Simulation of Flow over Ogee and Stepped Spillways and Comparison of Finite Volume and Finite Element Methods

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Abstract-In this study, a comparison of two discretization methods in the numerical modeling of the free surface flow over stepped and ogee spillways has been presented. FLUENT has been used for the numerical solution via Finite Volume Method (FVM), and for the Finite Element Method (FEM) ADINA has been used in literature. These two methods have been compared with experimental results of free surface flow over spillways. It is worth mentioning that k-ε model has been used for modeling the turbulence of flow. According to study results, an acceptable match has been observed between the two methods and experimental results. Nevertheless FVM has presented more acceptable results in some regions compared to FEM.

Keywords- Stepped and ogee spillways; Navier - Stokes Equations; FVM – FEM

I. INTRODUCTION

Dams and large hydraulic structures can control large volumes of water. Energy of water in such structures whether water discharged from the outlet or through the spillway is very high.

Spillways are hydraulic structures used to release excessive water in the reservoir. Analysis of flow over the spillway is one of the major problems in hydraulic engineering, so the United States Army Corps has studied water flow behaviour in spillways [1] and they offered design chart of spillway that has been recently updated [2]. Due to advances in numerical modelling, the calculations combined with experiments to understanding the complexity of the flow on the spillway. Many researchers have worked in the area of experimental and numerical modelling of flow over spillways [3-9]. Physical models have been implemented on several dams (e.g. Upper Stillwater Dam [10] and Monksville Dam [11]). Cassidy (1965) has taken the first step in the numerical solution by using flow potential theory and transforming it into complex potential pages [12]. He used his method to calculate water surface and pressure profiles in the crest. He also could observe agreement between numerical and experimental results in the limited number of points. Also, Assy (2001) simulated flow over a spillway by using the finite difference method for the first time [13]. Song and Zhou (1999) presented a numerical method paid to the finite volume method for studying geometry effect on the water surface profile over spillway [4]. They compared the numerical results with experimental results in some points. Olsen and Kjellesvig (1998) modelled two and three-dimensional flow over the spillway by solving Navier-Stokes equations for different geometries and applying k-ε turbulence model [14]. Some researchers also have used the finite element method for flow modelling [14-17]. Tsai and Yue (1996) also have examined the advantages and disadvantages of different methods for calculating water surface profiles over spillway [18]. Chatila and Tabbara (2004) investigated numerically water surface profile of ogee spillway via FEM (ADINA software). They observed close agreement with measured and numerical free surface profiles [19]. Tabbara et al. (2005) simulated flow over stepped spillway in different cases by using ADINA. They predicted water surface profiles over the entire length of the spillway in close agreement with experimental results [20].

In this study, flow over stepped and ogee spillways is simulated via FLUENT and water surface results will be compared with experimental and numerical results of ADINA (from literature).

II. GOVERNING EQUATIONS

Generally, finite element and finite volume methods are widely used in CFD. Both methods discrete flow field to many finite elements or finite volume and then solve the governing equations, i.e., Navier-Stokes equations. Navier-Stokes equations in two-dimensional and laminar flow presented as below. Where, u and v are the velocities in x and y direction respectively. Also p and ρ stand for pressure and density respectively. Eqs (1) and (2) are derived by momentum conservation rule, and Eq. (3) describe mass conservation (continuity equation),

$$-\frac{\partial}{\partial x}\left(\rho v \frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(\rho v \frac{\partial u}{\partial y}\right)+\rho u \frac{\partial u}{\partial x}+\rho v \frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=0 \tag{1}$$

$$-\frac{\partial}{\partial x}\left(\rho v \frac{\partial v}{\partial x}\right)-\frac{\partial}{\partial y}\left(\rho v \frac{\partial v}{\partial y}\right)+\rho u \frac{\partial v}{\partial x}+\rho v \frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}=0 \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

in which the first and second terms in Eqs. (1) and (2) describe diffusion, whereas the third and fourth terms in these equations handles convection. Also the fifth term in Eqs. (1) and (2) presents variation of pressure in the x and y direction respectively.

In both methods of FVM and FEM, a system of algebraic equations is formed and then reparative procedure is applied to solve governing equations. Numerical methods used in the derivation and defining this algebraic equations are different. Finite element method uses simple piece functions (e.g. linear or quadratic) to define local changes in undefined flow variables. If piece approximation mechanism functions cannot be replaced in the equations, accurate answers cannot be achieved and residuals will be formed for error measurement. Then residuals multiplied by weighting and integral functions can be minimized. As a result, a series of algebraic equations are achieved to obtain the unknown coefficients of approximation mechanism functions. In the finite volume method, integration of Navier-Stokes relations is performed on all control volumes of solving domain. Various types of finite difference approximation are applied to the terms made in the integration of flow such as convection, diffusion and source terms. This operation turns integral equations to the system of algebraic equations. Then algebraic equations can be solved formed by repeating procedure and calculating the residuals. This process will be continued to reach a minimum value for the residuals and the model convergence.

