# Concept of Quantum Fermi Liquid and Spin-Charge Separation Effect in 1D Systems

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***Abstract*-The concept of the quantum Fermi liquid for the description of (quasi)-1D electronic systems is recovered. The model of the (quasi)-1D quantum Fermi liquid is developed on the example of the *trans*-polyacetylene and it is the generalization of the well-known model of organic (quasi)-1D conductors, elaborated by Su, Schrieffer and Heeger (SSH-model). It is shown, that the spin-charge separation effect can be realized in (quasi)-1D quantum Fermi liquids. It has a topological soliton origin in distinction from a spinon-holon spin-charge separation effect, predicted by the Tomonaga-Luttinger liquid model. The model presented allows to extend the limits of the applicability of the SSH-model to the electron-electron correlated (quasi)-1D-systems without any restriction on an electron-electron interaction force. The (quasi)-1D-systems with a strong electron-phonon interaction and/or a strong electron-photon interaction can be also described within the framework of the 1D quantum Fermi liquid model proposed. The practical significance of the model proposed consists in the clarification of the nature of charge and spin carriers and in the clarification of the origin of mechanisms of the quasiparticles' interaction in the (quasi)-1D-systems. The model predicts the formation of longlived coherent, squeezed and entangled states of phonons in (quasi)-1D electronic systems with strong electron-phonon interaction at room temperature and even higher. It seems to be significant for the elaboration of the devices of the nanoelectronics, of the spintronics and of the other modern nanotechnology branches, based on the use of (quasi)-1D-systems, in particular, the nanotubes.**

***Keywords- Electron-Phonon Interaction; Electron-Photon Interaction; Spin-Charge Separation; Fermi Liquid***

## I. INTRODUCTION

There seems to be very essential for the tasks of nanoelectronics, spintronics and for the other branches of nanotechnology the knowledge of the nature of charge and spin carriers in the nano-devices, especially for the knowledge of mechanisms of a carrier transport and interactions of charge and spin carriers both between themselves and with phonons and photons. In particular, great hopes are pinned in modern nanotechnology on carbon nanotubes (NTs), that is, in using of carbon NTs for the production of the main devices for nanoelectronics and related nanotechnology branches. There is existing in the theory of (quasi)-1D electronic systems, in particular, in the theory of conducting NTs, the following concept, which was starting with the work of Tomonaga in 1950 [1] and with the works of Luttinger in 1960 [2] and in 1963 [3], when it has become clear that the electron-electron interaction destroys the sharp Fermi surface and leads to a breakdown of the Landau Fermi liquid (LFL) theory. The resulting non-LFL state was called Luttinger liquid (LL), or in some publications Tomonaga-Luttinger liquid (TLL). It was suggested, that TLL describes the universal low-energy properties of 1D conductors. TLL behavior is characterized theoretically by pronounced power-law suppression of the transport current and the density of states, and by an effect of a spin-charge separation. The idea of a spin-charge separation in the TLL model was originally introduced by Anderson in 1987 [4, 5, 6] for the doped Mott-Hubbard insulator in the metallic regime. The similar spin-charge separation effect was mathematically realized in the so-called slave-particle representation [7] of the $t-J$ model. The nature of spin and charge carriers in the TLL concept is not familiar. They are in Anderson approach chargeless spin 1/2 quasiparticles - spinons and spinless quasiparticles with the charge $\pm e$ - holons. The result of the mathematical realization of Anderson spin-charge separation effect in the slave-particle representation [7] of the $t-J$ model is $e_{i\sigma}=h_i^+ f_{i\sigma}$, where $h_i^+$, $f_{i\sigma}$ are holon and spinon fields, respectively. The occupancy constraint, reflecting the Hubbard gap in its extreme limit, is handled by an equality $h_i^+ h_i+\sum_{(\sigma)} f_{i\sigma}^+ f_{i\sigma}=1$, which commutes with the Hamiltonian. It is seen the close relation of the spin-charge separation and the constraint condition through the counting of the quantum numbers. But the spin-charge separation acquires a new meaning here. If holon and spinon fields indeed describe elementary excitations, the hole (electron) is no longer a stable object and must decay into a holon-spinon pair once being injected into the system. The instability of holes (electrons), being to be free quasiparticles in the solid state physics theory (within the frames of the effective mass method) is referred in the literature, see, for instance, [8] to be the deconfinement, in order to distinguish it from the narrow meaning of the Anderson mechanism of a spin-charge separation about elementary excitations. It seems to be appropriate to accentuate once again, that the TLL description is argued to be universal. The universality of the TLL description means in its turn, that the physical properties do not depend on details of the model, the interaction potential, and so on, but instead they are only characterized by a few parameters - critical exponents. Moreover, the TLL concept is believed to be true for arbitrary statistical properties of the particles, that is, both for fermions and bosons. It provided a paradigm for non-Fermi liquid physics. It seems to be appropriate to remark, that it is argued in many works, that the single-wall carbon nanotubes (SWCNTs), considered to be 1D objects (it is not always correct, especially for standard NTs with diameter in several nanometers) can be described the only within the

frames of the TLL concept. Moreover, SWCNTs are considered to be the best model systems of the TLL state demonstration. The given viewpoint does not have, however, reliable experimental grounds. Really, power-law behaviour was observed experimentally by measuring the tunneling conductance of SWNTs in dependence on temperature and voltage. Electron force microscopic measurements showed also the ballistic nature of transport in conducting SWNTs. At the same time, the most prominent feature of the model - the spin-charge separation by the spinon-holon mechanism - has not been observed so far. Moreover, the power-law behaviour of the conductance of SWNTs in dependence on temperature and voltage and the ballistic nature of the carriers' transport are rather universal physical properties. They can be obtained within the frameworks of the other theories. For instance, the authors of [9] have shown that 40-nanometre-wide graphene nanoribbons epitaxially grown on silicon carbide are single-channel room-temperature ballistic conductors on a length scale greater than ten micrometres, which is similar to the performance of metallic carbon nanotubes. However, the authors do not connect the given 2D ballistic transport with the Luttinger liquid, they compare the results obtained with theoretical predictions for perfect grapheme [10], based on the quite other model (see further). The authors of [11] have represented a unified analytic model for ballistic and quasi-ballistic silicon nanowires. Starting from the classical flux method and using the Lundstrom/Natori approaches, they enhanced them by taking into account a carrier degeneracy and original modelling of a short channel effect and drain induced barrier lowering. The model has been validated by a comparison with numerical simulations. The authors implemented the model in a Verilog-A environment and applied it to simulate simple circuit elements to evaluate potential performance of a silicon nanowire architecture. The next example is the work of [12], where the dynamical implications of the proof that for a quantum particle in a random potential on a regular tree graph absolutely continuous spectrum occurs non-perturbatively through rare fluctuation-enabled resonances is discussed. They have shown that absolutely continuous spectrum implies ballistic transport for quantum particles in a random potential on tree graphs.

Consequently, it can be concluded, that the existing viewpoint of the TLL concept applicability for SWNTs seems to be insufficiently experimentally confirmed.

## II. SPIN-CHARGE SEPARATION - BRIEF REVIEW

Concerned with the other 1D systems, the spinon-holon mechanism of the spin-charge separation seems not to be proved experimentally either. Only a few experiments have attempted to detect the spin-charge separation in (quasi)-1D systems directly [13-17]. The angle resolved photoemission spectroscopy (ARPES) was used with the given aim. It seems to be appropriate to analyze the explanation of the results in the work [17], where the attempt is undertaken to prove experimentally the existence of the spinon-holon spin-charge separation quite recently. Angle resolved photoemission spectroscopy studies in quasi-1D compound $Li_{0.9}Mo_6O_{17}$ have shown that observed lineshapes are asymmetric. The authors ascribe the asymmetry observed to the manifestation of the spin-charge separation by the holon-spinon mechanism. However, they have observed the only one-peaked spectrum. At the same time the two-line spectrum is predicted by a realization of the given mechanism [18]. The prediction has been obtained in [18] for a radiospectroscopy range, however, qualitatively, by ARPES measurements the same picture has to be observed. The authors of [17] believe, that there is the only dispersing holon peak and the spinon edge instead of a peak. In the given suggestion, the observed lineshape can be partly fitted by the ratio of holon $v_c$ to spinon $v_s$ velocities, being to be equal 2. It seems to be appropriate to remark, that the propagation of the electrically charged quasiparticles with the velocity, strongly exceeding the velocity of electrically neutal spin 1/2 quasiparticles contradicts the results, obtained for many quasi-1D systems, for instance, for *trans*-polyacetylene (t-PA) [19-21], carbynoids [22], quasi-1D NTs [23, 24]. It has been shown in above cited works [19-24], in which the spin-charge separation effect is surely experimentally established, that electrically neutal spin 1/2 quasiparticles are very mobile, whereas, charged spinless quasiparticles can at all be pinned [21]. Admittedly, the spin-charge separation effect has the quite another origin in the given case (see the next Section). The much more simple explanation for an ARPES lineshape observed in [17] can be proposed. It can be, for instance, the realisation of the optical analogue of the Dyson effect [25]. The second very possible explanation of the line asymmetry observed consists in the appearance of a distribution of hot photoelectrons in their kinetic energy by moving to the surface. It is seen from the results, represented in [17], that the asymmetry observed corresponds well to the presence of the energy distribution of the particles in Fermi gas, (or quasiparticles in Fermi liquid). It seems to be appropriate to remark, that quite similar asymmetric lineshapes of the lines were observed by an optical absorption on excitons both on free excitons [26], and condensed in Fermi liquid [27], in a number of dielectrics and semiconductors and the given result was explained theoretically just by the distribution of moving excitons on their kinetic energy in a very good agreement with experimental lineshapes [26]. Therefore, the connection of the asymmetry of lineshapes in ARPES measurements with a spinon-holon charge separation seems to be not proved in the works above cited.

It has to be also remarked, that both the models TLL and LFL are the models of ideal quantum liquids, moreover, they are oversimplified, since they do not take into account the nonlinearity of the fermion spectrum on the one hand and it is especially critical, that they do not take into account relaxation mechanisms, including the mechanisms, realised by means of electron-phonon interactions, on the other hand. In fact, both the models describe not adequately the real physical processes (like to an image in a distorting mirror). Actually, the changes in a charge state of an arbitrary atom in a 1D chain in the result of an electron-electron interaction are always accompanied by the changes in a phonon subsystem (and vice versa). It is the consequence of generic coupling between operators of a creation and annihilation in an electron subsystem and in a phonon

field (see for details the next Section). Consequently, the models, which do not take into account the electron-phonon interaction, seem to be strongly oversimplified and very restricted models in their applications. They can never explain the full set of experimental results. The following result seems to be substantial. The development of the 1D TLL model by means of including of an electron-phonon interaction has been done in [28] through the calculation of one-electron spectral functions in terms of the cluster perturbation theory together with an optimized phonon approach. It was found, that the retardation effect, which is the consequence of the finiteness of phonon frequency, suppresses the spin-charge separation by the spinon-holon mechanism and eventually makes it invisible in the spectral function. It is the strong additional indication, that fitting of a spectral function on the base of the TLL theory by authors of [13-17] can be physically incorrect. Quantum critical (QC) scaling in the lineshapes has been also studied in [17]. The authors themselves have been found the departures of the lineshapes and QC scaling from the TLL-theory. The authors of [17] have been found substantial differences relatively to expectations from the one-band TLL-model, specifically in the exponent of the temperature prefactor and in the lack of the full sharpening predicted by TLL-theory for decreasing of the temperature. The authors have phenomenologically included momentum broadening of the TLL spectral function, origin of which remained to be unknown. The discrepancies became smaller, however, they are remained.

