Size Dependent Shear Strength Of Reinforced Concrete Deep Beams Based On Refined Strut-And-Tie Model

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Abstract- **This paper reports on analytical investigations on the development of shear strength expression applicable to reinforced concrete (RC) deep beams. The shear strength of RC deep beams provided with shear reinforcement incorporating size dependent parameters has been studied. The development of the shear strength expression is based on the shear transfer mechanism of deep beams idealized through a refined strut-and-tie model according to the modified Bazant's size effect law and a large experimental data. The accuracy of the proposed expression has been validated with other existing models including the ACI Strut-and-Tie model. By incorporating various influencing parameters such as compressive strength of concrete, shear span-to-depth ratio, effective depth of beam and percentage of flexural steel reinforcement, the estimation of the shear strength of RC deep beams has been observed to be relatively better.**

Keywords- Deep Beam; Shear Strength; Strut-and-tie Mode; Size Effect; Reinforced concrete

I. INTRODUCTION

Several experimental and analytical studies on the shear strength of reinforced concrete (RC) beams have been reported [1-23]. Primarily there are two types of shear failures reported in RC deep beams *viz.*, diagonal tension and diagonal splitting or shear compression failure, depending on the ratio of the shear span-to-effective depth (*a/d*). If the *a/d* ratio is 2.5 or more, the diagonal tension failure can occur due to the inclined tensile cracking propagated into the entire depth of the compression portion. On the other hand, if the *a/d* ratio is less than 2.5, the diagonal splitting or shear compression failures can occur due to the arch action after flexural cracking. Apart from the above, there can also be other modes of failure including concrete web crushing, crushing of concrete underneath the supports and anchorage failure. The classical flexural beam theory is not applicable for the design of RC deep beams due to complex state of stress as described through St. Venant's principle. The prediction of shear strength of RC deep beams using strut-and-tie models at *a/d* ratio less than 2.5 appears to be reasonable. Procedures for the design of deep beams using strut-and-tie models are being incorporated in various national codes of practice, and also in technical standards such as ACI 318 [24], AASHTO LRFD [25], Canadian Code [26], and CEB-FIP [27]. The generic form of the strut-and-tie model to deep beams requires an iterative procedure for solution. Further, the results obtained by such models seem to be not satisfactory for RC deep beams made of high strength concrete due to their high brittleness. The influence of the flexural tensile reinforcement and the web reinforcement in resisting the applied shear can be significant. These shortcomings can be overcome using the refined strut-and-tie model.

After the flexural cracking, RC deep beams can further resist the loading as compared to the slender beams due to the redistribution of internal stresses, and its direct transfer of loads to the supports through the arch action. This load transfer mechanism distinguishes the behavior of RC deep beams significantly from that of the slender beams. For slender RC beams with the shear span-to-depth (*a/d*) ratio greater than 2.5, the nominal shear stress at failure decreases as the depth increases due to the size effect. In analyzing the reported experimental data, the size effect has been properly accounted for in the prediction of the shear strength of RC beams [28-30]. The dependence of the nominal stress at failure on the beam depth can be explained through the release of the stored elastic strain energy in the bulk of the crack front in concrete called the fracture process zone and the progressive nature of failure. Hence, the complex failure of concrete structures can be described by the energy approach using fracture mechanics than the strength based approaches. Walraven and Lehwalter [31] investigated the behavior of RC beams by varying the beam effective depth from 160mm (6.3 in.) to 930mm (36.61 in.) with *a/d* ratio less than 2.5. It has been observed that the crack propagation was rapid in large size beams. A significant size effect in beams without shear reinforcement while non negligible size effect in beams with shear reinforcement, has been observed. The strut-and-tie mechanism was significantly observed in RC beams with *a/d* ratio less than 2.5. Despite the fact that the results confirm the existence of size effect on the shear strength of deep beams. However, no accurate prediction of the shear strength of deep beams is reported. For RC deep beams, the size effect law could be applicable to the component of strut-and-tie alone, and hence the size effect factor for the strut-and-tie mechanism in RC deep beams becomes inevitable.

II. RESEARCH SIGNIFICANCE

The strut-and-tie model has been observed to be very useful for the development of the shear strength models for RC deep beams. Such popular proposals include Zhang and Tan [32], and strut-and-tie model by ACI 318 [24]. A study on predicting the size dependent shear strength of RC deep beams with shear reinforcement needs to be undertaken using the existing large experimental data. The accuracy of prediction by these models needs comparison with the reported models.

