Delamination of Bimaterial and Critical Curves of Quasi-Brittle Fracture in the Presence of Edge Cracks

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***Abstract*-Critical stresses for quasi-brittle bimaterial with edge cracks have been obtained. Cracks along the plane interface of the bimaterial are simulated as bilateral cuts. The critical stresses correspond to necessary and sufficient fracture criteria and those are lower-bound and upper-bound estimates for critical states of a system. The embrittlement effect is revealed by comparison of fracture between bimaterial and homogeneous material. Estimates for effective diameters of grain separation have been obtained. It is noted that T -stress affects critical stresses for very short cracks, so for cracks occupying the most of a specimen width. Critical stresses have been derived above which specimens are broken apart. Subareas on the plot of external load versus crack length are singled out where damage accumulation in material takes place after single loading. Comparison of critical stresses with the subareas mentioned above for homogeneous materials and bimaterials has been performed.**

Keywords- Brittle and Quasi-Brittle Fracture; Structured Materials; Edge Crack; Bimaterial

I. INTRODUCTION

In the general case, there is no unambiguous relation between force, strain, and strain-energy characteristics of crack toughness within the context of nonlinear fracture mechanics (NLFM) [1]. When linear fracture mechanics (LFM) is used in calculating critical stresses of specimens with cracks, the critical stress intensity factor (SIF) K_{Ic} is a unique governing parameter. The equivalence of force and strain-energy fracture criteria has been proved only in LFM. Simplified calculations in the framework of LFM describe well fracture of brittle materials with rather long cracks [1]. When studying fracture of quasi-brittle materials [2-4], some dimensionless parameter characterizing ultimate relative elongations of elastic perfectly-plastic material can be obtained from the classical $\sigma-\varepsilon$ diagrams. This parameter essentially affects the value of critical stresses of specimens with crack-like defects during quasi-brittle fracture.

It is beyond any doubt that for description of fracture in the presence of short cracks, it is necessary to take into account the effect of smooth parts of stress fields in the vicinity of crack tips. In order to consider fracture of solids with cracks within the wide range of variation in relative crack lengths, the effect of T-stresses is taken into consideration [1-4]. However, if analytical representations of SIF may be borrowed from hand-books, then analogous reference literature for constructing T -stresses is not currently available.

In the review of literature [1] and in papers [5, 6], the particular attention is given to the cohesive fracture model (CFM). The review [1] includes about three hundred references to works directly related to the CFM. It is necessary to pay attention on the fact that when making solutions in this model, the Khristianovich hypothesis [7] is used. English-language version of this hypothesis is discussed in [8]. The Khristianovich hypothesis resolves itself into the following: in the classical CFM, the zone length where cohesive forces can be adjusted so that the singular members of stress fields can be fully compensated.

The area of investigations being considered (fracture of quasi-brittle bodies) is intermediate between LMF for brittle bodies and the classical CFM. The comparison of models for brittle, quasi-brittle, and quasi-ductile bodies is given in the form of a table. Shown in Fig. 1 are schemes of loads (upper row), stress fields (middle row), and crack opening (lower row) for brittle (left column a), quasi-brittle (middle column b), and quasi-ductile (right column c) materials. The classical case of CFM [1, 5, 6] considered by Leonov-Panasyuk [9] and Dugdale [10] corresponds to the right column for quasi-ductile materials. In what follows, problems for brittle (a) and quasi-brittle (b) materials are the left and middle columns, respectively. Pay attention to the fact that for quasi-brittle (b) materials [2-4], the force fracture holds at the point 1 marked in the middle row in Fig. 1, and at the point 2 marked in the lower row in Fig. 1, the deformation fracture criterion holds (this condition corresponds to crack tip opening displacement [1]). Thus, the proposed fracture criteria [2-4] are two-parameter. The employed crack tip opening displacement (CTOD) is directly related to the pre-fracture zone width that allows arrangement of plasticity zones in bimaterial and their effect on fracture process to be taken into account [11] (bimaterial is a composite material with the plane interface).

When studying fracture of quasi-brittle bodies [1], it is proposed to use the critical SIF K_{Ic} drawing no distinctions between homogeneous and composite materials. Below are theoretical constructions of fracture curves for specimens with

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edge cracks just as in a homogeneous material, so in bimaterial. It is proposed for the bimaterial that a crack is situated on the plane interface between materials. Delamination in the bimaterial occurs along this interface. The aim of this work is as follows: *i)* to derive critical stresses above which specimens are broken apart; *ii)* to single out subareas on the plot of external load versus crack length where damage accumulation in material takes place after single loading; *iii)* comparison between critical stresses for homogeneous materials and bimaterials; *vi)* comparison between above mentioned subareas for homogeneous materials and bimaterials.

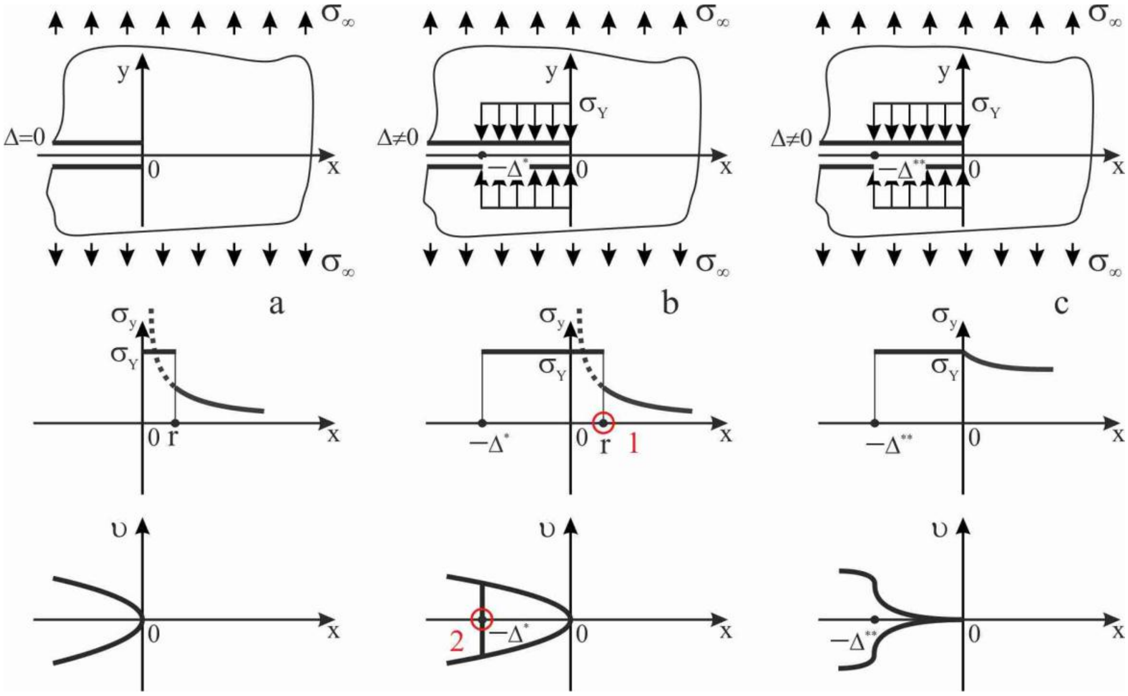

Fig. 1 Loading schemes, stress fields, and crack openings for brittle (a), quasi-brittle (b), and quasi-ductile (c) materials

II. CRITICAL FRACTURE CURVES FOR BRITTLE AND QUASI–BRITTLE HOMOGENEOUS MATERIALS

To derive necessary and sufficient fracture criteria, the Neuber-Novozhilov [12, 13] approach for structured materials is applied. The critical stresses [2-4] have been derived in the context of the modified Leonov-Panasyuk-Dugdale model [9, 10], which involves the width of a pre-fracture zone in addition to its length. Introduction of the supplementary parameter allows failure of material structure located at the tip of a real crack to be assessed. At the moment when the relative elongation of material of this structure situated in the pre-fracture zone reaches its maximum value, the material transforms to critical state [2-4]. The ultimate relative elongation of material is one of parameters of the standard $\sigma-\varepsilon$ diagram of the material. The derived sufficient fracture criteria [2-4] admit the passage to the limit to necessary criteria when the pre-fracture zone length vanishes. The necessary and sufficient criteria describe brittle and quasi-brittle fracture, respectively.