So, it is seen that the reliable experimental confirmation of the spinon-holon spin-charge separation mechanism in quasi-1D systems by means of ARPES is in fact absent. The given viewpoint is in the agreement with the opinion of the authors of [29]. They remark, that in these experiments TLL is "both probe and subject, so an independent study - in a different geometry - of the excitation spectrum is vital to be sure of the interpretation". The given remark of the authors of [29] refers also to the works [30-32], in which the authors discuss the evidence for a spinon-holon spin-charge separation in tunneling between two parallel quantum wires at a cleaved edge of a double–quantum-well heterostructure. For instance, in [31], two approaches have been used - one based on mapping out the elementary-excitation dispersions by measuring the conductance $G$ in dependence on the magnetic field $B$ applied perpendicular to the plane connecting the wires and the voltage bias $V$, and the other - focusing on the conductance oscillation pattern, in the $(V, B)$ plane, arising to be the result of the finite length of the tunnel junction. It seems to be appropriate to remark, that the authors of [30-32] and the authors of [29] too were dealing with elements of a 2D systems, since the 1D-1D tunnelling process determines the second direction, being to be transversal to the direction, along which two parallel quantum wires were placed. It means that the TLL model has to be generalized for the 2D case before its numerical application for the explanation of the results obtained. It is in principle correct task. Really, Haldane [33] has presented the generalization of the "bosonization" description, which is the key moment in the TLL model, to be the general treatment of the Fermi surface dynamics in any dimension. Generally from the mathematical viewpoint, the Luttinger liquid behaviour could be observed independently on the dimensionality for the systems, for which the energy at the Fermi surface is not extremal and, consequently, the linear term has to be preserved in its Taylor expansion about the Fermi surface points. However, any experimental results, indicating on Luttinger liquid behaviour in 2D or 3D systems are absent. Moreover, if to restrict the consideration of the task for the systems, for which the energy at the Fermi surface is not extremal with the accuracy to the linear term in the Taylor expansion, then the quasiparticles *will be massless*, since the effective mass is appeared in the quadratic term of the Taylor expansion. It takes really place, however for the task, describing the properties of graphene when to consider only the nearest-neighbour hopping interaction between the atoms, which is for mathematically quite other task (see further for details). Therefore, the interpretation of the results, obtained by tunneling between two parallel quantum wires or on related systems has to take into consideration the given remark. A priori, it can be suggested, taking into account the description of graphene physical properties (see further), that the mathematical treatment for the task considered in [29-32] will follow the theory developed for graphene, that is, the treatment is expected to be different in comparison with the treatment, proposed in [29-32] [Nevetheless, the experimental results, presented in given works, seem to be very interesting].

It seems to be appropriate thereupon to give some details, concerning the description of the physical properties of the graphene. In the pristine graphene, the Fermi level lies just at the touching (crossing) point (the Dirac or charge neutrality point) of $\pi^*$ and $\pi$ bands and graphene has a character of zero-band-gap semiconductor (semimetal). Graphene band structure on some distance from Fermi level in a standard tight-binding approach and by considering the only nearest-neighbour hopping consist of six symmetric Dirac cones with vertices, which produce regular hexagon [10]. Close to a given crossing (touching) point, the electronic bands are nearly linear and practically rotationally symmetric. In other words, the carrier dispersion relation takes a simple form

$$E_{\pi}^{*} = -E_{\pi} \approx v_F \hbar |\vec{k}|, \tag{1}$$

where the momentum $\vec{k}$ is measured with respect to $K(K')$ point. The parameter $v_F$, having the dimension of a velocity, is directly related to the coupling strength (hopping integral) between the nearest carbon atoms: $v_F = \sqrt{3} a_0 \gamma_0 / (2\hbar)$. According to Haldane [33] the linearity of bands in graphene (in the vicinity of the $K$ and $K'$ points) implies that the dispersion relation (1) is the key relation for a TLL-behaviour of an electronic system. Therefore, in the first approximation the electronic system of graphene seemingly has to be considered in the literature to be the 2D-Tomonaga-Luttinger liquid system.

However, the quite other way is used. It is taken into account, that the charge carriers' behaviour in pristine graphene is like to the behaviour of relativistic particles, with the zero rest mass and with the constant velocity $v_F$, equaled to $\approx 10^6 cms^{-1}$. In other words, the charge carriers in pristine graphene are attributed to massless Dirac fermions, and their behaviour is described by the effective Hamiltonian [10].

$$\hat{H} = v_F \begin{bmatrix} 0 & \hat{p}_x - i\hat{p}_y \\ \hat{p}_x + i\hat{p}_y & 0 \end{bmatrix} = v_F \hat{\vec{\sigma}} \hat{\vec{p}}, \tag{2}$$

Which is equivalent to the Hamiltonian in the Weyl equation for real relativistic particles with a zero rest mass (originally for neutrinos) derived from the Dirac equation. Therefore, the formalism of Tomonaga-Luttinger liquid is not used for the description of the physical properties of graphene, despite on the linear band structure of the given system, which from the viewpoint of the Tomonaga-Luttinger liquid concept is the most appropriate candidate for the TLL description. In favour of a foregoing conclusion indicate also the results of studies of the spin-charge separation effect in the spatial dimension $D = 2$ for some specific systems. It has been proposed, in particular, that the key feature underlying the anomalous behaviour of the cuprate high-$T_c$ materials is precisely a separation of spin and charge, and concrete scenarios, based on $Z2$ or $U(1)$ gauge theories, without using of the TLL concept, have been put forward [34]. In the paper [35] the quantum Hall (qH) regime, which is relevant for 2D electrons in strong magnetic fields, is considered. In particular, the separation of spin and charge in the given regime is discussed. Specifically, it was proposed a series of paired spin-singlet qH states, of filling fraction $\nu$ = 2/2m+1. The fundamental excitations over these states proposed to be spinons (with spin 1/2 and zero charge) and holons (with zero spin and fractional charge $\pm 1/2m+1$, in units of the charge of the electron). The statistics of these excitations is non-abelian, and thereby the paired spin-singlet states fall in the category of 'non-abelian qH states'. The prediction of the spin-charge separation with the fractional charges by the spinon-holon mechanism requires, naturally, its experimental confirmation, which, to our knowledge, is also not provided. It is interesting, that the more conventional 'abelian' spin-singlet qH states like to the Halperin states with label (m + 1, m + 1, m) do not predicted even theoretically to be exhibiting a spin-charge separation [35].

Further, the key argument for an insertion of the notion "Luttinger liquid" itself is based in fact also on the oversimplification, connected (it seems to be appropriate to accentuate once again) with the linearization of the generic spectrum of particles in a neighborhood of Fermi points in k-space. Some additional details concerning the given non-Fermi liquid physics paradigm can be given. Tomonaga idea [1], that the low-energy degrees of freedom of a 1D Fermi gas are completely collective, has allowed the development of the "bosonization" technique. At the same time, the conceptual starting point for the bosonization of the Fermi surface is the Luttinger theorem [3], from here arose the term "Luttinger liquid", introduced by Haldane [36]. To "Luttinger liquids" was referred the universality class of 1D quantum liquids, formed by gapless interacting 1D-systems at low energies. It is entirely characterized by the velocity of density waves $v$ and the Luttinger parameter $K$, which depends on the interaction strength. The given concept was based on the following analysis. For low energies, the spectrum $\varepsilon(k)$ of the physical fermions can be linearized around the two Fermi points $\pm k_F$. The interacting fermionic theory can then be mapped onto a theory of noninteracting bosons [37] and all correlation functions can be calculated exactly [36]. However, how it was remarked in [38], even for a linear spectrum, the bosonic or fermionic languages may be used equally comfortably and both offer their particular benefits. The advantage of the former is the direct relation between the bosonic modes and the density response functions. On the other hand, the fermionic description connects to the well-known physics of the Fermi edge problem. Moreover, the authors of the work [38] have shown, that in order to calculate the dynamic response functions in the case of the nonlinearity of the fermion spectrum, it is convenient to translate the bosonic spin and charge modes into fermionic quasiparticles. So, it is seen that if to take into account the only nonlinearity of the generic spectrum of particles in the neighborhood of Fermi points in k-space, then the Fermi liquid description of 1D systems becomes to be more convenient, although it remains distorted without including into consideration the electron-phonon interaction.

When concerning the Landau Fermi liquid theory, it seems to be appropriate to accentuate once again, that just the given simplification, that is, the linearization of the generic spectrum of particles in a neighbourhood of Fermi points in k-space, has led to divergences arising in the perturbation theory in a 1D-case, that is in LFL theory. However, it does not mean, that the 1D Fermi liquid description is incorrect in a general case.

It will be shown that the concept of the adequate description (not distorting the real processes) of 1D correlated electronic systems within the framework of 1D Fermi liquid (FL) can be recovered, at that, the FL concept can be applied just to 1D carbon NTs, that is to very perspective materials in applications in many branches of the modern nanotechnology. It is the aim of the presented work.

It will be considered the concept of 1D FL on the example of the well-known 1D system - *trans*-polyacetylene, that is, it will be in fact the generalization of the well-known model, proposed by Su, Schrieffer, Heeger (SSH-model), which, in distinction from LFL and TLL models, takes into account the electron-phonon interaction (however, it does not take into

consideration the electron-electron correlations in an explicit form). The subsequent generalization, for instance, for quasi-1D carbon zigzag shaped nanotubes (CZSNTs) can be easily obtained by using of the hypercomplex number theory like to the description of quantum optics effects, considered in [24, 39].

It seems to be appropriate to concern now briefly the history of the spin-charge separation effect. The idea of a spin-charge separation was explicitly treated for the first time already in 1974 by Luther and Emery [40] in the context of a continuum limit of the $1D$ electron gas theory. They have shown that the Hamiltonian $\hat{H}_{1DEG}$ of the 1D electron gas can be represented in the form of:

$$\hat{H}_{1DEG} = \hat{H}_c[\phi_c] + \hat{H}_s[\phi_s] + \hat{H}_{irr}[\phi_c, \phi_s], \tag{3}$$

where $\hat{H}_c[\phi_c]$ and $\hat{H}_s[\phi_s]$ are, respectively, the Hamiltonians, which govern the dynamics of the spin field $\phi_s$ and the charge field $\phi_c$ respectively. The Hamiltonian $\hat{H}_{irr}[\phi_c, \phi_s]$ consists of terms that can be neglected in the long wave-length limit. The related model, which describes the spin-charge separation in 1D systems, is the model of the formation of solitons with the fractional fermion number. The general idea belongs to Jackiw and Rebbi [41]. They have drawn attention to the field theories, especially in one spatial dimension, which lead to the soliton formation with the fractional fermion number. However, the concrete realization of the given idea in the condensed matter physics belongs to Su, Schrieffer, and Heeger [19, 20]. The model, proposed by Su, Schrieffer, and Heeger with the spin-charge separation to be the basis phenomenon, is the model of conjugated organic 1D-conductors.

Specifically, what Su, Schrieffer, and Heeger showed, is that, when an electron is added to a neutral *trans*-polyacetylene chain, it can break up into two pieces, one of which carries the electron's charge and the other its spin. The real significance of the SSH-soliton model of t-PA consists in that, that it introduced a new paradigm into the field that, in its turn, was the evidence of the triumph of the model. The triumph of the SSH-model is not occasional. The formulation of the model is very simple from the mathematical viewpoint and the simplicity itself is the great advantage of the model. At the same time it demonstrates the deep physical insight of Su, Schrieffer and Heeger in the field, that was argued in [24, 39, 42], and the consequence of which is the possibility of the extension of the model. So, in [43] was shown, that SSH-model can be extended even on some polymers with $-C-C-$ ordinary bonds, which are not possessing by any $\pi$-poly conjugation.