A. The Governing Equations on Determination of Water Surface Profiles by Volume-of-Fluid Method (VOF)

Since both methods of determining water surface profiles (FEM and FVM) are based on the VOF method, in this section, the governing equations of this method have been presented. Governing rules of flow are mass and momentum conservation that in the case of turbulent flow and in time-averaged mode, Navier-Stokes equations are derived from them as following:

$$\frac{\partial(\rho u_i)}{\partial x_i}=0 \tag{4}$$

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\mu \frac{\partial}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]+\frac{\partial(-\rho \overline{u_i' u_j'})}{\partial x_j} \tag{5}$$

where u_i: velocity components (for x and y directions, respectively), p: pressure, ρ: density, μ: dynamic viscosity, $-\rho \overline{u_i' u_j'}$: Reynolds stresses.

Since these equations contain unknown components of velocity in two directions (u, v), pressure, Reynolds stress, the above system of equations is not closed and must be calculated using the turbulence Reynolds stress model. According to the research of Cheng et al. (2005), Renormalization Group k-ε (RNG) turbulence model to simulate flow through a stepped spillway is very efficient [20].

A variable (α) is used in the VOF method that is known as an area fraction of water in the computational cell. If α equals 1, it means that the cell is full of water but when it equals zero, the cell is filled with air. $0<\alpha<1$ shows the percentage of water in computational cells.

$$\frac{\partial \alpha}{\partial t}+u \frac{\partial \alpha}{\partial x}+v \frac{\partial \alpha}{\partial y}=0 \tag{4}$$

Therefore, the free surface of the flow can be determined considering a certain value for α. In the current study, this fraction has been considered 0.5. By solving equation of continuity for volume fraction, α in total flow field is determined.

III.   PHYSICAL MODEL GEOMETRY OF STEPPED AND OGEE SPILLWAYS

Tabbara et al. (2005) and Chatila and Tabbara (2004) investigated stepped and ogee spillway respectively [19, 20]. The experiments have been done in a long flume with glass walls in the hydraulic laboratory. Two pumps were used to supply the

water flow, and the flow is measured by orifice meter in the feeding tubes. Flow control was done by two independent valves. The flume slope was approximately horizontal during experiments. Water depth has been measured by the point gauge with millimetre accuracy. In the case of oscillating water surface profile, measuring is performed several times.

The models of spillway profiles have been designed based on the smooth spillway. Joint line of step's edge of steeped spillway will follow chute profile in ogee spillway. Hydraulic Design Chart 111-2 - 1 / USACE-WES has been used to design spillway profiles [1]. Upstream of profile, before spillway crest, includes a vertical plane continues with two circular arcs with radii $0.2H_d$ and $0.5H_d$. H_d is spillway design head. Downstream of crest profile has been divided to three parts. The first part will follow of $x^n = KH_d^{1-n} y$ equation. Whereas x and y coordinates is placed in spillway's crest. Equation $y = 0.1265\, x^{1.85}$, is obtained with respect to that $K=2$, $n=1.85$ and H_d=5.08cm. The second part is a straight line with a slope angle of 60 degrees (1.73V: 1H). The third part is a circular arc that provides transformation between steep straight line, and horizontal line corresponds to the stilling basin.

Four types of the stepped spillway with different configurations have been used under water head $1.5H_d$. Every spillway has been formed by assembling two parts, which is made of Plexiglas's. The upper part of the crest that its height is 1/3 times of the spillway's height and the lower part includes toe area and its height equals 2/3 times of the spillway's height. Total height of the toe to crest of the spillway is equal to 380 mm. Curvature of the upper and lower parts of the spillway profile will follow the ogee spillway profile. In each section, large steps heights are equal to 1.20 times of spillway height (19 mm), and height of small steps is equal to 1.40 times of the spillway's height (9.5 mm). Configurations of steps on each types of spillway are presented in table 1. In the experiments, some measurements have been repeated to ensure the accuracy of data with possible minimal error by Tabbara et al. (2005) and Chatila and Tabbara (2004). The measurements have been recorded during the tests, are namely, flow discharge, water head in upstream of spillway, water depth in crest, free surface profiles, depth of floor or toe of spillway, and depth at end point of hydraulic jump and jump's length.

TABLE I CONFIGURATION OF STEPS IN STEEPED SPILLWAYS [20].

Type of steeped spillway	Number and height of steps in upper part	Number and height of steps in down part
1	6 @ 19 mm	12 @ 19 mm
2	12 @ 9.5 mm	24 @ 9.5 mm
3	6 @ 19 mm	24 @ 9.5 mm
4	12 @ 9.5 mm	12 @ 19 mm

IV. NUMERICAL MODEL

Numerical methods have three important advantages: Reducing expenses of experimental models, calculating and measuring complicated phenomenon such as air-water mixed phases in water surface profile of stepped spillway, and evaluating numerical methods' performance.

Computational fluid dynamics models for ogee and stepped spillways of Chatila and Tabbara (2004) and Tabbara et al. (2005) (by ADINA software) has been compared with numerical results of the current study (by FLUENT). ADINA is a general finite element code, which is capable of modelling a wide range of fluid flow. A major difference with FLUENT is that FLUENT uses finite volume method for modelling flow. One of the important steps in numerical modelling is determining water level at upstream before spillway crest, over the spillway and downstream of the toe. Since free surface is unknown before solving, a simple logical profile that composed of three straight lines has been used for the initial guess of water surface profiles with ADINA code [19 20], (see Fig. 1a). In FLUENT code, such initial guess has been done by two straight lines vertically (see Fig. 1b). With regard to the problem modelling in the laboratory under steady condition, it seems logical that a steady and unsteady analysis can be used. As the process continued; it was observed that the unsteady analysis did not converge to steady state. Thus, the unsteady flow option was used for numerical modelling. Selecting laminar flow option versus turbulent flow depends on the regime of real flow. Reynolds number criteria acknowledged the existence of turbulent flow in nature. The K–ε (RNG) turbulence model was adopted by available default parameters in the ADINA code (by [19, 20]) and FLUENT (current study), for modelling turbulence.