III. ENERGY BASED SIZE EFFECT LAWS

The generic size effect law for beams made of brittle materials was proposed by Bazant and Kim [28] in the form of $\sigma_N=Bf_t'/(1+d/(\lambda_0 d_a))^{1/2}$, in which σ_N = nominal stress at failure, B = a non-dimensional constant, f_t' = direct tensile strength of concrete, d = depth of the beam, λ_0 = a constant and d_a = maximum size of aggregate. In spite of the fact that the size effect law is in good agreement with the experimental test results, there exists some discrepancy as stated by Kim and Park [30] between the prediction of Bazant's size effect Law and the experimental data, particularly for large size beams. To reduce the discrepancy, a modified size effect law has been proposed by Kim and Eo [33] based on the principle of dissimilar initial cracks, which was used by [30] in the following form:

$$\sigma_N = \frac{k_1\sigma_r}{(1+k_4 d)^{1/2}} + k_2\sigma_r \tag{1}$$

Where σ_N is the size dependent nominal strength of the beam at failure, σ_r is the size independent stress, and k_1, k_2, and k_4 are the empirical constants. In order to ensure the dimensional homogeneity and also to maintain the originality of Bazant's size effect law, the modified Bazant's law in Eq. (1) can be rewritten in the form below:

$$\sigma_N = k_1\sigma_r\left(\frac{k_2}{k_1} + \frac{1}{(1+d/(\lambda_0 d_a))^{1/2}}\right) \tag{2}$$

IV. REFINED STRUT-AND-TIE MODEL

Based on the refined strut-and-tie model, it has been undertaken to formulate the size independent shear strength of deep beams with the shear span-to-overall depth or height (a/h) ratio less than or equal to 1.0. The internal forces in the refined strut-and-tie model of a deep beam on the application of external forces are shown in Fig. 1. The tensile stress (f_t) acting normal to the diagonal (AB), can be determined from the following equation

$$f_t = \frac{V}{2\sin\alpha}\left(\frac{\sin\alpha}{bz}\right) = \frac{V}{2bz}$$

From the above relationship, the shear strength can be written as

$$V = 2 f_t bz \tag{3}$$

Where V is the shear strength in the beam cross-section, b is the breadth of the beam, and z is the lever arm between the centroid of the flexural steel reinforcement and the line of the horizontal compression strut of the RC deep beam.

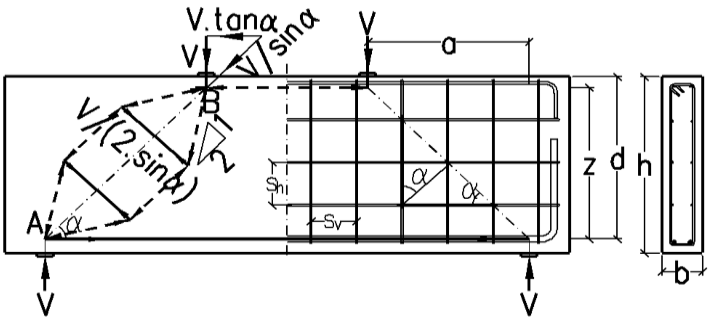


Fig. 1 Refined Strut-and-Tie Model for RC Deep Beam.

V. FACTORS INFLUENCING SHEAR STRENGTH OF RC DEEP BEAMS

The shear strength of RC beams depends on the uncracked depth of the concrete, friction between the cracked surfaces or the aggregate interlocking action, dowel action of the flexural reinforcing bars, support conditions and class of beam. The following sections can describe the influence of various parameters on the shear strength of RC deep beams. It is important to understand how each of the parameters influences the shear strength carefully from the large experimental data base. It is very well understood that the important factors influencing the shear strength of RC beams include inclination of the concrete strut which depends on the a/d ratio, quantity of the flexural reinforcement, quantity of the longitudinal web reinforcement.

5.1 *Influence of Inclination of Concrete Strut*

The flexural cracking formed in concrete in the tension region of the beam can penetrate into the diagonal concrete strut at larger *a/d* ratios. In other words, if the diagonal concrete strut is relatively flat, the possibility of the flexural cracks propagating into the diagonal concrete struts is relatively high as shown in Figs. 2(a) and 2(b). In beams with larger a/d ratio, the flexural cracking is very severe and the beams can fail in shear-tension or shear-compression modes, whereas in the beams with small a/d ratios, the cracking is predominantly flat due to the shear with inclined cracking making an angle greater than 60^0 with the horizontal. In such cases, the intensity of the flexural cracking in the shear span is relatively low. In any case, the cracking in the shear span of the beam can reduce the carrying capacity of the diagonal concrete struts significantly. Hence, the strength of RC deep beams is inversely proportional to the a/d ratio.

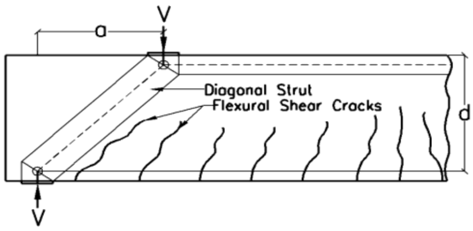


Fig. 2(a) Flexural Shear Cracks in Deep Beams with smaller a/d ratio.

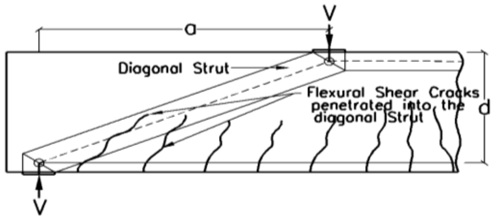


Fig. 2(b) Flexural Shear Cracks in Deep Beams with larger a/d ratio.

