Linear Inclined-Head-Discharge Equation for Flow Through Inclined Inverted V Notch

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Abstract: **Inverted V-Notch, as a measuring device, is found to have a near linear head-discharge relationship with the flow through the weir beyond a base-flow depth. This paper is concerned with the analysis of the flow over an Inclined Inverted V-Notch (IIVN) from a different perspective. Based on experimental work on 0^0 (normal), 15^0, 30^0, 45^0 and 60^0 weirs inclinations with the vertical plane across the flow direction, a generalized inclined-head-discharge equation was established for the flow through the weir. It was shown that the IIVN retains near linear discharge-head characteristics even when it is inclined to the channel bed. The discharge estimated by the linear discharge-head relationship was found to be in good agreement with the actual discharge and this was validated by the near unity regression coefficients of the same experimental data. Furthermore, the advantages of Inward Trapezoidal Weir as a flow measuring and flow control device were highlighted. With the linear head-discharge relationships and increased discharging capacity, the weir can be used in flood situations for flow measurements as the discharge can be directly read-off from the Piezometric graduations. In addition, the maintenance of uniform velocity, independent of discharge variation in sedimentation tanks or grit chambers, is an absolute necessity to settle the grit. The IIVN can maintain the uniform velocity irrespective of the variation of discharge and is highly useful in such situations.**

Keywords: Weirs; IVN; IIVN; Inclined-weir-discharging-index; Discharge Coefficient; Flow measurement and Afflux.

I. INTRODUCTION

In the fields of Hydraulics, Environment, Irrigation and Chemical engineering, weirs are most widely used as flow measuring/flow control devices. Conventional rectangular, triangular, and trapezoidal weirs are among the oldest weirs. In particular, triangular weirs are very accurate to measure low flow discharges in laboratories [1,2].

Keshava Murthy and Giridhar [2] have proposed the use of Inverted V-Notch (IVN) for flow measurement for nearly 72 % of the depth of IVN within a prefixed permissible error of ±1.5%. They developed two optimization procedures to obtain the linear head-discharge relationship through the weir. Using the same optimization procedures, they improved the linearity range by introducing a rectangular weir at an optimum height over the IVN.

Later, Keshava Murthy and Shesha Prakash [4,5] presented a general numerical optimization procedure to obtain the proportionality range for any sharp crested weir to develop any type of head-discharge relationship. Subsequently, the same authors presented a general algebraic optimization procedure to obtain the linear characteristics of an inverted semi-circular weir [6].

Furthermore, Udayasimha et al. [7] have analysed the effect of the crest height and width of IVN on the discharge coefficient. It has been the subject of importance for hydraulic researchers to develop flow-measuring structures that give higher coefficient of discharge and thereby higher discharging weirs. This is evident from the flow of literature, which shows that the upstream and/or downstream weir profiles have been modified for the above said purpose. Hydrofoil weirs were also an example of this type [8]. Ramamurthy and Vo Ngoc-Diep [9] tried to improve discharge coefficient by providing different upstream and downstream slope for circular crested weirs. Furthermore, USBR [10] studies include various profiles of Ogee-weir with inclined upstream and downstream faces. These are attempts to improve the discharge coefficient of the weir. The analysis was carried out by considering the upstream profile as a sharp crested weir [10]. Hence, Shesha Prakash and Shivapur [11,12,13,14] attempted to analyze the flow through inclined sharp-crested weirs, which have increased relative discharge capacity. The additional weight of the wedge of water above the weir opening increases the discharge capacity of the weir, thereby increasing the flow rate.