V. BOUNDARY CONDITION

Initial computational domain for type 4 of stepped spillway (as a sample), is given in Fig. 1 for both the ADINA code FLUENT. Other types of stepped and ogee spillways will follow the same trend. According to Fig. 1a, the lines marked with *A* represent the fixed wall boundary condition. The primary free surface of water is shown with three straight lines labelled *B* in Fig.1-a.

C as a uniform velocity inlet boundary is defined according to Table 2 for stepped spillways equivalent, and to 0.1065 m/s for ogee spillways. This value is the result of discharge division to depth of flow of upstream obtained from experimental observations. Such default definitions include all four types of stepped spillways and all three phases of ogee spillways.
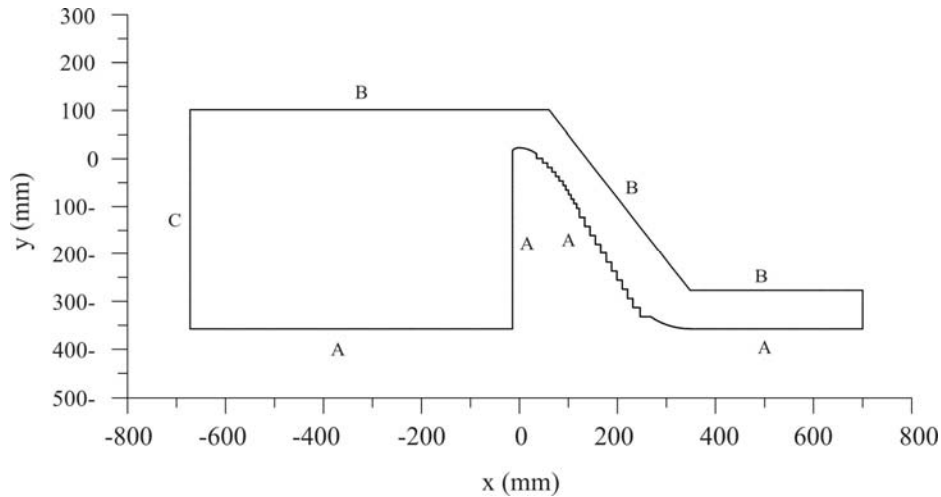
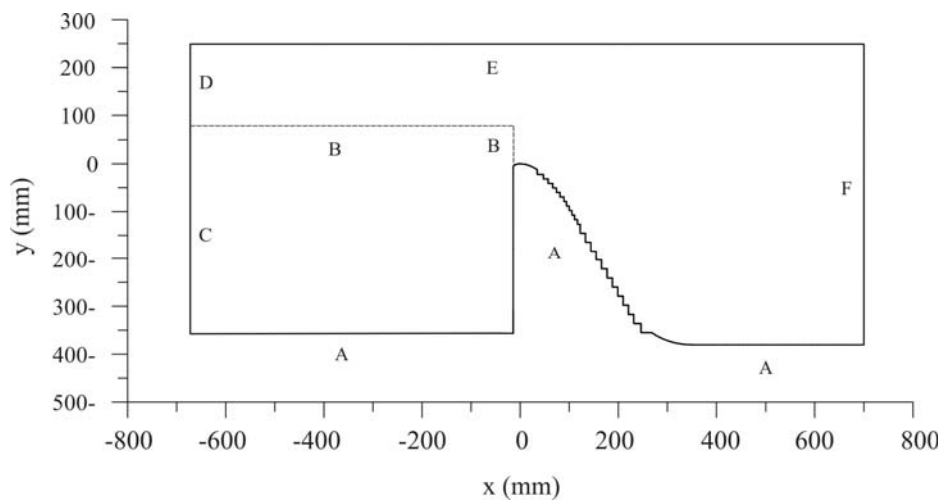Fig. 1a Computational domain and boundary conditions ADINA code [20]

Fig. 1b Computational domain and boundary conditions FLUENT Code

Fig. 1b represents lines marked with *A* indicating the fixed wall boundary condition. The primary free surface of water is shown with two straight dash lines labelled *B* in Fig. 1b. *C* boundary as a uniform velocity inlet boundary and for stepped spillways is calculated according to what was said for the ADINA code. *D* boundary is considered as a uniform air velocity inlet boundary that its velocity is equivalent to 0.00001 m/s and *E* and *F* boundaries are considered as the pressure inlet and pressure outlet boundaries, respectively. Table 2, shows the uniform velocity rate at *C* boundary, for ogee spillways for both FLUENT and ADINA code.

TABLE II VALUE OF UNIFORM VELOCITY IN THE C BOUNDARY FOR OGEE SPILLWAYS [19].