It seems to be appropriate to remark, that the term, which takes into consideration the static electron-electron correlations is not presented in SSH Hamiltonian in an explicit form, it, in fact, is represented in an implicit form. Really, the static electron-electron interaction can be taken into account in the model by means of its renormalization into an electron-phonon interaction with an effective coupling parameter. It was undertaken in [44, 45]. It is very interesting, that the very similar theoretical result on the possibility to renormalize electron-phonon coupling into an equivalent electron-electron static interaction was realised independently many years later (in 2006) in [28]. It was shown, that, the spin-1/2 Holstein model could be mapped onto the negative-$U$ Hubbard model with an effective dynamical attraction $U_{eff}(\omega)$, the dependence of which on the frequency $\omega$ is given by the relation $U_{eff}(\omega) = \lambda/(1-\omega^2/\omega_0^2)$, where $\lambda$ is the electron-phonon coupling constant in energetic units, $\omega_0$ is the bare phonon frequency. At the same time, although the model, used in [44] is the standard continuum model of an one-dimensional electron gas with short-range (that is screened) electron-electron repulsions, and a nearly half-filled band, however, the very essential simplification has been done in the given model. It consists in the linearization of the one-electron spectrum about the Fermi surface that seems to be the oversimplification. Moreover, the idea itself of a full renormalization of an electron-electron interaction into an electron-phonon interaction with an effective coupling parameter seems to be correct the only partly (see further). It concerns also in the principle the inverse task [28] above cited. The same extended Hubbard model that in [44], was used in [45], in which the exact bare phonon propagator at the renormalized electronic energy scale was obtained and applied. It leads to different physical conclusions concerning the possibility of an observation of crossover - charge density wave (CDW) singlet superconductivity (SS) - in t-PA in comparison with the work [44], where the approximate form of the phonon propagator was used. Voit [45] has concluded that a CDW-SS crossover does not occur in the interacting SSH model in distinction from the opposite conclusion in [44]. It seems to be appropriate to remark, that in both the works above cited the potential energy of electron-electron correlations is considered to be constant relatively the dimerization coordinate by the renormalization procedure, it is just oversimplification, on which was indicated above, since in real physical processes the dimerization coordinate *derivative* of the potential energy of electron-electron correlations seems to be the most essential.

The merit of SSH-model, consisting in the choose of the only dimerization coordinate $u_n$ of the $n$-th $CH$-group, $n = \overline{1, N}$, along the chain molecular-symmetry axis $x$ for the determination of main physical properties of the material and neglecting by the other five degrees of freedom, that is, the degrees of freedom, which are relevant to the bonds with the

directions not coinciding with the chain molecular-symmetry axis direction, seems to be substantial. Given moment was commented in [39] and in [24]. The possibility to neglect by five degrees of freedom is the consequence of the general principle, which was proposed by Slater at the earliest stage of the quantum physics era, already in 1924 [46]. It is – "Any atom may in fact be supposed to communicate with other atoms all the time it is in stationary state, by means of virtual field of radiation, originating from oscillators having the frequencies of possible quantum transitions...". The given idea has obtained its development in [47], where the origin of the virtual field of the radiation was clarified. It was shown, that Coulomb field in 1D-systems or 2D-systems can be quantized, that is, it has the character of the radiation field and it can exist without the sources, which have created the given field. Consequently, the Slater principle can be applied to t-PA. The Coulomb field can be considered in t-PA being to be "virtual" field with a propagation direction the only along a t-PA chain. In other words, it produces the preferential direction in an atom communication the only in one direction (being to be the consequence of the quasi-one-dimensionality), and the given direction remains to be preferential by an interaction with an external EM-field. It explains qualitatively the success of SSH-model in the sense that degrees of freedom, realized by bonds, which are not coinciding with a chain molecular-symmetry axis direction, can really be not taken into consideration for experiments with the participation of an external EM-field and indicating thereby on the deep physical insight of Su, Schrieffer, and Heeger in the field. However, the most merit of SSH-model, which demonstrates the very deep insight of authors in the field, was not commented up to now. In fact, the only given model in the condensed matter physics of dynamic electronic systems takes into consideration in an explicit form generic coupling between operators of a creation and an annihilation of two quantum fields - between the operators of the field corresponding to the electronic subsystem and the operators of the field of the lattice deformation system, that is, the phonon field. The simplest static analogues of taking into account generic coupling between the given two fields are quantum chemistry calculations of the structure of point centres in crystals. It is well known, that by the change of the charge state of any point centre in a crystal lattice, the atomic relaxation of the neighbourhood host lattice atoms has to be taken into account. In a dynamical case it corresponds to a phonon absorption or emission. It seems to be evident, that in SSH-model the operators of a phonon subsystem are represented through operators of an electronic subsystem taking into account given coupling in an explicit form. It seems to be appropriate to remark, that usually given operators are considered independently on each other that can lead to a distortion of the description of real physical processes.

It seems to be interesting, that there are fundamental qualitative differences by the description of spin-charge separation effects in 1D systems between the SSH mechanism and the Anderson mechanism, which is resulting of the TLL theory. The short review above represented indicates, that the applicability of the Anderson mechanism to correlated electronic systems seems to be not experimentally confirmed in distinction from the SSH mechanism. The main difference between both mechanisms consists in the role of phonon effects in the spin-charge separation phenomenon. Let us accentuate once again, that an inclusion of the electron-phonon interaction in the TLL-model suppresses the spinon-holon spin-charge separation effect [28] at all. At the same time, the electron-phonon interaction plays the essential role for the spin-charge separation presence in the SSH-model. It is appropriate to remark, that there is in an existing variant of SSH-model an upper limit on the value of the electron-phonon coupling constant. It is the consequence of the treatment of electron-phonon coupling to be the linear term in an expansion of the only hopping integral of tight-binding model about the undimerized state. The given restriction was discussed in [48] and the maximum for allowed value of the electron-phonon coupling constant $\alpha$ was evaluated to be $\approx 1.27$. It is shown, that the given restriction can be remitted in the generalised variant of SSH-model, based on the quantum Fermi liquid concept. Su, Schrieffer, Heeger [19, 20] describe mathematically the chain of t-PA by considering it to be the Fermi gas in the sense that the electron-electron interaction is not taken into consideration in an explicit form, although the electron-phonon interaction is taken into account. It is seen therefore, that the SSH-model takes, more strictly speaking, the intermediate place between Fermi gas and Fermi liquid quantum models. The main task of our work is the development of the SSH-model within the framework of completely 1D Fermi liquid description, in accordance with the aim above formulated, and to clarify, whether the phenomenon of the spin-charge separation, established in the SSH-model, will be preserved in 1D quantum Fermi liquid model.

## III. RESULTS AND DISCUSSION

Starting Hamiltonian is

$$\hat{\mathcal{H}}(u) = \hat{\mathcal{H}}_0(u) + \hat{\mathcal{H}}_{\pi,t}(u) + \hat{\mathcal{H}}_{\pi,u}(u). \tag{4}$$

Like to works [19], [20], Born-Oppenheimer approximation will be considered. The first term in (4) is the following

$$\hat{\mathcal{H}}_0(u) = \sum_m \sum_s \left(\frac{\hat{P}_m^2}{2M^*}\hat{a}_{m,s}^+\hat{a}_{m,s} + Ku_m^2\hat{a}_{m,s}^+\hat{a}_{m,s}\right). \tag{5}$$

It represents itself the sum of the operator of the kinetic energy of a CH-group motion (the first term in (5)) and the operator of the $\sigma$-bonding energy (the second term). The coefficient $K$ in (5) is the effective $\sigma$-bonds' spring constant, $M^*$ is the total mass of a CH-group, $u_m$ is the configuration coordinate for the $m$-th CH-group, which corresponds to the

translation of the $m$-th CH-group along the symmetry axis $z$ of the chain, $m=\overline{1,N}$, $N$ is a number of CH-groups in the chain, $\hat{P}_m$ is the operator of the impulse, conjugated to the configuration coordinate $u_m$, $m=\overline{1,N}$, $\hat{a}^+_{m,s}$, $\hat{a}_{m,s}$ are creation and annihilation operators of a creation or an annihilation of a quasipartile with the spin projection $s$ on the $m$-th chain site in the $\sigma$-subsystem of t-PA. The second term in (4) can be represented in the form of two components and it is

$$\begin{aligned}&\hat{\mathcal{H}}_{\pi,t}(u)=\hat{\mathcal{H}}_{\pi,t_0}(u)+\hat{\mathcal{H}}_{\pi,\alpha_1}(u)=\\&\sum_m\sum_s[t_0(\hat{c}^+_{m+1,s}\hat{c}_{m,s}+\hat{c}^+_{m,s}\hat{c}_{m+1,s})+\\&(-1)^m 2\alpha_1 u(\hat{c}^+_{m+1,s}\hat{c}_{m,s}+\hat{c}^+_{m,s}\hat{c}_{m+1,s})],\end{aligned} \tag{6}$$

where $\hat{c}^+_{m,s}$, $\hat{c}_{m,s}$ are creation and annihilation operators of a creation or an annihilation of a quasipartile with the spin projection $s$ on the $m$-th chain site in the $\pi$-subsystem of t-PA. It is the resonance interaction (the hopping interaction in the tight-binding model approximation) of quasiparticles in the $\pi$-subsystem of the t-PA electronic system, which is considered to be the Fermi liquid, and in which the only constant and linear terms in the Taylor series expansion of the resonance integral about the dimerized state with coefficients $t_0$ and $\alpha_1$ correspondingly are taking into account. The operator $\hat{\mathcal{H}}(u)$ is invariant under spatial translations with the period $2a$, where $a$ is the projection of spacing between two adjacent CH-groups in the undimerized lattice on the chain axis direction, which is equal to 1.22 A°. It means that all various wave vectors $\vec{k}$ in $\vec{k}$-space will be in the reduced zone with the module of $\vec{k}$ in the range $-\frac{\pi}{2a}\le k\le\frac{\pi}{2a}$ [20]. It can be considered, like to usual semiconductors, to be consisting of two subzones - the conduction $(c)$ band and the valence $(v)$ band. Then it seems to be convenient to represent the operators $\{\hat{c}^+_{m,s}\}$, $\{\hat{c}_{m,s}\}$, $m=\overline{1,N}$, in the form

$$\begin{aligned}\{\hat{c}_{m,s}\}&=\{\hat{c}^{(c)}_{m,s}\}+\{\hat{c}^{(v)}_{m,s}\},\\\{\hat{c}^+_{m,s}\}&=\{\hat{c}^{+(c)}_{m,s}\}+\{\hat{c}^{+(v)}_{m,s}\},\end{aligned} \tag{7}$$

related to $\pi-c$- and $\pi-v$-band correspondingly, and to define $\vec{k}$-space operators

$$\begin{aligned}\{\hat{c}^{(c)}_{k,s}\}&=\{\frac{i}{\sqrt{N}}\sum_m(-1)^{m+1}\exp(-ikma)\hat{c}^{(c)}_{m,s}\},\\\{\hat{c}^{(v)}_{k,s}\}&=\{\frac{1}{\sqrt{N}}\sum_m\exp(-ikma)\hat{c}^{(v)}_{m,s}\},\end{aligned} \tag{8}$$

$m=\overline{1,N}$. The principle, like to MO LCAO is used in fact to build the operators $\{\hat{c}^+_{k,s}\}$ and $\{\hat{c}_{k,s}\}$, consisting in that the antibonding character of $\pi-c$-band orbitals is taken into account by means of factor $i(-1)^{m+1}$. The inverse to (8) transform is

$$\begin{aligned}\{\hat{c}^{(c)}_{m,s}\}&=\{\frac{1}{\sqrt{N}}\sum_k\exp i[m(ka+\pi)-\frac{\pi}{2}]\hat{c}^{(c)}_{k,s}\},\\\{\hat{c}^{(v)}_{m,s}\}&=\{\frac{1}{\sqrt{N}}\sum_k\exp(ikma)\hat{c}^{(v)}_{k,s}\},\end{aligned} \tag{9}$$