Now consider a flat edge mode I macro crack located perpendicular to the surface when tensile stresses σ_∞ acting normally to the crack plane are specified at infinity. Aside from the straight edge crack-cut of length l_0, let us consider a model crack-cut of length $l = l_0 + \Delta$, the pre-fracture zone Δ of which is situated on the real crack continuation (l and Δ are lengths of model cracks and pre-fracture zones, respectively). The nonlinear problem of stress distribution and displacements of the mode I crack for elastic-plastic material is proposed to be essentially simplified making use of the classical LFM when the I mode model crack is represented as a bilateral cut and the problem nonlinearity is governed only by description of the pre-fracture zone.

Consider behavior of material just as in a plasticity zone, so during nonlinear deformation using a standard $\sigma-\varepsilon$ diagram of material. The real $\sigma-\varepsilon$ diagram of material can be presented as a diagram of perfectly elastic-plastic material which has an ultimate relative elongation of material in the inelastic region. The two-link broken line (curve one) approximates the real $\sigma-\varepsilon$ diagram (curve 2) of a structured material in Fig. 1: σ_Y is the conditional yield point and constant stresses acting in pre-fracture zone; ε_0 and ε_1 are ultimate relative material elongations in elastic and inelastic regions, respectively. The approximation of original $\sigma-\varepsilon$ diagram in Fig. 1 is adjusted so that areas under curves 1 and 2 should be coincident.

When considering mode I internal and edge cracks, sufficient fracture criteria are proposed for use [2-4, 14]

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$$\frac{1}{r}\int_0^r \sigma_y(x,0)dx \le \sigma_Y , \tag{1}$$

$$2\nu(x,0) \le \delta , \; -\Delta \le x < 0 . \tag{2}$$

Here $\sigma_y(x,0)$ are normal stresses on the continuation of model cracks; Oxy is the Cartesian coordinate system with x-axis and y-axis aligned parallel and normal to the crack plane, respectively; r is the linear size characterizing the diameter of a regular breakdown structure [4]; $2\nu = 2\nu(x,0)$ is model crack opening (CTOD); $2\nu^*(-\Delta^*,0) = \delta^*$ are critical model crack openings; superscript "$*$" corresponds to a critical state via the sufficient fracture criterion. In the critical state, relations (1) and (2) change into equalities. Proposed criterion (1)–(2) is sufficient, for $\Delta = 0$, criterion (1)–(2) being transformed into criterion (1), critical states for which are marked with superscript "^{0}". Proposed criteria (1), (2) describe brittle ($\Delta \equiv 0$) and quasi-brittle ($\Delta > 0$) fractures when

$$\Delta^* \ll l_0 . \tag{3}$$

Sufficient two-parameter criterion (1)–(2) includes two conditions, the first of which holds at the point 1 in Fig. 1 and the second one holds at the point 2. If the first equality (1) corresponds to the force fracture criterion, then the second equality (2) is typical deformation criterion for CTOD [1].

In order for sufficient discrete-integral fracture criteria (1), (2) to be used, analytical expressions [2-4, 14] are needed: *i)* for normal stresses $\sigma_y(x,0)$ on the continuation of internal and edge cracks at $x \ge 0$, *ii)* for model crack opening $2\nu = 2\nu(x,0)$ for $x \le 0$, *iii)* for critical opening δ^* of a model crack corresponding to the real crack tip when the pre-fracture zone width coincides with the plasticity zone width at the real crack tip.

The relations for $\sigma_y(x,0)$, $2\nu = 2\nu(x,0)$, δ^* in the case of plane stress for internal cracks are as follows [2-4, 14]

$$\sigma_y(x,0) = K_{\mathrm{I}} / (2\pi x)^{1/2} + \sigma_\infty , \; K_{\mathrm{I}} = K_{\mathrm{I}\infty} + K_{\mathrm{I}\Delta} , \; K_{\mathrm{I}\infty} = \sigma_\infty \sqrt{\pi l} > 0 , \; K_{\mathrm{I}\Delta} < 0 , \tag{4}$$

$$2\nu(-x,0) = \frac{8}{E} K_{\mathrm{I}} \sqrt{\frac{-x}{2\pi}}, \quad K_{\mathrm{I}} > 0, \tag{5}$$

$$\delta^* = (\varepsilon_1 - \varepsilon_0) \frac{5\left(K_{\mathrm{I}\infty}\right)^2}{4\pi\left(\sigma_Y\right)^2} . \tag{6}$$

Here $K_{\mathrm{I}} = K_{\mathrm{I}}(l,\Delta)$ is the total SIF at the model crack tip; $K_{\mathrm{I}\infty}$ is SIF generated by stresses σ_∞, $K_{\mathrm{I}\Delta 1}$ is SIF generated by constant stresses σ_Y acting in pre-fracture zones, E is the modulus of elasticity. The first and second summands in relation (4) are singular and smooth parts of solution, respectively. The second summand σ_∞ in relation (4) is approximation of T-stress. Here the first class of solutions [2-4, 14] is considered, for which $K_{\mathrm{I}} > 0$.

Now fracture of edge cracks is considered in the case when loading conditions are appropriate to the plane strain and plane stress states. When deriving stresses $\sigma_y(x,0)$ on the edge crack continuation $x \ge 0$, three cases (three types of specimens) are under consideration: 1) a half-plane with the edge crack perpendicular to its boundary; 2) axial tension of a plain specimen with single edge crack [15, 16]; 3) axial tension of a plain specimen with two collinear edge cracks [15, 16]. Let the width of the plain specimens be equal to b. Introduce numbers $i = 1, 2$, and 3 for studied types of specimens with edge cracks. Then additional subscripts appear in relations (1) – (6) for early introduced notations: l_i, l_{0i}, Δ_i, $\sigma_{yi}(x,0)$, $K_{\mathrm{I}i} = K_{\mathrm{I}i}(l_i,\Delta_i)$, $K_{\mathrm{I}\infty i}$, $K_{\mathrm{I}\Delta i}$, $\sigma_{\infty i}$, $2\nu_i(x,0)$, δ_i for $i = 1, 2$, and 3. Expression should be constructed in such a way that critical fracture parameters for the second ($i = 2$) and third ($i = 3$) cases be coincident in the limit for $b \to \infty$ with analogous parameters for the first case ($i = 1$). When edge cracks are under consideration, stresses $\sigma_{yi}(x,0)$ on the continuation of cracks, crack openings $2\nu_i = 2\nu_i(x,0)$, and δ_i^* for $i = 1, 2$, and 3 can be represented as

$$\sigma_{yi}(x,0) = K_{\mathrm{I}i} / (2\pi x)^{1/2} + \sigma_{\infty i} , \; K_{\mathrm{I}i} = K_{\mathrm{I}\infty i} + K_{\mathrm{I}\Delta i} , \; K_{\mathrm{I}\infty i} = \sigma_{\infty i} \sqrt{\pi l_i} > 0 , \; K_{\mathrm{I}\Delta i} < 0 , \tag{7}$$