Type of ogee spillway	Average velocity in C boundary (m/s)
1	0.1025
2	0.05539
3	0.03566

VI.   MESH SIZE

Mesh used in ADINA code is triangular [19, 20]. In FLUENT code quadrilateral mesh is selected according to comparison of results of different mesh sizes (Due to the geometry of spillway Map mesh was used in some areas and Pave mesh was used in some other areas). The mesh's size for each case is presented in table 3. The obtained values are selected by using trial and error method with regard to experimental results. According to existence of the eddy flow in the neighbourhood of steps in both codes, it is used the smaller mesh. Fig. 2 shows details of mesh size used in both the ADINA [20] and FLUENT codes as example in stepped Spillway type 4. Also, in ogee spillway the finer mesh has been used on the ogee profile. Free surface in ADINA is a movable boundary which acts as a common boundary between the liquid and the gas with negligible density [19 20].
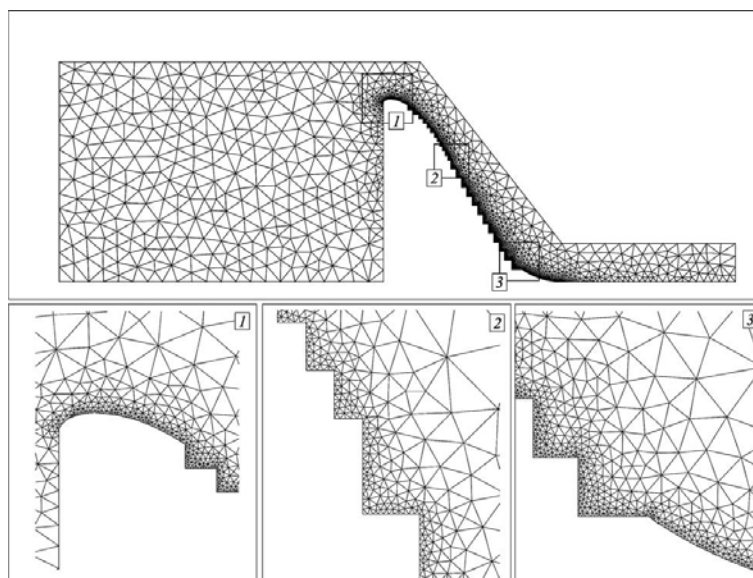
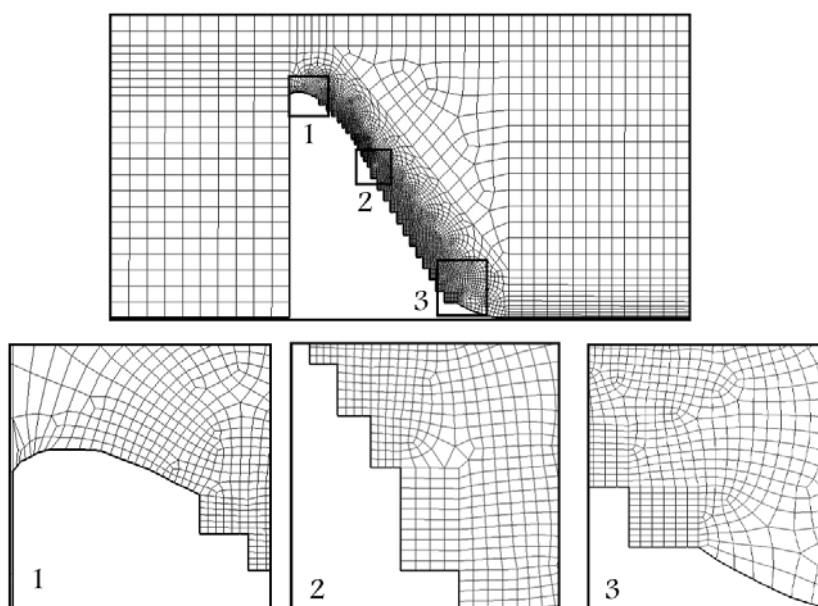Fig. 2a The triangular mesh used in the ADINA code and stepped Spillway [20]

Fig. 2b The quadrilateral mesh used in FLUENT code and stepped Spillway

TABLE III NUMBER OF MESHES

Type of spillway	Number of Meshes
Ogee Type 1	2454
Ogee Type 2	2454
Ogee Type 3	2454
Stepped Type 1	6236
Stepped Type 2	9254
Stepped Type 3	8949
Stepped Type 4	5644

VII. NUMERICAL SIMULATION AND RESULTS

  ADINA code consists of two major phases in the domain of calculation [19, 20]. The first phase consists of the part in which the water level is kept constant (with pre-defined wall boundary condition), and the primary calculations are done. The second phase is when nodal values resulting from the first phase were used as initial conditions and water levels may act as a free surface. Free surface in ADINA is a moving boundary and works as a common boundary between the liquid and the air with small density. The first phase of ADINA consists of 100 steps with interval 0.01 seconds per step (for both stepped and

ogee spillways). During this phase, the input velocity has been gradually applied with a ramp-shape rising function increasing until it reaches the unit. The second phase calculations include 200 steps and 100 steps with the time interval 0.005 and 0.01 seconds per step for stepped spillway and ogee spillways, respectively. During this phase, the velocity at the boundary *C* is kept constant [19, 20].