$m=\overline{1,N}$. The $\sigma$-operators $\hat{a}^+_{m,s}$ and $\hat{a}_{m,s}$, $m=\overline{1,N}$ can also be represented in the form like to (7) for $\pi$-operators and analogous to (8), transforms can be defined. Then the expression for the operator $\hat{\mathcal{H}}_0(u)$ can be rewritten

$$\hat{\mathcal{H}}_0(u) = \hat{\mathcal{H}}_0^{\sigma,c}(u) + \hat{\mathcal{H}}_0^{\sigma,v}(u) = \sum_m \sum_s (\frac{\hat{P}_m^2}{2M^*} + Ku_m^2) \times \frac{1}{N} \sum_k (\hat{a}_{k,s}^{+\sigma,c} \hat{a}_{k,s}^{\sigma,c} + \hat{a}_{k,s}^{+\sigma,v} \hat{a}_{k,s}^{\sigma,v}), \tag{10}$$

where $\hat{a}_{m,s}^{+\sigma,c}$, $\hat{a}_{m,s}^{\sigma,c}$ and $\hat{a}_{m,s}^{+\sigma,v}$, $\hat{a}_{m,s}^{\sigma,v}$ are $\sigma$-operators of creation and annihilation, related to the $\sigma-c$-band and to the $\sigma-v$-band correspondingly. The independence of $|u_m|$ on $m$, $m=\overline{1,N}$, means, that the expression $(\frac{\hat{P}_m^2}{2M^*} + Ku_m^2)$ is independent on $m$. Then the expression (10) can be rewritten in the form

$$\hat{\mathcal{H}}_0(u) = \sum_k \sum_s (\frac{\hat{P}^2}{2M^*} + Ku^2)(\hat{n}_{k,s}^{a,c} + \hat{n}_{k,s}^{a,v}), \tag{11}$$

where $\hat{n}_{k,s}^{a,c}$ and $\hat{n}_{k,s}^{a,v}$ are operators of a number of $\sigma$-quasiparticles in the $\sigma-c$-band and in the $\sigma-v$-band correspondingly. The expression for $\hat{\mathcal{H}}_{\pi,t_0}(u)$ in terms of $\{\hat{c}_{k,s}^+\}$ and $\{\hat{c}_{k,s}\}$ is coinciding with the known corresponding expression in [19, 20] and it is

$$\hat{\mathcal{H}}_{\pi,t_0}(u) = \sum_k \sum_s 2t_0 \cos ka(\hat{c}_{k,s}^{+(c)} \hat{c}_{k,s}^{(c)} - \hat{c}_{k,s}^{+(v)} \hat{c}_{k,s}^{(v)}). \tag{12}$$

The expression for the second part of operator $\hat{\mathcal{H}}_{\pi,t}(u)$ in terms of $\{\hat{c}_{k,s}^+\}$ and $\{\hat{c}_{k,s}\}$ is also coinciding in its form with the known corresponding expression in [19, 20] and it is given by

$$\hat{\mathcal{H}}_{\pi,\alpha_1}(u) = \sum_k \sum_s 4\alpha_1 u \sin ka(\hat{c}_{k,s}^{+(v)} \hat{c}_{k,s}^{(c)} + \hat{c}_{k,s}^{+(c)} \hat{c}_{k,s}^{(v)}), \tag{13}$$

where subscript $\alpha_1$ in the Hamiltonian designation indicates on taking into account the part of an electron-phonon interaction, connected with resonance interaction (hopping) processes. The expression for the Hamiltonian $\hat{\mathcal{H}}_{\pi,u}(u)$, which describes the part of an electron-phonon interaction, determined by an interaction between quasiparticles in the Fermi liquid state of the $\pi$-subsystem in terms of $\{\hat{c}_{k,s}^+\}$ and $\{\hat{c}_{k,s}\}$ can be represented in the form of

$$\hat{\mathcal{H}}_{\pi,u}(u) = \sum_k \sum_{k'} \sum_s \alpha_2(k,k',s) \hat{c}_{k',s}^{+(c)} \hat{c}_{k',s}^{+(v)} \hat{c}_{k,s}^{(v)} \hat{c}_{k,s}^{(c)}. \tag{14}$$

The constant independent on $u$ static term, which is determined by an electron-electron interaction on different atomic sites in a chain, that is, the constant term in a Taylor series expansion of the potential energy of the electron-electron interaction about the dimerization coordinate was omitted in its explicit form from the Hamiltonian in the given work, in order to establish the role of the phonon assisted part. The independent on $u$ static term is taking, however, into consideration by the calculation of the coefficient $\alpha_2(k,k',s)$ in the phonon assisted term. Physically, the identification of linear on the displacement $u$ parts of both the resonance interaction (hopping) and the pairwise interaction of quasiparticles in the $\pi$-subsystem between themselves with the electron-phonon interaction is understandable, if to take into account, that by atomic $CH$-group displacements the phonons are generated, which in its turn can by a release of the place on, for instance, $(CH)_m$ group, to deliver the energy and the impulse, which are necessary for the transfer of the quasiparticle (electron) from adjacent $(m-1)$-th or $(m+1)$-th position in a chain to $m$- th position in the case of the resonance interaction (hopping). For the case of the pairwise interaction of quasiparticles, it means, that its linear on the displacement $u$ part is realized by means of the phonon field, which transfers the energy and the impulse from one quasiparticle to another (which cannot be inevitable adjacent). It can be proved mathematically in the following way. The processes of the interaction in $c$ and $v$ bands can be considered to be independent on each other. It means, that the transition probability from the $|k_{i,s}\rangle$-state to the $|k_{j,s}\rangle$-state in

$c$-band and from the $\left|k'_{r,s}\right\rangle$-state to the $\left|k'_{n,s}\right\rangle$-state in $v$-band, which is proportional to the coefficient $\alpha_2(k,k',s)$, can be expressed in the form of the product of the real parts of corresponding matrix elements, that is, in the form

$$\alpha_2(k,k',s) \sim Re\left\langle k_{i,s}\left|\hat{V}^{(c)}\right|k_{j,s}\right\rangle Re\left\langle k'_{i,s}\left|\hat{V}^{(v)}\right|k'_{j,s}\right\rangle = \sum_{k_{ph}} Re\left\langle k_{i,s}\left|\hat{V}^{(c)}\right|k_{ph}\right\rangle\left\langle k_{ph}\left|k_{j,s}\right\rangle\times \sum_{k_{ph}} Re\left\langle k'_{r,s}\left|\hat{V}^{(v)}\right|k_{ph}\right\rangle\left\langle k_{ph}\left|k'_{n,s}\right\rangle\right., \quad (15)$$

where $\hat{V}^{(v)} = V_{0(v)}\hat{e}$ ($\hat{e}$ is the unit operator) is the first term in the Taylor expansion of a pairwise interaction of quasiparticles, for instance, with wave vectors $k'_r$, $k'_n$ and the spin projection $s$ in $v$-band, that is, in the ground state, $\hat{V}^{(c)} = V_{1(c)}u\hat{e}$ is the second term in the Taylor expansion of a pairwise interaction in an excited state (in c-band), that is, it is the product of the configuration coordinate $u$ and the coordinate derivative at $u=0$ of the operator of a pairwise interaction of quasiparticles with wave vectors $k_l$, $k_j$ and the spin projection $s$ in $c$-band, $k_{ph}$ is the phonon wave vector, and the summation is realized over all the phonon spectrum. At that, since the linear density of a pairwise interaction is independent on $k$, which is the consequence of the translation invariance of the chain, $V_{0(v)}$, $V_{1(c)}$ are constants. Therefore, a pairwise interaction is considered to be accompanying by a process of a phonon emission, when electronic quasiparticles are already in an excited state, that is, in $c$-band (the retardation effect of a phonon subsystem is taken into account).

A number of variants are possible along with the process of the phonon generation, corresponding to states of electronic quasiparticles in $c$-band above described. The result will mathematically be quite similar, if to change the energetic place of an excitation, that is, if to interchange the role of $c$ and $v$ bands for the given process. There seems to be possible the realization of both the stages (that is a phonon generation and an absorption) for electonic quasiparticles in single $c$ or $v$ band states and a simultaneous realization both the stages in both the bands. For distinctness we will consider the first variant only. For simplicity it will also be considered the processes, in which the spin projection is keeping to be the same. It is evident also, that in the $z$-direction the impulse distribution is quasi-continuous (since the chain has the macroscopic length $L=Na$). Consequently, the standard way $\sum_{k_{ph}} \to \frac{L}{2\pi}\int_{k_{ph}}$ can be used. Further, phonon states can be described by wave functions $\left|k_{ph}\right\rangle = v_0 exp(ik_{ph}z)$, where $z\in[0,L]$, $k_{ph}\in[-\frac{\pi}{2a},\frac{\pi}{2a}]$, $v_0$ is constant. Therefore, from (15) we have the expression

$$\alpha_2(k,k',s) = b\,|v_{0v}|^2|v_{0c}|^2 V_{0(c)}uV_{0(v)}\,|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi(q_l-q_j)(q_r-q_n)} Re\{\exp[i(k_l m_l - k_j m_j)a]\exp(ika)\}\times Re\{\exp[i(k'_r m_r - k'_n m_n)a]\exp(ik'a)\}, \quad (16)$$

where $|\phi_{0cs}|^2$, $|\phi_{0vs}|^2$ are squares of the modules of the wave functions $\left|k_{j,s}\right\rangle$ and $\left|k'_{n,s}\right\rangle$ respectively, $k = k_{ph}(q_l-q_j)$, $k' = k'_{ph}(q_r-q_n)$ $q_l,q_j,q_r,q_n\in N$ with additional conditions $(q_l-q_j)a\le L$, $(q_r-q_n)a\le L$, $b$ - is the aspect ratio, which, in principle, can be determined by a comparison with an experiment. Here the values $(q_l-q_j)$, $(q_r-q_n)$ determine the steps in a pairwise interaction with the phonon participation and they are considered to be fixed. We will consider the processes for which $k=k'$, consequently, $(q_r-q_n) = (q_l-q_j)$. The relation (16) by $k_l m_l = k_j m_j$ and by $k_r m_r = k_n m_n$ transforms into the following expression:

$$\alpha_2(k,k',s)=b\,|\,v_{0v}\,|^2|\,v_{0c}\,|^2\,V_{0(c)}uV_{0(v)}\,|\,\phi_{0cs}\,|^2|\,\phi_{0vs}\,|^2\,\frac{N}{2\pi[(q_l-q_j)]^2}\sin ka\sin k'a. \tag{17}$$

I t is convenient to designate

$$b\,|\,v_{0v}\,|^2|\,v_{0c}\,|^2\,V_{0(c)}V_{0(v)}\,|\,\phi_{0cs}\,|^2|\,\phi_{0vs}\,|^2\,\frac{N}{2\pi[(q_l-q_j)]^2}=4\alpha_2(s) \tag{18}$$

Then, taking into account that the spin projection $s$ is fixed, the dependence on $s$ can be omitted, consequently, $\alpha_2(s)=\alpha_2$. So we have

$$\hat{\mathcal{H}}_{\pi,u}(u)=\sum_k\sum_{k'}\sum_s 4\alpha_2 u\sin ka\sin k'a\hat{c}^{+(c)}_{k',s}\hat{c}^{+(v)}_{k',s}\hat{c}^{(v)}_{k,s}\hat{c}^{(c)}_{k,s}. \tag{19}$$