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$$2v_i(-x,0) = \frac{\eta+1}{G} K_{\mathrm{I}i} \sqrt{\frac{-x}{2\pi}}, \quad K_{\mathrm{I}i} > 0, \quad \eta_d = 3-4\mu, \quad \eta_s = \frac{3-\mu}{1+\mu}, \tag{8}$$

$$\delta_{id}^* = (\varepsilon_1 - \varepsilon_0) a_{id}, \; \delta_{is}^* = (\varepsilon_1 - \varepsilon_0) a_{is}, \tag{9}$$

$$a_{id} = \frac{(K_{\mathrm{I}\infty i})^2}{2\pi(\sigma_Y)^2} \left[\frac{3}{2} + (1-2\mu)^2 \right], \; a_{is} = \frac{5(K_{\mathrm{I}\infty i})^2}{4\pi(\sigma_Y)^2}, \; i = 1, 2, 3.$$

Here η_d and η_s are coefficients corresponding to the plane strain and plane stress states, respectively; $G = E/2(1+\mu)$ is the shear modulus; a_{id} and a_{is} are the zone plasticity widths at the real crack tips in the plane strain and plane stress states, respectively. Imaginary crack opening for the plane stress state $2v_s(-x,0)$ nearly always is more than analogous opening for the plane strain state $2v_d(-x,0)$, i.e. $2v_s(-x,0) > 2v_d(-x,0)$ as $\eta_s - \eta_d = 4\mu^2/(1+\mu) > 0$ for $\mu > 0$. Relations (8) and (9) derived for edge cracks differ from relations (5) and (6) for internal cracks. The second summand $\sigma_{\infty i}$ in (7) is the approximation of T -stress.

The smooth part of solution $\sigma_{\infty i}$ from relation (7) can be given as

$$\sigma_{\infty i} = \left[\chi_{0i} + \chi_{1i} \left(1 - \frac{2x}{b-l} \right) \right] \sigma_\infty, \; i = 1,2,3. \tag{10}$$

When calculating coefficients χ_{0i} and χ_{1i} in (10), equilibrium conditions for forces and moments for a band of the finite width is used, i.e., contribution of singular components is disregarded [16]: $\chi_{0i}\sigma_\infty$ are averaged stresses in specimens with account of the presence of crack-cuts; $\chi_{12}\left[1 - 2x/(b-l_2)\right]\sigma_\infty$ are stresses due to bending in a specimen with a single crack $0 \le x \le b - l_2$ when tensile stresses σ_∞ are specified as $y \to \pm\infty$

$$\chi_{01} = 1, \; \chi_{02} = \frac{b}{b-l_2}, \; \chi_{03} = \frac{b}{b-2l_3}, \; \chi_{11} = 0, \; \chi_{12} = \frac{b/l_2}{(b/l_2 - 1)^2}, \; \chi_{13} = 0. \tag{10}$$

Now consider singular parts of solution in relation (7). These singular parts of solution are derived with account of the smooth part of the solution $\sigma_{\infty i}$. Select representations of SIF $K_{\mathrm{I}\infty i}$ generated by stresses $(\chi_{0i} + \chi_{1i})\sigma_\infty$ in the form [15-19].

$$K_{\mathrm{I}\infty 1} = Y_1 \chi_{01} \sigma_\infty \sqrt{\pi l_1}\,; \; K_{\mathrm{I}\infty i} = Y_i (\chi_{0i} + \chi_{1i}) \sigma_\infty \sqrt{\pi l_i}, \quad i = 2,3, \tag{11}$$

$$Y_1 = 1{,}12\,; \; Y_2 = Y_1 + Y_{12}, \; Y_3 = Y_1 + Y_{13}\,; \; \lim_{b\to\infty} Y_{1i} = 0, \; i = 2, 3.$$

Consider the summand $K_{\mathrm{I}\Delta i}$ in relation (7). For the half-plane with an edge crack [18, 19]), SIF $K_{\mathrm{I}\Delta 1}$ generated by constant stresses σ_Y is calculated as follows:

$$K_{\mathrm{I}\Delta 1} = -\sigma_Y \sqrt{\pi l_1} \left[1 + f\left(1 - \frac{\Delta_1}{l_1} \right) \right] \left[1 - \frac{2}{\pi} \arcsin\left(1 - \frac{\Delta_1}{l_1} \right) \right], \; i = 1. \tag{12}$$

Here $f(1-\Delta_1/l_1)$ is some function, the estimate $0 \le f(1-\Delta_1/l_1) \le 0{,}0138$ being applied when $\Delta_1/l_1 \le 0{,}1$ [18, 19]. Neglect the value of this secondary function $f(1-\Delta_1/l_1)$ at quasi-brittle approximation $\Delta_1/l_{01} << 1$ (3). When quasi-brittle fracture ($\Delta_1/l_1 \ll 1$) is considered within the accuracy of the higher order of infinitesimals for the summand $\arcsin(1-\Delta_1/l_1)$ in relation (12), then

$$\arcsin(1-\Delta_1/l_1) \simeq \pi/2 - \sqrt{2\Delta_1/l_1}, \; \Delta_1/l_{01} << 1. \tag{13}$$

Relations (12) and (13) are assumed to be used for the second and third cases when a crack length is essentially less than a specimen width, more exactly, $l_{0i} < b/4$ for $i = 2$ and 3. It is obvious that for $b \to \infty$, relations (10)-(13) for $i = 2$ and 3 coincide with corresponding relations for $i = 1$.

Thus, analytical expressions for normal stresses $\sigma_{yi}(x,0)$ on the continuation of edge cracks $x \ge 0$, for model crack opening $2v_i = 2v_i(x,0)$ for $x \le 0$, and for critical crack opening δ_i^* in (7)-(12) have been derived. Emphasize that all the

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values $\sigma_{yi}(x,0)$, $2v_i = 2v_i(x,0)$, and δ_i^* ($i=1,2,3$) for edge cracks in (7)-(12) differ from analogous values in relations (4)-(6) for internal cracks in plates of the infinite width. These distinctions are related to the effect of finite specimen sizes [17] and occurring bending in specimens of the second kind [16].

For deriving critical values $\sigma_{\infty i}^*$ and Δ_i^* in sufficient criterion (1) and (2), equality signs are used. When transformations are performed in equalities (1) and (2) with account of relations (7)–(12), members with factors $\sqrt{\Delta_i / l_i}$ are kept and members with factors $\Delta_i / l_i \ll 1$ are suppressed in accordance with strong inequality (3). Finally, for three cases $i=1,2$, and 3 considered here, analytical expressions for critical stresses $\sigma_{\infty id}^*$, $\sigma_{\infty is}^*$ and critical pre-fracture zone lengths Δ_{id}^*, Δ_{is}^* of quasi-brittle materials can be written as:

for the plane strain state

$$\frac{\sigma_{\infty id}^*}{\sigma_Y} = \left[\left(1 - \frac{3+2(1-2\mu)^2}{8\pi(1-\mu^2)} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right) Y_i \sqrt{\frac{2l_{id}^*}{r_d}} + (\chi_{0i} + \chi_{1i})\right]^{-1}, \; 2l_{id}^* = 2l_{0i} + 2\Delta_{id}^*, \tag{14}$$

$$\frac{\Delta_{id}^*}{l_{id}^*} = \frac{\left[3+2(1-2\mu)^2\right]^2}{2^9(1-\mu^2)^2} \left(\frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right)^2 \left(\frac{\sigma_{\infty id}^*}{\sigma_Y}\right)^2, \; i = 1, 2, 3, \tag{15}$$

for the plane stress state

$$\frac{\sigma_{\infty is}^*}{\sigma_Y} = \left[\left(1 - \frac{5}{8\pi} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right) Y_i \sqrt{\frac{2l_{is}^*}{r_s}} + (\chi_{0i} + \chi_{1i})\right]^{-1}, \quad 2l_{is}^* = 2l_{0i} + 2\Delta_{is}^*, \tag{16}$$

$$\frac{\Delta_{is}^*}{l_{is}^*} = \frac{5^2}{2^9} \left(\frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right)^2 \left(\frac{\sigma_{\infty is}^*}{\sigma_Y}\right)^2, \; i = 1, 2, 3. \tag{17}$$