Modelling trend in FLUENT code has been used in this method in which the initial boundary of free surface is specified (Fig. 1-b) and then equations with a time interval 0.001 second are continued until convergence. Convergence criterion for each variable based on the residual value of the relative error is selected equal to 0.0001. FLUENT considers total flow field as a separate control volume. The governing equations of flow on the volume control have been integrated, and algebraic equations are obtained by using various discrete schemes. In order to providing and meshing geometry of the flow domain, GAMBIT pre-processor software is used. In addition, PRESTO scheme to discrete pressure, Quick scheme for discrete momentum equations transport sentences, and ultimately PISO algorithm for coupling the pressure and velocity has been used.
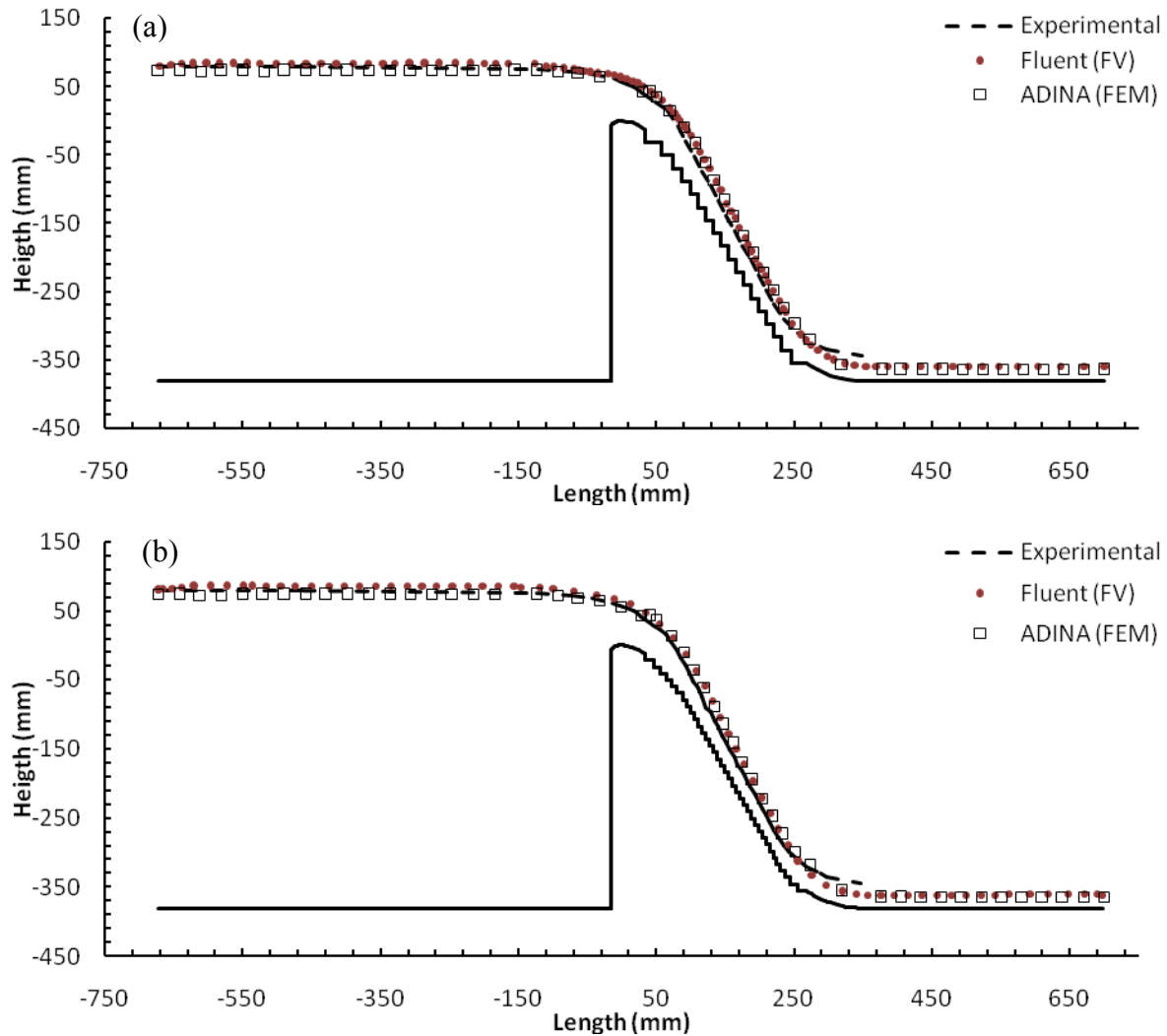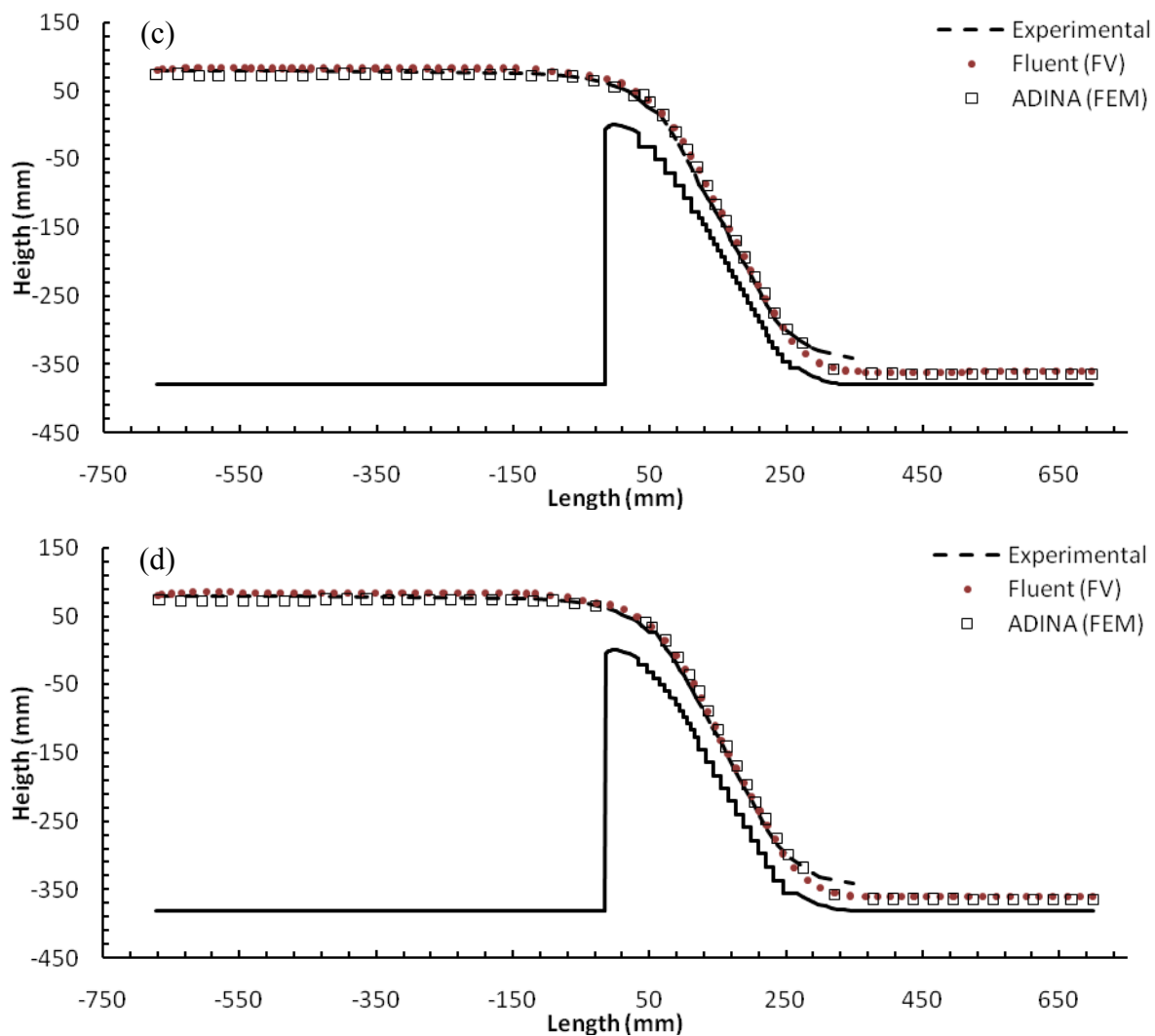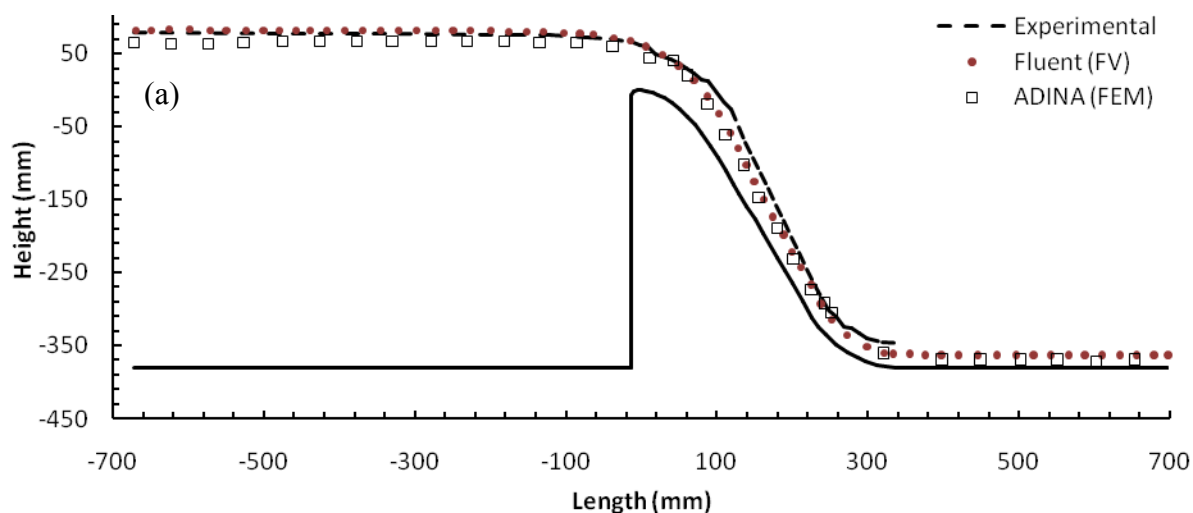
Fig. 3 Comparison of results of two ADINA and FLUENT numerical models with experimental results of stepped spillway, (a) First type, (b) Second type, (c) Third type, (d) Fourth type
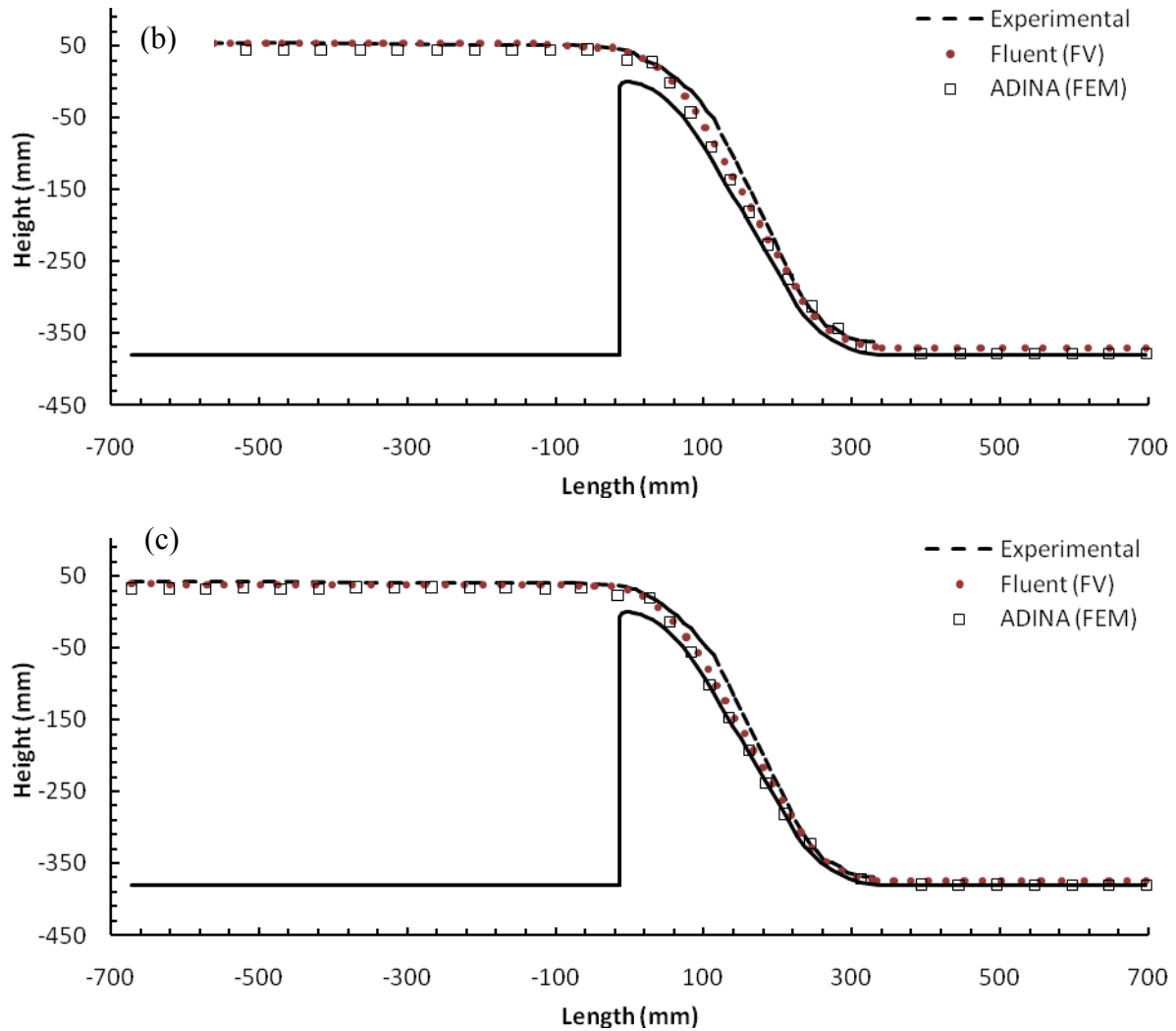
Fig. 4 Comparison of results of FLUENT and ADINA numerical models with experimental results of ogee spillway, (a) First type, (b) Second type, (c) Third type