5.2 *Influence of Longitudinal Flexural Reinforcement on Shear Strength*

The longitudinal flexural tensile reinforcement can provide resistance against the shear strength in the beams even after formation of the diagonal cracking through dowel action. The shear strength of RC beams increases as the quantity of the longitudinal flexural reinforcement increases. Hence, the expression for the estimation of the shear strength of RC beam has to be incorporated into the longitudinal flexural tension reinforcement term. In this study, the influence of the longitudinal flexural reinforcement has been taken into account. The shear strength of RC beams varies as a function of the square root of the percentage of the flexural tensile reinforcement.

Influence of Flexural Reinforcement Ratio on Lever Arm Distance: The lever arm distance, designated as z in Eq. (3), can be expressed as $z = j_0 d$, in which j_0 is a constant, defined as the location of the resultant of the compression force at the end of the shear span, a. According to the classical bending theory of RC beams with only flexural tensile reinforcement ignoring the tensile strength of concrete, the constants can be determined from the following relationships

$$j_0 = 1 - \frac{k}{3}, \quad k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{4}$$

where k is the ratio of the depth of the neutral axis(c) to the effective depth, (d). Eq. (4) may be replaced by a simpler form as $j_0 = C_1/\rho^{b1}$, n is the modular ratio.

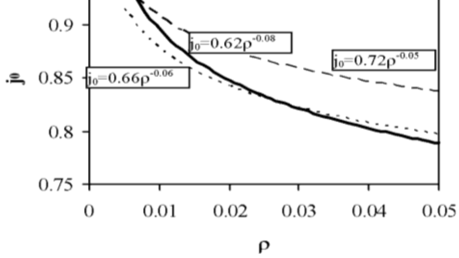


Fig. 3 Relation between j_0 and ρ.

The lever arm distance depends on the percentage of the flexural reinforcement and the compressive strength of concrete. Fig. 3 shows the variation of the lever arm depth constant with the percentage of the flexural reinforcement. The lever arm distance decreases as the percentage of the flexural reinforcement decreases. In practice, the range of coefficient n varies between 5.0 and 10.0, and that of the flexural reinforcement ratio ρ between 0.005 and 0.05. For the above ranges of n and ρ, we can relate the power function for j_0 as $0.62\rho^{-0.08}$ based on the similar proposed equations [34]. Fig. 3 shows the variation of the lever arm for all practical ranges of coefficients n and ρ. For the RC deep beams with smaller shear span, the linear elastic theory provides better estimation of j_0 because the inclined shear cracks form when the load on the beam is far below its full flexural capacity. The tensile strength of concrete, f_{ct}, is assumed to be a function of the square root of the compressive strength of concrete as shown in Eq. (5).

$$f_{ct} = 0.5\sqrt{f_c^{'}} \tag{5}$$

Substituting the value of f_{ct} for f_t in Eq. (3) and expressing the lever arm (z) in terms of the effective depth as $z = j_0 d$, we can obtain the nominal shear stress of concrete(v_c) as

$$v_c = \frac{V}{bd} = j_0\sqrt{f_c^{'}} \tag{6}$$

In order to account for the effect of a/d ratio and the longitudinal flexural reinforcement ratio, the nominal shear strength of concrete (v_c) in Eq. (6) can now be modified into the following form

$$v_c = \frac{A_1(\rho)^{a1} j_0\sqrt{f_c^{'}}}{1+B_1\left(a/d\right)} \tag{7}$$

According to Eq. (7), the nominal shear strength has been observed to be independent of the depth of the beam. Hence, the prediction of the shear strength of concrete is size independent. In order to account for the effect of the beam depth on the shear strength of concrete, the term v_c in Eq. (7) should be substituted for σ_r in Eq. (2). Now, the size dependent shear strength can be rearranged by replacing the lever arm distance, $j_0 = 0.62\rho^{-0.08}$

$$v_c = \frac{A_1 k_1 0.62(\rho)^{a1-0.08}\sqrt{f_c^{'}}}{1+B_1\left(a/d\right)}\left(\frac{k_2}{k_1}+\frac{1}{(1+(1/\lambda_0)(d/d_a))^{1/2}}\right)$$

Replacing the constants ($0.62A_1k_1$) and (a_1-0.08) by D_1 and d_1, we can rewrite the above expression as

$$v_c = \frac{D_1(\rho)^{d1}\sqrt{f_c^{'}}}{1+B_1\left(a/d\right)}\left(\frac{k_2}{k_1}+\frac{1}{(1+(1/\lambda_0)(d/d_a))^{1/2}}\right) \tag{8}$$

5.3 Influence of Web Steel Reinforcement

In RC deep beams, a large portion of the applied shear strength can be resisted by the tensile strength of concrete due to strut-and-tie mechanism through concrete strut. The ultimate failure of the beam is predominantly caused by the crushing of the concrete along the diagonal strut than the yielding of the web reinforcement. In general, the reinforcement in RC deep beams consists of vertical stirrups and longitudinal reinforcement. When the smaller shear span-to-depth ratios are adopted, the horizontal reinforcement has been observed to be more effective than the vertical reinforcement. When the beam is provided with vertical and horizontal reinforcement, the shear strength offered by the web reinforcement can be assumed to be in the form expressed in [35]

$$v_s = F_1\rho_h f_{yh}\frac{a}{d} + G_1\rho_v f_{yv}\frac{a}{d} \tag{9}$$

where F_1 and G_1 are constants; ρ_{yh} and ρ_{yv} are the horizontal and the vertical web reinforcement ratios respectively; f_{yh} and f_{yv} are the yield strengths of the horizontal and the vertical web reinforcements respectively.

