Combination Prediction of Aviation Fuel Consumption in Dominance Matrix Method

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***Abstract*-Aviation fuel consumption is the major component of costs for air transport enterprises. The paper chooses the aviation fuel consumption volume as the research object and grey prediction, linear regression prediction and time series prediction as the individual prediction methods. Based on dominance matrix method, a combination prediction model is set up in the paper to forecast the aviation fuel consumption volume with better prediction results. It provides necessary ground for aviation fuel order and storage.**

Keywords- Aviation Fuel Consumption Volume; Combination Prediction; Dominance Matrix Method

I. INTRODUCTION

In the coming few years, Asia will continue to drive the rapid development of the international aviation business. Statistics show the growth of China's aerial passenger and cargo transportation accounts for one third of global growth and China will become Asia's busiest aviation market. Aviation fuel consumption is a major cost for air transport enterprises. It not only covers a large proportion of overall costs but it remains costly itself. Effective prediction of aviation fuel consumption has an important significance for the study of aviation transportation costs. Currently, many individual prediction methods have been applied to the study of aviation fuel consumption. Due to differences in the environmental requirements for each individual prediction, the prediction is far from accurate. Therefore combination prediction method based on individual predictions becomes the focus of this research. Its essence is to give different weight to the prediction result of individual competing model and produce a single prediction. With less errors and higher accuracy, combination prediction method provides more reliable theoretical support for policy-makers [1]. Scientific determination of weight is key to combination prediction. Based on the three individual prediction results of grey prediction model, linear regression prediction and time series prediction, the paper determines the weight of combination prediction in dominance matrix method to forecast aviation fuel consumption so as to provide the necessary basis for decision-making on aviation fuel order and storage.

II. MODEL OVERVIEW

A. Basic Models of Combination Prediction

Various methods have been applied to quantitative prediction of development tendency of objects and each method is built on certain assumptions. Nevertheless, any single assumption can not accurately describe the complexity of the real world. With different principles, each prediction method provides useful information from different perspectives. If a single prediction method is applied, an appropriate selection of which will be particularly important. If several individual prediction methods are rationally combined and information provided by these methods is synthesized, the combined prediction results will be insensitive to any poor single prediction method and will be accurate and reliable [2].

Linear combination prediction is based on a weighted linear combination of certain individual prediction results. Let m sorts of individual prediction models be used to predict, the combination prediction model composed of these models will be

$$\hat{y}_t = \sum_{i=1}^{m} w_i \hat{y}_{it} \ , \quad \text{and} \quad \sum_{i=1}^{m} w_i = 1, w_i \geq 0, i = 1,2,\cdots,m \tag{1}$$

Where $\hat{y}_t$ is the prediction value of combination prediction model at t period of time, $\hat{y}_{it}$ is the prediction value of *ith* prediction model at t period of time; w_i is the weight of *ith* prediction model. Depending on the methods of determining the weight w_i, combination prediction generally contains equal weight combination prediction, optimal combination prediction and variable weight combination prediction. Equal weight combination prediction does not require the accuracy of individual prediction values and doesn't obtain the co-relationship between individual prediction errors, either. It treats each individual prediction equally. Optimal combination prediction determines the weight of each prediction method based on the principle that errors of combination prediction are the smallest at the past phase of time. In both equal weight combination prediction and optimal combination prediction, weight is constant while weight varies with conditions and environments in individual prediction model. As the performance of individual prediction model is unstable, the use of variable weight method is more scientific.

B. Dominance Matrix Method

Dominance matrix method is a steady method for determining weight proposed by Gupta and Wilton in 1987 [3]. Let there be m sorts of individual prediction models. For any two competing models *i* and *j*, *i, j*=1,2,…, m, let z_{ij} represent the times the prediction results of model i are better than those of model j in the valid data set and set up m order dominance matrix $Z=(z_{ij})$, where $z_{ii}=0$, $i=1,2,\cdots,m$. So the weight of *No.i* individual prediction model is

$$w_i=\frac{\sum_{j=1}^{m} z_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} z_{ij}} \qquad i=1,2,\cdots,m \tag{2}$$

The basic principle of determining weight by dominant matrix method is that good prediction method should be given greater weight. The indices of evaluating prediction effect mainly include absolute error, mean absolute error, mean absolute percentage error, mean square error, prediction error sum of squares, etc. Eq. (2) is based on time series and is the weight of dominant frequency of the ith sort prediction method. So it can reflect the effect of each prediction in general.

