Hartree-Fock Theory for Neutron Matter with Bare Two-body Interaction and Three-body Interaction

Jinniu Hu[1], Hiroshi Toki[2], Wu Wen[3], Hong Shen[4]

[1,2]Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan
[3]Institute of Applied Physics and Computational Mathematics (IAPCM), Beijing 100094, China
[4]Department of Physics, Nankai University, Tianjin 300071, China
[1]jinniuhu@rcnp.osaka-u.ac.jp; [2]toki@rcnp.osaka-u.ac.jp; [3]wwsemir@gmail.com; [4]songtc@nankai.edu.cn

***Abstract*-The properties of neutron matter is studied in the Hartree-Fock theory with a bare meson-exchange nucleon-nucleon interaction and Urbana IX three-body interaction. We find that the energy contribution from tensor interaction is very small in neutron matter, which consists of only the two-body isospin triplet channel ($T=1$). The strong repulsive interaction between two nucleons in the short distance is treated by unitary correlation operator method (UCOM). The equation of state (EOS) in our calculation only with the Bonn potential agrees with the one of AV18 potential including the relativistic boost effect in the variational method for neutron matter. The three-body interaction is also introduced in this framework to improve the properties of neutron matter in higher density region.**

Keywords-Hartree-Fock; Neutron Matter; UCOM; Three-body Interaction

I. INTRODUCTION

The properties of neutron matter play a very important role in the study of supernovae and neutron stars. After a gravitational core collapse, supernova explosion takes place and leads to the formation of a neutron star or a black hole in the central part of a massive star. For this purpose, we need the equation of state (EOS) of nuclear matter, especially of neutron matter up to high density for the description of astrophysical objects as supernovae and neutron stars [1, 2].

The EOS's of nuclear and neutron matter have been predicted by many different models of various assumptions. One kind of models uses phenomenological Lagrangians which are able to describe finite nuclei by fitting about 10 parameters. Due to this construction, the properties of symmetric nuclear matter around and below nuclear matter are well constrained but those of neutron matter and of symmetric nuclear matter at high density are not strongly constrained. Actually, experiments on heavy ion collisions give some constraints for the behavior of nuclear matter at high density [3]. From this view point, the microscopic calculations are better providing the EOS of symmetric nuclear matter and neutron matter which adopt a realistic nucleon-nucleon (NN) interaction, which is obtained by fitting the phase shifts of NN scattering.

There is a strong short range repulsive interaction in the realistic NN interaction due to their quark structure [4, 5]. The basic task of microscopic theories is to take into account the short range correlation in the many-body problem and hence to introduce high momentum components in the many-body wave function. Therefore, two kinds of correlation method for short range repulsive interaction were developed in parallel about 50 years ago. One method is to introduce the short range correlation through the correlated wave function with variational approach by Jastrow [6]. Pandharipande *et al*. pursued this method as a variational chain summation (VCS) approach, based on hyper-netted chain-summation techniques in nuclear matter system in 1970's [7]. Recently, Akmal *et al*. used this method and a realistic NN interaction as the Argonne V18 potential, which excellently fits the NN scattering data, to obtain the EOS of symmetric nuclear matter and neutron matter [8]. These EOS's are in accordance with the constraint by the experimental data of heavy ion collision [3].

At the same time, Brueckner *et al*. introduced the hole-line expansion method and used the G-matrix instead of the bare NN interaction to treat the short range correlation [9]. This method is extended to the relativistic Brueckner Hartree-Fock (RBHF) theory by Brockmann and Machleidt for nuclear matter with Bonn potential [10]. It is the first time to give reasonable saturation properties of symmetric nuclear matter from the microscopic approach. This result illuminates that the relativistic effect is very important in the many-body calculation which provides the repulsive contribution in nuclear matter.

We are aware of the fact that the G-matrix includes the high momentum components due to the short range correlation and additionally the tensor correlation. There are several studies on the role of the tensor interaction, which is extremely important to provide large binding energy for symmetric nuclear matter and finite nuclei [11-15]. The tensor interaction arises from the pion exchange interaction. The effect of the tensor interaction in neutron matter is suppressed largely for the T=1 isospin channel. In fact, the calculation of Krastev and Sammarruca in the RBHF method has shown that the EOS's of Bonn-A, Bonn-B, and Bonn-C are identical for neutron matter [16]. On the other hand, the difference of these three potentials is distinct due to the different tensor effects. Hence, we are motivated to look into neutron matter by just considering explicitly only the short range correlation. If this is the case, we may find some other many-body methods to treat the many-body system in more efficient way for various phases of astrophysics.

For the short range correlation, there is a very attractive method developed recently by Feldmeier *et al*. in terms of the unitary correlation operator method (UCOM) [17, 18]. The UCOM was demonstrated extremely good to provide binding energies and wave functions for light nuclei by taking an effective NN interaction with only the central interaction including the short range repulsive interaction [17]. Therefore, we discussed the properties of neutron-rich matter in the framework of relativistic Hartree-Fock model with UCOM (RHFU) by using the realistic NN interaction, Bonn potential [19]. We found that the EOS in our calculation could completely reproduce the one of pure neutron matter in RHBF model with Bonn-A potential. This achievement drives us to consider whether we can also obtain the similar results as the variational method in a simpler theory like the Hartree-Fock UCOM model.

It is necessary to introduce the three-body interaction to provide enough repulsive contribution in high density region in the non-relativistic framework of the microscopic calculation. Li *et al*. obtained reasonable saturation properties of symmetric nuclear matter by including the microscopic meson-exchange three-body interaction in Brueckner-Hartree-Fock approach [20]. However, Akmal used a phenomenological three-body interaction, Urbana model IX (UIX) to improve the EOS of nuclear matter, especially in high density part [8]. Therefore, it is very interesting to study the effect of a three-body interaction in the Hartree-Fock framework for neutron matter.

Furthermore, the realistic NN potential is obtained from the center of mass framework where the total momentum $\mathbf{P}_{ij}$ is zero. To take the effect of non-zero $\mathbf{P}_{ij}$ into account, the relativistic boost correction was applied in the variational method with the AV18 potential. However, Forest *et al*. [21] pointed out that the meson exchange potential contains the relativistic boost correction. Hence, we will discuss the properties of nuclear matter with Bonn potential which is constructed by using the meson exchange model and not include the relativistic boost correction term.