VI. NOMINAL SHEAR STRENGTH

The nominal shear strength (v_n), offered by the concrete and the web reinforcement, shall be calculated by summing up the shear strength of concrete with that of the web reinforcement in Eqs. (8) and (9).

$$\begin{aligned} v_n &= v_c + v_s \\ &= \frac{D_1(\rho)^{d1}\sqrt{f_c^{'}}}{1+B_1\left(a/d\right)}\left(\frac{k_2}{k_1}+\frac{1}{(1+(1/\lambda_0)(d/d_a))^{1/2}}\right) + F_1\rho_h f_{yh}\frac{a}{d} + G_1\rho_v f_{yv}\frac{a}{d} \end{aligned} \tag{10}$$

In the above equation, the constants $B_1, D_1, F_1, G_1, k_1, k_2, d_1,$ & λ_0 need to be determined from the non-linear regression analysis of the experimental data base. For the determination of these constants, the experimental data consisting of 314 deep beam tests from [1,6,7,9,13,14,17,18,20,36-40] have been analyzed. Using the nonlinear optimization (Levenberg-Marquardt algorithm) and on trial and error procedure, the constants have been determined. Eq. (10) can now be expressed in terms of various constants incorporated

$$v_n = \frac{60\sqrt{f_c^{'}\rho}}{1+8(a/d)}\left(0.07+\frac{1}{\sqrt{1+(d/(100d_a))}}\right)+0.35\left(\frac{a}{d}\right)\rho_h f_{yh}+0.25\left(\frac{a}{d}\right)\rho_v f_{yv} \tag{11}$$

where $0.35(a/d) \le 0.43$, $0.25(a/d) \le 1$, and $\min(0.35(a/d)\rho_h f_{yh}, 0.43\rho_h f_{yh}) + 0.25(a/d)\rho_v f_{yv} \le \sqrt{f_c^{'}}$.

For representing the experimental data points on the graph, the following relations are used.

$$v_{\exp} = V_{\exp}/(bd)$$

$$v_s = \min\left(0.35\left(a/d\right)\rho_h f_{yh}, 0.43\rho_h f_{yh}\right) + 0.25\left(a/d\right)\rho_v f_{yv}$$

$$C = \frac{60\sqrt{f_{ck}\rho}}{1+8(a/d)}$$

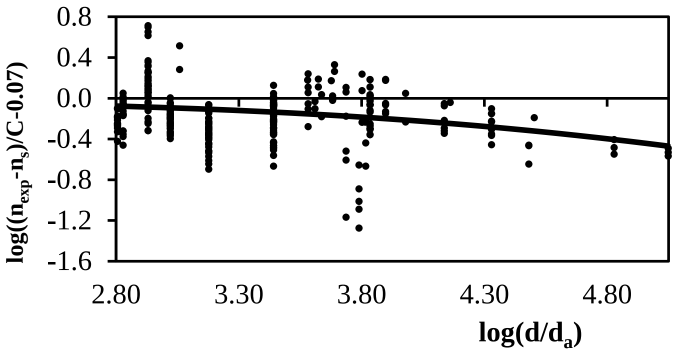


Fig. 4 Nominal Shear Stress verses Relative Size.

Fig. 4 shows the variation of the logarithm of nominal shear strength $\frac{(v_{\exp} - v_s)}{(C-0.07)}$ and the logarithm of relative size $\frac{d}{d_a}$. Though the size effect is clearly demonstrated, the scatter is wider due to the geometric dissimilarities such as size, shear span-to-depth ratio, and clear cover-to-main reinforcement, and also due to material properties like compressive strength of concrete, quantity of the reinforcement and also due to different testing procedures adopted in various laboratories. Fig. 4 demonstrates that there has been a strong size dependency of shear strength, which is addressed in the present form.

VII. EXISTING SIZE EFFECT MODELS

The strut-and-tie models are effectively adopted for predicting the shear strength of reinforced concrete deep beams[11,22,41]. ACI [24], Canadian [26] codes and AASHTO [25] recommended the use of strut-and-tie models for the design of deep beams. Though the above models can predict the size independent shear strength of deep beams, there are only few models, which accounted for the size effect with limitations [30,32,42]. Description of the size dependent models is given in the following sections:

7.1 Kim and Park Model

For deep beams without shear reinforcement for shear-span-to-depth ratio ranging between ($1.0 \le a/d \le 3.0$), Kim and Park [30] proposed the following model for estimation of the shear strength of RC deep beams

$$V_u = 19.4 f_c^{'\alpha/3}\rho^{3/8}(0.4 + d/a)\left(\frac{1}{\sqrt{d}} + 0.07\right)bd$$

in which $d \ge 250$mm (9.84 in.), and

$$\alpha = 2 - \frac{a/d}{3}$$

7.2 Tan and Cheng Model

Tan and Cheng [42] proposed the following model to predict the shear strength of deep beams.