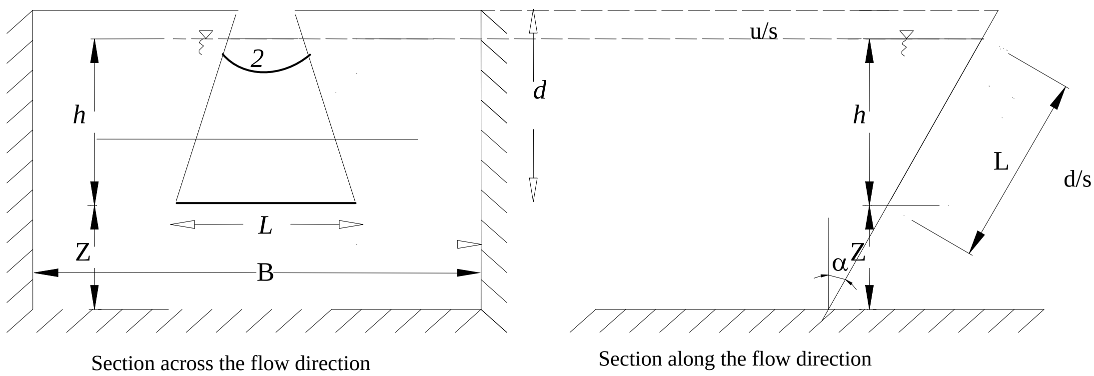

Fig. 1. Arrangement of Inclined Inward Trapezoidal Weir (IITW) or Inclined Inverted V-notch (IIVN)

Shesha Prakash and Shivapur [15] showed that the general head-discharge equation proposed on the basis of an optimization procedure can estimate the discharge through the inclined Inverted V-notch (IIVN) with an accuracy of 5%. They introduced a parameter β to compensate for the non-parallel stream-lined behaviour and showed that the equation for the flow through the inclined weir is still a linear equation.

In the present study, the flow through an IIVN was analyzed. The weir plane was inclined along the flow direction, with reference to the normal plane perpendicular to the general flow direction in the channel (vertical). The main objective of this work is to improve the analysis of flow through the weir, thereby to obtain an expression for the inclined-discharge-head relationship using a newly developed mathematical model.

II. MATHEMATICAL EQUATION

The discharge through the inverted V-notch along the general flow direction in the channel along vertical projection of the weir, i.e. *x*-axis is given by

$$q = C_d\left[\frac{2}{3}\sqrt{2g}\,(2w)\,h^{3/2} - \frac{8}{15}\sqrt{2g}\tan\theta\, h^{5/2}\right], \quad (0 \le h \le d), \tag{1}$$

where
q is the discharge through the weir,
C_d is the coefficient of discharge, treated to be constant with respect to device,
g is the acceleration due to gravity,
w is the half crest-width of the IIVN,
h is the head over the weir crest, and
$2q$ is the vortex angle of the IIVN.
Eq. 1 can be non-dimensionalised as

$$Q = \frac{2}{3}H^{3/2} - \frac{4}{15}H^{5/2}, \qquad (0 \le H \le 1), \tag{2}$$

where $Q = q \big/ 2C_d\sqrt{2g}\,d^{5/2}\tan q$, $\quad H = \frac{h}{d}$.

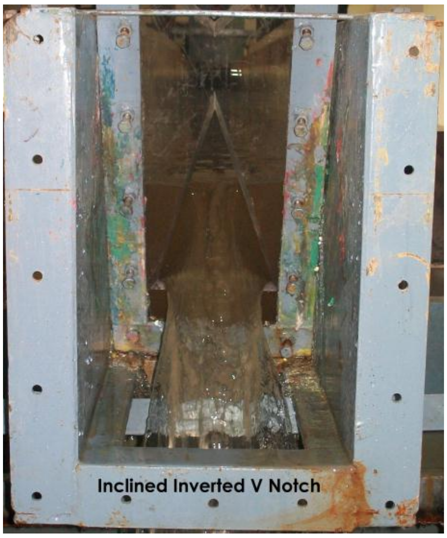

Fig. 2 The Flow through an inclined inverted V-notch

III. III EXPERIMENTS

Experiments were carried on an inclined inverted V-notch fixed normal to the flow directions of 0^0, 15^0, 30^0, 45^0, and 60^0 inclinations with respect to the normal plane (vertical) along the flow axis. The experimental channel was rectangular in section, and 0.28 m in 0.45 m in depth and 11.5 m in length. The channel was constructed of Plexiglass (Perspex) and had smooth walls and bed with nearly horizontal bed to reduce the boundary frictional force. It was connected to a head tank of dimensions 1.5mx1.5mx1.5m. The inverted V-notch was made of 8 mm Plexiglass (Perspex) with a crest thickness of 1 mm and a 45^0 chamfer given on downstream side to get a springing nappe. The inverted V-Notch used for the experiments was of 150 mm crest width and 30^o subtended angle at the top.

