The Universal Speed Limit as an Attribute of the Space-Time Symmetry

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***Abstract*-Relativistic kinematic effects are deduced immediately from the space-time symmetry.**

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The theory of relativity [1, 2], that began primarily with explanations of concrete physical effects, is now assumed [3, 4] to be just a reflection of the space-time symmetry resulting in the relativistic coordinate-time transform [1, 2] without using a priori limitation on speeds of material bodies. Historically, the latter approach was initiated by Proclus [5], who abolished the fifth element of Aristotle (the ether) and, so, converted both the motion and the rest from absolute to relative ones.

The present comment to [3-5] reproduces the author’s talk [6] and exercises with physics students. Following the antique methodology [5], start with definitions. Launch two identical crystals into the infinite vacuum along a common direct line (Fig. 1). In each reference frame, measure any distance, ξ or ξ', with an unit equal to *N* crystal periods, and measure any time interval, τ or τ', with an unit equal to *M* electron “rotations” around any local nucleus.

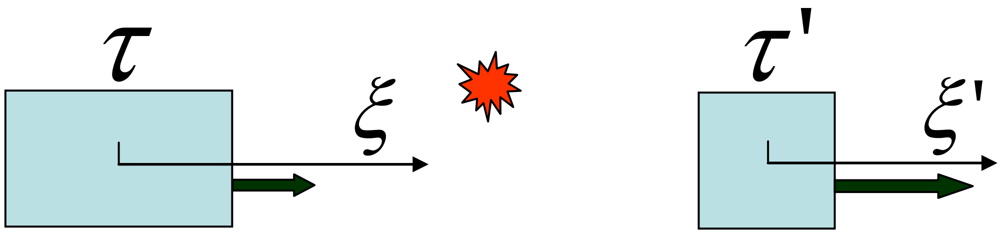

Fig. 1 Inertial reference frames representing crystals moving with different speeds (denoted with different thick arrows). From the external viewpoint, the identical crystals seem squeezed to different lengths - because of different Lorenz “length contractions” [1, 2]

To meet the aesthetic “ancient-Greek” symmetry (συμμετρείν = symmetrein means “to measure together"), assume the ratio *N/M* being adjusted so that both coordinates ξ, ξ' and times τ, τ' are measured with a common uni-dimensional etalon*. Under this convention, any registration of any event (indicated by a flash in Fig. 1 and Fig. 2) in any of the reference frames should be invariant relative to the commutation

$$\begin{array}{ll} \xi \to \tau & \qquad \xi' \to \tau' \\ \tau \to \xi & \qquad \tau' \to \xi' \end{array} \tag{1}$$

In addition, with account of the counter-directional mutual reciprocity of the identical reference frames, the space-time isotropic interrelation is to be invariant relative to the commutation

$$\begin{array}{ll} \xi \to -\xi' & \qquad \xi' \to -\xi \\ \tau \to \tau' & \qquad \tau' \to \tau \end{array} \tag{2}$$

Put $\xi_0 = \tau_0 = \xi_0' = \tau_0' = 0$ for a primary event and, to meet the symmetry invariance conditions (1)-(2), interrelate subsequent events $(\xi, \tau \quad \xi', \tau')$ by equations

$$\begin{array}{lcl} \xi = \gamma\xi' + \kappa\tau' & \Rightarrow & \xi' = \gamma\xi - \kappa\tau \\ \tau = \kappa\xi' + \gamma\tau' & \Leftarrow & \tau' = -\kappa\xi + \gamma\tau \end{array} \tag{3}$$

*At this point, progressive modern students propose: “Let both *N* periods and *M* rotations be equal to 1$”.

By putting the right equations into the left ones and by putting the left equations into the right ones, derive equations of the following type: $\xi = \gamma(\gamma\xi - \kappa\tau) + \kappa(-\kappa\xi + \gamma\tau) = (\gamma^2 - \kappa^2)\xi,$ which result in the common condition

$$\gamma^2 - \kappa^2 = 1 \tag{4}$$

(converted to the Pythagoras theorem in Appendix 1).