$$V_n = \frac{1}{\dfrac{\sin 2\theta_s}{f_t A_c} + \dfrac{1}{\nu f_c^{'} A_{str} \sin\theta_s}}$$

where θ_s is the inclined angle of diagonal strut as shown in Fig. 5, A_c is the beam cross-sectional area, A_{str} is the cross-sectional area of strut, $f_c^{'}$ is the cylindrical compressive strength and

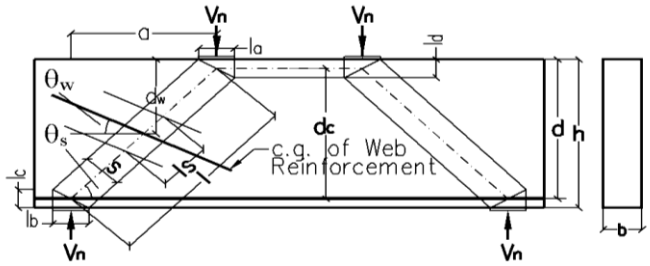


Fig. 5 Tan and Cheng's Model for simply supported deep beams.

$$\nu = \xi\zeta \; ; \xi = 0.8 + \frac{0.4}{\sqrt{1+(l-s)/50}}$$

$$\zeta = 0.5 + \sqrt{\frac{kd_s}{l_s}} \le 1.2 \; ; k = \frac{\sqrt{\pi}}{2}\sqrt{\frac{f_y}{0.5\sqrt{f_c^{'}}}}$$

$$f_t = \frac{2A_s f_y \sin\theta_s}{A_c / \sin\theta_s} + \sum \frac{2A_w f_{yw} \sin(\theta_s + \theta_w)}{A_c / \sin\theta_s}\frac{d_w}{d} + 0.5\sqrt{f_c^{'}}$$

in which A_s and A_w are areas of the longitudinal and the web reinforcement respectively, f_y and f_{yw} are the yield strengths of the longitudinal and the web reinforcement respectively, θ_w is the inclined angle intercepted by the web reinforcement with respect to the horizontal line, d_s is the diameter of the web reinforcement bar; when the web reinforcement is not provided, d_s is taken as the minimum diameter of the bottom longitudinal reinforcement bars and d_w is taken from the beam top to the intersection of the web reinforcement with the centerline of inclined strut as shown in Fig. 5.

7.3 Zhang and Tan's Model

Zhang and Tan [32] proposed the following model for predicting the shear strength of RC deep beams.

$$V_n = \frac{1}{\frac{2\sin 2\theta_s}{f_t A_c} + \frac{\sin\theta_s}{\nu f_c^{'} A_{str}}}$$

where θ_s is the angle of inclination of diagonal strut as shown in Fig. 6; $f_c^{'}$ is the cylindrical compressive strength; A_c is the cross-sectional area of the deep beams; A_{str} is the cross sectional area of the diagonal strut and f_t is the maximum tensile capacity of the bottom nodal zone computed as

$$f_t = \frac{4A_s f_y \sin\theta_s}{A_c / \sin\theta_s} + \sum \frac{f_{yw} A_{sw} \sin(\theta_s + \theta_w)}{A_c / \sin\theta_s} + 0.31\sqrt{f_c^{'}}\left(\frac{\varepsilon_{cr}}{\varepsilon_1}\right)^{0.40}$$

where A_s and A_{sw} are the total areas of the longitudinal and the web reinforcement respectively; f_y and f_{yw} are the yield strengths of the longitudinal and the web reinforcement respectively; θ_w is the inclined angle of the web reinforcement with respect to the horizontal line as shown in Fig. 6; ε_{cr} is the concrete strain at cracking, assumed as 0.00008; ε_1 is the principal tensile strain in the concrete strut calculated as $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + \varepsilon_2)\cot^2\theta_s$, where ε_s and ε_2 are the tensile strain in the longitudinal reinforcement and the peak compressive strain in the concrete strut at crushing respectively. The value of ε_2 is assumed as 0.002 for the normal strength concrete. The term ν is the efficiency factor, which is the product of ξ and ζ, where ξ is accountable for the effect of strut geometry and ζ for the effect of boundary condition influenced by the web reinforcement. These parameters can be expressed as

$$\xi = 0.8 + \frac{0.4}{\sqrt{1+(l-s)/50}}$$

$$\zeta = 0.5 + \sqrt{\frac{kd_s}{l_s}} \le 1.2$$

where l and s are the strut length and the width respectively in Fig. 6. The term d_s is the diameter of the web reinforcement bar. When the web reinforcement is not provided, d_s is taken as the minimum diameter of the bottom longitudinal reinforcement bars and the material factor incorporating the yield strength of the reinforcement bar, f_y and the concrete tensile strength f_{ct} is given as

$$k = \frac{1}{2}\sqrt{\frac{\pi f_y}{f_{ct}}}$$

When the web reinforcement is not provided, it can be taken as half of the above value. The term, l_s, is the maximum spacing of web reinforcement intercepted by inclined strut.