Determining weight by the use of dominance matrix has the following advantages: firstly, weight is insensitive to changes in dominance ratio, eliminating the need for substantial prior data; secondly, with constant change of obtained data, weight can be updated simultaneously; thirdly, the simple operation makes it realistic to predict aviation consumption with improved accuracy [1, 4].

III. CONSTRUCT INDIVIDUAL PREDICTION MODELS

Grey system prediction model adopts grey system analysis method and discriminate the similarity or dissimilarity of development tendency between elements within a system. Continuous differential equations are established based on discrete data. Grey system prediction model can capture the inherent rules of objects and is apt to be applied to medium-and-long term project prediction. Regression prediction model adopts regression analysis method to find out the causal relationship between the subject being predicted and the factors influencing the subject and establishes a series of regression prediction models. Time series prediction model is used to arrange the historical data of study subject to time-series in the order of time and analyze its developing tendency over time. Time series prediction is used to predict the future value of the study subject.

A. Grey System Prediction Model

Grey system prediction avoids the complicated correlation in a system and focuses on the grey information in the system itself. By processing the raw data and establishing grey model, grey system prediction detects the development rules of system and make scientific quantitative prediction for the future state of system to whiten grey system. Grey prediction is based on grey prediction model GM (1,1) [5, 8].

Let the original sequence be $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$, the accumulated generating sequence can be written as,

$$X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\}$$

Set up GM (1,1) albino equation: $\frac{dx^{(1)}}{dt}+ax^{(1)}=b$, based on least squares estimation, the parameter list $\hat{a}=(a,b)^T$ meets

$$\hat{a}=(a,b)^T=(B^TB)^{-1}B^TY \tag{3}$$

Where

$$Y=(x^{(0)}(2),x^{(0)}(3),\cdots,x^{(0)}(n))^T$$

$$B=\begin{pmatrix} -\frac{x^{(1)}(2)+x^{(1)}(1)}{2} & -\frac{x^{(1)}(3)+x^{(1)}(2)}{2} & \cdots & -\frac{x^{(1)}(n)+x^{(1)}(n-1)}{2} \\ 1 & 1 & \cdots & 1 \end{pmatrix}^T$$

Therefore

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \qquad k = 1,2,\cdots,n \tag{4}$$

The results thus obtained is in turn counted down and comes to the prediction results,

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \qquad k = 1,2,\cdots,n \tag{5}$$

B. Regression Prediction Model

Based on the concrete state of correlation, regression prediction approximately conveys the average change in the relationship between variables with appropriate selection of mathematical model. Predictions that are based on a single factor and involve only two variables are called simple regression predictions while those involving more than one factor are called multiple regression analysis. The steps are as follows:

(1) Identify the causal relationship between variables and determine prediction target, dependent variables and independent variables by analysis of historical and realistic data. Dependent variables are prediction contents and independent variables are various factors that cause changes in prediction target;

(2) Build prediction model according to the causal relationships between variables;

(3) Test the prediction model, detect errors and work out the prediction values.