Here $l_{id}^* = l_{0id} + \Delta_{id}^*$ or $l_{is}^* = l_{0is} + \Delta_{is}^*$ are critical model macro crack lengths in the plane strain and plane stress states, r_d or r_d is diameter of a regular structure induced at fracture load [4] in the plane strain and plane stress states. Expressions (14), (15) and (16), (17) make sense if

$$1 - \frac{3+2(1-2\mu)^2}{8\pi(1-\mu^2)} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0} > 0, \quad i = 1, 2, 3, \tag{18}$$

$$1 - \frac{5}{8\pi} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0} > 0, \quad i = 1, 2, 3. \tag{19}$$

Inequalities (18) and (19) are restrictions for the the plane strain and plane stress states, respectively. These restrictions are valid only for brittle and quasi-brittle materials of the type of ceramics and high-strength metal alloys [4].

It is obvious a passage to the limit for relations (14), (15) and (16), (17) as $\varepsilon_1 / \varepsilon_0 \to 1$. That allows consideration of fracture of brittle materials (pre-fracture zone for brittle materials is absent $\Delta^0 \equiv 0$). In this case, critical stresses $\sigma_{\infty i}^0$ are calculated via necessary criterion (1) as follows (now the tensile strength for brittle material is meant by stress σ_Y):

$$\frac{\sigma_{\infty i}^0}{\sigma_Y} = \left(Y_i \sqrt{\frac{2l_{0i}}{r_0}} + (\chi_{0i} + \chi_{1i})\right)^{-1}, \quad i = 1, 2, 3. \tag{20}$$

Relation (20) holds for any plane problem, r_0 is the diameter of a regular structure induced at fracture of brittle material [4]. The effect of T-stress on critical stresses $\sigma_{\infty id}^*$, $\sigma_{\infty is}^*$, and $\sigma_{\infty i}^0$ in relations (14), (16), and (20) is reflected by the members in a small round parenthesis $(\chi_{0i} + \chi_{1i})$. Dimensionless critical stresses $\sigma_{\infty i}^0 / \sigma_Y$ in (20) obtained via necessary

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criterion (1) depend on two parameters: *i*) dimensionless length of the initial crack $2l_{0i}/r_0$ and *ii*) specimen width b_i. The dimensionless critical stresses $\sigma^*_{\infty di}/\sigma_Y$ or $\sigma^*_{\infty si}/\sigma_Y$ in relations (14) or (16) corresponding to sufficient fracture criterion (1) and (2), depend on three parameters: *i)* dimensionless crack length $2l^*_{id}/r_d$ or $2l^*_{is}/r_s$ with $2l^*_{id}/r_0 > 2l_{0i}/r_0$ or $2l^*_{is}/r_0 > 2l_{0i}/r_0$, *ii)* width of specimen b_i, and *iii)* relation $(\varepsilon_1-\varepsilon_0)/\varepsilon_0$ characterizing material quasi-brittleness (besides, there is of Poisson's ratio μ dependence for the plane strain state). Thus, relations (20) and (14) or (16) describe brittle and quasi-brittle fracture of specimens, respectively.

In the modified Leonov-Panasyuk-Dugdale model, the two-sheeted solution is considered: over the whole plane with a bilateral cut, the solution corresponding LFM is defined, and the solution corresponding NLFM is applied only in pre-fracture zones (rectangles with sides Δ^*_i and δ^*_i). In the pre-fracture zone, tension of a fiber bundle may be considered as only tensile stresses take place on the real crack mode I continuation. The thickness of these fibers is equal to diameters r_0, r_d, and r_s of regular breakdown structures [4].

III. ESTIMATES OF EFFECTIVE DIAMETERS OF BREAKDOWN STRUCTURES

All speculations about dependence of critical stresses $\sigma^0_{\infty i}/\sigma_Y$, $\sigma^*_{\infty di}/\sigma_Y$, and $\sigma^*_{\infty si}/\sigma_Y$ in relations (14), (16) and (20) on dimensionless parameters rest on the fact that dimensional approximation parameters of $\sigma-\varepsilon$ diagrams for brittle and quasi-brittle materials, as well as effective diameters of breakdown structures r_0, r_d, and r_s are known. Explanations to estimates of dimensional approximation parameters of the $\sigma-\varepsilon$ diagrams are given in Fig. 2. Pass on to deriving estimates of effective diameters of structures induced at fracture. These estimates are of crucial importance when the Neuber-Novozhilov approach [12, 13] is used for structured materials by virtue of the fact that sufficient fracture criterion (1) and (2) include averaging over some specific linear size. In LFM and NLFM [1], experimental procedures of the critical SIF K_{Ic} determination are worked out well. Assume that the critical SIF K_{Ic} is obtained experimentally for a quasi-brittle material. Specify the procedure of derivation of estimates for effective diameters of breakdown structures based on above mentioned theoretical developments for internal cracks.

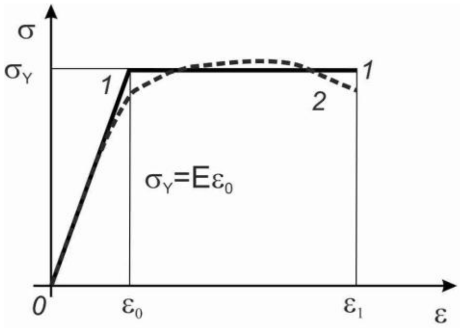

Fig. 2 Original $\sigma-\varepsilon$ diagram of a real elastic-plastic material (curve 2) and its two-link approximation (curve 1)

For the sake of simplicity, an internal crack is considered. Make use of relations (4)-(6) for the plane stress state and analogous relations for the plane strain state. After appropriate transformations, analytical expressions for critical stresses σ^*_∞ and critical pre-fracture zone lengths Δ^* for quasi-brittle materials (sufficient fracture criterion) and critical stresses σ^0_∞ for brittle materials via the necessary criterion have the form

$$\frac{\sigma^*_{\infty d}}{\sigma_Y}=\left[1+\left(1-\frac{3+2(1-2\mu)^2}{8\pi\left(1-\mu^2\right)}\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right)\sqrt{\frac{2l^*_d}{r_d}}\right]^{-1},\ \frac{\sigma^0_\infty}{\sigma_Y}=\left[1+\sqrt{\frac{2l_0}{r_0}}\right]^{-1},\ 2l^*_d=2l_0+2\Delta^*_d,$$

$$\frac{\Delta^*_d}{l^*_d}=\frac{\left[3+2(1-2\mu)^2\right]^2}{2^9\left(1-\mu^2\right)^2}\left(\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right)^2\left(\frac{\sigma^*_{\infty d}}{\sigma_Y}\right)^2. \tag{21}$$

Relations (21) describe the critical state of an infinite plane with a crack-cut for the plane strain state when $\mu\neq 0$ and for the plane strain state when $\mu=0$.