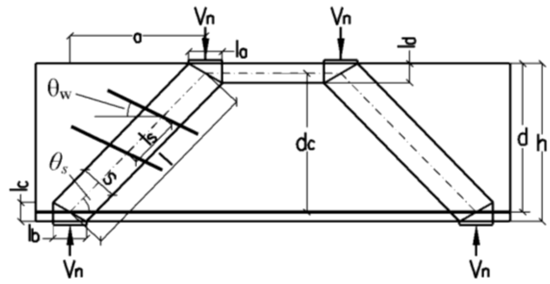
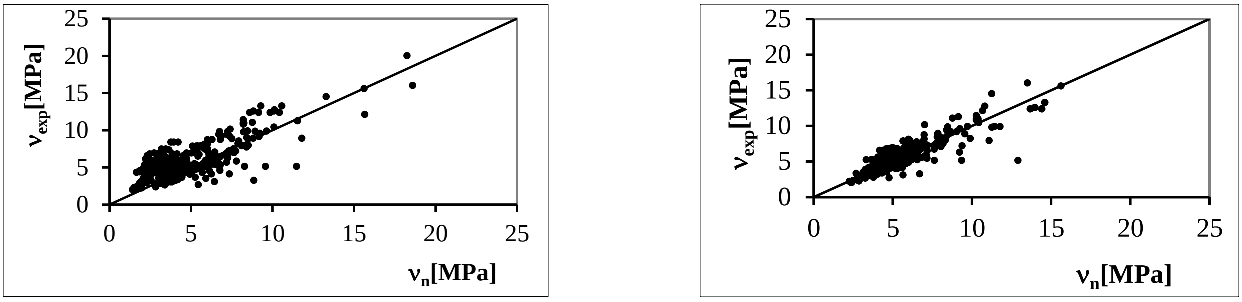


Fig. 6 Zhang and Tan's Model for simply supported deep beams.

VIII. VALIDATION OF PROPOSED MODEL

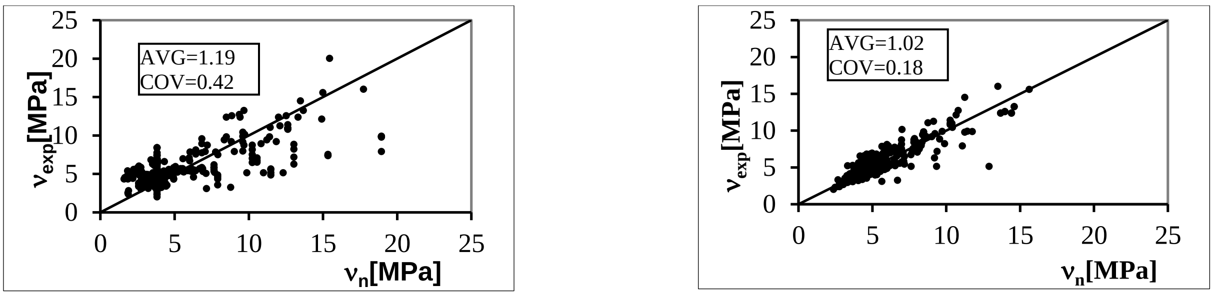
The nominal shear strength (v_n), calculated from experimental data base of 314 deep beam tests, has been evaluated by comparing with the proposed equation and also with the iterative procedure suggested by Zhang and Tan [32]. The calculated nominal shear strength (v_n) versus the experimental shear strength (v_{exp}) is plotted for the models in Fig. 7. The coefficient of variation of the proposed equation is 0.18 which is 51% less than that of Zhang and Tan. Hence the proposed equation seems to predict the nominal shear strength of deep beams more uniformly as compared to the latest size effect model.



(a) Zhang and Tan's Model (b) Proposed Equation

Fig. 7 Nominal shear stress from (a) Zhang and Tan's Model [32] and (b) Proposed equation versus experimental stress for 314 reinforced concretedeep beams. *Note 1 MPa = 145 psi; 1mm = 0.0394 in.*

For comparison with the Strut-and-Tie Model of ACI 318 [24], the deep beams with a/d ratio less than or equal to 2.0 are only considered to satisfy the minimum angle of inclination of the diagonal strut with the horizontal tie. The nominal shear strength calculated on 272 deep beams has been calculated by an iterative procedure to satisfy the equilibrium requirements stipulated in the ACI Code. For these beams, the calculated nominal strength versus the shear strength obtained from experiments is plotted for the Strut-and-Tie model adopted by the ACI code [24] and proposed equation as shown in Fig. 8.



(a) ACI Code 318-08 (b) Proposed Equation

Fig. 8 Nominal shear stress from (a) ACI 318-08 [24] and (b) Proposed equation versus experimental stress for 272 reinforced concrete deep beams. *Note 1 MPa = 145 psi; 1mm = 0.0394 in.*

