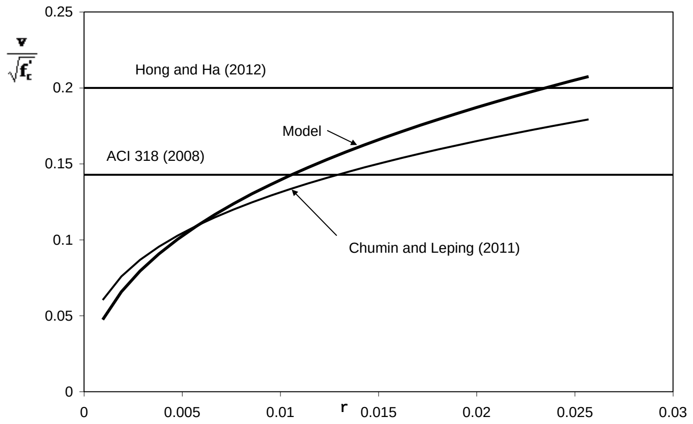


Fig. 3 Variation in shear strength due to bond splitting versus geometrical ratio of steel bars

The resistance contribution in the arch mechanism can be evaluated, with reference to the mechanism shown in Fig. 1, by relating the shear force V2 to the variation in j and by using the following relation:

$$V_2 = T \cdot d \cdot \frac{d\,j}{d\,x} \tag{35}$$

To define Eq. (35) it is necessary to establish a possible variation law of j with variation in x. As originally suggested in [11], the following can be assumed:

$$j = j_0 \cdot \left(\frac{x}{a}\right)^{\alpha} \tag{36}$$

x being measured starting from the support. Deriving Eq. (36) with respect to x, the following relation can be obtained:

$$\frac{d\,j(x)}{d\,x} = j_0 \cdot \frac{\alpha}{a} \cdot \left(\frac{x}{a}\right)^{\alpha\text{-}1} \tag{37}$$

In Eq. (37) if α= 1 is assumed, as suggested in [18], this gives the following:

$$\frac{d\,j(x)}{d\,x} = \frac{j_0}{a} \tag{38}$$

The steel contribution is

$$T_s = \sigma_s \cdot \rho \cdot b \cdot d \tag{39}$$

with σs the stress in the longitudinal bar.

This stress can be related to the bond residual resistance by considering the equilibrium of internal forces in the crack spacing s_{rm} (see Fig. 3).

$$s_{rm} \cdot \pi \cdot \sum q_{resi} \cdot D_i = \sigma_s \cdot \frac{\pi}{4} \cdot \sum D_i^2 \tag{40}$$

$$\sigma_s = \frac{4 \cdot s_{rm} \cdot q_{resi}}{\sum_i^n \cdot D_i} \tag{41}$$

s_{rm} being the cracking spacing determined as in [14]:

$$s_{rm} = \left(50 + 0.1 \cdot \frac{D}{\rho_{eff}}\right) \tag{42}$$

with

$$\rho_{eff} = \frac{3.14 \cdot \sum D_i^2}{4} \cdot \frac{1}{b \cdot (h - x_c)} \tag{43}$$

Substituting Eq. (43) in Eq. (42) and utilizing Eq. (41) gives:

$$v_2 = \frac{j_o}{b \cdot d} \cdot \frac{d}{a} \cdot \sigma_s \cdot \rho \cdot b \cdot d \tag{44}$$

Introducing Eq. (41) and Eq. (29) in Eq. (44) gives:

$$v_2 = \frac{3.54 \cdot j_o \cdot s_{rm} \cdot q_{resi}}{\sqrt{b \cdot d \cdot \rho}} \cdot \rho \cdot \frac{d}{a} \tag{45}$$