Hence, the purpose of the present study is to develop the Hartree-Fock theory with UCOM and three-body interaction (HFUT) to investigate the properties of neutron matter, which can deal with a bare NN interaction. We want to find a simpler way to discuss neutron matter in the many-body framework. In Section Ⅱ, we will illuminate first that the tensor effect of the pion exchange interaction in neutron matter is very weak using the perturbation theory. In Section Ⅲ, we will construct a theoretical framework of the Hartree-Fock theory with UCOM (HFU) for the nuclear matter system. In Section Ⅳ, a phenomenological three-body interaction is included in the HFU model. In Section Ⅴ, we present numerical results for nuclear matter. We will discuss the effect of short range correlation and three-body interaction. Section Ⅵ is devoted to the summary of the present study.

II. THE TENSOR CONTRIBUTION IN THE PERTURBATION THEORY

In this section, we would like to estimate the contributions of the tensor interaction both for neutron matter and symmetric nuclear matter in the perturbation theory. In the one-boson-exchange model of the NN interaction, the tensor contribution mainly comes from the one-pion exchange potential (OPEP). For the pseudo-vector coupling between pion and nucleon, the OPEP in momentum space is given by

$$V_\pi(\mathbf{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2}\frac{\boldsymbol{\sigma}_1\cdot\mathbf{q}\boldsymbol{\sigma}_2\cdot\mathbf{q}}{q^2+m_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \tag{1}$$

where $f_{\pi NN}$ is the pion-nucleon coupling constant and $\mathbf{q}$ is the exchanged momentum between two nucleons. The operators, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, represent the spin and isospin operators, respectively. This interaction can be separated into the spin-spin central and tensor parts

$$V_\pi(\mathbf{q}) = -\frac{1}{3}\frac{f_{\pi NN}^2}{m_\pi^2}\left(\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 q^2}{q^2+m_\pi^2}+\frac{S_{12}(\hat{\mathbf{q}})q^2}{q^2+m_\pi^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2. \tag{2}$$

Here, $S_{12}(\mathbf{q})$ is the tensor operator

$$S_{12}(\hat{\mathbf{q}}) = 3\boldsymbol{\sigma}_1\cdot\mathbf{q}\boldsymbol{\sigma}_2\cdot\mathbf{q}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2. \tag{3}$$

As we know, the expectation value of the tensor operator is zero at the Hartree-Fock level for a spin-saturated system. We should discuss the tensor effect of pion in the higher-order terms of the perturbation theory. They are called as the iterated one-pion-exchange Hartree and Fock terms in [11], which can be expressed in the following Feynman diagrams.

To evaluate the energy contribution from the iterated one-pion-exchange term, we would like to write the matrix elements of OPEP as,

$$\begin{aligned}V^\pi_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_1',\mathbf{k}_2'} &= \frac{1}{(2\pi)^3}\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_1'-\mathbf{k}_2')V_\pi(\mathbf{q})\\ &= -\frac{1}{3(2\pi)^3}\frac{f_{\pi NN}^2}{m_\pi^2}\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_1'-\mathbf{k}_2')\\ &\quad [C\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+TS_{12}(\hat{\mathbf{q}})]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\ ,\end{aligned} \tag{4}$$

where $\mathbf{q}=\mathbf{k}_1'-\mathbf{k}_1=\mathbf{k}_2-\mathbf{k}_2'$ and

$$C = T = \frac{q^2}{q^2+m_\pi^2}. \tag{5}$$

Therefore, the energy per particle arising from the Hartree contribution of iterated one-pion-exchange is,

$$E_H^{[2]} = \frac{1}{2\rho V}\int_\Gamma \frac{d^3k_1d^3k_2d^3k_1'd^3k_2'}{\varepsilon_{k_1}+\varepsilon_{k_2}-\varepsilon_{k_1'}-\varepsilon_{k_2'}}\sum_{\sigma,\tau}|V^\pi_{k_1,k_2,k_1',k_2'}|^2, \tag{6}$$

where ρ is the baryon density and V the volume of nuclear matter. The single particle energy $\varepsilon_{\mathbf{k}}$ occurring in the energy denominator is simple kinetic energies $k^2/2M$. M is the nucleon mass, while the integrated region Γ is limited by the Pauli principle, $|\mathbf{k}_1|,|\mathbf{k}_2|<k_F$ and $|\mathbf{k}_1'|,|\mathbf{k}_2'|>k_F$ which means that two states are above the Fermi surface and the other two states are below the Fermi surface. After taking the summation of spin and isospin operators with plane wave functions in Eq. (6), the energy $E_H^{[2]}$ can be rewritten as

$$E_H^{[2]} = \frac{4MT_H f_{\pi NN}^4}{3(2\pi)^8 m_\pi^4\rho}\int_0^\infty (C^2+2T^2)I_H(q)q^2dq. \tag{7}$$

Here, T_H is the result of summation of the isospin operator. Its value depends on the isospin channel.

$$T_H = \begin{cases}12 & \text{for symmetric nuclera matter}\\ 1 & \text{for pure neutron matter}\end{cases} \tag{8}$$

and $I(q)$ is the integration related with the Pauli principle,

$$I_H(q) = \int_\Gamma \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{\mathbf{q}\cdot(\mathbf{q}+\mathbf{k}_1-\mathbf{k}_2)}, \tag{9}$$

where the region of integration about Γ is generally defined by

$$\Gamma = \begin{cases}|\mathbf{k}_i\pm\mathbf{q}|>k_F\\ |\mathbf{k}_i|<k_F\end{cases}\quad i=1,2, \tag{10}$$

where k_F is the Fermi momentum. The integration about $I(q)$ can be expressed in the analytical form,

$$
\begin{aligned}
I_H(q) &= \frac{\pi^2 k_F^4}{x}\left[\frac{(58-80\ln 2)x^2}{15}+\frac{8}{15}\ln\left(1-x^2\right)\right. \\
&\left. -\frac{2x^4}{5}+\left(x-\frac{2x^3}{3}+\frac{x^5}{5}\right)\ln\left(\frac{1+x}{1-x}\right)\right], \text{ if } \quad x\le 1 \\
&= \frac{\pi^2 k_F^4}{x}\left[\frac{44x}{15}+\left(-\frac{8x^3}{3}+\frac{8x^5}{15}\right)\ln\left(1-\frac{1}{x^2}\right)\right. \\
&\left. +\frac{8x^3}{15}+\left(\frac{8}{15}-\frac{8x^2}{3}\right)\ln\left(\frac{x+1}{x-1}\right)\right], \text{ if } \quad x>1.
\end{aligned}
\tag{11}
$$

