Design of a Flight Test Matrix and Dynamic Relocation of Test Points

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***Abstract*-A computational methodology for designing an experimental test matrix is presented based on the concept of potential and repulsive fields. The problem consists in the optimal distribution of test points in a two-dimensional domain, pursuant to hard constraints (permitted boundaries of the domain) and soft constraints (minimization of potential). Each test point is assumed to be the source of different fields which expose all other points to repulsive forces, thus accelerations, acting in different directions. The result of the mutual repulsive forces is a dynamic evolution of the configuration of test points in the domain, which eventually converges to a condition of minimum potential. An iterative process is adopted to find a numerical solution where residual accelerations are below a desired threshold. The method has been extended to the additional task of dynamically relocating the remaining test points, after an initial subset has been performed and a need to change (either increase or reduce) the number of test points has arisen. The proposed technique allows for an easy accomplishment of the task with minor modifications to the algorithm. A large degree of flexibility in the algorithm is allowed to tune the relative weights to attribute to the different requirements. The method proved effective and computationally efficient, exhibiting satisfactory results in both the test matrix design task and the dynamic relocation task.**

***Keywords*- Flight Test; Flutter; Envelope Expansion; Field Theory; Optimization; Spatial Location**

I INTRODUCTION

When designing a set of experiments, it is important to find an optimal test point distribution in order to maximize efficiency and minimize costs. In some cases, i.e. flutter or envelope expansion testing, the test organization deals with unique prototype. For this type of aircraft, limiting the total flight hours of testing is a mandatory requirement not only for the significant cost associated, but also for the consideration of minimizing the risk of failure/loss of the highly valuable asset; in the last decade, a fatal mishap to an F-22 prototype fully instrumented brought to almost one year of delay in the Raptor Development Program and, unfortunately, to the loss of the experimental crew. Cost wise is useful to highlight that the average cost of a $4^{th}/5^{th}$ generation fighter type aircraft could range between 50 K\$ and 100 K\$. Therefore, when testing is required in the entire flight envelope, it is essential to find out a way to distribute the test points efficiently in order to gather all the required data, but at the same time saving time and reducing the number of test points. This means that given some test constraints and key parameters to be evaluated, all efforts should be spent in order to optimize test points distribution, covering the entire envelope following the rules imposed by the objective functions, whose aim is populating the areas where the test execution has a higher priority based on engineering requirements.

While the optimality criteria are dictated by the specific problems at hand, the optimization process itself is applicable to wide classes of problems. Many techniques can be found in the literature, although only few deal with the problem of spatial location [1-6]. The latter has been approached as a non-cooperative game in the companion paper [7]. In general terms the proposed method aims at locating points in a two-dimensional space according to soft constraints (minimization of potential) and hard constraints (boundaries of the permitted domain). The method makes use of the concept of potential of a point immersed in the field generated by other points [8, 9], producing mutual repulsive forces, in line with other optimization methods based on the analogy with physical systems, such as the Simulated Annealing.

The idea has been tested against a practical case: the definition of a flight test matrix for the evaluation of the aero-elastic and environmental characteristics of an aircraft. The goal is to distribute flight test points in the flight envelope in such a way to satisfy the requirements of structural engineers, interested in an optimal distribution in terms of airspeed and Mach, and systems engineers, more interested in the altitude and airspeed distribution. The method provides means to combine all objectives in a single test campaign, through an optimization of the test point distribution, being the result of a compromise of all needs.

More specifically in [7] we have tested the idea against a practical problem: the design of a flight test matrix for a combined aero-elastic and environmental flight test campaign. The objective is to locate a predefined number of points in the classic Mach-Altitude envelope, in order to simultaneously maximize the mutual distances of test points in the envelope and optimize the distributions of the three major parameters (Mach number, altitude and dynamic pressure, or equivalent airspeed)[1] according to desired engineering requirements.