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Figs. 3a and 3b demonstrate diagrams of quasi-brittle fracture in log-log coordinates within the wide range of variation in relative crack length $2l/r$: curve 1 corresponds to critical stresses via the necessary σ_∞^0/σ_Y fracture criterion when $\varepsilon_1-\varepsilon_0=0$ in relation (21) and curves 2 and 3 correspond to the sufficient $\sigma_{\infty d}^*/\sigma_Y$ and $\sigma_{\infty s}^*/\sigma_Y$ fracture criteria from relation (21) when $(\varepsilon_1-\varepsilon_0)/\varepsilon_0=2.5$ for $\mu=0,3$. It has been obtained by calculations $\sigma_\infty^0/\sigma_Y<\sigma_{\infty d}^*/\sigma_Y<\sigma_{\infty s}^*/\sigma_Y$ when an assumption is made that $\sigma-\varepsilon$ diagrams of a real elastoplastic material are independent of the kind of a stress state. The critical curve σ_∞^0/σ_Y is a unified curve for the arbitrary type of a stress state, more exactly, that is approximation of this curve. Inasmuch as $\sigma_{\infty d}^*/\sigma_Y<\sigma_{\infty s}^*/\sigma_Y$ ($0<\mu<0,5$), then some material embrittlement takes place in passing from the plane stress state to the plane strain state. The critical curves $\sigma_\infty^0/\sigma_Y, \sigma_{\infty d}^*/\sigma_Y$, and $\sigma_{\infty s}^*/\sigma_Y$ in the log-log coordinates in Fig. 3b most closely resemble straight lines. This fact may be used in processing results of full-scale experiments.

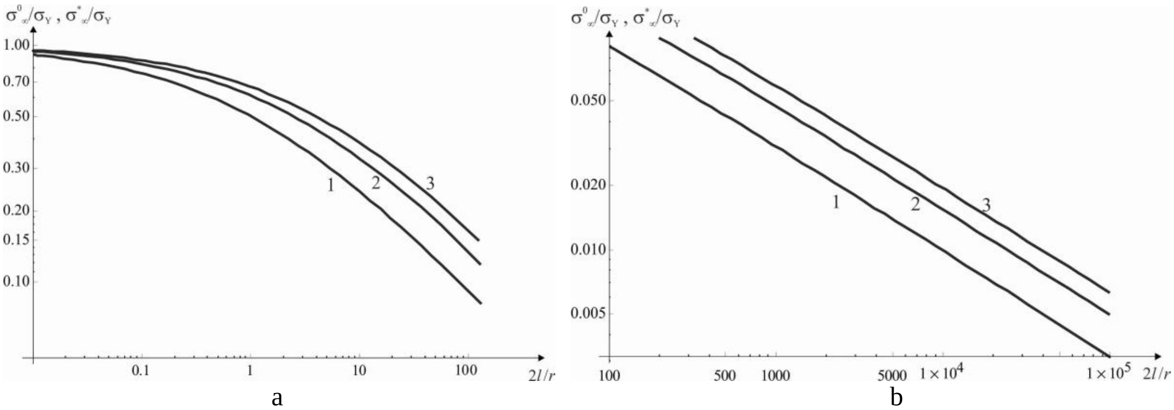

Fig. 3 Diagrams of quasi-brittle fracture for the plane strain and plane stress states

In the area between curves σ_∞^0/σ_Y and $\sigma_{\infty d}^*/\sigma_Y$ or σ_∞^0/σ_Y and $\sigma_{\infty s}^*/\sigma_Y$, damage accumulation occurs under repeated loading conditions. The pairs of curves 1 and 2 or 1 and 3 are diagrams of quasi-brittle fracture in the plane strain and plane stress states, respectively, the critical stresses σ_∞^0/σ_Y being a lower-bound estimate and the critical stresses $\sigma_{\infty d}^*/\sigma_Y$ and $\sigma_{\infty s}^*/\sigma_Y$ being an upper-bound estimate for appropriate problems.

Then the critical SIFs are considered. The common viewpoint [1] is as follows: the critical SIF of a material is a material constant. In the foregoing, critical stresses via the necessary σ_∞^0/σ_Y and sufficient $\sigma_{\infty d}^*/\sigma_Y, \sigma_{\infty s}^*/\sigma_Y$ fracture criteria have been derived. Consider the critical SIF's $K_{\mathrm{I}\infty}^0$, $K_{\mathrm{I}\infty d}^*$, and $K_{\mathrm{I}\infty s}^*$ for plane states being studied (critical values of SIFs derived via necessary and sufficient fracture criteria are marked with superscripts "^{0}" and "*" and subscripts "d" and "s" correspond to the plane strain and plane stress states)

$$K_{\mathrm{I}\infty}^0=\sigma_\infty^0\sqrt{\pi l_0},\quad K_{\mathrm{I}\infty d}^*=\sigma_{\infty d}^*\sqrt{\pi l_d^*},\quad K_{\mathrm{I}\infty s}^*=\sigma_{\infty s}^*\sqrt{\pi l_s^*}\,. \tag{22}$$

As it is obvious from relations (22), the critical SIF $K_{\mathrm{I}\infty}^0$, when constants of a brittle material r_0,σ_Y are known, and critical SIFs $K_{\mathrm{I}\infty d}^*$ or $K_{\mathrm{I}\infty s}^*$ for a particular quasi-brittle material with known $r_d,\sigma_Y,(\varepsilon_1-\varepsilon_0)/\varepsilon_0,\mu$ or $r_s,\sigma_Y,(\varepsilon_1-\varepsilon_0)/\varepsilon_0$ are not, in general case, constants since these SIFs depend on crack lengths apart from material constants mentioned above.

The limited values of critical SIFs $K_{\mathrm{I}\infty}^0$ and $K_{\mathrm{I}\infty d}^*, K_{\mathrm{I}\infty s}^*$ for infinitely long cracks have the form for brittle material

$$\lim_{l_0\to\infty}K_{\mathrm{I}\infty}^0=\sqrt{\pi/2}\,\sigma_Y\sqrt{r_0}\,, \tag{23}$$

for quasi-brittle materials

$$\lim_{l_d^*\to\infty}K_{\mathrm{I}\infty d}^*=\sqrt{\pi/2}\,\sigma_Y\sqrt{r_d}\left[1-\frac{3+2(1-2\mu)^2}{8\pi(1-\mu^2)}\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right],\ \lim_{l_s^*\to\infty}K_{\mathrm{I}\infty s}^*=\sqrt{\pi/2}\,\sigma_Y\sqrt{r_s}\left(1-\frac{5}{8\pi}\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0}\right). \tag{24}$$

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The limited values of SIFs $K_{I\infty}^{0}$, $K_{I\infty d}^{*}$, and $K_{I\infty s}^{*}$ are independent of crack lengths. Derived relations (23) and (24) allow estimation of effective diameter r of breakdown structures: *i*) for brittle material if elastic strength σ_Y and critical SIF $K_{Ic} = K_{I\infty}^{0}$ are known, *ii*) for quasi-brittle material if the conditional yield strength σ_Y, the parameter $(\varepsilon_1 - \varepsilon_0)/\varepsilon_0$, Poisson's ratio μ, and critical SIF $K_{Ic} = K_{I\infty d}^{*}$ or $K_{Ic} = K_{I\infty s}^{*}$ are known. As a rule, values of the critical SIF K_{Ic} for quasi-brittle material are given without specifying what state is implemented during experiment: the plane strain state or the plane stress one. Let experimental values of the critical SIF K_{Ic} have been obtained for rather long cracks, and then the following relations are valid for the effective diameter r of breakdown structures for brittle and quasi-brittle materials

$$r_0 = \frac{2}{\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2, \; r_d = \frac{2}{\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2 \left[1 - \frac{3+2(1-2\mu)^2}{8\pi(1-\mu^2)} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right]^{-2}, \; r_s = \frac{2}{\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2 \left(1 - \frac{5}{8\pi}\frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right)^{-2}, \tag{25}$$

In calculating effective diameters of breakdown structures (25), asymptotic representations of the SIF for long cracks are applied.