Finally, the shear resistance is obtained by the sum of Eq. (30) and Eq. (45):

$$v_c = \left(0.886 \cdot j_o \cdot \sqrt{\rho} \cdot \sqrt{\frac{d}{b}}\right) \cdot \sqrt{f_c^{'}} + \left(\frac{3.54 \cdot j_o \cdot s_{rm} \cdot q_{resi}}{\sqrt{b \cdot d \cdot \rho}}\right) \cdot \rho \cdot \frac{d}{a} \tag{46}$$

Substituting expression of qres in Eq. (46) gives:

$$v = \left(0.886 + 1.17 \cdot \frac{s_{rm}}{a}\right) \cdot j_o \cdot \sqrt{\rho} \cdot \sqrt{\frac{d}{b}} \cdot \sqrt{f_c^{'}} \tag{47}$$

If for example we consider a beam with d=600 mm, minimum a/d=2, s_{rm}=50 mm, the second term of Eq. (47) proves to be 0.09, which is negligible if compared with 0.886. To take the size effect into account the expression of ζ proposed in [8] (see Eq. (17)) was assumed.

It has to be stressed that if in the arch model the resistance is governed by the crushing of concrete, an upper limit for the shear strength (Eq. (47)) has to be taken into account. By imposing the failure of the arch in compression and by relating the axial force in the arch with the shear force at the support it is possible to derive the ultimate shear strength related to the crushing of the concrete arch.

The ultimate axial force in the concrete strut is:

$$N_u = \upsilon \cdot f_c^{'} \cdot b \cdot x_c \cdot \cos\alpha \tag{48}$$

υ being the softening coefficient assumed as in the Swiss design code [19] in the form:

$$\upsilon = 0.6 \cdot \left(\frac{30}{f_c^{'}}\right)^{0.33} \tag{49}$$

and

$$\alpha = \arctan\left(\frac{j \cdot d}{a}\right) \tag{50}$$

The equilibrium of the forces at the support gives:

$$V_u = N_u \cdot \sin\alpha \tag{51}$$

and therefore the ultimate shear stress related to the arch crushing:

$$v_{uc} = \frac{\upsilon \cdot f_c^{'}}{2} \cdot \frac{x_c}{d} \cdot \sin(2\alpha) \tag{52}$$

At least, the shear resistance given by Eq. (47) should be lower than the value given by Eq. (52) to avoid premature crushing of the compressed arch.

The values derived from Eq. (52) are in agreement with the values suggested by ACI 318 [9] for the upper limit of the shear strength of $v_{uc} = 0.3 \cdot \sqrt{f_c}$ as shown in Fig. 4.

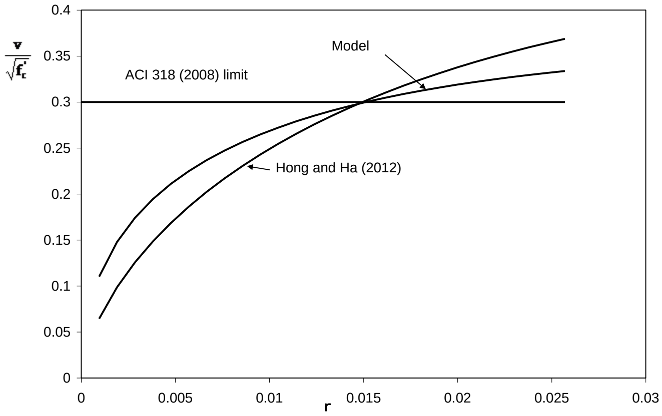


Fig. 4 Variation in shear strength due to concrete crushing versus geometrical ratio of steel bars

By utilizing the approach of Hong and Ha [2] the shear strength associated with the crushing of the compressed strut (assumed to be reduced in area) proves to be:

$$v_{uc} = 0.75 \cdot f_c^{'} \cdot \left[\xi^2 \cdot (1-\xi) \cdot (1+\xi-\xi^2)\right] \cdot \frac{d}{a} \tag{53}$$

with the neutral axis position expressed as:

$$\xi = 1.7 \cdot \sqrt[3]{\rho} \tag{54}$$

Eq. (53), for a practical range of a/d between 2 and 6 and ρ up to 3%, gives results in the same range as Eq. (52) and the 0.3 limit value given by ACI 318 [9] proves to be the upper limit for ρ<0.025.