Here we propose a different method, which is computationally less demanding or more efficient, and we expand the results to the problem of dynamically relocating points at a given stage of the test program, when contingencies require a revision of the amount of the overall number of experiments. Specifically in Section 2, we describe the location problem to distribute the flight test points and the relocation one, in Section 3 the algorithm is presented together with some test cases, in Section 4 some remarks conclude the paper.

II THE LOCATION PROBLEM

The objective of the specific problem is the identification of the test matrix to simultaneously gather all relevant data supporting the evaluation of aero-elastic and environmental response characteristics of an aircraft. This leads to an optimization process for distributing points according to the requirements of structural engineers, mostly interested in airspeed and compressibility effects, and systems engineers, mostly interested in altitude and airspeed effects.

The key idea of the proposed method is to consider each point as the source of a different field for each parameter to be controlled and let the points move as a result of the mutual repulsive forces generated by the fields. Eventually the points will come to a nominal state of rest when the nominal equilibrium of forces is reached, which corresponds to the condition of minimum potential energy.

More precisely, let n be a fixed natural number ($n > 5$) that is the number of the prescribed flight tests. Each test point is defined by a pair (Mi, Hi), where i is an integer number, $i \in [1; n] = \{1,..., n\}$, Mi and Hi are real numbers chosen in the following sets: $Mi \in [ML, MU]$ and $Hi \in [HL, HU]$, where the nonnegative constants $0< ML< MU$ and $0< HL<HU$ define the bounds of Mach number and altitude choices.

Definition 1. We define Flight Test Matrix the following 2n dimensional vector $(Mi, Hi)i$, $i \in [1;n]$.

An additional hard constraint on the test points (Mi, Hi) is the condition that the equivalent airspeed is bounded: $Vi \in [VL, VU]$ ($0< VL<VU$): the equivalent airspeed can be computed as a function of Mi and Hi under the assumption of International Standard Atmosphere ([10])

$$V_i(M_i, H_i)=a\, M_i\,(1-b\, H_i)^c \tag{1}$$

with a, b, c positive real constants.

Definition 2. The domain $[ML, MU] \times [HL, HU]$, with the additional hard constraint $Vi \in [VL, VU]$ is called flight envelope (FE).

A graphical depiction of the domain (flight envelope) is shown in Fig. 1, where $[M_L, M_U]=[0,1]$ and $[H_L, H_U]=[0, 35000]$ ($V_i \in [100, 600]$). In line with some previous papers dealing with experimental design problems [11], we decide an optimal configuration of the test points as the solution of a facility location problem in the admissible domain, which is the flight envelope. In general, the problem is to decide for two variables x and y the values of n available experiments. So we want to settle n points $P_1, P_2,..., P_n$ in the admissible region Ω of the plane in such a way that they are far as possible from the rest and from the boundary of the region. This implies to maximize the dispersion of the points.

[1]Equivalent Airspeed in knots = KEAS or EAS is the airspeed at sea level in the International Standard Atmosphere at which the dynamic pressure is the same as the dynamic pressure at the true airspeed (TAS) and altitude at which the aircraft is flying. It is useful for predicting aircraft handling, aerodynamic loads, stalling and other flying characteristics.

Indicated Airspeed in knots = KIAS or IAS is the airspeed read directly from the airspeed indicator on an aircraft, driven by the Pitot-static system. It uses the difference between total pressure and static pressure, provided by that system, to either mechanically or electronically measure dynamic pressure. The dynamic pressure includes terms for both density and airspeed. Since the airspeed indicator cannot know density, it is by design calibrated to assume the sea level standard atmospheric density when calculating airspeed. Not least since the actual density will vary considerably from this assumed value as the aircraft changes altitude, IAS varies considerably from true airspeed (TAS), the relative velocity between the aircraft and the surrounding air mass.

Calibrated Airspeed in knots = KCAS or CAS is the IAS corrected for instrument and position error. When flying at sea level under International Standard Atmosphere (ISA) conditions (15°C, 1013 hPa, 0% humidity) calibrated airspeed is the same as equivalent airspeed (EAS) and true airspeed (TAS). If there is no wind it is also the same as Ground Speed (GS). Under any other conditions, CAS may differ from the aircraft's TAS and GS.