Using the relaxation factors smaller than one for the pressure, momentum and Reynolds stress prevented from divergence. In Figs. 3 and 4, the results of FLUENT numerical analysis have been compared with results of ADINA, Chatila and Tabbara numerical and experimental results [19, 20]. Comparing the figures shows that FLUENT also has provided acceptable results compared with experimental results in regard to ADINA code. In some regions, the result of the FLUENT code was closer to the experimental results.

Figs. 5 and 6 have analysed the difference between water surface profiles in FVM and FEM in regard to experimental results, respectively, during the ogee and stepped spillways model. Whatever the plotted values are closer to zero, the related model shows more accurate results. So by these figures, comparing at any location would be possible.
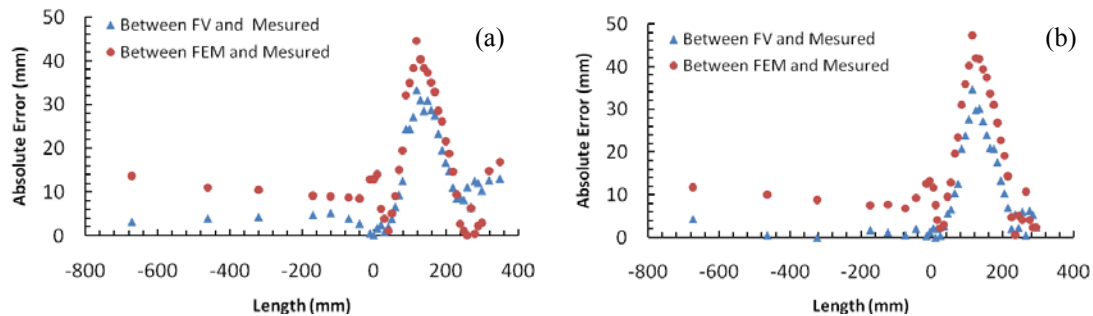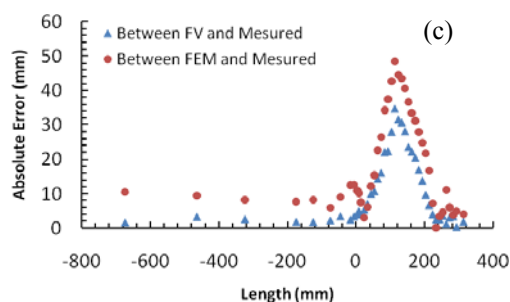
Fig. 5 Comparison of difference between the ogee spillway water surface profiles in finite volume and finite element methods. (a) First type, (b) Second type, (c) Third type
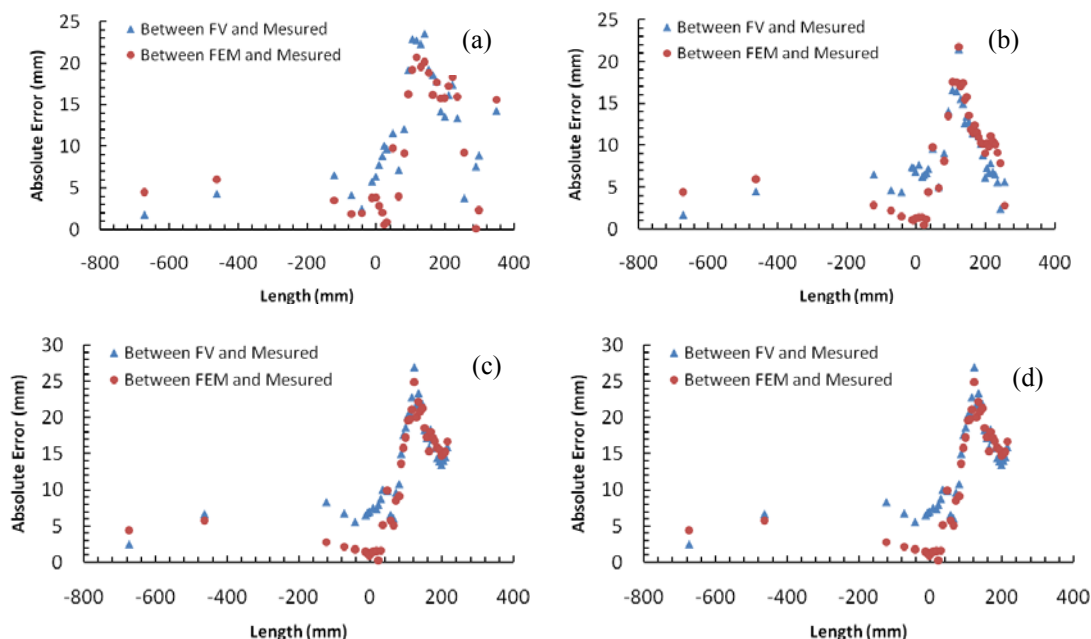