In the presence of stirrups the shear strength contribution due to the stirrups is calculated, as in ACI 318 [9], in the form:

$$v_{st} = \frac{A_{st}}{b \cdot s} \cdot f_{ys} = \rho_{sw} \cdot f_{yw} \tag{55}$$

f_{yw} being the yielding stress of the stirrups, A_{st} the area of the two legs of one stirrup and $\rho_{sw} = \dfrac{A_{st}}{b \cdot s}$.

III. EXPERIMENTAL VALIDATION

In this section, a validation of the equations mentioned for flexural strength is first of all provided, followed by a validation of the shear strength with reference to available experimental data.

For the experimental validation of flexural capacity, 48 data from [20-23] were utilized. No safety factors were utilized in the application of code expressions. Using Eq. (7) it emerges that all data refer to under-reinforced beams. The data utilized had: - compressive concrete strength between 36 and 107 MPa; - effective depth between 215 and 264 mm; - ρ between 0.76 and 3.61%; - f_y between 300 and 579 MPa.

Table 2 gives the mean, standard deviation, correlation factor, minimum, maximum and median factors for all cases examined. From the results it emerges that all the expressions utilized underestimate the effective flexural capacity of HSC beams and in most of the cases examined conservative results are obtained.

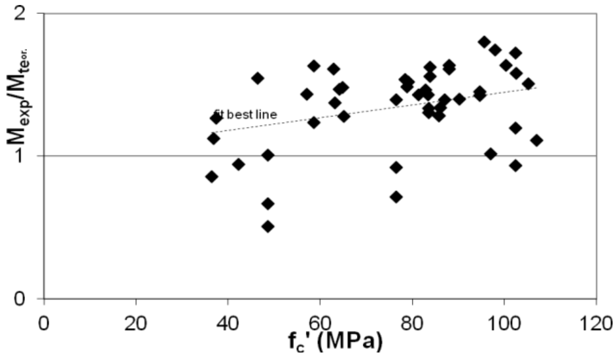
TABLE 2 FLEXURAL STRENGTH PREVISION

Code	Mexp./Mteo.	St.dev	Cov.	Min.	Max.	Mediana
A318-08	1.17	0.49	0.42	0.54	2.58	1.05
CAN3-A23-3-M24 (1994)	1.34	0.29	0.22	0.50	1.8	1.39
Eurocode 2 (1994)	1.38	0.29	0.22	0.50	1.80	1.40
Bae and Bayrak (2003)	1.34	0.29	0.22	0.50	1.80	1.39

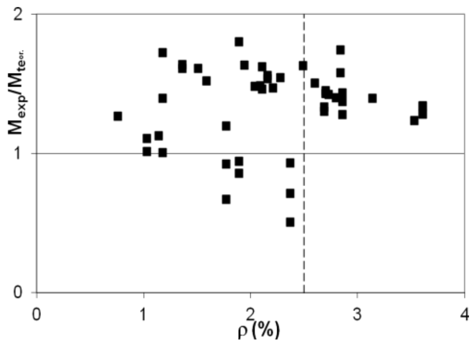
Fig. 5a) shows the variation in ultimate experimental moment referring to an analytical moment calculated with Bae and Bayrak's coefficients [24], with variation in the compressive strength of the concrete. It has to be stressed that as the concrete strength increases a progressive increase in underestimation of moment capacities is observed (see best line fitting), as already observed in [25] and explained by early cover spalling in HSC columns.

Fig. 5b) shows the variation in ultimate experimental moment and analytical moment versus the geometrical ratio. The upper limit of 0.0025 indicated by ACI 318 [9] is also given. The results obtained show that the limit of 0.0025 is much too conservative to ensure that steel bars yield before crushing of concrete and higher values could be utilized. But the latter choice has to be checked very carefully if members have to be designed in seismic areas.