Here we have defined $x=q/2k_F$. However the energy per particle in Eq. (7) is divergent. It is necessary to introduce the form factor for the OPEP to regularize this integration. We choose a monopole form factor for the vertex between pion and nucleon,

$$F_\pi(q)=\frac{\Lambda^2-m_\pi^2}{\Lambda^2+q^2}, \tag{12}$$

where Λ is the cut off momentum. From Eq. (7), we find that the tensor contribution of pion related with T is twice of spin-spin central force related with C in nuclear matter for the Hartree diagram of iterated one-pion-exchange term. Furthermore, the tensor effect in neutron matter is much smaller than the one in symmetric nuclear matter due to the isospin factor.

In the similar way, we can discuss the Fock energy contribution per particle from the right diagram as shown in Fig. 1.

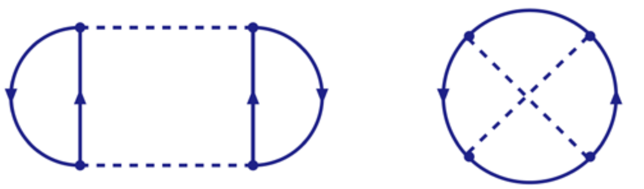

Fig. 1 The iterated one-pion-exchange Hartree and Fock diagrams: the left-hand figure is the Hartree diagram and the right hand one is the Fock diagram. The dashed line represents the propagator of pion, while the solid one is the propagator of nucleon

It is written as

$$E_F^{[2]}=\frac{1}{2\rho V}\int_\Gamma \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_1' d^3\mathbf{k}_2'}{\varepsilon_{\mathbf{k}_1}+\varepsilon_{\mathbf{k}_2}-\varepsilon_{\mathbf{k}_1'}-\varepsilon_{\mathbf{k}_2'}}\sum V^\pi_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_1',\mathbf{k}_2'}V^\pi_{\mathbf{k}_2',\mathbf{k}_1',\mathbf{k}_1,\mathbf{k}_2}, \tag{13}$$

where the exchange momentum for $V^\pi_{\mathbf{k}_2',\mathbf{k}_1',\mathbf{k}_1,\mathbf{k}_2}$, is changed as $\mathbf{q}'=\mathbf{q}+\mathbf{k}_1'-\mathbf{k}_2'$. After taking the spin and isospin sum and using the momentum conservation in intermediate states in Eq. (13), we obtain

$$
\begin{aligned}
E_F^{[2]} &= \frac{MT_F f_{\pi NN}^4}{2(2\pi)^9 m_\pi^4\rho}\int d^3\mathbf{q}\int_\Gamma \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{\mathbf{q}\cdot(\mathbf{q}+\mathbf{k}_1-\mathbf{k}_2)} \\
&\left[4TT'(\mathbf{q}\cdot\hat{\mathbf{q}}')^2-\frac{2}{3}(CC'+2TT')\right].
\end{aligned}
\tag{14}
$$

Here, C',T' are the same expressions as C,T, but with $\mathbf{q}$ replaced by $\mathbf{q}'=\mathbf{q}+\mathbf{k}_1-\mathbf{k}_2$ and the isospin factor T_F is

$$T_F=\begin{cases}-6 & \text{for symmetric nuclera matter} \\ 1 & \text{for pure neutron matter}\end{cases} \tag{15}$$

$\mathbf{q}\cdot\hat{\mathbf{q}}'$ represents the cosine of the angle between $\mathbf{q}$ and $\mathbf{q}'$. The second integration over the region Γ in Eq. (14) cannot be written in an analytical form like Hartree term any more in the case of the Pauli principle on exchanging momentum $\mathbf{q}'$. However, we can get it by numerical calculation in the cylindrical coordinates which will reduce to a 6-dimensional integration about the radius, height, and angles. We reproduce the results of the analytical expression for the case of the abrupt momentum cutoff [11]. Now the relation between the tensor contribution and the spin-spin central contribution is not so obvious. We can only discuss them based on the numerical results.

We show the total tensor contributions from the iterated one-pion-exchange Hartree and Fock diagrams for neutron matter and symmetric nuclear matter as shown in Fig. 2.

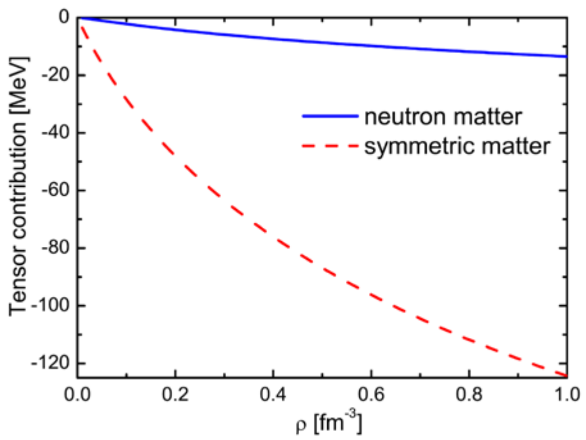

Fig. 2 The tensor contribution of iterated one-pion-exchange terms: the solid curve denotes the tensor contribution for neutron matter, while the dashed curve for symmetric nuclear matter. We take Λ = 1000 MeV in the form factor.