The following materials fall into the class of quasi-brittle materials (see restrictions (18) and (19)): *i)* steels, when tests are conducted at temperatures below the fracture appearance transition temperature, *ii)* some high-strength aluminum alloys, many casting alloys, and *ii)* all rocks and high-strength ceramics [4].

IV. DELAMINATION OF QUASI-BRITTLE BIMATERIALS

In what follows, delamination of quasi-brittle composite materials will be considered. Let an internal crack of the finite length $2l_0$ or an edge crack of the length l_0 (Fig. 4 a, b) is in the composite piecewise-homogeneous plane or half-plane as a right line of the interface between two media. Normal tensile stresses σ_∞ specified at infinity act normally to the crack plane, i.e., mode I fracture is implemented. Let each material have its own regular structure, the parameter r_j being a specific linear size of the structure of j-th material $j = 1, 2$. The approximate equalities are assumed to satisfy

$$E_1 \approx E_2, \; \mu_1 \approx \mu_2, \; r_1 \approx r_2, \tag{26}$$

where E_1, μ_1 and E_2, μ_2 are Young's modulus and Poisson's ratio of materials 1 and 2, respectively.

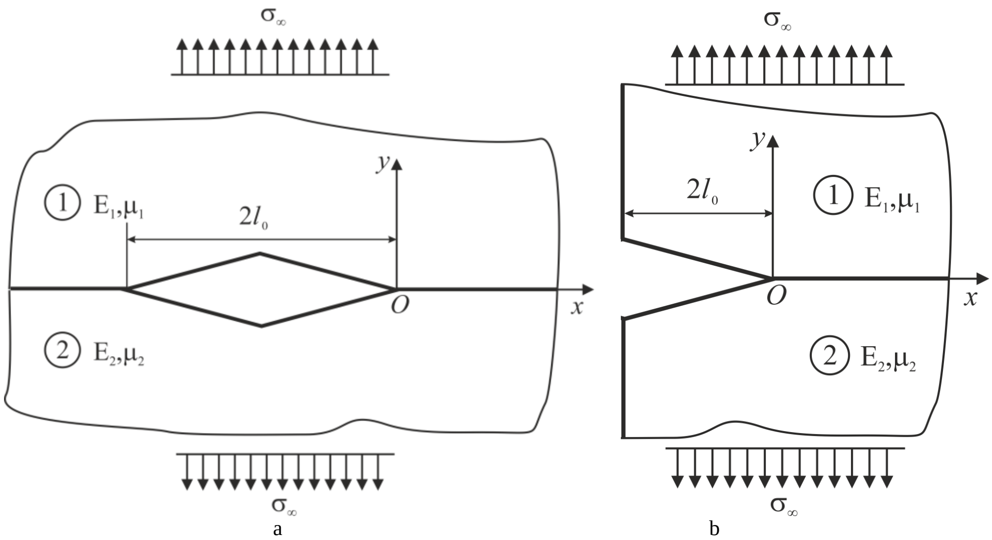

Fig. 4 Crack between two media: a) internal crack, b) edge crack

The first two restrictions from relations (26) are of fundamental importance since in the general case for

$$E_1 \neq E_2, \; \mu_1 \neq \mu_2. \tag{27}$$

Significant disagreement is observed. The point of such disagreement is described in detail in hand-books [18, 19]: in the vicinity of crack tips, the sign of stresses changes infinitely often and crack flanks overlap. Thus the derived solution turns to be senseless [18, 19] for the vicinity of crack tips.

Remark. The following is some plausible reasoning that allows one to understand where this disagreement arises from.

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When deriving analytical relations for the SIF $K_{\mathrm{I}}, K_{\mathrm{II}}$ for composite bodies, the hypothesis is used that variables in the plane problem in the elasticity theory are separable. Consider a related problem of tension of a composite half-plane such as in Fig. 4b, but in the absence of an edge crack. If conditions (27) are satisfied, a boundary value problem is stated for the equation of elliptic type. For the linear equation of elliptic type, the principle of maximum is valid. Then boundary-layer solutions are derived for the vicinity of an angular point in the interface between two materials. In the presence of components of the type of boundary-layer in the smooth part of solution, variables in this solution are not separated. Now consider a problem for composite bodies in the presence of a crack on the interface. In this case, the solution includes smooth and singular parts. As it was mentioned above, the smooth part of the solution contains, in the general case, boundary-layer for composite bodies. Therefore, the assumption that singular parts of solutions for homogeneous and composite bodies look out identically is no more than a hypothesis.

Turn back to study of delamination of quasi-brittle materials when restrictions (26) are valid. Let materials of the upper and lower half-planes in Fig. 4a or those in the upper and lower quarters of the plane in Fig. 4b differ only in conditional yield points

$$\sigma_{Y1} < \sigma_{Y2}, \tag{28}$$

i.e., initial sections of $\sigma_j - \varepsilon_j$ diagrams in Fig. 2 are nearly coincident for different materials $j = 1, 2$, all the difference is related to mismatch of conditional points. Stresses on the interface may not exceed the conditional yield point of the first material σ_{Y1}.

The disagreement between yield points $\sigma_{Y1} \neq \sigma_{Y2}$ results in profound rearrangement of plasticity zones (pre-fracture zones) in the vicinity of a crack tip [11] located along the interface. The excellent illustration of plasticity zones and possible mechanisms affecting energy dissipating in homogeneous materials under repeated loadings are given in Figs. 2 and 3 in [20]. It may be well to point out the unsuccessful interpretation of energy dissipating micro mechanisms in a composite material in Fig. 2 from [21] when plasticity zones are arranged simultaneously in metal and ceramics. Fig. 5 presents the plasticity zone located in the vicinity of a crack tip (emphasize that only single loading is considered). In accordance with inequality (28), this plasticity zone is only in the first material, $a^{\otimes}$ being the plasticity zone (pre-fracture zone) width. If inequalities (28) are valid, the pre-fracture zone width $a^{\otimes}$ for bimaterial is two times narrower than the pre-fracture zone width a, i.e., $2a^{\otimes} = a$.