For experimental validation of shear strength, two sets of data were utilized. The first set of data was the same as utilized in [8] and not given here for brevity's sake; the other set of data collected used were from those of [1, 2, 5, 25, 26, 27]. Reference [8] consider 164 data with: - compressive concrete strength between 12.8 and 103 MPa; - effective depth between 95 and 1200 mm, - a/d between 2.4 and 5.05; ρ between 0.5 and 4.54%; - ρv between 0.06 and 0.84; - f_y between 242 and 844 MPa. 51 data collected in the current paper had: - compressive concrete strength between 40 and 94 MPa, - effective depth between 160 and 655 mm; -a/d between 2 and 6; - ρ between 0.6 and 6.64 %, ρv between 0.08 and 1; f_y between 370 and 430 MPa.



a)



b)

Fig. 5 Variation in experimental moment versus analytical moment with: a) compressive strength; b) steel percentage

Fig. 6 shows the variation in ultimate experimental shear strength versus analytical shear strength calculated with all the models mentioned here. Table 3 gives mean, standard deviation, correlation factor, minimum, maximum and median factors for all cases examined. From the results it emerges that most of the expressions proposed predict the shear strength of HSC beams accurately.

TABLE 3 SHEAR STRENGTH PREVISIO

Code	vexp./vteo.	St.dev	Cov.	Min.	Max.	Mediana
A318-08	1.27	0.21	0.16	0.70	1.84	1.29
CAN3-A23-3-M24 (1994)	1.43	0.36	0.25	0.36	2.52	1.43
Eurocode 2 (1994)	2.42	0.99	0.40	0.98	7.98	2.27
Arslan (2008)	1.32	0.38	0.29	0.73	2.25	1.40
Russo et al. (2013)	1.12	0.211	0.189	0.64	1.61	1.12
Current model	1.24	0.193	0.145	0.78	1.74	1.27