The pion coupling constant is fixed as $f_{\pi NN}^2/4\pi=0.08$ and $\Lambda=1000$ MeV for the form factor of the pion-nucleon vertex. We find that the tensor contribution in neutron matter is much smaller than the one in symmetric nuclear matter. There are two reasons which cause these large differences. The first one is the isospin factor, which is 1/12 for neutron matter to symmetric nuclear matter. The second one is that the tensor contribution in Fock term is repulsive which is opposite to the Hartree term in neutron matter, while the Fock contribution in the Hartree term is attractive in symmetric nuclear matter. Therefore, we may drop the tensor effect in the discussion of neutron matter and treat the many body system in a simple framework as the Hartree-Fock theory.

III. THE HARTREE-FOCK THEORY WITH UCOM

In this section, we would like to construct the Hartree-Fock theory with unitary correlation operator method (UCOM) in the non-relativistic framework. The nuclear matter can be considered as a system of identical nucleons (in a large box of volume V) which satisfies a non-relativistic Schroedinger equation. The Hamiltonian of this system is given by the summation of the kinetic energy of all particles and the two-body interactions between them,

$$\mathcal{H} = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V_{ij}. \tag{16}$$

The kinetic energy operator in non-relativistic case can be written as,

$$T_i = \frac{p_i^2}{2M} \tag{17}$$

and two-body interaction comes from the Bonn potential, which is the one-boson-exchange potential defined as a sum of one-particle amplitudes of six bosons, with π and η pseudoscalar, σ and δ scalar, and ρ and ω vector particles. These amplitudes can be obtained from the following Lagrangian,

$$\begin{aligned}\mathcal{L}_{\text{int}} = \Big[& \bar{\psi} - g_\sigma \sigma - g_\delta \tau_a \delta^a - \frac{f_\eta}{m_\eta} \gamma_5 \gamma_\mu \partial^\mu \eta - \frac{f_\pi}{m_\pi} \gamma_5 \gamma_\mu \tau_a \partial^\mu \pi^a \\ & -g_\omega \gamma_\mu \omega^\mu + \frac{f_\omega}{2M} \sigma_{\mu\nu} \partial^\nu \omega^\mu - g_\rho \gamma_\mu \tau_a \rho^{a\mu} + \frac{f_\rho}{2M} \sigma_{\mu\nu} \partial^\nu \tau_a \rho^{a\mu} \psi \Big] \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} \partial_\mu \delta^a \partial^\mu \delta^a - \frac{1}{2} m_\delta^2 \delta^{a2} \\ & + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \frac{1}{2} m_\eta^2 \eta^2 + \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a - \frac{1}{2} m_\pi^2 \pi^{a2} \\ & - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{a\mu},\end{aligned} \tag{18}$$

where

$$W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, R = \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a. \tag{19}$$

The field ψ is the nucleon field and M the nucleon mass. Meanwhile, we use a monopole form factor,

$$F_\alpha(q^2) = \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + q^2}, \tag{20}$$

For each meson-nucleon vertex denoted by α. There exists a strong repulsive effect in the short range region of the realistic nucleon-nucleon interaction. Nuclear matter cannot get enough binding energy in the Hartree-Fock approximation with a realistic NN interaction. We should treat the short range correlation to reduce the effect of the strong repulsive interaction. In the variational method, one chooses the two-body correlation function on the wave function [8] to treat this short range effect. In the Brueckner theory, it is considered by the G -matrix [10].

Here, we will follow the method of Neff-Feldmeier [17, 18], which is named UCOM. They adopt a unitary operator C to transform the trial many-body wave function

$$|\Psi\rangle = C|\Phi\rangle = (C_{i,j} + C_{i,j,k} + ...)|\Phi\rangle, \tag{21}$$

where $|\Psi\rangle$ indicates the full wave function and $C_{ij}, C_{ijk}, C_{ij...n}$ corresponds the two-body correlator, three-body correlator and many-body correlator of unitary operator C, respectively. For simplification in this work, we just consider the two-body correlator, C_{ij}. Finally the ground-state energy of A nucleons can be written as,

$$\varepsilon = \langle \Phi | C^\dagger \mathcal{H} C | \Phi \rangle. \tag{22}$$

It is a good approximation to take terms up to two-body correlation in the UCOM. In an actual calculation, the operator C modifies the Hamiltonian $\mathcal{H}$ and we shall use the modified Hamiltonian. Therefore, it would be better to define a correlated NN interaction $\widetilde{V}_{ij}$ instead of the bare interaction V_{ij},

$$\widetilde{V}_{ij} = C_{ij}^{\dagger} V_{ij} C_{ij} + C_{ij}(T_i + T_j) C_{ij} - (T_i + T_j). \tag{23}$$

In this correlated potential, the first term comes from the short range correlation on the bare NN interaction and the last two terms from the correlation on the kinetic energy. The emphases of this work are focused on the properties of neutron matter where the tensor effect is very small as shown in Section Ⅱ. We just treat the short range correlation on the spin-spin central part of the realistic potential. Furthermore, we also assume that the UCOM operator C is independent of the spin and isospin channel. These effects will be included in the variational calculation for the total energy of system with the short range effect.

When the unitary correlation operator C is applied on the two-body NN interaction, we just need to take the following transformation in the potential $V(r_{ij})$ as,

$$C_{ij}^\dagger V(r_{ij}) C_{ij} = V(R_+(r_{ij})), \tag{24}$$

where $R_+(r_{ij})$ is a parameterized function of the unitary operator C_{ij} [17],

$$R_+(r) = r + \alpha \left(\frac{r}{\beta} \right)^\eta \exp(-\exp(r/\beta)). \tag{25}$$

It does not have so simple form for the short range correlation on the kinetic energy part. Because the unitary operator C just has an effect of short range correlation for the relative momentum, we need to separate the kinetic energy of two particles into relative and center of mass energy,

$$T_i + T_j = \frac{\mathbf{q}^2}{M} + \frac{(\mathbf{p}_i + \mathbf{p}_j)^2}{4M}. \tag{26}$$

The UCOM operator just correlates the part which is related with the relative momentum, $\mathbf{q} = (\mathbf{p}_i - \mathbf{p}_j)/2$. More details can be found in Section Ⅱ in [17]. Here, we directly give the kinetic energy operator modified by the correlation operator.