The experimental setup is shown in Fig. 2. Water was supplied to the channel by an inlet valve on the supply pipe. The overhead tank was provided with an overflow arrangement to maintain constant head. Smooth, undisturbed and steady-uniform flow was obtained by making the water to flow through graded aggregates and the surface waves were dampened by tying gunny bags at the surface near the tank. The head over the weir was measured using an electronic point gauge placed in the piezometer located at a distance of about 1.40 m on the upstream of the inclined rectangular notch. The least count of the point gauge was 0.1 mm. A collecting tank, 1.465 m in length, 1.495 m in width and 1.5 m in depth, was provided with a piezometer. The water through the experimental setup was collected by an underground sump from which it is re-circulated by lifting it back to the overhead tank with a pump.

In the present study, the conventional method of volumetric discharge measurement was used improve the accuracy. The measurements were done through an electronic point gauge with an accuracy of 1 mm, which automatically detects the water level and records the gauge reading. The volumetric measurement was done by a self-regulated timer for a fixed rise of water level automated through sensors.

The variations of the parametric values of the actual head and discharge are given in Table 1.

TABLE 1 EXTREME EXPERIMENTAL VALUES OF THE ACTUAL HEAD AND DISCHARGE

Particulars	H (m)	Q (m^3/s)	Q (LPS)
Minimum	0.0011	0.0058	5.8069
Maximum	406.52	0.1037	103.66

IV. MATHEMATICAL MODELING

This paper proposes a programmable iterative algorithm to intermittently improve the efficiency of the interpolation by the well-known simple and popular ***Newton-Gregory Forward Interpolation Formula***, statistical perspective of ***Reduced-Bias***. The impugned formula uses the values of the simple forward differences using values of the unknown function $f(x)$ at equidistant-points knots in the Interpolation-interval $[x_0, x_n]$. It is a finite difference identity capable of giving an interpolated value between the tabulated points $\{f_k\}$ in terms of the first value f_0 and powers of the forward difference "Δ".

$$f_k = f_0 + k\Delta_1 + \frac{k(k-1)}{2!}\Delta_2 + \frac{k(k-1)(k-2)}{3!}\Delta_3 + ... + \frac{k(k-1)(k-2)....(k-n+1)}{n!}\Delta_n,$$

where, k is any real number; $\Delta_1, \Delta_2, \Delta_3, ...\Delta_n$ are respectively the first, second, third,… n^{th} forward differences.

The discharge-head-inclination equation can be expressed as

$$Q = f(a)H + \phi(\alpha). \tag{3}$$

The modeling part was divided into two stages.

In the first stage, the actual head-discharge data values were statistically fitted with the exponential equation. $Q = mH + C$.

TABLE 2 CALIBRATED HEAD-DISCHARGE EQUATIONS FOR VARIOUS ANGLES α

S. No	α	$Q=mH+C$	C_{di}	Afflux
1	0	$Q = 0.503H - 0.0095$	1.00	0.0%
2	15	$Q = 0.544H - 0.0113$	1.03	6.4%
3	30	$Q = 0.580H - 0.0108$	1.20	13.3%
4	45	$Q = 0.692H - 0.0172$	1.34	21.6%
5	60	$Q = 0.840H - 0.0133$	1.89	37.6%

In the second stage, the ***Newton-Gregory Forward Interpolation Formula*** was adopted to develop the model for the present problem. It was observed that there were 5 values and hence only value up to n-1, i.e. 4^{th} order polynomial curve can be fitted with the data. By simplifying the equations, we get the final general head-discharge-angle expression for any given notch and inclination was obtained as below:

$$Q = mH + C, \ (0 \le H \le 1), \tag{4}$$

where m is the slope of the proposed linear head-discharge relationship and C is the intercept.

$$m = 5.74\text{x}10^{-07}\alpha^3 - 2.68\text{ x}10^{-06}\alpha^2 + 1.25\text{ x}10^{-03}\alpha + 0.281, \tag{5}$$

$$C = 2\times10^{-8}\alpha^4 - 2\times10^{-6}\alpha^3 + 8\times10^{-5}\alpha^2 - 9\times10^{-5}\alpha - 9.1\times10^{-4}, \tag{6}$$

where $Q = \dfrac{q}{\frac{4}{5}\sqrt{2g}\tan q d^{\frac{3}{2}}}$; $H = \dfrac{h}{d}$.