Something otherwise has to be treated the participation of the phonons in the linear on $u$ part of the pairwise interaction, if the phonon generation is an accompanying process of the quasiparticle transition from the $v$-band into the $c$-band, that is, when the retardation effect of the phonon subsystem can be neglected. It is the case of the strong electron-photon interaction, described in [24, 39, 49]. By the strong electron-photon interaction, the absorption process of photons is a long time process. It is accompanied by a quantum Rabi waves' or Rabi wave packets' formation and by their space propagation, that is, by the formation of the long-lived coherent state of the joint photon-electron system. In the given case, the expression for the density of the electron-phonon coupling parameter $\alpha_2(k,k',s)$, which is related to the part of the electron-phonon interaction, determined by the interaction between quasiparticles in the $\pi$-system Fermi liquid, is the following

$$\begin{aligned}&\alpha_2(k,k',s)\sim Re\left\langle k_{l,s}\,|\hat{V}\,|\,k'_{j,s}\right\rangle=|\,v_{0v}\,|^2|\,v_{0c}\,|^2\,uV_1\,|\,\phi_{0s}\,|^2\times\\&\frac{N^2}{[2\pi]^2}\int_{k_{ph}}\exp[i(k_{ph}qa-k_lm_la)]\{\int_{k'_{ph}}\exp[i(k'_{ph}-k_{ph})q'a]\times\\&\exp[-i(k'_{ph}q'a-k'_jm_na)]dk'_{ph}\}dk_{ph},\end{aligned} \tag{20}$$

where $q=m_j-m_l$, $q'=m_r-m_n$ are integers, satisfying foregoing relations, subscripts in the left part are omitted, since a fixed step is considered. Then, taking into account that in the continuous limit by the integration the modules $k$ and $k'$ of wave vectors $\vec{k}$ and $\vec{k'}$ are running all the $k$- and $k'$-values in $k$- and $k'$-spaces, the following designations $(k_{ph}qa-k_lm_la)=ka$, $(k'_{ph}q'a-k'_jm_ja)=k'a$, where the subscripts are omitted, will be correct. In a result, it is obtained

$$\begin{aligned}&\alpha_2(k,k',s)\sim Re\left\langle k_{l,s}\,|\hat{V}\,|\,k'_{j,s}\right\rangle=|\,v_{0v}\,|^2|\,v_{0c}\,|^2\,uV_1\,|\,\phi_{0s}\,|^2\times\\&\frac{N^2}{[2\pi]^2}(\sin ka\sin k'a+\cos ka\cos k'a).\end{aligned} \tag{21}$$

It was taken into account, that by a $v$-band $\rightarrow$ $c$-band transition of a quasiparticle, the impulse of an emitted phonon by a vibronic system with an electronic quasiparticle in the $v$-band is equal to the impulse of an absorbed phonon by a vibronic system with an electronic quasiparticle in the $c$-band.

The system of operators $\hat{c}^{+(c)}_{k',s}$, $\hat{c}^{+(v)}_{k',s}$, $\hat{c}^{(v)}_{k,s}$, $\hat{c}^{(c)}_{k,s}$ corresponds to non-interacting quasiparticles, and it is understandable, that in the case of interacting quasiparticles their linear combination has to be used

$$\begin{bmatrix}\hat{a}^{(v)}_{k,s}\\ \hat{a}^{(c)}_{k,s}\end{bmatrix}=\begin{bmatrix}\alpha_{k,s} & -\beta_{k,s}\\ \beta_{k,s} & \alpha_{k,s}\end{bmatrix}\begin{bmatrix}\hat{c}^{(v)}_{k,s}\\ \hat{c}^{(c)}_{k,s}\end{bmatrix}, \tag{22}$$

where matrix of transformation coefficients $||\,A||$ is

$$||A||=\begin{bmatrix}\alpha_{k,s} & -\beta_{k,s}\\ \beta_{k,s} & \alpha_{k,s},\end{bmatrix}. \tag{23}$$

It is the unimodular matrix with the determinant $det||A||=\alpha_{k,s}^2+\beta_{k,s}^2=1$. Since $det||A||\neq 0$, it means, that the inverse transformation exists and it is given by the matrix

$$||A||^{-1}=\begin{bmatrix}\alpha_{k,s} & \beta_{k,s}\\ -\beta_{k,s} & \alpha_{k,s}\end{bmatrix}. \tag{24}$$

Then the Hamiltonian $\hat{\mathcal{H}}_{\pi,\alpha_1}(u)$, which corresponds to the SSH one-electron treatment of electron-phonon coupling, can be represented in the form

$$\begin{aligned}\hat{\mathcal{H}}_{\pi,\alpha_1}(u)=\sum_k\sum_s\Delta_k[&\alpha_{k,s}^2\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(c)}-\alpha_{k,s}\beta_{k,s}\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(v)}\\ &+\beta_{k,s}\alpha_{k,s}\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(c)}-\beta_{k,s}^2\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(v)}+\alpha_{k,s}^2\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(v)}\\ &+\alpha_{k,s}\beta_{k,s}\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(c)}-\beta_{k,s}\alpha_{k,s}\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(v)}-\beta_{k,s}^2\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(c)}],\end{aligned} \tag{25}$$

where $\Delta_k=4\alpha_1 u\sin ka$. The diagonal part $\hat{\mathcal{H}}_{\pi,\alpha_1}^{d}(u)$ of the operator $\hat{\mathcal{H}}_{\pi,\alpha_1}(u)$ is

$$\hat{\mathcal{H}}_{\pi,\alpha_1}^{d}(u)=\sum_k\sum_s 2\Delta_k\alpha_{k,s}\beta_{k,s}(\hat{n}_{k,s}^{(c)}-\hat{n}_{k,s}^{(v)}), \tag{26}$$

where $\hat{n}_{k,s}^{(c)}$ is the density of the operator of a quasiparticles' number in the $c$-band, $\hat{n}_{k,s}^{(v)}$ is the density of the operator of a quasiparticles' number in the $v$-band. The part of the pairwise interaction, which is linear in the displacement coordinate $u$ and leads to the participation of the phonons, is given by the Hamiltonian

$$\begin{aligned}\hat{\mathcal{H}}_{\pi,u}(u)=\sum_k\sum_{k'}\sum_s 4\alpha_2 u\sin ka\sin k'a(&\alpha_{k',s}^2\hat{a}_{k',s}^{+(c)}\hat{a}_{k',s}^{(v)}-\beta_{k',s}^2\hat{a}_{k',s}^{(v)}\hat{a}_{k',s}^{+(c)}\\ +\alpha_{k',s}\beta_{k',s}\hat{a}_{k',s}^{(c)}\hat{a}_{k',s}^{+(c)}-\beta_{k',s}\alpha_{k',s}\hat{a}_{k',s}^{(v)}\hat{a}_{k',s}^{+(v)})&(\alpha_{k,s}^2\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(v)}-\beta_{k,s}^2\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(c)}\\ +\alpha_{k,s}\beta_{k,s}\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(c)}-\beta_{k,s}\alpha_{k,s}\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(v)}).\end{aligned} \tag{27}$$

The diagonal part $\hat{\mathcal{H}}_{\pi,u}^{d}(u)$ of the operator $\hat{\mathcal{H}}_{\pi,u}(u)$ is

$$\begin{aligned}\hat{\mathcal{H}}_{\pi,u}^{d}(u)=4\alpha_2 u\sum_k\sum_{k'}\sum_s\alpha_{k'}\beta_{k'}(\hat{n}_{k',s}^{(v)}-\hat{n}_{k',s}^{(c)})\\ \times\alpha_{k,s}\beta_{k,s}(\hat{n}_{k,s}^{(v)}-tn_{k,s}^{(c)})\sin k'a\sin ka.\end{aligned} \tag{28}$$

The Hamiltonian $\hat{\mathcal{H}}_{\pi,t_0}(u)$ in terms of operator variables $\hat{a}_{k,s}^{(c)}$ $\hat{a}_{k,s}^{(v)}$ is

$$\begin{aligned}\hat{\mathcal{H}}_{\pi,t_0}(u)=\sum_k\sum_s 2t_0\cos ka[(\alpha_{k,s}^2-\beta_{k,s}^2)(\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(c)}-\\ \hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(v)})-2\alpha_{k,s}\beta_{k,s}(\hat{a}_{k,s}^{+(v)}\hat{a}_{k,s}^{(c)}+\hat{a}_{k,s}^{+(c)}\hat{a}_{k,s}^{(v)})].\end{aligned} \tag{29}$$

The diagonal part $\hat{\mathcal{H}}^{d}_{\pi,t_0}(u)$ of the operator $\hat{\mathcal{H}}_{\pi,t_0}(u)$ is given by the relation

$$\hat{\mathcal{H}}^{d}_{\pi,t_0}(u)=\sum_k\sum_s \varepsilon_k(\alpha^2_{k,s}-\beta^2_{k,s})(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s}), \tag{30}$$

where $\varepsilon_k = 2t_0\cos ka$. The operator transformation for the $\sigma$-subsystem, analogous to (22) shows, that the Hamiltonian $\hat{\mathcal{H}}_0(u)$ is invariant under the given transformation, that is, it can be represented in the form, given by (11).

To find the values of elements of matrices $||A||$ and $||A||^{-1}$, the Hamiltonian $\hat{\mathcal{H}}(u)$ has to be tested for the conditional extreme on the variables $\alpha_k$, $\beta_k$ with the condition $\alpha^2_{k,s}+\beta^2_{k,s}=1$. The corresponding Lagrange operator function $\hat{E}^L(u)$ is

$$\begin{aligned}\hat{E}^L(u)&=\sum_k\sum_s(\frac{\hat{P}^2}{2M^*}+Ku^2)(\hat{n}^{\sigma,c}_{k,s}+\hat{n}^{\sigma,v}_{k,s})+\sum_k\sum_s\varepsilon_k(\alpha^2_{k,s}-\beta^2_{k,s})(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s})\\&+\sum_k\sum_s 2\Delta_k\alpha_{k,s}\beta_{k,s}(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s})+4\alpha_2 u\sum_k\sum_{k'}\sum_s\alpha_{k',s}\beta_{k',s}(\hat{n}^{(c)}_{k',s}-\hat{n}^{(v)}_{k',s})\alpha_{k,s}\beta_{k,s}\\&\times(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s})\sin k'a\sin ka+\lambda_{k,s}(\alpha^2_{k,s}-1+\beta^2_{k,s}).\end{aligned} \tag{31}$$

Then, the necessary condition for the extremum is determined by Lagrange equations

$$\begin{aligned}\frac{\partial\hat{E}^L(u)}{\partial\alpha_k}&=2\alpha_{k,s}\varepsilon_k(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s})+2\Delta_k\beta_{k,s}(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s})\\&\times[1+\frac{\alpha_2}{\alpha_1}\sum_{k'}\sum_s\alpha_{k',s}\beta_{k',s}\sin k'a(\hat{n}^{(c)}_{k',s}-\hat{n}^{(v)}_{k',s})]+2\lambda_{k,s}\alpha_{k,s}=0,\end{aligned} \tag{32}$$

$$\begin{aligned}\frac{\partial\hat{E}^L(u)}{\partial\beta_{k,s}}&=2\beta_{k,s}\varepsilon_k(\hat{n}^{(v)}_{k,s}-\hat{n}^{(c)}_{k,s})+2\Delta_k\alpha_{k,s}(\hat{n}^{(c)}_{k,s}-\hat{n}^{(v)}_{k,s})\\&\times[1+\frac{\alpha_2}{\alpha_1}\sum_{k'}\sum_s\alpha_{k',s}\beta_{k',s}\sin k'a(\hat{n}^{(c)}_{k',s}-\hat{n}^{(v)}_{k',s})]+2\lambda_{k,s}\beta_{k,s}=0\end{aligned} \tag{33}$$

and

$$\frac{\partial\hat{E}^L(u)}{\partial\lambda_{k,s}}=\alpha^2_{k,s}-1+\beta^2_{k,s}=0. \tag{34}$$