True Airspeed in Knots = KTAS or TAS of an aircraft is the speed of the aircraft relative to the airmass in which it is flying.

Ground Airspeed in Knots = GS is the horizontal speed of an aircraft relative to the ground. Ground speed can be determined by the vector sum of the aircraft's true airspeed and the current wind speed and direction; a headwind subtracts from the ground speed, while a tailwind adds to it. Winds at other angles to the heading will have components of either headwind or tailwind as well as a crosswind component.

Problem 1. Experimental Design The problem of deciding the values of two variables for n assigned experiments is to choose P1,...,Pn in Ω maximizing the dispersion(P1,..., Pn), where the dispersion function is defined in a suitable way.

In our paper, we use the idea of potential energy, given by a repulsive forces field in the flight envelope in order to maximize the dispersion of the test points in the flight envelope. For the specific problem, three fields are introduced, associated with Mach number, pressure altitude and equivalent airspeed. The intensity of each field is a function of the value of the related parameters at the specific position of the point; this allows shaping the distribution of each parameter as desired (i.e. adjusting field intensities and thus repulsive forces to concentrate points where needed). Engineering considerations suggest that high Mach numbers and airspeeds are more critical for aero-elastic issues, whereas high airspeeds and low altitudes are more severe for environmental aspects; thus test points should concentrate at the highest speed boundary curve of the envelope to best compromise the requirements of both structural and systems engineers. This is achieved by establishing field intensity laws reflecting the following objective: Mach field intensity decreases with Mach number, altitude field intensity increases with altitude and airspeed field intensity decreases with airspeed. The relative importance of different parameters is attributed by properly scaling the intensities of the three fields.

Moreover Mach and pressure altitude fields act only along the corresponding direction (Mach and altitude respectively), while airspeed field acts radially. Thus the airspeed field plays the dual role of distributing points in airspeed and spreading points over the envelope.

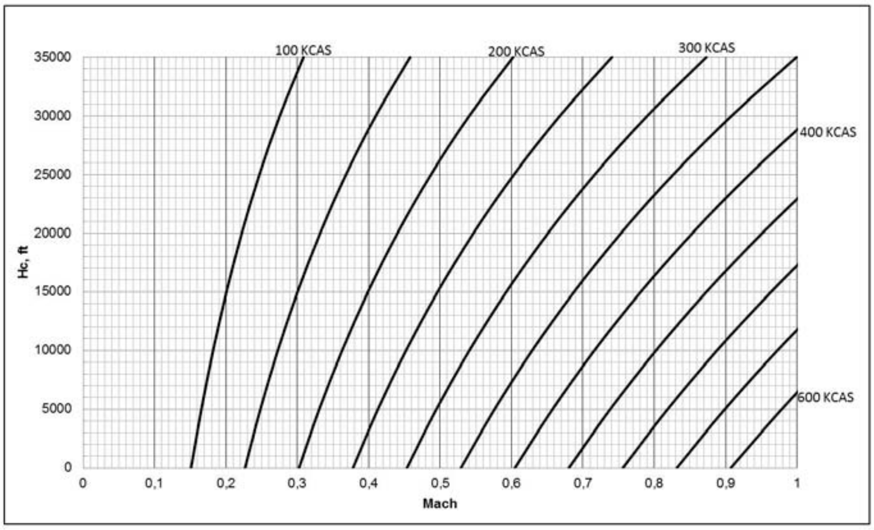

Fig. 1 The flight envelope

Let each test point be the source of three distinct fields, whose intensities are:

$$m_{M_i} = W_M \left(1 + K_M \frac{M_i - M_L}{M_U - M_L}\right)$$

$$m_{H_i} = W_H \left(1 + K_H \frac{H_i - H_L}{H_U - H_L}\right)$$

$$m_{V_i} = W_V \left(1 + K_V \frac{V_i - V_L}{V_U - V_L}\right)$$

respectively, where W_M, W_H, W_V are positive real numbers (defining relative weight of the three fields), while K_M, K_H, K_V are real numbers (prescribing the desired distribution trend of the corresponding parameters).