$$\begin{aligned} T_{[2]} &= C_{ij}^\dagger (T_i + T_j) C_{ij} - (T_i + T_j) \\ &= q_r^{\dagger 2} \frac{1}{M} \left[\frac{1}{R_+^{'2}}(r) - 1 \right] q_r + \frac{1}{M} \left[\frac{1}{R_+^2(r)} - \frac{1}{r^2} \right] \mathbf{L} + w(r), \end{aligned} \tag{27}$$

where q_r is the radial component of relative momentum, $q_r = \frac{\mathbf{r}}{r} \cdot \mathbf{q}$, $\mathbf{L} = \mathbf{r} \times \mathbf{q}$ the angular momentum and $\mathbf{r} = |\mathbf{r}_1 - \mathbf{r}_2|$ the relative distance. The function $w(r)$ comes from the commutation between the momentum operator and $R_+(r)$,

$$w(r) = \frac{1}{M R_+^{'\,2}(r)} \left[2 \frac{R_+^{''}(r)}{r R_+^{'}(r)} - \frac{5}{4} \left(\frac{R_+^{''}(r)}{R_+^{'}(r)} \right)^2 + \frac{1}{2} \frac{R_+^{'''}(r)}{R_+^{'}(r)} \right]. \tag{28}$$

Now, we can obtain the ground-state energy per particle in

Hartree-Fock approximation with UCOM correlation,

$$\frac{\varepsilon_{HFU}}{A}=\frac{3}{10}\frac{k_F^2}{M}+\frac{1}{A}\sum_{i<j}^{A}\langle ij|\widetilde{V}_{ij}|ij\rangle_A. \tag{29}$$

Here $\langle ij|\widetilde{V}_{ij}|ij\rangle_A$ means the anti-symmetrized two-body matrix element of operator $\widetilde{V}_{ij}$ taken with the single particle plane wave functions,

$$|i\rangle=\frac{1}{\sqrt{V}}\exp(i\mathbf{k}_i\cdot\mathbf{r})\otimes|\chi_s\rangle\otimes|\chi_t\rangle, \tag{30}$$

where $|\chi_s\rangle$ and $|\chi_s\rangle$ represent the eigenstates of spin and isospin.

There are three parameters, α,β and η, when we consider the short range correlation of realistic NN interaction. We would like to determine them by minimizing the energy per particle of whole system with variational principle,

$$\frac{\partial^3(\varepsilon_{HFU}/A)}{\partial\alpha\partial\beta\partial\eta}=0. \tag{31}$$

We have constructed the non-relativistic Hartree-Fock model with UCOM (HFU), where we do not have any free parameters.

IV. THREE-BODY INTERACTION IN THE HFU MODEL

The three-body interaction is very important to provide reasonable saturation properties for the microscopic calculation in the non-relativistic framework [8, 21]. We shall introduce the Urbana three nucleon interaction (TNI) [22], which contains two terms: the two-pion exchange part $V_{ijk}^{2\pi}$ from the Fujita-Miyazawa model [23], and the repulsive part V_{ijk}^{R} due to the relativistic effect,

$$V_{ijk}=V_{ijk}^{2\pi}+V_{ijk}^{R}. \tag{32}$$

Now, the Hamiltonian of the whole system is changed as,

$$\mathcal{H}=\sum_{i=1}^{A}T_i+\sum_{i<j}^{A}V_{ij}+\sum_{i<j<k}^{A}V_{ijk}^{2\pi}+\sum_{i<j<k}^{A}V_{ijk}^{R}. \tag{33}$$

More explicit form about the three nucleon interaction part can be written as,

$$\begin{aligned}V_{ijk}^{2\pi}&=A_{2\pi}\sum_{cyclic}\Big[\{\mathbf{X}_{ij},\mathbf{X}_{ik}\}\{\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k\}\\&\quad+\frac{1}{4}[\mathbf{X}_{ij},\mathbf{X}_{ik}][\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k]\Big]\\V&=A_R\sum_{cyclic}[T(r_{ij})]^2[T(r_{ik})]^2\end{aligned} \tag{34}$$

with

$$\mathbf{X}_{ij}=Y(r_{ij})\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j+T(r_{ij})S_{ij}. \tag{35}$$

Here, $\{,\}$ and $[,]$ denote the anti-commutator and commutator. The functions $Y(r)$ and $T(r)$ are radial functions related with the Yukawa and tensor part of the one-pion exchange interaction which take into account the form factor,

$$\begin{aligned}Y(r)&=\frac{e^{-m_\pi r}}{m_\pi r}[1-\exp(-br^2)],\\T(r)&=\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)\frac{e^{-m_\pi r}}{m_\pi r}[1-\exp(-br^2)]^2,\end{aligned} \tag{36}$$

where $b=2.0\ \ \mathrm{fm}^{-2}$.

Now, the correlated potential after short range correlation is written as,

$$\begin{aligned}\widetilde{V}&=C_{ij}^{\dagger}V_{ij}C_{ij}+C_{ijk}V_{ijk}C_{ijk}\\&\quad+C_{ij}^{\dagger}(T_i+T_j)C_{ij}-(T_i+T_j).\end{aligned} \tag{37}$$

For the three-body interaction, we take the UCOM to the two-body correlation. The Urbana TNI V_{ijk} can be separated as the product of two two-body interactions, $V_{ij}^{[3]}$ and $V_{ik}^{[3]}$. Therefore, the short range correlation on the TNI can be approximately written as,

$$\widetilde{V}_{ijk}\sim\widetilde{V}_{ij}^{[3]}\widetilde{V}_{ik}^{[3]}, \tag{38}$$

where we have used the unitary property of short range operator C. Finally, the energy per particle in the HFU model with three-body interaction is obtained as

$$\begin{aligned}\frac{\varepsilon_{HFUT}}{A}&=\frac{3}{10}\frac{k_F^2}{M}+\frac{1}{A}\sum_{i<j}^{A}\langle ij|\widetilde{V}_{ij}|ij\rangle_A\\&\quad+\frac{1}{A}\sum_{i<j<k}^{A}\langle ijk|\widetilde{V}_{ijk}|ijk\rangle_A.\end{aligned} \tag{39}$$

Because there is a cyclic symmetry in the interaction of $V_{ijk}^{2\pi}$ and V_{ijk}^{R}, the expectation value of three-body interaction in Eq. (39) can be simplified,

$$\begin{aligned}\sum_{i<j<k}^{A}\langle ijk|\widetilde{V}_{ijk}|ijk\rangle_A&=\sum_{i,j,k}^{A}\Big[\frac{1}{6}\langle ijk|\widetilde{V}_{ijk}|ijk\rangle-\frac{1}{2}\langle ijk|\widetilde{V}_{ijk}|ikj\rangle\\&\quad+\frac{1}{3}\langle ijk|\widetilde{V}_{ijk}|kij\rangle\Big].\end{aligned} \tag{40}$$

More obvious expression about the energy contribution of the Urbana TNI in nuclear matter with plane wave function is described in detail in [24]. Only the difference appears in the treatment of the short range correlation on the three-body interaction in this work.