Introducing the designation

$$[1+\frac{\alpha_2}{\alpha_1}\sum_{k'}\sum_s\alpha_{k',s}\beta_{k',s}\sin k'a(atn^{(c)}_{k',s}-\hat{n}^{(v)}_{k',s})]=\hat{Q}, \tag{35}$$

and passing on to observables in the Lagrange equations (32)-(34), it is easily to obtain for $\beta^2_{k,s}$, $\alpha^2_{k,s}$ and for the product $\alpha_{k,s}\beta_{k,s}$ the relations

$$\beta^2_{k,s}=\frac{1}{2}(1\pm\frac{\varepsilon_k}{\sqrt{\varepsilon^2_k+Q^2\Delta^2_k}}), \tag{36}$$

$$\alpha_{k,s}^2 = \frac{1}{2}(1 \mp \frac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}}), \tag{37}$$

$$\alpha_{k,s}\beta_{k,s} = \frac{1}{2}\frac{Q\Delta_k}{\sqrt{n_k^2 + Q^2\Delta_k^2}}, \tag{38}$$

where $Q$ is the eigenvalue of the operator $\hat{Q}$. The equation for the factor $Q$ is

$$[1 + \frac{\alpha_2}{2\alpha_1}\sum_k\sum_s \frac{Q\Delta_k \sin ka}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}}(n_{k,s}^{(c)} - n_{k,s}^{(v)})] = Q, \tag{39}$$

where superscript ' is omitted and $n_{k,s}^{(c)}$ is the eigenvalue of the density operator of a quasiparticles' number in the $c$-band, $n_{k,s}^{(v)}$ is the eigenvalue of the density operator of a quasiparticles' number in the $v$-band. It is evident, that at $Q=1$ in (36) - (38) we will have the case of the SSH-model. The relationship $Q=1$ will be fulfilled, if $\frac{\alpha_2}{\alpha_1}\sum_k\sum_s[\frac{1}{2}\frac{\Delta_k}{\sqrt{\varepsilon_k^2+\Delta_k^2}}\sin ka(n_{k,s}^{(c)} - n_{k,s}^{(v)})] \to 0$, which is realized, in particular, if $\alpha_2 \to 0$. Consequently, it seems to be interesting to consider the opposite case, when $|\frac{\alpha_2}{\alpha_1}\sum_k\sum_s[\frac{1}{2}\frac{\Delta_k}{\sqrt{\varepsilon_k^2+\Delta_k^2}}\sin ka(n_{k,s}^{(c)} - n_{k,s}^{(v)})]| \gg 1.$ Passing on to a continuum limit, in which $\sum_k\sum_s \to 2\frac{Na}{\pi}\int_0^{\frac{\pi}{2a}}$, and assuming $n_{k,s}^{(v)} = 1$, $n_{k,s}^{(c)} = 0$, the relation (39) transforms into the integral equation

$$\frac{2Nua\alpha_2}{\alpha_1\pi t_0}\int_0^{\frac{\pi}{2a}} \frac{\sin^2 ka}{\sqrt{1 - \sin^2 ka[1 - (\frac{2uQ}{t_0})^2]}}dk = 1. \tag{40}$$

In the case $|\frac{2uQ}{t_0}| < 1$ the relation (40) can be rewritten in the form

$$K\left\{\sqrt{1 - \left(\frac{2\alpha_1 uQ}{t_0}\right)^2}\right\} - E\left\{\sqrt{1 - \left(\frac{2\alpha_1 uQ}{t_0}\right)^2}\right\} = \frac{\pi[t_0^2 - (2uQ)^2]}{2Nu\alpha_2}, \tag{41}$$

where $K\left\{\sqrt{1 - \left(\frac{2\alpha_1 uQ}{t_0}\right)^2}\right\}$ and $E\left\{\sqrt{1 - \left(\frac{2\alpha_1 uQ}{t_0}\right)^2}\right\}$ are the complete elliptic integrals of the first and the second kind, respectively. Expanding of given integrals into the series and restricting by the terms of the second-order of smallness leads to the relation

$$Q \approx \frac{t_0}{6u}\sqrt{25 - 32\frac{t_0\alpha_1}{Nu\alpha_2}}. \tag{42}$$

It is evident, that the condition $\left(\frac{2uQ}{t_0}\right) < 1$ is held true by $\frac{1}{3}\sqrt{25 - 32\frac{t_0\alpha_1}{Nu\alpha_2}} < 1$. In the case $|\frac{2uQ}{t_0}| > 1$ the expression (40) can be represented in the form

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2 y}{\sqrt{1-\sin^2 y\left[1-\left(\frac{t_0}{2uQ}\right)^2\right]}}dy = -\frac{\pi Q\alpha_1}{\alpha_2 N}, \tag{43}$$

where $y = \frac{\pi}{2} - ka$. It is equivalent to the equation

$$\left(\frac{t_0}{2uQ}\right)F\left\{\frac{\pi}{2}, \sqrt{1-\left(\frac{t_0}{2uQ}\right)^2}\right\} - E\left\{\frac{\pi}{2}, \sqrt{1-\left(\frac{t_0}{2uQ}\right)^2}\right\} = \frac{\pi Q\alpha_1}{\alpha_2 N}\left[1-\left(\frac{t_0}{2uQ}\right)^2\right], \tag{44}$$

where $F\left\{\frac{\pi}{2}, \sqrt{1-\left(\frac{t_0}{2uQ}\right)^2}\right\}$ is the complete elliptic integral of the first kind. The approximation of elliptic integrals, like to the approximation, given by (42), leads to the expression

$$Q \approx \frac{-3\alpha_2 N}{16}\left[1 \pm \sqrt{1+\frac{80\alpha_1 t_0}{9Nu\alpha_2}}\right]. \tag{45}$$

In the case $\frac{2uQ}{t_0} = 1$ the relation (40) is

$$\int_0^{\frac{\pi}{2}} \cos^2 y dy = -\frac{\pi\alpha_1 Q}{\alpha_2 N}, \tag{46}$$

where $y = \frac{\pi}{2} - ka$. It is seen, that in the given case the value of the parameter $Q$ is calculated exactly and it is

$$Q = \frac{\alpha_2 N}{4\alpha_1} \tag{47}$$

The values of the energy of $\pi$-quasiparticles in the $c$-band $E_k^{(c)}(u)$ and in the $v$-band $E_k^{(v)}(u)$ can be obtained in the following way

$$E_k^{(c)}(u) = \frac{\partial E^L(u)}{\partial n_{k,s}^{(c)}}, E_k^{(v)}(u) = \frac{\partial E^L(u)}{\partial n_{k,s}^{(v)}}, \tag{48}$$

where $E^L(u)$ is the energy of the $\pi$-subsystem of a t-PA chain, which is obtained from the Lagrange operator function (31) by passing on to observables. Then the energy of $\pi$-quasiparticles in the $c$-band $E_k^{(c)}(u)$ and in the $v$-band $E_k^{(v)}(u)$ can be represented by the relations

$$\begin{aligned} &E_k^{(c)}(u) = \varepsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) + 2\Delta_k\alpha_{k,s}\beta_{k,s} + 8\alpha_2 u \sin ka \times \\ &\sum_{k'}\sum_{s}\alpha_{k,s}\beta_{k,s}\alpha_{k',s}\beta_{k',s}(\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)})\sin k'a = \varepsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) + 2\Delta_k\alpha_{k,s}\beta_{k,s}Q \end{aligned} \tag{49}$$

and

$$E_k^{(v)}(u) = -\varepsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) - 2\Delta_k\alpha_{k,s}\beta_{k,s} - 8\alpha_2 u \sin ka \times$$
$$\sum_{k'}\sum_{s}\alpha_{k,s}\beta_{k,s}\alpha_{k',s}\beta_{k',s}(\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)})\sin k'a = -\varepsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) - 2\Delta_k\alpha_{k,s}\beta_{k,s}Q. \quad (50)$$

It is seen from (49) and (50), that $E_k^{(v)}(u) = -E_k^{(c)}(u)$. Taking into account the relations (36) - (38), the following expressions for the energy of $\pi$ -quasiparticles are obtained

$$E_k^{(c)}(u) = \mp\frac{\varepsilon_k^2}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}} + \frac{Q^2\Delta_k^2}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}}, \quad (51)$$

$$E_k^{(v)}(u) = \pm\frac{\varepsilon_k^2}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}} - \frac{Q^2\Delta_k^2}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}}. \quad (52)$$

Therefore, there are two values for the energy of quasiparticles, indicating on the possibility of the formation of the quasiparticles of two kinds. Upper sign in the first terms in (51), (52) corresponds to the quasiparticles with the energy

$$E_k^{(c)}(u) = \frac{Q^2\Delta_k^2 - \varepsilon_k^2}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}},$$
$$E_k^{(v)}(u) = \frac{\varepsilon_k^2 - Q^2\Delta_k^2}{\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2}} \quad (53)$$

in the $C$ -band and in the $V$ -band, respectively. Lower sign in the first terms in (51), (52) corresponds to the quasiparticles with the energy

$$E_k^{(c)}(u) = \sqrt{\varepsilon_k^2 + Q^2\Delta_k^2},$$
$$E_k^{(v)}(u) = -\sqrt{\varepsilon_k^2 + Q^2\Delta_k^2} \quad (54)$$

in $C$ -band and $V$ -band, respectively. The quasiparticles of the second kind at $Q = 1$ are the same quasiparticles that were described in [20].

It was used the only necessary condition for the extremum of the functions $E(\alpha_{k,s}, \beta_{k,s}, \lambda_{k,s})$. It was shown in [24], that for the SSH-model the sufficient conditions for the minimum are substantial, they change the role of both solutions. The sufficient conditions for the minimum of the functions $E(\alpha_{k,s}, \beta_{k,s}, \lambda_{k,s})$ can be obtained by a standard way, which was used in [24]. It consists in that, that the second differential of the energy being to be the function of three variables $\alpha_{k,s}$, $\beta_{k,s}$ and $\lambda_{k,s}$ has to be a positively defined quadratic form. From the condition of a positiveness of three principal minors of quadratic form coefficients the three groups of sufficient conditions for the energy minimum of the set of the functions $E(\alpha_{k,s}, \beta_{k,s}, \lambda_{k,s})$ has been obtained. They are the following:

*A. The First Conditions*

The first conditions are given by the expressions

$$\left\{\varepsilon_k(1-\frac{\varepsilon_k}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}})<\frac{(Q\Delta_k)^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\,|\,(n_{k,s}^c-n_{k,s}^v)<0\right\},$$
$$\left\{\varepsilon_k(1-\frac{\varepsilon_k}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}})>\frac{(Q\Delta_k)^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\,|\,(n_{k,s}^c-n_{k,s}^v)>0\right\} \quad (55)$$

for the second solution which coincides with the SSH-solution at the value $Q=1$ (SSH-like solution) and

$$\left\{\varepsilon_k(1+\frac{\varepsilon_k}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}})<\frac{(Q\Delta_k)^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\,|\,(n_{ks}^c-n_{ks}^v)<0\right\},$$
$$\left\{\varepsilon_k(1+\frac{\varepsilon_k}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}})>\frac{(Q\Delta_k)^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\,|\,(n_{ks}^c-n_{ks}^v)>0\right\} \quad (56)$$

for the first solution. It is seen, that the first conditions are realizable for the quasiparticles of both the kinds, at that for both the near equilibrium state $(n_{ks}^c-n_{ks}^v<0)$ and for the strongly nonequilibrium state $(n_{ks}^c-n_{ks}^v>0)$.

*B. The Second Conditions*

The second conditions are the same for both the solutions and they are

$$\left(\frac{\varepsilon_k^2}{\sqrt{n_k^2+Q^2\Delta_k^2}}-2\frac{(Q\Delta_k)^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\right)^2-\varepsilon_k^2+\frac{1}{4}(Q\Delta_k)^2>0 \quad (57)$$

The second group of conditions is realizable for the quasiparticles of both the kinds.