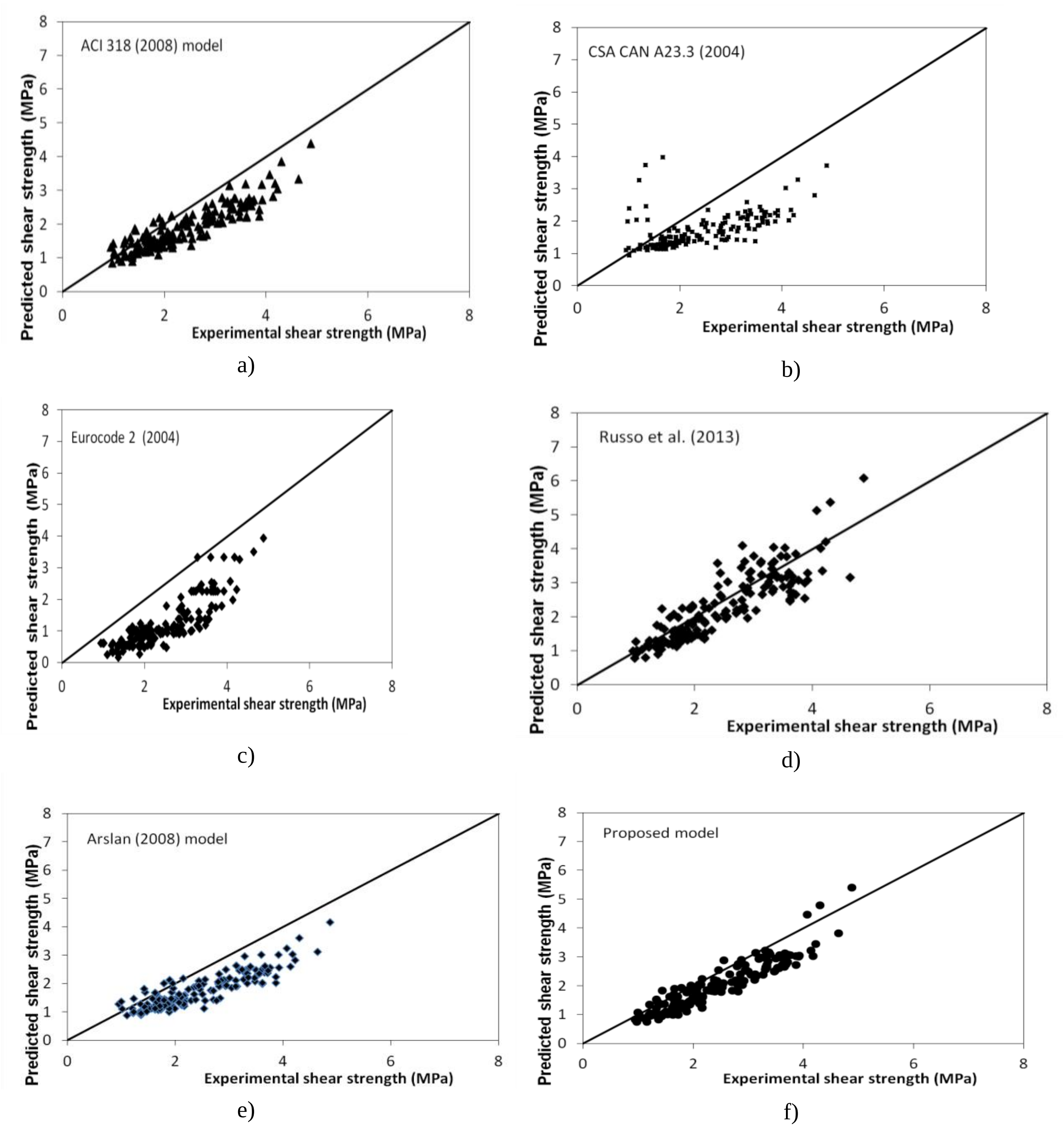


Fig. 6 Variation in experimental shear stress versus analytical stress according to the following models: a) ACI 318 [9]; b) CSA CAN A23.3 (2004); c) Eurocode 2 (2004), d) Russo et al. (2013); e) Arslan (2008); f) proposed model

Fig. 7 shows the variation in ultimate dimensionless shear stress with respect to $\sqrt{f_c}$ versus the geometrical ratio of the longitudinal bars. Only cases of beams without stirrups are considered. Analytical prediction with the proposed model and with ACI 318 [9] and Hong and Ha [2] models are also given. The results obtained highlight that for a geometrical ratio up to 0.025 all models are quite conservative and specifically the current model gives safer prediction.

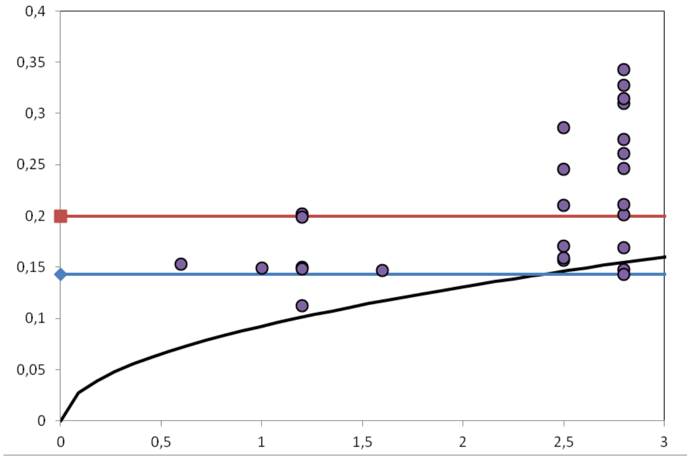


Fig. 7 Variation in experimental shear stress versus analytical stress with ρ (%)

Fig. 8 shows the variation in ultimate dimensionless shear stress with respect to $0.2\sqrt{f_c}$ versus the mechanical ratio of the stirrups. The choice of $0.2\sqrt{f_c}$ was related to the results obtained in the absence of stirrups to be compared with the bond failure mode calculated as in [2].