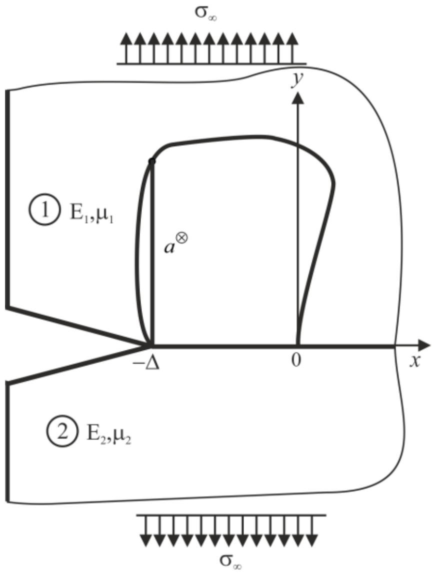

Fig. 5 Pre-fracture zone (plasticity zone) in bimaterial

The delamination problem of composite quasi-brittle materials is described by sufficient fracture criterion (1) and (2). However, when this criterion is implemented, some refinements should be introduced. There are no changes in relation (1): averaging takes place within the interval $0 \le x \le r$ (compare Figs. 1 and 5). Relation (2) such as $2v^{*}(-\Delta^{*}, 0) = \delta^{**}$ is the critical crack opening of model cracks (δ^{**} is critical crack opening for bimaterial). Emphasize that left-hand and right-hand sides of this equation are derived taking into account the fact that a composite material is considered in the vicinity of a crack tip: *i)* for the left-hand side $2v^{*}(-\Delta^{*}, 0)$, relations (5) or (8) hold inasmuch as approximate equalities (26) hold, *ii)* for the right-hand sides, critical crack openings for bimaterial δ^{**} have the form

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$$\delta_d^{**} = (\varepsilon_1 - \varepsilon_0) a_d^{\otimes},\ \delta_s^{**} = (\varepsilon_1 - \varepsilon_0) a_s^{\otimes},\ \delta_{id}^{**} = (\varepsilon_1 - \varepsilon_0) a_{id}^{\otimes},\ \delta_{is}^{**} = (\varepsilon_1 - \varepsilon_0) a_{is}^{\otimes}, \tag{29}$$

$$a_d^{\otimes} = \frac{(K_{\mathrm{I}\infty})^2}{\pi(\sigma_Y)^2}\left[\frac{3}{2} + (1-2\mu)^2\right],\ a_s^{\otimes} = \frac{5(K_{\mathrm{I}\infty})^2}{2\pi(\sigma_Y)^2};\ a_{id}^{\otimes} = \frac{(K_{\mathrm{I}\infty i})^2}{\pi(\sigma_Y)^2}\left[\frac{3}{2} + (1-2\mu)^2\right],\ a_{is}^{\otimes} = \frac{5(K_{\mathrm{I}\infty i})^2}{2\pi(\sigma_Y)^2},\ i = 1, 2, 3.$$

Here $a_d^{\otimes}, a_s^{\otimes}$ and $a_{id}^{\otimes}, a_{is}^{\otimes}$ are the widths of plasticity zones at real crack tips in the plane strain (subscript d), plane stress states (subscript s) for internal and edge cracks, respectively (three kinds of specimens $i = 1, 2, 3$). The distinction in relation (29) for the pre-fracture zone in a bimaterial from relations (6) and (9) for a homogeneous material is related to the fact that the pre-fracture zone width identified with that of the plasticity zone is less for the bimaterial. Inasmuch as stresses on the interface cannot exceed the conditional yield point of the first material σ_{Y1} in accordance with relation (28), the plasticity zone in the bimaterial cannot be in the second material.

After appropriate transformations, analytical expressions for critical stresses $\sigma_{\infty d}^{**}$ and $\sigma_{\infty id}^{**}$ are derived, which describe delamination of bimaterial when specimens with internal and edge cracks are tested in the case when testing is performed under conditions of the plane strain state

$$\frac{\sigma_{\infty d}^{**}}{\sigma_{Y1}} = \left[\left(1 - \frac{3 + 2(1-2\mu)^2}{4\pi(1-\mu^2)} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right)\sqrt{\frac{2l_d^*}{r_d}} + 1\right]^{-1}. \tag{30}$$

$$\frac{\sigma_{\infty id}^{**}}{\sigma_{Y1}} = \left[\left(1 - \frac{3 + 2(1-2\mu)^2}{4\pi(1-\mu^2)} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0}\right) Y_i \sqrt{\frac{2l_{id}^*}{r_d}} + (\chi_{0i} + \chi_{1i})\right]^{-1}. \tag{31}$$

The following restriction is valid for a biomaterial

$$1 - \frac{3 + 2(1-2\mu)^2}{4\pi(1-\mu^2)} \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0} > 0. \tag{32}$$

This restriction (32) does not depend on whether internal crack or edge one is considered. The obtained restriction (32) is not such strict as compared with a homogeneous material (compare relations (32) and (18)).

The critical stresses $\sigma_{\infty s}^{**}$ and $\sigma_{\infty is}^{**}$ for bimeterial under conditions of the plane stress state in problems of internal and edge cracks can be derived using relations (30) and (31), if $\mu = 0$ is set in these relations. Effect of T -stress on the critical stresses $\sigma_{\infty d}^{**}$ and $\sigma_{\infty id}^{**}$ for bimaterial in relations (30), (31) reflects the last members in square brackets.

In relations (30) and (31), there are no analytical expressions for the critical stresses σ_{∞}^{0} and $\sigma_{\infty i}^{0}$ derived via necessary fracture criterion (1) for a homogeneous material and bimaterial inasmuch as these critical stresses have already been given in relations (21) and (20). Recall that a pre-fracture zone does not exists at brittle fracture.

Compare the critical stresses for homogeneous materials $\sigma_{\infty d}^{*}$, $\sigma_{\infty s}^{*}$ and bimaterials $\sigma_{\infty id}^{**}$, $\sigma_{\infty is}^{**}$ when $\sigma_Y = \sigma_{Y1}$ and the rest parameters coincide in relations (14), (16), (21), (30) and (31). After obvious correlations, these critical stresses are expressible in the form

$$\sigma_{\infty d}^{*} > \sigma_{\infty id}^{**},\ \sigma_{\infty s}^{*} > \sigma_{\infty is}^{**}. \tag{33}$$

Embrittlement of bimaterial takes place as compared with a homogeneous material: a structure involving a bimaterial with a crack fails earlier than that containing of a homogeneous material with a crack under other conditions being equal. Pay attention to the fact that the class of bimaterials possessing quasi-brittle fracture is essentially broader than that of homogeneous materials possessing quasi-brittle fracture (compare relation (32) with relations (18) and (19)).

Given in Fig. 7 a, b are diagrams of quasi-brittle fracture of bimaterial of specimens with edge cracks: for plotting curves 1, relations (20) for critical stresses $\sigma_{\infty 2}^{0}$ were used, curves 2 and 3 were plotted via relations (31) for critical stresses $\sigma_{\infty 2d}^{**}$ and $\sigma_{\infty 2s}^{**}$, respectively, for the plane strain and plane stress states ($\mu = 0.4$, $(\varepsilon_1 - \varepsilon_0)/\varepsilon_0 = 2{,}36$, specimens of the second kind). In relation (31), the expression for parameter Y_2 borrowed from hand-book [18] is used. This expression

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$$Y_2 = 1{,}12 - 0{,}23(l_0/b) + 10{,}55(l_0/b)^2 - 21{,}72(l_0/b)^3 + 30{,}39(l_0/b)^4,\ l_0 = l_{2d}^*,\ l_0 = l_{2s}^*.$$

corresponds to the problem of a band with the edge crack shown in Fig. 6 when tension at infinity is applied (Fig. 6 differs from Fig. 3.54 borrowed from [18] only in agreed notations, b is specimen width).