The same was obtained as an excel curve fitted as shown in Fig. 5.

The comparison with the equation developed by Shesha Prakash and Shivapur [14,15] was attempted as below.

$Q = mH + \beta C$,

where $\beta = 0.193\ \alpha + 0.108$.

However, the expression obtained could not be compared for two reasons. The value of m is a constant as 0.04481 and the regression to find β has been for dimensional and the limitation is that it has to be computed for similar dimensional of experimental setup and weir constants. It is visible that even though the estimated discharge values were dependent on m and their scope was limited to only C by retaining m to be independent of α and relied only on β and in turn C.

It can be seen that the equation developed by the model agrees with the one obtained by Excel and furthermore, the regression coefficients in both the cases are exactly unity. That improves the credibility of the analysis and practical usage of the notch. Even though the obtained discharge-head-inclination equation is complicated, it reduces to a simple equation once the α values are substituted and simplified.

V. DISCHARGING CAPACITY AND AFFLUX

The discharging capacity of the inclined inverted V-notch relative to normal position is shown in Fig. 4. The variation of the Inclined-Weir-Discharging Index (C_{di}), defined as the ratio of the discharge coefficient of normal weir to that of the inclined weir, with the various inclinations of the weir is shown in Fig. 5.

$$C_{di} = 5\text{x}10^{-07}\alpha^4 - 5\text{x}10^{-05}\alpha^3 + 1.85\text{x}10^{-03}\alpha^2 - 1.53\text{x}10^{-03}\alpha + 1 \text{ and the corresponding } R^2 = 1. \tag{7}$$

From Fig. 4, it is seen that the discharging capacity was increased by over 189% comparing with normal position of the notch, validating the theory and experiments.

Afflux is the heading-up of water on the upstream of any obstruction, like weir, that restricts the usage of the device due to additional problem of creating additional height of the channel. That can be avoided in inclined weir as it can relatively reduce the afflux developed on the upstream side of an inclined weir due to its additional discharging capacity.

It can be seen in Table 2 that the afflux of the inclined inverted V-notch was decreased by 37.6% comparing with its corresponding normal position.

VI. PRACTICAL APPLICATION

Due to the linear head-discharge relationship, the inclined inverted V-Notch can be used in rectangular channels in order to directly read off the discharge by having a suitable scale in the Piezometer attached to the channel. The ease of use can motivate the farmers to use the same approach in their channels to know the discharge, which will be in the numerical scale without going through any computations.

In case of water treatment plants in environmental engineering, and chemical mixing plants as dosing device in chemical engineering, currently a device with varying cross-sectional area is being used to maintain constant velocity, which relies on discharge and head. However, the proposed device can be used without any monitoring as it maintains constant velocity that is independent of discharge as shown in section 11.