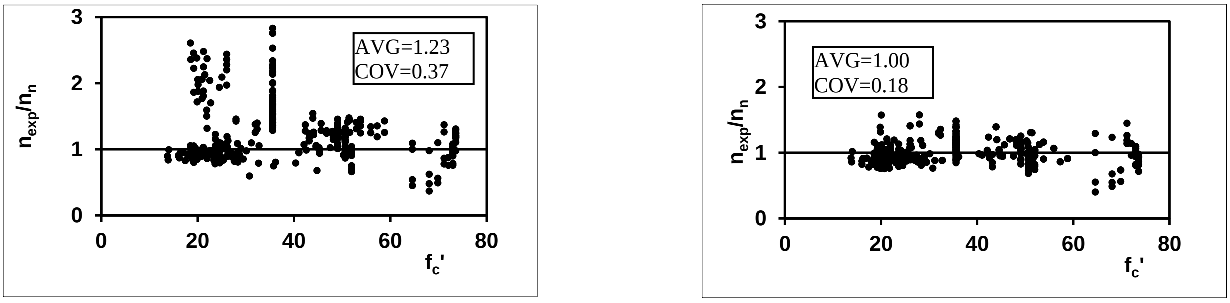
The parameters such as the cylindrical compressive strength of concrete, a/d ratio and size of the member influence the shear strength. The effects of these variables on the ratio v_{exp}/v_n are shown in Figs. 9-12. The degree of accuracy is ascertained from the location of the data points with reference to the line corresponding to the ratio $v_{exp}/v_n = 1$. If relatively more data points are lying either above or below the above line with reference to any of the parameters like cylindrical compressive strength of concrete, a/d ratio, size of the member, it is considered that the prediction is not uniform with respect to that parameter. This means the particular parameter has not been addressed correctly in the expression.

8.1 Shear Strength Ratio versus Compressive Strength of Concrete

The effect of cylindrical compressive strength of concrete on the shear strength ratio is shown in Fig. 9. According to Zhang and Tan's model, the large scatter of shear strength ratios using low strength concrete has been observed compared with the medium and high strength concrete. The shear strength ratio for the proposed equation seems to be less affected by the range of concrete strengths leading to more uniform prediction of the shear strength with reference to the strength of concrete.

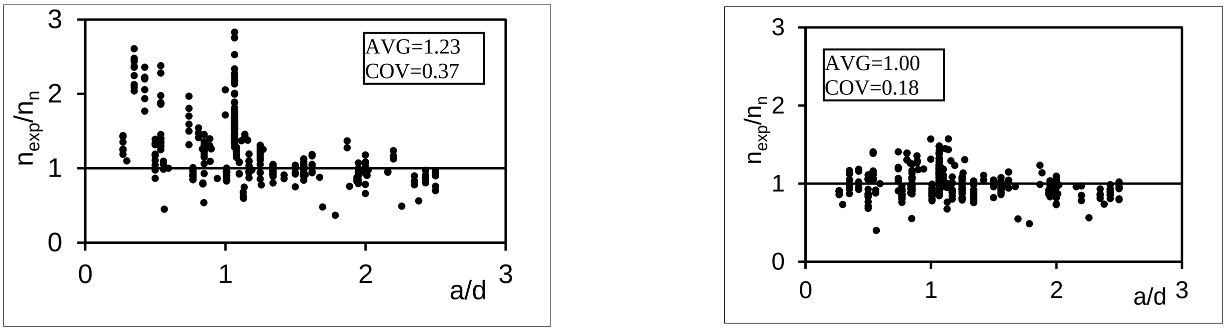
8.2 Shear Strength Ratio versus a/d Ratio

As observed from Fig. 10, Zhang and Tan's model relatively underestimates the shear strength of RC beams at failure when the *a/d* ratio is less than 1.0. The proposed equation predicts the shear strength more uniformly for the entire range of a/d ratios. The influence of a/d ratio has been accounted for accurately in the proposed expression.



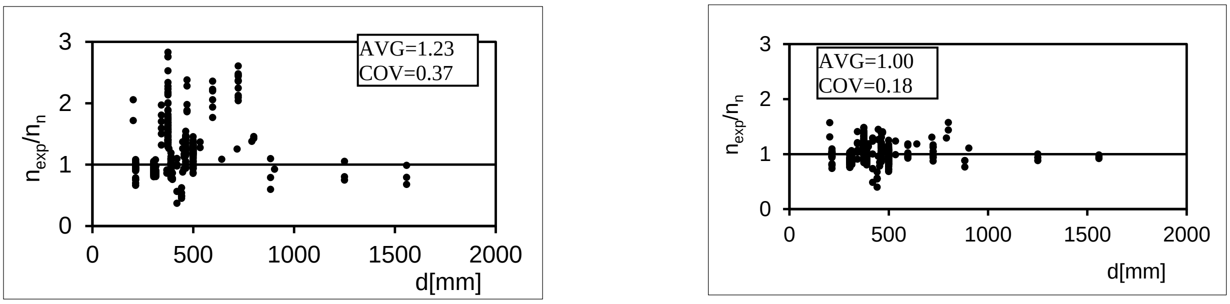
(a) Zhang and Tan's Model (b) Proposed Equation

Fig. 9 Effect of concrete strength $f_c^{'}$ on shear stress: (a) Zhang and Tan's Model [32] and (b) Proposed expression for 314 reinforced concrete deep beams. *Note 1 MPa = 145 psi; 1mm = 0.0394 in.*



(a) Zhang and Tan's Model (b) Proposed Equation

Fig. 10 Effect of *a/d* on shear stress: (a) Zhang and Tan's Model [32] and (b) Proposed expression for 314 reinforced concrete deep beams *Note 1 MPa = 145 psi; 1mm = 0.0394 in.*