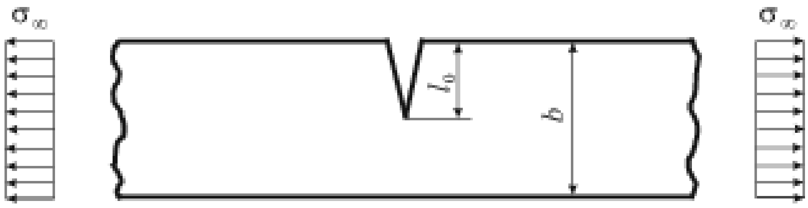

Fig. 6 The band with the edge crack when tension at infinity is applied

Curves 1, 2 and 3 in Fig. 3 a and b for critical stresses $\sigma_{\infty 2}^0$, $\sigma_{\infty 2d}^{**}$, $\sigma_{\infty 2s}^{**}$ in the plot of log-log coordinates almost coincide with straight lines ($2l/r \gg 1$). This fact may be used for processing large-scale experiments. Note that T-stress substantially affects the critical stresses $\sigma_{\infty 2}^0$, $\sigma_{\infty 2d}^{**}$, and $\sigma_{\infty 2s}^{**}$ just as for very short cracks as $l/r \to 0$, so for cracks occupying almost all the specimen width for $l/b \to 1$.

Diagrams of quasi-brittle material in Fig. 3 a and b and bimaterial in Fig. 7 a and b on the plot of external load versus crack length is represented as two pairs of curves 1 – 2 or 1 – 3 for the plane strain and plane stress states, respectively. These pairs of curves divide the first quadrant of the above-mentioned plane into three subareas: *i)* below curve 1 (subarea I), *ii)* above curves 2 or 3 (subarea III), *iii)* between curves 1 –2 or 1 –3 (subarea II). Let a crack of certain length and loading level be assigned, then: *i)* in subarea I, there are no fracture and damage accumulation, *ii)* in subarea III, fracture occurs under single loading (specimen is disaggregated), *iii)* in subarea II, damage accumulation takes place under single loading, but the system is not broken.

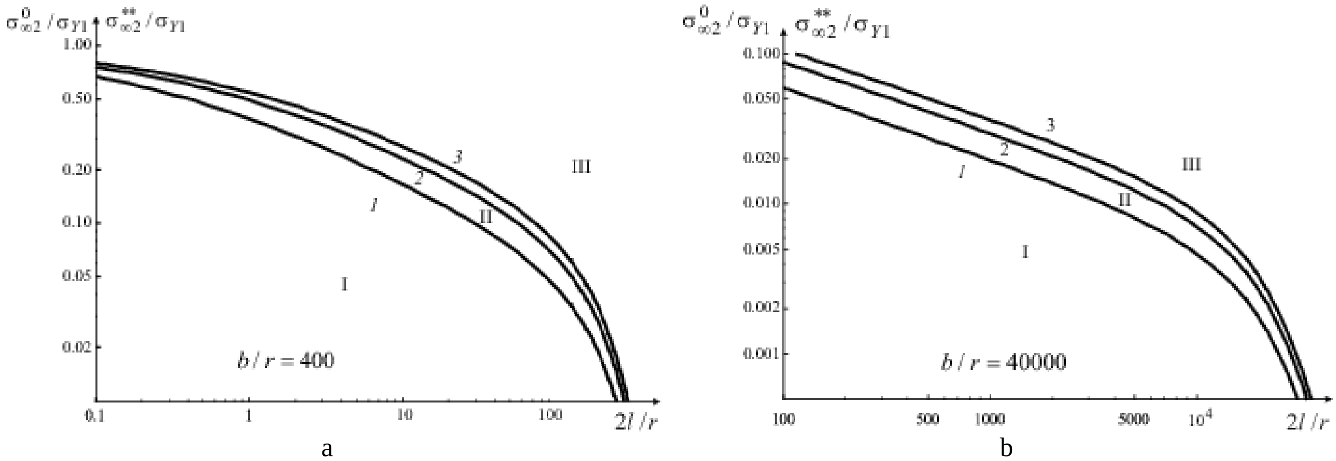

Fig. 7 Diagrams of quasi-brittle fracture of bimaterial: a) specimen width $b/r = 400$, b) specimen width $b/r = 40000$

The mutual arrangement of subareas I, II and III on the plot of external load versus crack length is responsible for behavior of a nonlinear system. Such arrangements of subareas for homogeneous materials and bimeterials are nearly coincident if restriction (26) for bimaterial holds and the lowest conditional yield point of bimaterial σ_{Y1} from relation (28) coincides with the conditional yield point of homogeneous material σ_Y. Inasmuch as the critical stresses for bimaterial are essentially less than similar stresses for homogeneous material ($\sigma_{\infty 2d}^{**} < \sigma_{\infty 2d}^*$, $\sigma_{\infty 2s}^{**} < \sigma_{\infty 2s}^*$), then subarea II for biomaterial is less than the subarea for homogeneous material. Thus, when quasi-brittle bimaterial is broken, embrittlement takes place as compared with the homogeneous material.

Emphasize the significant distinction of the proposed model from the classical CZM [1, 5, 6]: the proposed model takes into consideration the structure of a composite material (bimaterial) in the vicinity of a crack tip. A pre-fracture (plasticity) zone is unilateral when restriction (26) and (28) hold for bimaterial.

V. DISCUSSION

The critical fracture stresses for quasi-brittle bodies derived in an explicit form describe decomposition of a specimen to the parts. Partly, the simulation results were validated in [22], when the elastoplastic problem of extension of a plate with the

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central crack was solved using the finite-element method. In this work, dimensions and shape of the plastic zone near the crack tip for different levels of loads corresponding to quasi-brittle and quasi-ductile fracture were studied. It has been established that critical stresses for a plate with the central crack derived just as analytically, so with the help of numerical modeling are nearly coincident for the parameter $(\varepsilon_1-\varepsilon_0)/\varepsilon_0 \le 3$ in the case of elastic-plastic material.

It would be desirable to validate the simulation results by performance of appropriate full-scale experiments. At first, two preliminary full-scale experiments ought to be done in accordance with the classical $\sigma-\varepsilon$ diagram and the critical SIF K_{Ic} for quasi-brittle material. Recall that it is recommended in review [1] to use only the critical SIF K_{Ic}. According to these full-scale experiments, the parameters $(\varepsilon_1-\varepsilon_0)/\varepsilon_0$, σ_Y, r_d or r_s are calculated. It is advisable for the first parameter that the restriction $(\varepsilon_1-\varepsilon_0)/\varepsilon_0 \le 3$ to be valid (refer to the numerical experiment performed in [22]). According to obtained model parameters $(\varepsilon_1-\varepsilon_0)/\varepsilon_0$, σ_Y , r_d or r_s , critical stresses $\sigma^*_{\infty d}/\sigma_Y, \sigma^*_{\infty S}/\sigma_Y$ for homogeneous materials or $\sigma^{**}_{\infty d}/\sigma_Y, \sigma^{**}_{\infty S}/\sigma_Y$ for bimaterials. At the final stage, full-scale experiments should be carried out in order to determine critical stresses of selected specimens having various lengths l_0 of initial cracks. Next critical stresses derived via theoretical calculations and those obtained in the full-scale experiments are compared.

VI. CONCLUSIONS

Critical stresses of specimens with edge cracks are derived in the explicit form when the specimens are made from quasi-brittle bimaterial. Critical stresses corresponding to the necessary and sufficient fracture criteria are lower and upper bounds for critical states of a system. When plotting quasi-brittle fracture diagrams, the critical SIF K_{Ic} and approximation of the classical $\sigma-\varepsilon$ diagram are taken to be governing characteristics. The embrittlement effect has been revealed when comparing fracture of bimaterial with that of homogeneous material. Estimates of effective diameters for breakdown structures have been derived. The substantial effect of T -stress on critical stresses just as for very short cracks, so for cracks occupying almost all the specimen width has been accentuated.

Applying the plotted fracture diagrams [23-25] provides a way of describing step-wise advance of the tips of fatigue cracks in both homogeneous materials and bimaterials.

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