The first two fields act along a single dimension (the respective parameter), while the third field acts radially. Assuming repulsive forces proportional to the inverse of the cubic distance from the field source, the resulting accelerations (in the two directions: M and H) to which all points are subjected (except the first 5 points fixed for initialization) are:

$$a_{M_i} = M_U \left[\sum_{\substack{j=1,3 \\ j\epsilon\,[6,n] \\ j\neq i}} \frac{m_{M_j}}{\left(\frac{M_i-M_j}{M_U}\right)^3} + \sum_{\substack{j\epsilon\,[1,n] \\ j\neq i}} \frac{m_{V_j}\left(\frac{M_i-M_j}{M_U}\right)}{\left[\left(\frac{M_i-M_j}{M_U}\right)^2 + \left(\frac{H_i-H_j}{H_U}\right)^2\right]^2} \right]$$

$$a_{H_i} = H_U \left[\sum_{\substack{j=1,3 \\ j \epsilon [6,n] \\ j \neq i}} \frac{m_{H_j}}{\left(\frac{H_i - H_j}{H_U}\right)^3} + \sum_{\substack{j \epsilon [1,n] \\ j \neq i}} \frac{m_{V_j} \left(\frac{H_i - H_j}{H_U}\right)}{\left[\left(\frac{M_i - M_j}{M_U}\right)^2 + \left(\frac{H_i - H_j}{H_U}\right)^2\right]^2} \right]$$

(where the first fixed point (j=1) is at the bottom left corner of the envelope (ML, HL) and the third fixed point (j=3) is at the top right corner of the envelope (MU, HU)).

The points are then allowed to move sequentially in the envelope in response to the respective accelerations. At each iteration the time step is chosen such that displacements are progressively smaller and smaller (as the distribution converges toward the optimal solution) while the Mach and altitude hard constraints are not violated.

Thus the potential energy of the configuration is:

$$J(M,H) = \sum_{i=1}^{n} \left[\left(\frac{a_{M_i}}{M_U}\right)^2 + \left(\frac{a_{H_i}}{H_U}\right)^2\right]$$

with (M,H)=(M1,…,Mn,H1,…,Hn).

The objective function is actually the cost function represented by the sum of the accelerations generated by all test points. To minimize the cost function is the goal of the proposed algorithm, achieved looking for an equilibrium condition, local or global. Each equilibrium condition of all the considered test points is represented by a local or global minimum value of the potential energy function. In fact, during the evolution of the transient location, part of the potential energy of the test points within the field generated by the other points is converted into kinetic energy, that eventually is dissipated instantaneously (a different approach could also introduce a friction coefficient). Therefore the equilibrium condition is characterized by a minimum value, local or global, of the potential energy function.

Our problem is summarized as the following:

Problem 2. Flight Test Distribution The problem is to decide the optimal Flight Test Matrix, i.e. the vector (M*, H*) in the flight envelope minimizing the potential energy J in the flight envelope FE.

A. The Execution Order Problem

Once the test matrix is defined, a preliminary chronological order of the test points must be established. To this end, several approaches can be followed depending on the particular application. In our example, we considered two requirements: safety and efficiency. Given the hazardous nature of flutter (aero-elastic phenomenon) testing, safety is the first and paramount priority. Thus efficiency can be sought only when safety is assured.

Assuming that a 20 KEAS (Knots Equivalent Air Speed) margin between test points is a cautious and safe approach to the envelope expansion task, test points are ordered with increasing airspeed; however, if more than a single point meet the 20 KEAS margin criterion, efficiency considerations suggest that points are ordered to best manage energy (either in ascending or descending order). The two forms of energy attributed to a flight condition represented by a point in the envelope are potential and kinetic energy, leading to the following expression for the specific energy (energy per unit weight):

$$SE = H + \frac{1}{2} \frac{V^2}{g}$$

where H is the pressure altitude, V is the equivalent airspeed (under the assumption of zero wind) and g is the acceleration of gravity.