(a) Zhang and Tan's Model (b) Proposed Equation

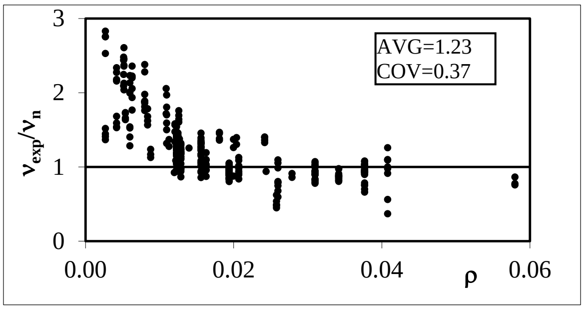
Fig. 11 Effect of effective depth, *d* on shear stress: (a) Zhang and Tan's Model [32] and (b) Proposed expression for 314 reinforced concrete deep beams. *Note 1 MPa = 145 psi; 1mm = 0.0394 in.*

8.3 Shear Strength Ratio versus Effective Depth

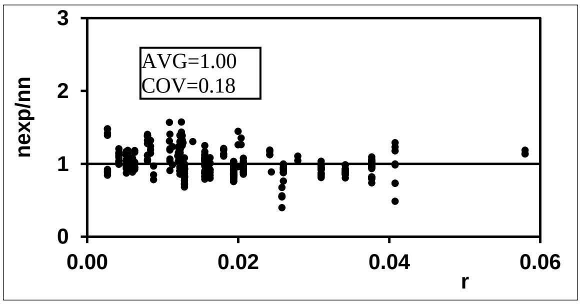
Fig. 11 shows the ratio of the experimental-to-calculated shear strength of the existing data with the effective depth, *d*. The large scatter of the data points of the shear strength ratio especially for small depths of RC deep beams has been observed in Zhang and Tan's model. The proposed equation seems to be not affected by the depth of beams for predicting the shear strength more uniformly as compared to that of Zhang and Tan.

8.4 Shear Strength Ratio versus Flexural Tensile Reinforcement Ratio

With reference to the flexural tensile reinforcement ratio, the scatter of the data points of the proposed equation has been observed to be relatively smaller and also more uniform as compared to the prediction by Zhang and Tan's model at low flexural reinforcement ratios in Fig. 12.



(a) Zhang and Tan's Model



(b) Proposed Equation

Fig. 12 Effect of tensile reinforcement ratio on shear stress: (a) Zhang and Tan's Model [32] and

(b) Proposed expression for 314 reinforced concrete deep beams. *Note 1 MPa = 145 psi; 1mm = 0.0394 in.*

IX. CONCLUSIONS

1) The size effect on shear strength of RC deep beams with shear reinforcement exists.
2) The proposed predictive equation based on the modified size effect law using strut-and tie model agrees well with the experimental data as compared to that of the traditional size independent strength expressions.
3) Even though the contribution of concrete in the tension zone is ignored in the refined strut-and-model, the extent of cracking, and propagation of cracking lead to the failure, which plays a key role in the size effect on shear strength. If the propagation of the cracking has been properly accounted for, the prediction of the strength looks more uniform with the size.
4) The proposed size dependent expression appears to be simple for estimating the shear strength as compared to the other existing size effect models, which requires laborious iterative procedure for solution.
5) The size effect on the strut-and-tie component of RC deep beam with shear reinforcement is now well understood from the above studies.

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NOTATION

σ_N = size dependent nominal stress at failure in MPa.

B = a non-dimensional constant

f_t' = direct tensile strength of concrete in MPa.

a = shear span *ie.,* distance from the load and nearby reaction

b = breadth of the beam

d = effective depth of beam in mm.

λ_0 = a constant

d_a = maximum size of coarse aggregate in mm.

σ_r = size independent nominal stress at failure in MPa.

k_1, k_2, *and* k_4 = empirical constants

f_t = actual tensile stress in concrete acting across a plane

f_{ct} = allowable tensile stress in concrete

V = shear strength in beam

v_n = $V/(bd)$ = Nominal shear stress of the beam

v_c = Nominal shear resistance of concrete in the beam

v_s = Nominal shear resistance of web steel reinforcement in the beam

V_{exp} = Experimentally observed shear strength in beam

v_{exp} = $V_{exp}/(bd)$ = Experimentally observed nominal shear stress of the beam

f_c' = characteristic cylindrical strength of concrete in *MPa*.

n = modular ratio of steel to concrete

E_s =Young's modulus for steel

E_c =Young's modulus for Concrete

ρ_h = $A_{sh}/(bs_h)$ = ratios of horizontal web reinforcement

A_{sh} = Area of horizontal reinforcement in one level

s_h =spacing of horizontal web reinforcement in vertical direction

ρ_v = $A_{sv}/(bs_v)$ = ratios of vertical web reinforcement

A_{sv} = Area of vertical reinforcement in one position

s_v =spacing of vertical web reinforcement in horizontal direction

α = angle between the line joining the points of load and reaction with the horizontal

z = lever arm between centroids of main steel reinforcement and the horizontal compression strut

j_0 = ratio of lever arm, z to the effective depth, d

k = ratio of the depth of neutral axis, c to the effective depth, d

ρ = $A_s/(bd)$ = main tensile reinforcement ratio

A_s = Area of main reinforcement

A_1 , B_1 , C_1 , D_1, F_1, G_1, a_1 , b_1 , c_1, d_1 = Constants

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