By putting $\xi' = 0$ or $\xi = 0$ into Eqs. (3), find the relative velocity of the reference frames

$$\beta = \pm\kappa / \gamma. \tag{5}$$

By using Eq. (5), convert the cyclic recurrence condition (4) to the form

$$\gamma = \left(1 - \beta^2\right)^{-1/2}, \tag{6}$$

resulting in the limitation

$$|\beta| \le 1 \tag{7}$$

for relative velocities of reference frames and, so, of any material bodies. Thus, the space-time-symmetry postulate (1)-(2) has given additional proof (7) for the 12th theorem of Proclus [5]: “Εν πεπερασμένω χρόνω το άπειρον κινείσθαι ουκ έστιν” = “During a limited time it is not possible to go an infinite distance”.

The space-time transform formulas (3)-(4) are followed by popular effects of the relativistic kinematics [1, 2]:

- substitutions τ=0 or τ'=0 into Eqs. (3) give, correspondingly, $\xi = \xi'/\gamma$ or $\xi' = \xi/\gamma$, which is called the Lorenz “length contraction” (shown in Fig. 1);
- substitutions ξ'=0 or ξ=0 into Eqs. (3) give, correspondingly, $\tau' = \tau/\gamma$ or $\tau = \tau'/\gamma$, which is called the “twin paradox”;
- if a body is moving with a velocity $\beta_b' = \xi'/\tau'$ relative to the $\{\xi', \tau'\}$ reference frame, then - according to Eqs. (3) - the body velocity relative to the $\{\xi, \tau\}$ frame is

$$\beta_b = \xi / \tau = \frac{\beta_b' + \beta}{1 + \beta\beta_b'}, \tag{8}$$

  which is called the “relativistic velocity summation” - consistent with the Proclus speed limitation (7);
- according to limitation (7) and Eq. (8), in all inertial reference frames all small vacuum perturbations propagate with a common – ultimate – speed equal to 1 (exemplified with the electromagnetic pulse propagation [1, 7] - Appendix 2).

In Appendix 3, formulas (3)-(4) are converted to the conventional relativistic kinematic transform [1, 2]. However, note that, economically compared to [1, 2], the commutation Eqs. (3)-(4) have been above deduced from the only postulate - of the aesthetic space-time-symmetry [3-5].

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APPENDIX 1 COORDINATE-TIME TRANSFORM IN COMPLEX VARIABLES

It may be pedagogical to undertake the Minkowski’s [1, 2] change of variables $\hat{\tau} = i\tau$, $\hat{\tau}' = i\tau'$, $\hat{\kappa} = i\kappa$ converting the Eqs. (3) to the form

$$\begin{aligned}\xi &= \gamma\xi' - \hat{\kappa}\hat{\tau}' \\ \hat{\tau} &= \hat{\kappa}\xi' + \gamma\hat{\tau}'\end{aligned} \quad \Leftrightarrow \quad \begin{aligned}\xi' &= \gamma\xi + \hat{\kappa}\hat{\tau} \\ \hat{\tau}' &= -\hat{\kappa}\xi + \gamma\hat{\tau}\end{aligned} \tag{A1.1}$$

and the cyclic recurrence condition (4) to the form

$$\det\begin{vmatrix}\gamma & \mp\hat{\kappa} \\ \pm\hat{\kappa} & \gamma\end{vmatrix} = 1. \tag{A1.2}$$

These formulas may be illustrated with Fig. 2: taking into account the Euclidean similarity of the triangles, the "Descartes coordinates" ξ, $\hat{\tau}$ and ς', τ' are mutually turned at the angle $\alpha = \arcsin\hat{\kappa} = \arccos\gamma$. Correspondingly, the Eq. (A1.2) takes the form of "Pythagoras theorem" $\cos^2\alpha + \sin^2\alpha = 1$, where, the angle α being imaginary, the "cathetuses" $\cos\beta = \gamma$ and $\sin\alpha = \hat{\kappa}$ may be longer than the "hypotenuse" 1.