Of course this is just a possible simple criterion to attribute an a priori execution order. Depending on the complexity of the problem, several additional constraints might apply and the actual execution order might need to be dynamically adjusted while in progress based on the results gathered from previous points. However different choices of the execution order do not invalidate the effectiveness of the proposed method for identifying the location of the test points.

Efficiency is pledged by the objective functions design, in fact the test points simulation as source of mutually repulsive forces allows for a well-spread distribution in the entire flight envelope; therefore, maximizing the mutual distances would maximize the test efficiency; however, the only flight envelope coverage would not ensure the positive solution of the optimization problem, because safety would not have been still properly addressed. That is the reason for the presence of hard constraints and different source/field intensities depending on the location in flight envelope, allowing the concentration of test points in the high risk test areas. This process leads to a more safe approach because airspeed and dynamic pressure test evolution steps are smaller in those areas where the risk of flutter is deemed to be higher, allowing for a better prediction between flights (data analysis and reduction with less uncertainty) and reducing the delta increment per each step (reduced extrapolation).

The choice to represent test points as mutually repulsive sources is in line with the actual decision process used by the two classical methods for flutter testing test matrix generation: the Standard one (very expensive) which is based on the assumption that test points must be located at prescribed distances, in terms of altitude and airspeed, depending on the value of dynamic pressure, increasing test points density in the areas where dynamic pressure is higher; and the Economy one that actually has a philosophy very similar to the first one, but does not foresee at low altitudes the execution of test points characterized by dynamic pressure values lower than the maximum value already tested at higher altitudes.

In order to evaluate the algorithm performance, the Potential Energy function J has been introduced: low value of J certifies an acceptable optimization process. The overall evaluation of the test points distribution and goodness of the final result is actually related somewhat to the experience of the test conductor and also to a comparison with the test matrix proposed by the Economy Method which is based on pre-canned steps of Mach and Altitude band. With the method we propose in the present paper we shoot for a new target: covering the same portion of the flight envelope with less test points or using the same number of test points, or increasing the test points concentration in the “hot spot area”, where dynamic pressure is the highest (low altitude, high speed/Mach).

B. The Relocation Game

Suppose that a test matrix has been designed and a certain amount of test points has been performed according to a predefined execution order. Suppose also that initially unforeseen events (partial test results, budget reviews, changes of the trial objectives) require a modification of the amount of test points. The relocation problem of the remaining test points (which may be either more or less than the original plan) can be approached similarly to the initial task described at the beginning of the Section. The only difference is that the remaining points must be distributed with an additional hard constraint: the presence in the envelope of the test points already performed along with their respective fields. With this minor adjustment, the same algorithm can be used for the relocation problem.

III ALGORITHM AND RESULTS

The solution of such a problem cannot be found analytically, thus an iterative process is adopted, letting the points evolve until numerical convergence is reached (the sum of all forces is below a given threshold). Repulsive forces are similar to those acting between electrical charges having the same sign, except that the intensity decreases with the cubic power of the distance (to reduce the effect of distant points compared to near ones). The acceleration to which a point is subject only depends on its position in the field and the field intensity (same as electric or gravitational fields). To improve convergence, momentum is not preserved from step to step: in other terms, the point is allowed to move according to the acceleration imposed by the fields, but at the next step it is assumed initially at rest and it further evolves only by virtue of the new acceleration produced by the new spatial configuration, regardless of the previous velocity. All points move sequentially and the time step for each point is chosen in such a way that the distance travelled (at the given step) exponentially decreases with elapsed time (to improve convergence) and the point is not allowed to exit the permitted domain (violate the hard constraints).