V. NUMERICAL RESULTS

Firstly, we would like to discuss the properties of nuclear matter with a two-body NN interaction. We adopt the Bonn-A potential as a realistic NN interaction which is constructed by exchanging six non-strange mesons with masses below 1 GeV. The meson masses, meson-nucleon coupling constants and cut-off masses in the form factor can be referred in Table VI in [10].

In Fig. 3, we show the EOS of symmetric nuclear matter with the Bonn-A potential. We compare the results of symmetric nuclear matter with and without the short range correlation in the Hartree-Fock theory. The realistic NN interaction cannot bind the system of symmetric nuclear matter due to the strong repulsive effect in the short distance. The UCOM effect can largely cut down the repulsive effect in the short distance and make the symmetric nuclear matter bound. This correlation effect becomes larger with the density.

However, the saturation properties of symmetric nuclear matter in the HFU model, $E/A = -12.66$ MeV and $\rho_0 = 0.33$ fm^{-3}, are far from the empirical values. It is caused by the omission of the tensor effect in the Hartree-Fock model, which is very important as shown in Section Ⅱ for symmetric nuclear matter

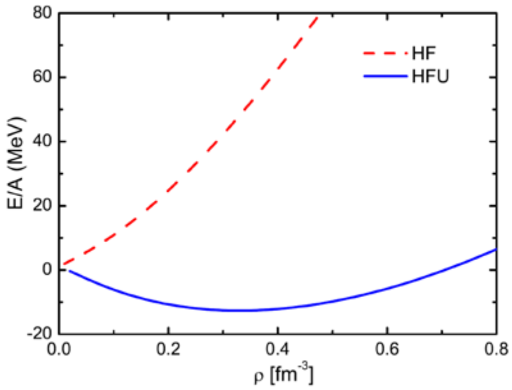

Fig. 3 The EOS of nuclear matter with the HFU model for symmetric nuclear matter: the dashed curve is the EOS of symmetric nuclear matter without the short range correlation in the Hartree-Fock approximation, while the solid curve is the result of the HFU model

We also give the corresponding UCOM parameters, α, β and η, as functions of density in Fig. 4 . These parameters are obtained by minimizing the ground state energy with variational principle. They change slightly in the low density region and become stable in the high density region. The minimization of the binding energy is obtained by the competition between the short range correlation on the kinetic energy and the potential energy. The short range correlation effect on the kinetic energy is repulsive, while it is attractive for the NN interaction energy. Finally, they cancel with each other and make the binding energy minimized.

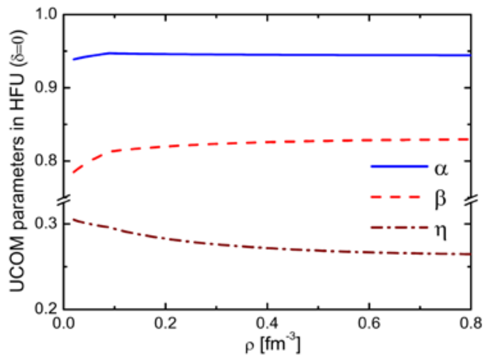

Fig. 4 The UCOM parameters as functions of density for symmetric nuclear matter

The tensor effect is very small in neutron matter. It should be a good approximation to consider only the short range correlation. We plot the EOS of pure neutron matter with the HFU model in Fig. 5. We find that it is compared very well with the calculation of the variational method with the AV18 potential including the relativistic boost correction (δv) [8]. This success is based on the following two points. The first one is that the UCOM takes the reasonable short range correlation into account for the Bonn-A potential and the contribution of the tensor interaction can be neglected for neutron matter. The other one is that the HFU model with the use of the Bonn potential includes the relativistic boost effect automatically.

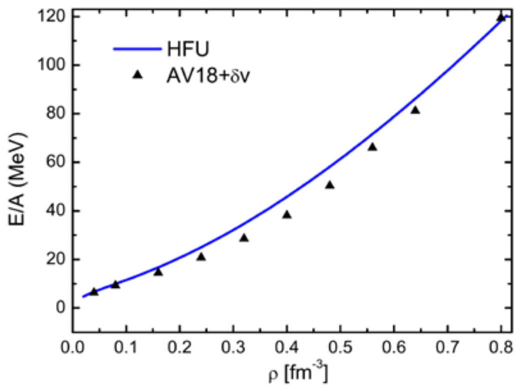

Fig. 5 The EOS of neutron matter with the HFU model as a function of density: the solid curve is the result of the HFU model with the Bonn-A potential, while the triangles are the EOS from the variational method with the AV18 potential

Actually, we have the similar framework with the relativistic mean field approximation in the HFU model except for the kinetic energy part. Forest *et al*. has proved that the boost corrections for the meson-exchange potential, obtained from the relativistic mean field model is in agreement with the results of δv [21]. The relativistic boost correction coming from the NN interaction is described in the framework where the total momentum $\mathbf{P}_{ij} = \mathbf{p}_i + \mathbf{p}_j$ is zero. However, there is no such constraint when we calculate the energy in the framework of the relativistic mean field approximation. Furthermore, the meson-exchange potentials contain the Dirac spinors which takes the relativistic effect.