Fig. 6 Comparison of difference between the profiles of water surface in the steeped spillway in finite volume and finite element methods. (a) First type, (b) Second type, (c) Third type, (d) Fourth type

Absolute errors value in both ogee and stepped spillways were been compared in table 4.

TABLE IV COMPARISON BETWEEN FVM AND FEM IN TERMS OF MEAN ABSOLUTE ERROR

Type of spillway	Mean Absolute Error (mm)	
	FEM	FVM
Ogee Type 1	16.43	12.58
Ogee Type 2	17.08	10.07
Ogee Type 3	17.47	10.39
Stepped Type 1	10.17	12.09
Stepped Type 2	8.83	9.63
Stepped Type 3	11.47	12.90
Stepped Type 4	8.06	7.71

According to Figs. 5 and 6 and Table 4, the closest results in all three types of ogee spillway and the fourth type of stepped spillway have been provided for the finite volume method (FLUENT). Results of finite element method are close to experimental results than the result of finite volume method in the three types of stepped spillway (first type to third type). But considering the small difference (approximately less than 2 mm) it can be stated that both methods have presented close and satisfactory results.

VIII.   CONCLUSION

Stepped and ogee spillways are one of the most common used spillways along the years. In this paper, these two types of spillways have been numerically modelled using code FLUENT as well water surface profiles have been obtained in four types of stepped spillway and three types of ogee spillway. The results obtained in water surface profiles have been compared with

the experimental results and the results of the ADINA code produced by Chatila and Tabbara [19] and Tabbara et al. [20]. As mentioned already in the recent paper, FLUENT code is based on finite volume method and the ADINA code is based on finite element method. By comparing the presented results, the relatively close match was observed between the water surface profiles in both the code. It should be noted that water surface profiles in the FLUENT code in some areas, is closer to the experimental results with regard to water surface profiles in the ADINA code.

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