A. The Algorithm

Step 1 Initializing. Let the first five points stay fixed in the 5 corners of the flight envelope (hard constraint). The remaining n-5 points are free to travel within the permitted envelope; let the initial distribution of those points be according the following:

$$M_i = \frac{M_L + M_U}{2} + \frac{M_U - M_L}{4} cos\left[\frac{2\pi \cdot (i-6)}{n-5}\right] \quad i\epsilon[6, n]$$

$$H_i = \frac{H_L + H_U}{2} + \frac{H_U - H_L}{4} sin\left[\frac{2\pi \cdot (i-6)}{n-5}\right] \quad i\epsilon[6, n]$$

Step 2 Displacement computation. The displacements consequent to the accelerations acting during the time step are computed as:

$$dM_i = \frac{1}{2} a_{M_i} dt^2$$

$$dH_i = \frac{1}{2} a_{H_i} dt^2$$

thus ignoring any velocity gathered in the previous time steps, in order to facilitate convergence. Here

$$dt = min \begin{cases} dt_{min} \\ \sqrt{\dfrac{2 \cdot dM_{max}}{|a_{M_i}|}} \\ \sqrt{\dfrac{2 \cdot dH_{max}}{|a_{H_i}|}} \end{cases}$$

where

$$dt_{min} = \frac{0.01}{n}$$

$$dM_{max} = \begin{cases} min \begin{cases} \dfrac{M_U - M_i}{2} \\ \dfrac{M_U - M_L}{2} e^{-\frac{t}{10}} \end{cases} & , a_{M_i} > 0 \\ min \begin{cases} \dfrac{M_i - M_L}{2} \\ \dfrac{M_U - M_L}{2} e^{-\frac{t}{10}} \end{cases} & , a_{M_i} \le 0 \end{cases}$$

$$dH_{max} = \begin{cases} min \begin{cases} \dfrac{H_U - H_i}{2} \\ \dfrac{H_U - H_L}{2} e^{-\frac{t}{10}} \end{cases} & , a_{H_i} > 0 \\ min \begin{cases} \dfrac{H_i - H_L}{2} \\ \dfrac{H_U - H_L}{2} e^{-\frac{t}{10}} \end{cases} & , a_{H_i} \le 0 \end{cases}$$

The displacements thus computed do not guarantee adherence to the last hard constraint: airspeed within the two permitted boundaries. An additional check must be performed: if computed time step and acceleration cause the airspeed to exceed the envelope boundary, then the new position is set at 90% of the distance between initial position and airspeed limit (along the direction of the calculated acceleration). Then acceleration is set to zero, because points constrained on the border are assumed to be subjected to a reaction force (acceleration) equal and opposite to the force (acceleration) which tends to push them out of the envelope.

Define

$$dM_{V_{max}} = 0.9 \cdot \frac{V_U - V_i}{\left(\frac{dV}{dM}\right)_i + \left(\frac{dV}{dH}\right)_i \left(\frac{dH}{dM}\right)_i}$$

$$dM_{V_{min}} = 0.9 \cdot \frac{V_L - V_i}{\left(\frac{dV}{dM}\right)_i + \left(\frac{dV}{dH}\right)_i \left(\frac{dH}{dM}\right)_i}$$

where, differentiating Eq. (1),

$$\left(\frac{dV}{dM}\right)_i = a(1 - b\, H_i)^c, \; \left(\frac{dV}{dH}\right)_i = -abc\, M_i\, (1 - b\, H_i)^{c-1}, \left(\frac{dH}{dM}\right)_i = \frac{dH_i}{dM_i}$$

then

$$if\ dM_i > dM_{V_{max}} \quad \rightarrow \quad dM_i = dM_{V_{max}} \quad \rightarrow \quad dH_i = dM_i \cdot \left(\frac{dH}{dM}\right)_i \quad \rightarrow \quad a_{M_i} = 0$$

$$if\ dM_i < dM_{V_{min}} \quad \rightarrow \quad dM_i = dM_{V_{min}} \quad \rightarrow \quad dH_i = dM_i \cdot \left(\frac{dH}{dM}\right)_i \quad \rightarrow \quad a_{H_i} = 0$$