*C. The Third Conditions*

For the SSH-like solution the third conditions are given by the expressions

$$\left(3\frac{(Q\Delta_k)^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}+4\frac{\varepsilon_k^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\right)(n_{ks}^c-n_{ks}^v)>0. \quad (58)$$

It means, that the SSH-like solution is inapplicable for the description of standard processes, passing near an equilibrium state by any parameters. The quasiparticles, described by the SSH-like solution, can be created the only in the strongly non-equilibrium state with the inverse population of the levels in $c$- and $v$-bands. At the same time, the first solution, which corresponds to upper signs in (51), (52), has to satisfy to the following conditions

$$\left(3\frac{Q^2\Delta_k^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}-4\frac{\varepsilon_k^2}{\sqrt{\varepsilon_k^2+Q^2\Delta_k^2}}\right)(n_{ks}^c-n_{ks}^v)>0, \quad (59)$$

which can be realized both in the near equilibrium and in the strongly non-equilibrium states of the $\pi$-subsystem of t-PA, which is considered to be a quantum Fermi liquid.

The continuum limit for the ground state energy of the $t$-PA chain with SSH-like quasiparticles will coincide with the known solution [20], if to replace $\Delta_k Q\to\Delta_k$. The ground state energy $E_0^{[u]}(u)$ of the $t$-PA chain with the quasiparticles' branch, which is stable near an equilibrium can be then calculated. Taking into account that in the ground state $n_{k,s}^c=0$, $n_{k,s}^v=1$, the corresponding expression in the continuum limit is

$$E_0^{[u]}(u) = -\frac{2Na}{\pi}\int_0^{\frac{\pi}{2a}} \frac{(Q\Delta_k)^2 - \varepsilon_k^2}{\sqrt{(Q\Delta_k)^2 + \varepsilon_k^2}}\,dk + 2NKu^2, \tag{60}$$

which by calculating the integral and using the complete elliptic integral of the first kind $F(\frac{\pi}{2},1-z^2)$ and the complete elliptic integral of the second kind $E(\frac{\pi}{2},1-z^2)$ can be rewritten in the form

$$E_0^{[u]}(u) = \frac{4Nt_0}{\pi}\{F(\frac{\pi}{2},1-z^2) + \frac{1+z^2}{1-z^2}[E(\frac{\pi}{2},1-z^2) - F(\frac{\pi}{2},1-z^2)]\} + 2NKu^2, \tag{61}$$

where $z^2 = \frac{2Q\alpha_1 u}{t_0}$. The approximation of (61) at $z \ll 1$ gives

$$E_0^{[u]}(u) = N\{\frac{4t_0}{\pi} - \frac{6}{\pi}\ln\frac{2t_0}{Q\alpha_1 u}\frac{4(Q\alpha_1)^2u^2}{t_0} + \frac{28(Q\alpha_1)^2u^2}{\pi t_0} + ...\} + 2NKu^2. \tag{62}$$

It is seen from (62), that the energy of quasiparticles, described by the solution, which corresponds to upper signs in (51), (52) has the form of Coleman-Weinberg potential with two minima at the values of the dimerization coordinate, equaled to $u_0$ and $-u_0$ like to the energy of quasiparticles, described by the SSH-solution [20]. It is understandable, that the subsequent consideration, including the electrically neutral S =1/2 soliton formation and the electrically charged spinless soliton formation, that is, the appearance of the phenomenon of the spin-charge separation, by the Fermi liquid description of 1D systems will be coinciding in its mathematical form with the starting SSH-model.

## IV. CONCLUSIONS

Thus, the possibility to describe the physical properties of (quasi)-1D systems within the frames of the (quasi)-1D quantum Fermi liquid including the mechanism of the appearance of the most prominent feature of (quasi)-1D systems - the phenomenon of the spin-charge separation - is proved. Therefore, the consideration of (quasi)-1D systems within the quantum Fermi liquid concept is recovered. It is shown, that all qualitative conclusions of the model proposed in [20] are holding by the quantum Fermi-liquid consideration of the $\pi$ -electronic subsystem of a t-PA chain (instead of a formal Fermi-gas consideration) for the quasiparticles, corresponding to the first-branch-solution. It seems to be substantial, that the Fermi-liquid treatment of an electron-phonon interaction extends the applicability limits of the SSH-model of (quasi)-1D conjugated conductors, allowing its use in the case of the strong electron-phonon interaction. It is achieved by taking into account the contribution in the electron-phonon interaction of the electron-electron correlations in the explicit form [by the second term in the Taylor expansion of an electron-electron pairwise interaction]. The quantum Fermi-liquid model proposed allows therefore to predict (in distinction from SSH-model, in which electron-phonon interaction strength is restricted by the value of the electron-phonon coupling constant $\alpha$ evaluated to $\approx 1.27$) the formation of long-lived both electron-photon and electron-phonon quantum states by the interaction with an external EM-field for the (quasi)-1D systems with the strong electron-photon and electron-phonon interactions. There can be formed coherent, squeezed and entangled states. It seems to be the main practical significance of the theory developed. It can be considered to be the starting base theory for the detailed theoretical description and practical realisations of long-lived both electron-photon and electron-phonon quantum coherent, squeezed and entangled states. [The proof for the applicability of the 1D quantum Fermi liquid model for (quasi)-1D-systems with strong electron-photon interactions by arbitrary electron-electron interactions can be obtained, if the expression (19) to replace by the expression (21) in the subsequent calculations].

It seems to be especially significant, that the (quasi)-1D quantum Fermi liquid model proposed can be easily generalised for the description of the properties of quasi-1D carbon nanotubes, which, according to [24] are characterised by the strong electron-photon interaction. Consequently, they are perspective materials for the nanoelectronics, for the spintronics and for the number of the other practical applications in the sense of practical using of quantum effects above predicted.

It is shown, that the mechanism of the phenomenon of the spin-charge separation in a (quasi)-1D Fermi-liquid is the topological soliton mechanism, analogous to the mechanism proposed by Jackiw and Rebbi. It means, like to the SSH-model, that when an electron is added to a neutral *trans*-polyacetylene chain (or similar (quasi)-1D system), it can break up into two pieces, one of which carries the electron's charge and the other its spin. The given result bears a clear family relation with the phenomenon of the spin-charge separation in the 1D electron gas theory of Luther and Emery [40], but it is quite different from the Anderson spinon-holon mechanism. The results obtained allow to make more accurate and to correct the prevalent viewpoint, that the spin-charge separation effect is an indication on a non-Fermi-liquid behaviour of electronic systems and

that it can be reasonably described within the framework of the Tomonaga-Luttinger liquid theory only. Generally, from the mathematical viewpoint, the Tomonaga-Luttinger liquid behaviour can be observed independently on the dimensionality and the aforesaid viewpoint could only be true for the systems with the function of the electronic energy in $k$-space, which is characterised by the absence of an extremum in the dependence on $k$ (and, consequently, the linear term has to be preserved in its Taylor expansion about the Fermi surface points) and for which the electron-phonon interaction could be neglected [we have to remark, that the existence of the similar systems in the Nature is questionable]. In other words, it is argued, that the model of the Tomonaga-Luttinger liquid in its existing form seems to be not sufficiently correct mapping of real processes, since it does not take into account the electron-phonon interaction, which always takes place by the change of the charge state in any 1D system. Moreover, the inclusion of the electron-phonon interaction term in the TLL Hamiltonian leads to the strong decrease of the display of the main peculiarity of the TLL - the spinon-holon spin charge separation – up to its full absence. Our analysis of experimental works shows, that the presence of the spinon-holon spin-charge separation effect remains to be experimentally unproved.

It will be represented now the most probable explanation of quite recently reported rather significant experimental results, concerning the organic spintronics. It is appropriate to remark, that the organic spintronics is a new direction. Its emerging is based on the long spin-lifetimes in organic materials which are attributed to the low mass value of their atoms. However, before the work [50] the progress was hindered by intrinsic problems associated with low conductivity of organic (polymer) materials. Watanabe and colleagues [50] have used a new route which allows to overcome the resistance mismatch problem. The authors have prepared the ferromagnet/polymer/non-magnetic metal tri-layer samples. Then the authors have provided spin pumping, an effect reciprocal to the well-known current-driven spin torque oscillator. A precession was externally driven by a radiofrequency field, leading to a pure spin current being pumped into the organic film part of the samples. After the spin current has traversed the organic film, within the non-magnetic electrode it was converted into a charge current by the inverse spin Hall effect. Watanabe and co-workers have used PBTTT film, and they ascribe spin transport to spin-1/2 polarons, that is, to quasiparticles, representing themselves localized electrons dressed up by a molecular deformation. The conduction, to authors' opinion occurs to be consequence of polarons' hopping between trapping sites.

Koopmans in its comment [51] to the work [50] writes "the validity of conventional spin diffusion theories, used by Watanabe and colleagues to describe the case of non-doped organic semiconductors in the space-charge limited hopping regime, is far from trivial. Clearly, understanding spin transport is still in its infancy". The authors themselves conclude also, that a more serious microscopic framework has yet to be developed. The polaron spin transport proposed seems to be not very probable, if to take into account the rather great value of a spin diffusion length of approximately 200 nm, and, its practical independence on temperature (in the range between 200 and 300 K of measurements). At the same time, given results can be explained by the model of 1D Fermi liquid, or even within the frames of SSH-model (it depends on the value of electron-phonon coupling constant, which is unknown in PBTTT film). In any case, by an arbitrary value of the electron-phonon coupling constant the 1D Fermi liquid model is applicable. The results aforegoing can be explained by the spin-charge separation effect, accompanying the formation of spin 1/2 topological solitons with zeroth charge. Just the given solitons are formed in *undoped* organic conductors. It explains also the high spin-carrier mobility established and its independence on temperature (in the range above indicated) by very low activation energy of the topological soliton motion. The dimerisation coordinate in PBTTT films seems to be the angle of the 5-atomic ring displacement from the ring plane position. Naturally, more detailed studies are necessary [like to those ones described, for instance, in [22-24] to confirm surely the explanation proposed.

So, the foregoing example shows that the properties (including the possibility of the observation of the phenomenon of the spin-charge separation) of many physical (quasi)-1D systems can be described within the framework of the (quasi)-1D quantum Fermi-liquid proposed. For instance, the model proposed seems to be the working model for a rather wide class of conjugated organic conductors.

The practical significance of the model proposed consists also in the clarification of the nature of charge and spin carriers and in the clarification of the origin of mechanisms of quasiparticles' interactions in (quasi)-1D-materials, that is, it can be the theoretical base for an elaboration of devices of the nanoelectronics, the spintronics and objects of the other nanotechnology branches along with devices working by the participation of external EM-field - the devices of more broad profile.

REFERENCES

[1] Sin-itiro Tomonaga, "Remarks on Bloch's method of sound waves applied to many-fermion problems," *Prog. Theor. Phys.*, vol. 5, iss. 4, pp. 544-569, 1950.

[2] J M Luttinger, "Fermi surface and some simple equilibrium properties of a system of interacting fermions," *Phys. Rev.*, vol. 119, pp. 1153-1163, 1960.

[3] J M Luttinger, "An exactly soluble model of a many-fermion system," *J. Math. Phys.*, vol. 4, pp. 1154-1162, 1963.

[4] P W Anderson, "The resonating valence bond state in La2CuO4 and superconductivity," *Science*, vol. 235, iss. 4, pp. 1196-1198, 1987.

[5] P W Anderson, In: Frontiers and Borderlines in Many Particle Physics, edited by Broglia P A and Schrieffer J.R, North-Holland, Amsterdam, 1987.

[6] P W Anderson and Y Ren, In: High Temperature Superconductivity, edited by K.S. Bedell et al., Addison-Wesley, Redwood City, 1990.