Although we reproduce the results of the variational calculation for neutron matter with the two-body realistic NN interaction, Bonn Potential, this EOS is still too soft in the high density region. An additional repulsive contribution should be introduced. It is obtained from the Z-graph of the σ meson exchange through nucleon-antinucleon excitation in the RBHF model. However, we would like to adopt a phenomenological three-body interaction, Urbana three-nucleon interaction (TNI), follow the work of the variational method. The EOS of pure neutron matter in the HFU model with a three-body interaction (HFUT) is given in Fig. 6. The strengths of the Urbana TNI, $A_{2\pi}$ and A_R, are chosen as the same values as the UIX* in [8], which have values $A_{2\pi} = -0.0293$ MeV and $A_R = 0.63 \times 0.048$ MeV.

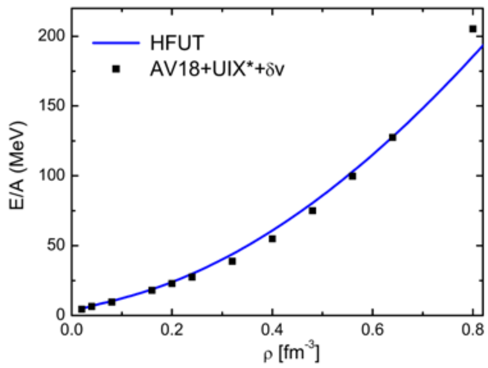

Fig. 6 The EOS of neutron matter with the HFUF model: the solid curve is the results of the HFUT model, while the square points are the EOS in the variational method with the AV18 potential and three-body interaction

In Section Ⅱ, we have shown that the tensor contribution of the pion and even the central spin-spin interaction of the pion are suppressed by the isospin factor in neutron matter. For the 2 π exchange part of the Urbana three-body interaction, the similar suppression effect of the pion matrix elements is present. Its effect can be neglected as compared with the contribution from the repulsive term. Therefore, we drop this contribution for the consistency in the three-body interaction case by the same reason as we have dropped the tensor interaction for the two-body interaction case. The solid curve in the left panel represents the EOS of neutron matter in the HUFT model without the term $V_{ijk}^{2\pi}$, which is a factor 20 smaller than that of the repulsive three-body interaction in the present model space calculation. We found our results almost reproduce those of the variational methods with the AV18 potential and the Urbana three-body interaction [8].

In Fig. 7, we also show the UCOM parameters with density in the HFUT model for neutron matter. In the high density region, they change gradually with the density and are not stabilized as compared with the case with only the two-body NN interaction. This is because the three-body interaction is influenced largely by the short range correlation. In the high density region, the repulsive contribution of the three-body interaction becomes large, where the UCOM plays a very important role.

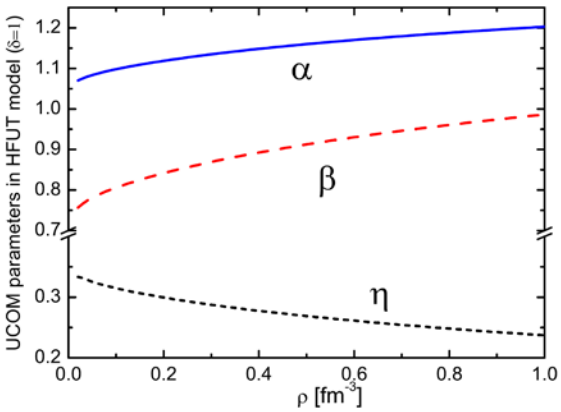

Fig. 7 The UCOM parameters in the HFUT model for neutron matter

VI. CONCLUSIONS

We have studied the properties of nuclear matter, especially pure neutron matter in the non-relativistic Hartree-Fock theory with three-body interaction in this work. We explain the role of tensor force in the different isospin channels of nuclear matter by evaluating the iterated one-pion-exchange diagram. The tensor force makes very large attractive contribution to symmetric nuclear matter, while its effect becomes very weak in neutron matter with only $T=1$ nucleon pairs. Therefore, it is sufficient to treat only the short range correlation induced by the strong repulsive interaction of a realistic NN interaction in neutron matter.

The unitary correlation operator method (UCOM) has been adopted to treat the short range correlation. We constructed the Hartree-Fock theory with UCOM (HFU) by using the two-body meson-exchange potential, Bonn-A potential, which is a realistic NN interaction constructed from the phase shifts of NN scattering. It is necessary to introduce a three-body interaction to improve the properties of nuclear matter in the microscopic calculation of nuclear matter with the non-relativistic framework. We choose a phenomenological one, Urbana three-nucleon interaction. The HFU model with three-body interaction was called as the HFUT model.

We have studied firstly the equation of state (EOS) of nuclear matter with Bonn-A potential in the HFU model. The UCOM can largely cut down the strong repulsive contribution of a realistic NN interaction for symmetric nuclear matter being bound together. However, the saturation properties are still bit far from the empirical values due to the lack of the tensor effect. The parameters, α, β and η in the UCOM are obtained by minimizing the ground energy of the total system with the variational principle. They changed somewhat at the low density region and become very stable in the high density region. For the pure neutron matter, we have obtained the similar EOS with the variational method using the AV18 potential and relativistic boost effect. This success is based on the fact that the tensor effect is very weak in neutron matter as we have shown before and the Bonn-A potential contains the relativistic boost effect in the framework of the HFU model.

To make the EOS harder, we also have performed calculations in the HFU model by including a three-body interaction, UIX*, for neutron matter. In this calculation, we include only the V_{ijk}^{R} three-body interaction and do not include the $V_{ijk}^{2\pi}$ three-body interaction for consistency with the treatment of the two-body interaction without inclusion of the tensor interaction for neutron matter. The HFUT calculation compares nicely with the one in the variational method for neutron matter. Therefore, we can claim that the three-body force is very important in the non-relativistic framework for neutron matter.

ACKNOWLEDGMENT

The authors are grateful to the members of the theory group of RCNP to clarify the treatment of pion in nuclear physics. This work is supported by the JSPS research grant: 21540267. J. Hu is supported by the MEXT fellowship.

REFERENCES

[1] H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Relativistic equation of state of nuclear matter for supernova and neutron star, *Nuclear Physics. A*, vol.637, pp. 435-450, May, 19, 1998.

[2] H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Relativistic Equation of State of Nuclear Matter for Supernova Explosion, *Progress of Theoretical Physics.* vol. 100, pp. 1013-1031, June, 29, 1998.