Step 3 Update points distribution. The position is updated according to the calculated displacements, the field intensities are updated pursuant with the new configuration

$$M_i^{k+1} = M_i^k + dM_i$$

$$H_i^{k+1} = H_i^k + dH_i$$

$$t^{k+1} = t^k + dt$$

Weights $m_{M_i}, m_{H_i}, m_{V_i}$ are also updated at each iteration and we assume that the EAS weight decreases with time: initially points must be quickly spread over the envelope and the weight is large, then the weight must decay with time to the desired final value. More precisely, we let

$$m_{M_i}^k = W_M \left(1 + K_M \frac{M_i^k \ - M_L}{M_U - M_L}\right)$$

$$m_{H_i}^k = W_H \left(1 + K_H \frac{H_i^k \ - H_L}{H_U - H_L}\right)$$

$$m_{V_i}^k = W_V \ (1 + 100 \ e^{-\frac{t^k}{10}}) \left(1 + K_V \frac{V_i^k - V_L}{V_U - V_L}\right)$$

Step 4 Iteration and stopping criterion. The process is reiterated until a convergence cost function decays below a predetermined threshold. The convergence cost function is a measure of the residual accelerations to which the test points are subject, thus the potential energy of the configuration J(M,H). Convergence is reached when J is less than a predefined value (dependent on the number of points).

B. Test Case

We present a test case with n=30 planned flight tests. The parameter choice is specified in the following. The flight envelope bounds are

$$[M_L, M_U]=[0.1, 0.8]; [H_L, H_U]=[0, 3\times10^4],$$

the weights are

$$W_M=1, K_M=-0.95; W_H=2, K_H=100; W_V=500, K_V=-0.8$$

and the constants in Eq. (1) are

$$a= 1116.46; b=6.87\times10^{-6}; c=2.62$$

Results for a thirty test points location problem are shown in Fig. 2. The test points are numbered in chronological order, in accordance with the criterion described in Section 3.

In Fig. 3, when the 25 points have been performed, the test management decides to increase the number of tests from 25 to 30 (for an overall number of thirty points): in the new configuration the added test points are denoted with white circles. In this case, we have a higher value of potential J with respect to the 30 test points case represented in Fig. 2.