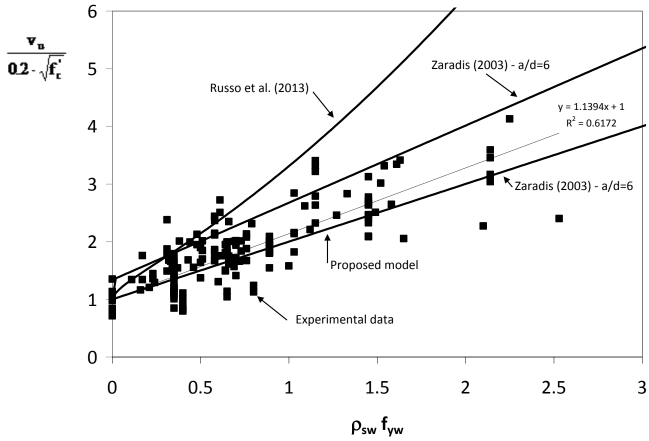


Fig. 8 Variation in shear stress ($\frac{v_u}{0.2\cdot\sqrt{f_c'}}$) versus $\rho_{sw}f_{sw}$

In the same graph prediction with the current model and with the Russo et al. [7] and Zararis [8] models are given. The best fitting line is also shown. It has to be stressed that all experimental data are in the range expected with the Zararis model [8], while in most cases the Russo et al. model [7] overestimates the real shear strength of beams. The proposed model gives the most conservative results.

IV. CONCLUSIONS

The existing analytical expressions for the calculation of the flexural and shear resistance of high strength reinforced concrete beams with stirrups are reviewed. An analytical model for the determination of the maximum shear resistance of reinforced concrete beams in the presence of stirrups is presented. The model is based on the determination of the resistance contributions due to beam and arch effects including bond splitting and concrete crushing failure, plus stirrup yielding. The model was validated on the basis of experimental data given in the literature, showing its capacity to predict the shear resistance of beams with good approximation.

ACKNOWLEDGMENTS

This work has benefited from material derived the 2010-2013 Research Project ReLUIS (Rete dei Laboratori di Ingegneria Sismica), AT 1, Task 1.1.2: Strutture in Cemento Armato ordinarie prefabricate.

REFERENCES

[1] S. H. Ahmad and D. M. Lue, “Flexural-shear interaction of reinforced high strength concrete beams,” *ACI Structural Journal*, Title no. 84-S36, 1987.

[2] S. G. Hong and T. Ha, “Effective capacity of diagonal strut for shear strength of reinforced concrete beams without shear reinforcement,” *ACI Structural Journal*, vol. 109, no. 2, pp. 139-148, 2012.

[3] M. Kunthia, B. Stojadinovic, and S. C. Goel, “Shear resistance of normal and high-resistance fiber reinforced concrete beams without stirrups,” *ACI Structural Journal*, vol. 965, no. 2, pp. 282-289, 1999.

[4] M. Kunthia and B. Stajadinovic, “Shear strength of reinforced concrete beams without transverse reinforcement,” *ACI Structural Journal*, vol. 98, no. 5, pp. 648-656, 2001.

[5] A. G. Mphonde and G. C. Frantz, “Shear tests of high and low strength concrete beams without stirrups,” *ACI Structural Journal*, vol. 81, no. 4, pp. 350-357, 1986.

[6] G. Arsaln, “Shear strength of reinforced concrete beams with stirrups,” *Material and Structures*, vol. 41, pp. 113-122, 2008.

[7] G. Russo, D. Mitri, and M. Pauletta, “Shear strength design formula for RC beams with stirrups,” *Engineering Structures*, vol. 51, pp. 226-235, 2013.

[8] P. D. Zararis, “Shear strength and minimum shear reinforcement of reinforced concrete slender beams,” *ACI Structural Journal*, vol. 100, no. 2, pp. 203-214, 2003.