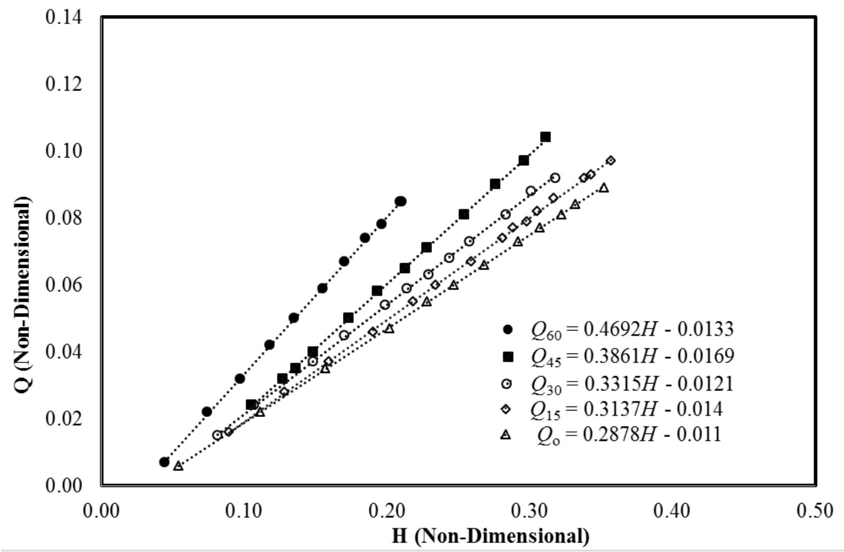

Fig. 4 Actual and Proposed Non-dimensional Discharge-Head relationship for various inclinations

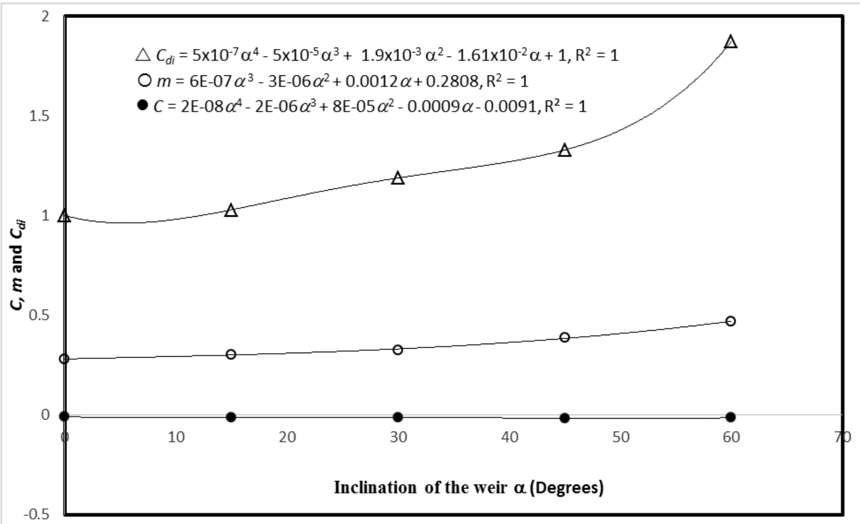

Fig. 5 Variations of m, C and C_{di} with respect to weir inclination angle α

VII. CONCLUSIONS

Following conclusions were drawn based on the experimental investigation and the subsequent analysis by the authors.

- The discharging capacity of the weir increases with the increase in the inclination of the plane of weir. In particular, the discharging capacity was found to be maximum at 60^0 inclination at 189% of that of the normal position.
- The characteristic that the discharge coefficient increases with the increase in inclination of weir plane can be used to discharge more water quickly without increasing the afflux on upstream side in pre-designed canal structure during flood season and without changing the pre-installed weir, which is very difficult in practice.
- The linearity of the head-discharge relationship for the flow through the Inclined Inward Trapezoidal Weir is independent of the weir's inclination with respect to its normal position.
- Due to the simple geometry and ease of construction, Inclined Inward Trapezoidal Weir finds its applications as a simple measuring device in irrigation, chemical and sanitary engineering for flow measurement and flow control. The discharge readings can be marked on a piezometer, which can be read directly by a farmer.
- The mathematical modeling results in a single head-discharge-inclination equation, which can be used for any trapezoidal notch of any desired inclination.

VIII. LIMITATION

The experiment can be done with larger discharge in larger channels and the Head-Discharge-Inclination equation can be improved by using the model. However, the equations that were developed by the mathematical modelling and then verified by regression analysis are dimensionless and hence can be used for any values based on the channel dimensions.

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X. PROOF OF CONSTANT VELOCITY INDEPENDENT OF DISCHARGE

Discharge through a rectangular channel or tank is given by $Q = AV$

Where $A = B \times y$

$Q = (By)V$

$Q \alpha yV$ (as B is a constant) (8)

The discharge through a Linear weir $Q \alpha y$ (9)

From Eqs 8 and 9, we get

$V =$ Constant

Hence independent of Discharge

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