[3] P. Danielewicz, R. Lacey, and W. G. Lynch, Determination of the Equation of State of Dense Matter, *Science*, vol. 637, pp. 1592-1596, Oct., 31, 2002.

[4] H. Toki, Short range behavior of the N-N potential within the quark Model, *Z. Physic A*, vol. 294, pp. 173-181, Oct., 17, 1980.

[5] M. Oka, and K. Yazaki, Short Range Part of Baryon-Baryon Interaction in a Quark Model. I, *Progress of Theoretical Physics*, vol. 66, pp. 556-571, Feb., 1981.

[6] R. Jastrow, Many-Body Problem with Strong Forces, *Physics. Rev.*, vol. 98, pp. 1479-484, June, 1955.

[7] V. R. Pandharipande, and R. B. Wiringa, Variations on a theme of nuclear matter, *Rev. Mod. Physics.* vol. 51, pp. 821-861, Oct., 1, 1979.

[8] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Equation of state of nucleon matter and neutron star structure, *Physics. Rev. C*, vol. 58, pp. 1804-1828, Aprl., 14, 1998.

[9] K. Brueckner, C. Levinson, and H. Mahmoud, Two-Body Forces and Nuclear Saturation. I. Central Forces, *Physics. Rev.*, vol. 95, pp. 217-228, Jul, 1954.

[10] R. Brockmann, and R. Machleidt, Relativistic nuclear structure. I. Nuclear matter, *Physics. Rev. C*, vol. 42, pp. 1965-1980, Nov., 1990.

[11] N. Kaiser, and W. Weise, Chiral dynamics and nuclear matter, *Nuclear. Physics. A*, vol. 697, pp. 255-276, Nov., 27, 2011.

[12] S.C. Pieper, and R. B. Wiringa, Quantum Monte Carlo calculation of light nuclei, *Annu. Rev. Nuclear. Part. Sci.*, vol. 51, pp. 53-90, Dec., 2001.

[13] J. Hu, Y. Ogawa, H. Toki, A. Hosaka, and H. Shen, Extended relativistic chiral mean field model for nuclear matter, *Phys. Rev. C*, vol. 79, pp. 024305, Feb., 9, 2009.

[14] T. Myo, H. Toki, and K. Ikeda, Tensor-Optimized Shell Model with Bare Nucleon-Nucleon Interaction for 4He, *Progress of Theoretical Physics*, vol. 121, pp. 511-531, May, 2009.

[15] Y. Ogawa, H. Toki, S. Tamenaga, and A. Haga, Relativistic Chiral Mean Field Model for Finite Nuclei, *Progress of Theoretical Physics*, vol. 122, pp. 477-498, Aprl., 21, 2009.

[16] P. G. Krastev and F. Sammarruca, Neutron star properties and the equation of state of neutron-rich matter, *Phys. Rev. C*, vol. 74, pp. 025808, Aug., 24, 2006.

[17] H. Feldmeier, T. Neff, R. Roth, and J. Schnack, A unitary correlation operator method, *Nucl. Phys. A*, vol. 632, pp. 61-95, Jun, 23, 1998.

[18] T. Neff, and H. Feldmeier, Tensor correlations in the unitary correlation operator method, *Nucl. Phys. A*, vol. 713, pp. 311-371, Oct., 23, 2003.

[19] J. Hu, H. Toki, W. Wen, and H. Shen, Relativistic Hartree-Fock model with bare nucleon-nucleon interaction for neutron-rich matter, *Phys. Lett. B*, vol. 687, pp. 271-274, Mar. 11, 2010.

[20] Z. H. Li, U. Lombardo, H. J. Schulze, W. Zuo, L. W. Chen, and H. R. Ma, Nuclear matter saturation point and symmetry energy with modern nucleon-nucleon potentials, *Phys. Rev. C*, vol. 74, 047304, Oct. 20, 2006.

[21] J. L. Forest, V. R. Pandharipande, and J. L. Friar, Relativistic nuclear Hamiltonians, *Phys. Rev. C*, vol. 52, pp. 568-575, Aug., 1, 1995.

[22] J. Carlson, and V. R. Pandharipande, Three-nucleon interaction in 3-, 4- and N-body systems, *Nuclear Physics. A*, vol. 401, pp. 59-85, May, 30, 1983.

[23] J. Fujita, and H. Miyazawa, Pion Theory of Three-Body Forces, *Prog. Theor. Phys.*, vol. 17, pp. 360-365, May, 1957.

[24] H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, and M. Takano, Variational calculation for the equation of state of nuclear matter at finite temperatures, *Nuclear Physics. A*, vol. 791, pp. 232-250, Jul., 1, 2007.

[25] J. Hu, H. Toki, W. Wen, and H. Shen, Relativistic Hartree-Fock model with bare nucleon-nucleon interaction for neutron-rich matter, Phys. Lett. B, vol. 687, pp. 271-274, Mar. 11, 2010.

[26] J. Hu, Y. Ogawa, H. Toki, A. Hosaka, and H. Shen, Extended relativistic chiral mean field model for nuclear matter, Phys. Rev. C, vol. 79, pp. 024305, Feb., 9, 2009.

[27] J. Hu, H. Toki, W. Wen, and H. Shen, The role of the form factor and short range correlation in the relativistic Hartree-Fock model for nuclear matter, Eur. Phys. J. A., vol. 43, pp. 323-334, Feb. 2, 2010.

Jinniu Hu was born in Sichuan Province, China in May, 1983. He obtained his Doctor Degree of Science in Osaka University, Japan. He used to work at RCNP, Osaka University and RIKEN, Japan as a postdoctoral researcher. Now, he is a postdoctoral researcher in Peking University, China. He published several papers in Phys. Lett. B, Phys. Rev. C and Eur. Phys. J. A.

His research interests are concentrated in the properties of nuclear matter with realistic nucleon-nucleon interaction, the chiral properties at finite density and hyperon nuclear matter.

Hiroshi Toki is a professor in RCNP, Osaka University, Japan.

Wu Wen is a researcher in the Institute of Applied Physics and Computational Mathematics in China.

Hong Shen is a professor in department of Physics, Nankai University, China.