[9] ACI Committee 318, “Bulding code requirements for structural concrete and commentary,” American Concrete Institute USA P.O.BOX9044-Framington Hills, MI 48333, 2008.

[10] T. C. Zsutty, “Beam shear strength prediction by analysis of existing data,” *ACI Structural Journal*, vol. 65, no. 11, pp. 943-951, 1968.

[11] Z. P. Bazant and S. S. Kim, “Size effect in shear failure of longitudinally reinforced beams,” *ACI Struct. J.*, vol. 81, no. 5, pp. 456-468, 1984.

[12] Canadian Standards Association. CAN CSA A23.3-04 Design of concrete structures. CSA, Rexdale, Ontario, 2004.

[13] E. C. Bentz and M. P. Collins, “Simplified form of the modified compression field theory (MCFT) for the analysis of beams,” ID 3-56 of Proc. of Second Int. Conf., Naples Italy, June 5-8, 2006.

[14] Eurocode 2: Design of Concrete Structures – Part 1: General rules and rules for buildings, European Committee for Standardization (CEN), 2005.

[15] R. Eligehausen, E. P. Popov, and V. V. Bertero, “Local bond stress–slip relationship on deformed bars under generalized excitations,” Report N.O.UCB/ERC-83/23, pp. 1-168, 1983.

[16] M. Harajli, M., “Comparison of Bond Strength of Steel Bars in Normal- and High-Strength Concrete,” *J. Mater. Civ. Eng.*, vol. 16, no. 4, pp. 365-374, 2004.

[17] D. Chunmin and N. Leping, “A general shear strength method for reinforced concrete beams,” proceedings of the International Conference on Electric Technology and Civil Engineering - ICETCE, 22-24, pp. 17-20, April 2011.

[18] R. N. Swamy, R. Jones, and A. T. P. Chiam, “Influence of steel fibers on the shear resistance of lightweight concrete I-beams,” *ACI Structural Journal*, vol. 90, no. 1, pp. 103-114, 1993.

[19] SIA 262. Concrete structures. Code, Swiss society of Engineers and Architects, Zurich, 90 pp, 2003.

[20] H. J. Pam, A. K. Kwan, and M. S. Islam, “Flexural strength and ductility of reinforced normal and high strength concrete beams,” *Structure and Building*, vol. 146, no. 4, pp. 381-389, 2001.

[21] S. Sarkar, O. Adwan, and J. G. Munday, “High strength concrete: an investigation of the flexural behavior of high strength R.C. beams,” *The structural Engineer*, vol. 75, no. 7, pp. 115-121, 1997.

[22] L. F. A. Bernardo and S. M. R. Lopes, “Neutral axis depth versus-flexural ductility in high strength concrete beams,” *ASCE Journal of Structural Engineering*, vol. 130, no. 3, pp. 425-459, 2004.

[23] S. A. Ashour, “Effect of compressive strength and tensile reinforcement ratio on flexural behavior of high strength concrete beams,” Engineering Structures, vol. 22, pp. 413-423, 2000.

[24] S. Bae and O. Bayrak, “Stress Block Parameters for High-Strength Concrete Members,” *ACI Structural Journal*, vol. 100, no. 5, pp. 626-636, 2003.

[25] A. H. Elzanaty, A. H. Nilson, and F. O. Slate, “Shear capacity of reinforced concrete beams using high strength concrete,” *ACI Structural Journal*, vol. 83, no. 2, pp. 290-296, 1986.

[26] Y. S. Yoon, W. D. Cook ,and D. Mitchell, “Minimum shear reinforcement in normal, medium, and high strength concrete beams,” *ACI Structural Journal*, vol. 93, no. 5, pp. 576-584, 1996.

[27] R. S. Pendyala and P. Mendis, “Experimental study on shear strength of high strength concrete beams,” *ACI Structural Journal*, vol. 97, no. 4, pp. 564-571, 2000.