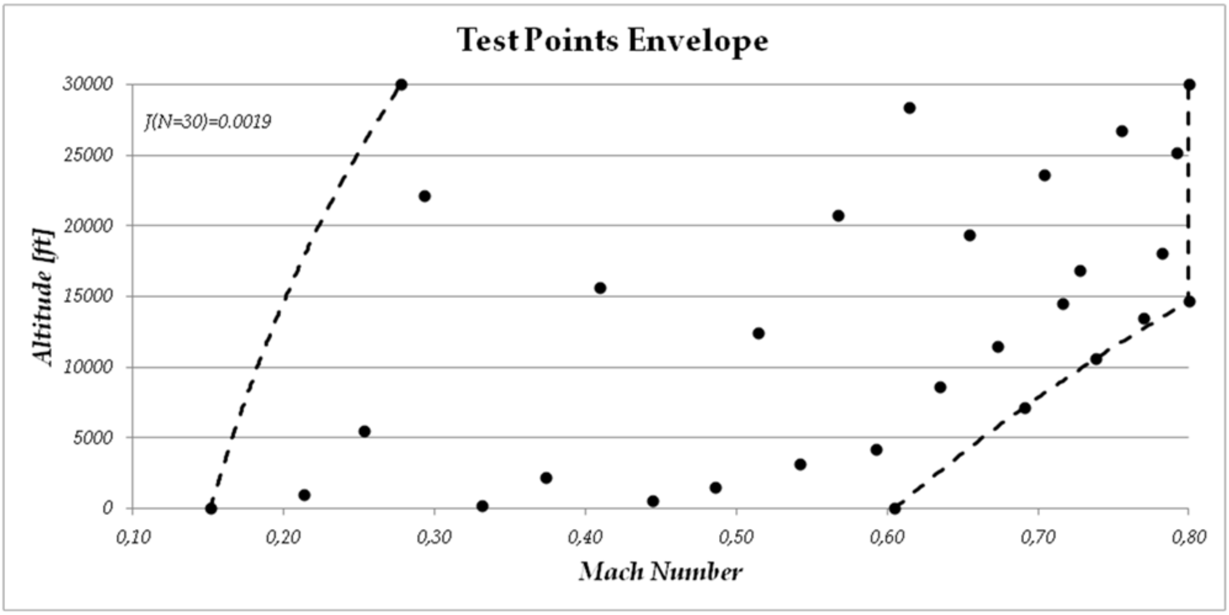

Fig. 2 Test points distribution n=30 and the energy value

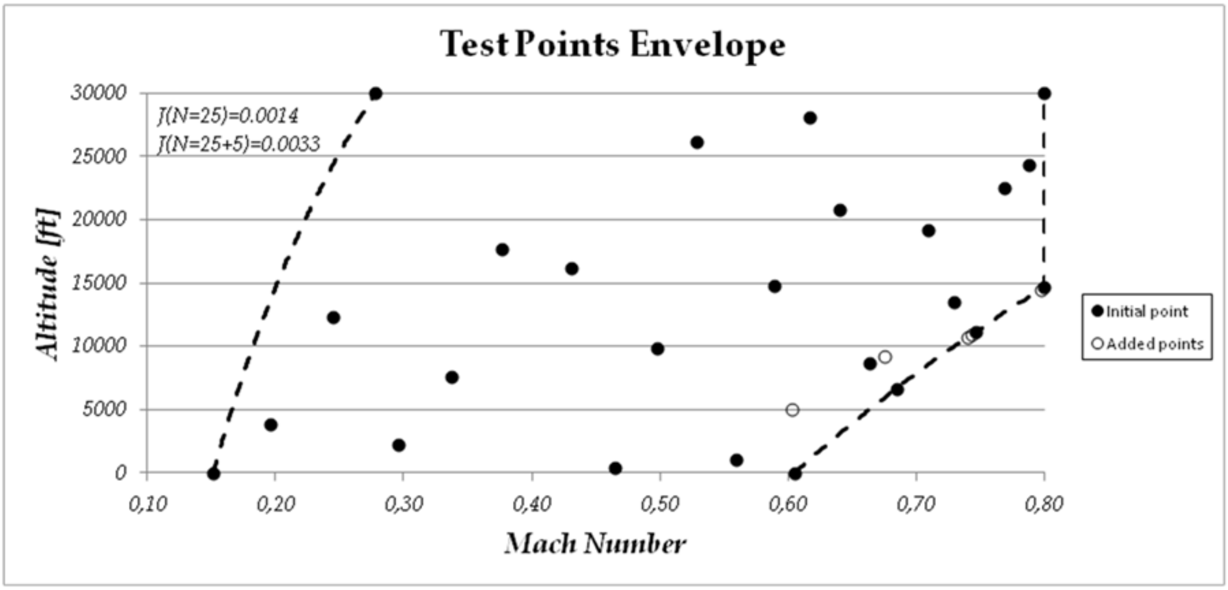

Fig. 3 Test points reallocation n=25 and 5 additional points

IV CONCLUSIONS

An optimization method based on the concept of fields is proposed for the identification of a two-dimensional test matrix. The experimental test point distribution is optimized according to tunable soft constraints and hard constraints. The method has been tested against a practical case: the simultaneous evaluation of aero-elastic and environmental characteristics of an aircraft. The method proved effective and computationally efficient: all the configurations tested came to a convergence in short time and the outcome was satisfactory.

The method was extended to the additional problem of relocating part of the test points after the execution of an initial subset of experiments and following the decision of the test management to increase the number of experiments. The results were satisfactory also for this additional task.

An alternative approach would be an entropy maximization to find the optimal test point distribution. In this case, a suitable computational procedure would be a genetic algorithm type [12-15]. This will be investigated in a future research.

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