Establish simple linear regression prediction model according to the characteristics of aviation fuel consumption.

$$y_i = a + bx_i \qquad i = 1,2,\cdots,n \tag{6}$$

Where x stands for the time factor affecting consumption and is independent variable; y stands for aviation fuel consumption and is dependent variable; a and b are regression coefficients. The estimated value of the parameters can be calculated by the least squares method

$$\hat{b} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \tag{7}$$

$$\hat{a} = \frac{\sum_{i=1}^{n} y_i}{n} - \hat{b}\frac{\sum_{i=1}^{n} x_i}{n} \tag{8}$$

Due to various effects of random factors, prediction target y is a normal random variable centering round the corresponding value on a regression line. Significance test methods commonly used in a linear regression model are correlation coefficient test method, F-test method and t-test method. The intensity of linear correlation between two variables in the model is determined by testing. If not passed, the regression model cannot be used to predict. Analyze the reasons and re-adjust the regression model [6, 9].

C. Time Series Prediction Model

Based on the development trend of the aviation industry and aviation fuel consumption in recent years, we establish the moving average prediction model for aviation fuel consumption. Moving average method is a method to reflect the long-term trend by calculating time average which contains a certain number of items in accordance with the passage of time series data [9, 10]. Moving average method contains simple moving average method, weighted moving average method and the trend moving average method.

Set time sequence is $y_1, y_2, \cdots y_t, \cdots$; simple moving average model is

$$M_t = \frac{y_t + y_{t-1} + \cdots + y_{t-N+1}}{N} \qquad t \geq N \tag{9}$$

among which M_t is the moving average at t time; N is the items of moving average.

The above equation indicates when t moves forward for a period of time, a new data will be added with the removal of a long-term data and a new average will be produced. Recursion formulas can be introduced $M_t = M_{t-1} + \frac{y_t - y_{t-N}}{N}$ and the prediction model is

$$\hat{y}_{t+1} = M_t \tag{10}$$

That is, take the moving average at t time as the prediction value at $t+1$ time. The selection of N is important, which can be decided by experience or selection of the optimal method through multiple sets of calculation.

IV. EACH INDIVIDUAL PREDICTION RESULT

Take the aviation fuel consumption of Xuzhou Guanyin Airport as study object, aviation fuel consumptions in the first six months of 2013 are included in Table 1.

TABLE 1 AVIATION FUEL CONSUMPTION FROM JANNUARY TO JUNE

Month	1	2	3	4	5	6
Consumption (Unit：Kiloton)	362.2	369.7	366.8	369.3	376.6	376.5

A. Grey Prediction Results

Take aviation fuel consumption from January to June as the original sequence and set grey prediction model based on Eqs. (3) to (5)

$$X^{(1)}(k+1) = 58085.2e^{0.0063k} - 57723$$

Prediction results obtained are shown in Table 2.

TABLE 2 RESULTS FROM GREW PREDICTION MODEL

Month	1	2	3	4	5	6	7
Prediction Value (Unit：Kiloton)	362.2	367	372	364.7	371.6	376.1	370.08

B. Simple Linear Regression Prediction Results

Let time series be independent variable x and the total aviation fuel consumption be dependent variable y, set simple linear regression prediction model based on Eqs. (6)-(8) $\hat{y} = 360.7032 + 2.7108x$.

Prediction results obtained are shown in Table 3.

TABLE 3 RESULTS FROM SIMPLE LINEAR REGRESSION PREDICTION MODEL

Month	1	2	3	4	5	6	7
Prediction Value（Unit：Kiloton）	363.4	366.12	368.83	371.54	374.24	376.95	379.68

C. Moving Average Prediction Results

By the use of Eqs. (9) and (10), Choose N=2, 3, 4 to predict and work out the mean square error of prediction in these three cases to determine the suitable value of N. The moving average prediction results are shown in Table 4.

TABLE 4 RESULTS FROM MOVING AVERAGE PREDICTION

Month		1	2	3	4	5	6	7
Consumption（Unit：Kiloton）		362.2	369.7	366.8	369.3	376.6	376.5	
Prediction Value（Unit：Kiloton）	N=2			365.95	368.25	368.05	372.95	376.55
	N=3				366.2	368.6	370.9	374.13
	N=4					367	370.6	372.3

The mean square errors of prediction are as follows:

When N=2,

$$S = \frac{1}{4}\sum_{i=3}^{6}(y_i - \hat{y}_i)^2 = \frac{1}{4} \times 87.53 = 21.88$$

When N=3,

$$S = \frac{1}{3}\sum_{i=4}^{6}(y_i - \hat{y}_i)^2 = \frac{1}{3}\times 104.97 = 34.99$$

When N=4,

$$S = \frac{1}{2}\sum_{i=3}^{6}(y_i - \hat{y}_i)^2 = \frac{1}{2}\times 126.97 = 63.48$$

The results show the value of S is comparatively small when N=2. So N=2 is chosen to predict [6].