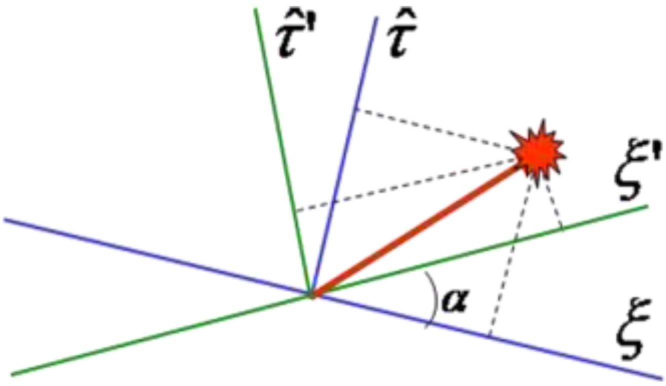

Fig. 2 Rotation of Descartes coordinates $\xi, \hat{\tau}$ and ξ', $\hat{\tau}'$.

APPENDIX 2 VACUUM PERTURBATIONS

Take the wave equations [1, 7]

$$\frac{\partial^2\varphi}{\partial\xi^2} = \frac{\partial^2\varphi}{\partial\tau^2} \Leftrightarrow \frac{\partial^2\varphi}{\partial\xi'^2} = \frac{\partial^2\varphi}{\partial\tau'^2} \tag{A2.1}$$

describing an electromagnetic perturbation φ of the vacuum (the unitary coordinate and time in these equations are related to the "usual" ones in Appendix 3). These equations satisfy the double symmetry conditions (1)-(2) and, so, should be invariant relative to the space-time commutation Eqs. (3)-(4). For checking, use Eqs. (3) to derive

$$\begin{aligned}\frac{\partial^2\varphi}{\partial\xi^2} &= \gamma^2\frac{\partial^2\varphi}{\partial\xi'^2} - 2\gamma\kappa\frac{\partial^2\varphi}{\partial\xi'\partial\tau'} + \kappa^2\frac{\partial^2\varphi}{\partial\tau'^2}, \\ \frac{\partial^2\varphi}{\partial\tau^2} &= \gamma^2\frac{\partial^2\varphi}{\partial\tau'^2} - 2\gamma\kappa\frac{\partial^2\varphi}{\partial\xi'\partial\tau'} + \kappa^2\frac{\partial^2\varphi}{\partial\xi'^2},\end{aligned} \tag{A2.2}$$

put these derivatives into the left (A2.1) and see how the cyclic recurrence formula Eq. (4) converts the left (A2.1) to the right (A2.1).

APPENDIX 3 INTERRELATION BETWEEN UNITARY AND "USUAL" SPACE-TIME SCALES

To relate the uni-dimensional symmetrized coordinate ξ and time τ to their "usual" (measured, for instance, with meters and seconds) equivalents χ and t, undertake the linear change of variables

$$x = a\xi, \quad t = b\tau \tag{A3.1}$$

converting the "ancient-Greek" commutation Eqs. (3)-(4) to the "usual" Lorenz transform [1, 2]

$$\begin{matrix} x = \gamma\left(x' + vt'\right) \\ t = \gamma\left(t' + vx'/c^2\right) \end{matrix} \Leftrightarrow \begin{matrix} x' = \gamma\left(x - vt\right) \\ t' = \gamma\left(t - vx/c^2\right) \end{matrix} \tag{A3.2}$$

$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

where $v = \beta c$ is the "usual" relative velocity of reference frames and $a/b = c$ is the "usual" speed limit common for all material bodies. Measured at the end of the previous millennium, this absolute limit - the speed of light – was proved to be $3 \cdot 10^8 m/s$ [1, 2].

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