[7] Z Zou and P W Anderson, "Neutral fermion, charge-e boson excitations in the resonating-valence-bond state and superconductivity in La2CuO4-based compounds," *Phys. Rev. B*, vol. 37, pp. 627, 1988.

[8] Weng Z.Y., D.N. Sheng and C.S. Ting, "Nature of spin-charge separation in the t-J model," *Phys. Rev. B*, vol. 61, pp. 12328, 2000.

[9] Jens Baringhaus, Ming Ruan, Frederik Edler, Antonio Tejeda, Muriel Sicot, Amina Taleb-Ibrahimi, An-Ping Li, Zhigang Jiang, Edward H. Conrad Claire, Berger Christoph, Tegenkamp Walt, A. de Heer, "Exceptional ballistic transport in epitaxial graphene nanoribbons," *Nature*, vol. 506, pp. 349-354, 2014, DOI: 10.1038/nature12952.

[10] A H Castro Neto, F Guinea, N M R Peres, K S Novoselov, and A K Geim, "The electronic properties of grapheme," *Rev. Mod. Phys.*, vol. 81, pp. 109-162, 2009.

[11] S. Martinie, D. Munteanu, G. Le Carval, M-A Jaud, J. L. Autran, "Analytical modelling of ballistic and quasi-ballistic nanowires: validation and application to CMOS architecture," Nanotech 2009, vol. 3, Nanotechnology 2009: Biofuels, Renewable Energy, Coatings, Fluidics and Compact Modeling, Chapter 9: Workshop on Compact Modeling, pp. 570-573.

[12] Michael Aizenman and Simone Warzel, "Absolutely continuous spectrum implies ballistic transport for quantum particles in a random potential on tree graphs," *J. Math. Phys.*, vol. 53, pp. 095205, 2012, http://dx.doi.org/10.1063/1.4714617 (15 pages).

[13] C. Kim, A. Y. Matsuura, Z.-X. Shen, N. Motoyama, H. Eisaki, S. Uchida, T. Tohyama, and S. Maekawa, "Observation of spin-charge separation in one-dimensional SrCuO2," *Phys. Rev. Lett.*, vol. 77, pp. 4054-4057, 1996.

[14] P. Segovia, D. Purdie, M. Hengsberger, Y. Baer, "Observation of spin and charge collective modes in one-dimensional metallic chains," *Nature*, vol. 402, pp. 504-507, 1999.

[15] R. Claessen, M. Sing, U. Schwingenschloegl, P. Blaha, M. Dressel, C.S. Jacobsen, "Spectroscopic signatures of spin-charge separation in the quasi-one-dimensional organic conductor TTF-TCNQ," *Phys. Rev. Lett.*, vol. 88, pp. 096402, 2002.

[16] B. J. Kim, H. Koh, E. Rotenberg, S.-J. Oh, H. Eisaki, N. Motoyama, S. Uchida, T. Tohyama, S. Maekawa, Z.-X. Shen & C. Kim, "Distinct spinon and holon dispersions in photoemission spectral functions from one-dimensional SrCuO2," *Nature Physics*, vol. 2, pp. 397-401, 2006.

[17] L Dudy, J D Denlinge, J W Allen, F. Wang, J. He, D. Hitchcock, A. Sekiyama, S. Suga, "Photoemission spectroscopy and the unusually robust one dimensional physics of lithium purple bronze," *Journal of Physics: Condensed Matter*, vol. 25, pp. 014007, 2013.

[18] A De Martino, R Egger, F Murphy-Armando and K Hallberg, "Spin-orbit coupling and electron spin resonance for interacting electrons in carbon nanotubes," *Journal of Physics: Condensed Matter*, vol. 16, pp. 1437-1452, 2004.

[19] W-P Su, J.R. Schrieffer, and A.J. Heeger, "Solitons in polyacetylene," *Phys. Rev. Lett.*, vol. 42, pp. 1698-1701, 1979.

[20] W-P Su, J.R. Schrieffer, and A.J. Heeger, "Soliton excitations in polyacetylene," *Phys. Rev. B*, vol. 22, pp. 2099-2111, 1980.

[21] A.J. Heeger, S. Kivelson, J.R. Schrieffer, W-P Su, "Solitons in conducting polymers," *Rev. Mod. Phys.*, vol. 60, pp. 781-850, 1988.

[22] D.P. Ertchak, Yu.P. Kudryavtsev, M.B. Guseva, A.F. Alexandrov, S.E. Evsyukov, V G Babaev, L.M. Krechko, Yu.A. Koksharov, A.N. Tikhonov, L.A. Blumenfeld, H-J v.Bardeleben, "Electron spin resonance and microwave photoconductivity in carbynoid films," *Journal of Physics: Condensed Matter*, vol. 11, pp. 855-870, 1999.

[23] D.P. Ertchak, V.G. Efimov, V.F. Stelmakh, Review, "ESR-spectroscopy of low dimension structures produced in natural diamonds and synthetic diamond films by ion implantation," *Zh. Prikladn. Spectr.*, vol. 64, iss. 4, pp. 421-449, 1997; *J. Appl. Spectr.*, vol. 64, iss. 4, pp. 433-460, 1997.

[24] D. Yerchuck, A. Dovlatova, "Quantum optics effects in quasi-1D and 2D carbon materials," *J. Phys. Chem. C*, vol. 116, iss. 1, pp. 63-80, 2012, DOI: 10.1021/jp205549b.

[25] F D Dyson, "Electron spin resonance absorption in metals. II. Theory of electron diffusion and the skin effect," *Phys. Rev.*, vol. 98, iss. 2, pp. 349-359, 1955.

[26] E F Gross, Uspekhi Fiz. Nauk, "Excitons and their motion in crystal lattices," *Soviet Physics Uspekhi*, vol. 5, iss. 2, pp. 195-218, 1962, DOI: 10.1070/PU1962v005n02ABEH003407.

[27] Physical Encyclopedia, (Ed. in-Chief Prokhorov A.M), vol. 5, pp. 502-503, M., 1998, 688 pp.

[28] Wen-Qiang Ning, Hui Zhao, Chang-Qin Wu and Hai-Qing Lin, "Phonon effects on spin-charge separation in one dimension," *Phys. Rev. Lett.*, vol. 96, pp. 156402-156405, 2006.

[29] Y Jompol, C J B Ford, J P Griffiths, I Farrer, G A C Jones, D Anderson, D A Ritchie, T W Silk and A J Schofeld, "Probing spin-charge separation in a Tomonaga-Luttinger liquid," *Science*, vol. 325, pp. 597-601, 2009, DOI: 10.1126/science.1171769.

[30] O.M Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, K. W. West, "Tunneling spectroscopy of the elementary excitations in a one-dimensional wire," *Science*, vol. 295, pp. 825-828, 2002.

[31] Yaroslav Tserkovnyak, Bertrand Halperin, Ophir Auslaender and Amir Yacoby, "Signatures of spin-charge separation in double--quantum-wire tunnelling," Proceedings of the NATO Workshop on Theory of Quantum Transport in Metallic and Hybrid Nanostructures, St. Petersburg, August 2003, Kluwer Academic Publishers B.V., Dordrecht, The Netherlands, edited by V. Kozub and V. Vinokur. O.M. Auslaender, H. Steinberg, A. Yacoby, Y. Tserkovnyak, B.I. Halperin, K.W. Baldwin, L.N. Pfeiffer, K.W. West, "Spin-charge separation and localisation in one dimension," *Science*, vol. 308, pp. 88-92, 2005.

[32] Yaroslav Tserkovnyak, Bertrand Halperin, Ophir Auslaender and Amir Yacoby, "Interference and zero-bias anomaly in tunneling between Luttinger-liquid wires," *Phys. Rev. B*, vol. 68, pp.125312-, 2003.

[33] F D M Haldane, "Luttinger's theorem and bosonization of the Fermi surface," Proceedings of the International School of Physics "Enrico Fermi", Course CXXI: "Perspectives in Many-Particle Physics," eds. Broglia R and Schrieffer J.R (North Holland, Amsterdam 1994), pp. 5-30.

[34] T Senthil and M P A Fisher, "Fractionalization and confinement in the U(1) and Z2 gauge theories of strongly correlated systems", *J.*

*Phys. A*, vol. 34, pp. L119-L125, 2001.

[35] E Ardonne, F J M van Lankvelt, A W Ludwig and K Schoutens, "Separation of spin and charge in paired spin-singlet quantum Hall states," *Phys. Rev. B*, vol. 65, pp. 041305, 2002, DOI: 10.1103/PhysRevB.65.041305.

[36] F D M Haldane, "Luttinger liquid theory of one-dimensional quantum fluids. I. Properties of the Luttinger model and their extension to the general 1D interacting spinless Fermi gas," *J. Phys. C: Solid State Phys.*, vol. 14, pp. 2585, 1981, DOI: 10.1088/0022-3719/14/19/010.

[37] D C Mattis and E H Lieb, "Exact solution of a many-fermion system and its associated boson field," *J. Math. Phys.*, vol. 6, pp. 304-312, 1965.

[38] Thomas L.Schmidt, Adilet Imambekov and Leonid I.Glazman, "Spin-charge separation in one-dimensional fermion systems beyond Luttinger liquid theory," *Phys. Rev. B*, vol. 82, pp. 245104, 2010.

[39] A. Dovlatova, D. Yearchuck, "QED model of resonance phenomena in quasi one dimensional multi chain qubit systems," *Chem. Phys. Lett.*, vol. 511, pp. 151-155, 2011.

[40] A Luther, V J Emery, "Backward scattering in the one-dimensional electron gas," *Phys. Rev. Lett.*, vol. 33, pp. 589-592, 1974.

[41] R Jackiw and C Rebbi, "Solitons with fermion number ½," *Phys. Rev. D*, vol. 13, pp. 3398-3409, 1976.

[42] S.A. Kivelson, "Electron fractionalization," *Synthetic Metals*, vol. 125, pp. 99-106, 2002.

[43] D. Yearchuck, Y. Yerchak, A. Alexandrov, "Antiferroelectric spin wave resonance," *Phys. Lett. A.*, vol. 373, pp. 489-495, 2009.

[44] G T Zimanyi, S.A. Kivelson, and A Luther, "Superconductivity from predominantly repulsive interactions in quasi one-dimensional systems," *Phys. Rev. Lett.*, vol. 60, pp. 2089-2092, 1988.

[45] J Voit, "Superconductivity in models of conducting polymers," *Phys. Rev. Lett.*, vol. 64, pp. 323-326, 1990.

[46] J C Slater, Radiation and Atoms, *Nature*, vol. 113, pp. 307-308, 1924.

[47] Alla Dovlatova and Dmitri Yerchuck, "Concept of fully dually symmetric electrodynamics," *Journal of Physics: Conference Series*, vol. 343, pp. 012133, 2012, DOI: 10.1088/1742-6596/343/1/012133.

[48] Rice M J, Philpot S R, Bishop A R, and Campbell D K, "Solitons, polarons, and phonons in the infinite polyyne chain," *Phys. Rev. B*, vol. 34, iss. 6, pp. 4139-4149, 1986.

[49] G.Ya. Slepyan, Y.D. Yerchak, A. Hoffmann, F.G. Bass, "Strong electron-photon coupling in a one-dimensional quantum dot chain: Rabi waves and Rabi wave packets," *Phys. Rev. B*, vol. 81, pp. 085115-085132, 2010.

[50] Watanabe Shun, Ando Kazuya, Kang Keehoon, Mooser Sebastian, Vaynzof Yana, Kurebayashi Hidekazu, Saitoh Eiji, Sirringhaus Henning, "Polaron spin current transport in organic semiconductors," *Nature Phys.*, 2014, http://dx.DOI.org/10.1038/nphys2901.

[51] Bert Koopmans, "Organic spintronics: Pumping spins through polymers," *Nature Phys.*, vol. 10, pp. 249-250, 2014, DOI: 10.1038/nphys2928.