V. COMBINATION PREDICTION BASED ON DOMINANCE MATRIX METHOD

It is known that the actual consumption of aviation fuel is 374.8 kiloton in July. Let the prediction value of grey prediction, single linear regression prediction and moving average prediction be $\hat{y}_{1t}$, $\hat{y}_{2t}$, and $\hat{y}_{3t}$ respectively, work out the absolute error（shown in Table 5) of each individual prediction with the prediction data of aviation fuel consumption from January to June in Table 1 to Table 4.

Compare the absolute error of individual prediction from March to June and determine the merits of each individual prediction by absolute error. The following can be obtained:

$$Z_{12} = 1;\ \ Z_{13} = 2;\ \ Z_{21} = 3;\ \ Z_{23} = 2;\ \ Z_{31} = 2;\ \ Z_{32} = 2$$

And the dominance matrix is $\begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}$. The weight of each individual prediction model can be worked out by the Eq. (2):

$$w_1 = \frac{3}{12}, w_2 = \frac{5}{12}, w_3 = \frac{4}{12},$$

The prediction model of aviation fuel consumption can be set up as follows by the use of combination prediction Eq. (1):

$$\hat{y}_t = \frac{3}{12}\hat{y}_{1t} + \frac{5}{12}\hat{y}_{2t} + \frac{4}{12}\hat{y}_{3t} \tag{11}$$

Predict the consumption of aviation fuel from March to June by the use of Eq. (11) and calculate absolute error (shown in Table 5).

TABLE 5 ABSOLUTE ERROR AND AVERAGE ABSOLUTE ERROR OF EACH PREDICTION METHOD

Month			1	2	3	4	5	6	Average Absolute Error
Absolute Error	Method 1	Grey Prediction	-	2.7	5.2	4.6	5.5	0.4	3.68
	Method 2	A Linear Regression Prediction	1.2	3.58	2.03	2.24	2.36	0.45	1.98
	Method 3	Moving Average Prediction	-	-	0.85	1.05	8.55	3.55	3.5
	Combination Prediction		-	-	1.86	0.57	4.2	1.1	1.93

As shown in Table 5, the average absolute error indicates combination prediction is superior to any individual prediction. So prediction Eq. (11) is chosen to predict the airport's aviation fuel consumption in July, 2013 and the prediction results are shown in Table 6.

TABLE 6 AVIATION FUEL CONSUMPTION PREDICTION OF JULY AND ERROR COMPARISON

Prediction Method	Actual Consumption (Unit: Kiloton)	Prediction Results (Unit: Kiloton)	Absolute Error	Relative Error (%)
Grey Prediction	374.8	370.08	2.72	0.73
Simple Linear Regression Prediction	374.8	379.68	4.88	1.30
Moving Average Prediction	374.8	376.55	1.75	0.47
Combination Prediction Based on Dominance Matrix Method	374.8	376.24	1.44	0.38

VI. CONCLUSIONS AND SUGGESTIONS

The comparison of absolute error and relative error between each prediction method indicates that both absolute error and relative error of combination prediction method are minimal which is based on dominance matrix. The prediction accuracy is far better than that of each individual prediction. Combination prediction provides the scientific basis for optimizing strategy of aviation fuel order and storage. Nevertheless, the combination prediction model established in this paper is only based on three individual predictions and data collected in this paper is also limited. Should more individual prediction methods be chosen and screened and more data be collected in this paper, the weight would be determined more rationally and the prediction results would be further improved.

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