Mathematical Modeling of Human Posture Balance When Standing on One Foot

A. V. Chigarev[1], A.V. Borisov[2]

[1] Department of Theoretical Mechanics, Byelorussian National Technical University,

65, Prospect Nezavisimosti, Minsk 220027, Republic of Belarus

[2] Department of Higher Mathematics, Smolensk branch of National Research University "MPEI" (Technical University),

1, Energeticheskiy proezd, Smolensk 214013, Russia

[1]chigarev@rambler.ru; [2]borisowandrej@yandex.ru

Abstract-**The given work deals with the model of stability of a human body as a multilink pivotal pattern with concentrated masses in the phase standing on a skid foot. The moments in joints, providing static posture of a human body, are modeled by the springs of a certain deflection rate. The zones of deflection rate of the springs, necessary for balance maintenance, are defined in the work. A possibility of practical realization of the appliance in the form of exoskeleton with a person inside and its vertical balance maintenance is considered hereinafter. A generalization of the obtained for the case of a bio-mechanical system consists of a random finite number of links.**

To construct the solution let us first record the potential energy of the system. Then it is expanded into Maclaurin series. After that, the notations, which suitably represent the potential energy for the application of the criterion of Sylvester are introduced. The solution of inequalities is done numerically. The graphs are constructed for a visual representation of the obtained solutions.

It was stated that the spring stiffness zones necessary for maintaining vertical posture a man in exoskeleton are large. Minimum required stiffness notations are of the main practical interest, since only these values can be sufficient while creating an exoskeleton enough these values. They can be determined graphically. These are the lower bound areas of the graphs.

This model can be applied to create anthropomorphic robots while resolving a question of their static posture balance and the necessary structural elements.

In practice, exoskeletons can be used in military technologies, medicine, gerontology, sports, industry, in everyday life, etc. wherever it is necessary to offload or enforce a natural human skeleton. Therefore, this work can be considered relevant for practical realization of human-machine system as well.

Keywords-Balance; Exoskeleton; Stability; Joints; Deflection Rate Of Springs; Single-support Phase; Statics

I. INTRODUCTION

First of all let us create a mathematical model of the stability of the three links of the supporting leg and human body influenced by internal forces in the joints. Then it is possible to estimate values necessary for balanced standing. Next step is the modeling with the use of spiral springs, located in joints. Let us use the method described in our previous research [2, pp 78-79] for the model of the support foot. Let the model be of the form which is shown in Fig. 1.

Here the OA is a foot conjoint with the support. Let us then designate: $OA = l_1$ – a foot, $AB = l_2$ – a shin, $BC = l_3$ – a hip, $CD = l_4$ – a body – and all these are lengths of the relevant links of the lower extremeties and the body of the mechanism.

The position during the single-support phase is uniquely determined by the angles φ_i and the lengths of the rods l_i ($i = 1... 4$), so the considered system has four degrees of freedom. The centres of the masses are situated in points: C_1 – of the foot of the support leg, C_2 – of the shin of the support leg, C_3 – of the hip of the support leg, C_4-of the body. Their positions will be fixed as the ratio of the length from the beginning of the appropriate link to its center of mass and the entire length of the link, through the multipliers n_i ($i = 1, ..., 4$), ($0 < 1 < n_i$) (if all the links are renumbered by appropriate angles indexes). This way of fixing of the centers of mass is more preferable, because a human-being the positions of centers of masses in extremities are defined empirically and are specified in percentage terms of one part of the link to the other [4, 5], and besides, it allows to take into account changes of the center of mass in the process of motion through the certain deformations. Masses: m_1, m_2, m_3, m_4, are of the foot, of the shin, of the thigh, of the body respectively. At the calculation and modeling of a biomechanical system all the above-mentioned characteristics are taken equally to the relevant experimental data of a human being [1].

Friction in horizontal cylindrical bearings is neglected. Spiral springs with deflection rates i k_i ($i = 1... 4$) in the upper vertical position of the links are in their natural undeformed state. Let us define the deflection rates of the springs k_i ($i = 1... 4$) so, that the balance of the system in the upper vertical position is sustainable.

The ties in the system are ideal, stationary and holonomic, and the active forces influencing the system, are conservative. Therefore Lagrange's theorem can be applied here. The position of the links will be determined by the angles φ_i ($i = 1... 4$). The potential energy Pof the system consists of the springs potential energy and potential energy of the gravity forces.
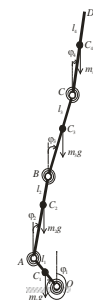

Fig. 1 A model the supporting leg in the form of 4-link rod-spring system

II. SOLUTION FORMATIONT

Potential energy P of the system is in this case is found this way:

$$P_1 = k_1\varphi_1^2/2 + k_2(\varphi_2 - \varphi_1)^2/2 +$$
$$+ k_3(\varphi_3 - \varphi_2 - \varphi_1)^2/2 + k_4(\varphi_4 - \varphi_3 - \varphi_2 - \varphi_1)^2/2,$$
$$P_2 = -m_1gl_1n_1(1-\cos\varphi_1) - m_2g[l_1(1-\cos\varphi_1) +$$
$$+ l_2n_2(1-\cos\varphi_2)] - m_3g[l_1(1-\cos\varphi_1) + \qquad (1)$$
$$+ l_2(1-\cos\varphi_2) + l_3n_3(1-\cos\varphi_3)] -$$
$$- m_4g[l_1(1-\cos\varphi_1) + l_2(1-\cos\varphi_2) +$$
$$+ l_3(1-\cos\varphi_3) + l_4n_4(1-\cos\varphi_4)].$$

Hence,

$$P = k_1\varphi_1^2/2 + k_2(\varphi_1^2 - 2\varphi_1\varphi_2 + \varphi_2^2)/2 +$$
$$+ k_3(\varphi_1^2 + 2\varphi_1\varphi_2 + \varphi_2^2 - 2\varphi_1\varphi_3 - 2\varphi_2\varphi_3 + \varphi_3^2)/2 +$$
$$+ k_4(\varphi_1^2 + 2\varphi_1\varphi_2 + \varphi_2^2 + 2\varphi_1\varphi_3 + 2\varphi_2\varphi_3 + \varphi_3^2 -$$
$$- 2\varphi_1\varphi_4 - 2\varphi_2\varphi_4 - 2\varphi_3\varphi_4 + \varphi_4^2)/2 - \qquad (2)$$
$$- (m_1n_1 + m_2)gl_1(1-\cos\varphi_1) - m_2gl_2n_2(1-\cos\varphi_2) -$$
$$- m_3g[l_1(1-\cos\varphi_1) + l_2(1-\cos\varphi_2) + l_3n_3(1-\cos\varphi_3)].$$

Using extantion in a McLaurin series, after grouping we get:

$$P = (1/2)\{[k_1 + k_2 + k_3 + k_4 - (m_1n_1 + m_2 + m_3 +$$
$$+ m_4)gl_1]\varphi_1^2 + (k_2 + k_3 + k_4 - (m_2n_2 + m_3 + m_4)gl_2)\varphi_2^2 +$$
$$+ (k_3 + k_4 - (m_3n_3 + m_4)gl_3)\varphi_3^2 + (k_4 - m_4n_4gl_4)\varphi_4^2 -$$
$$- 2(k_2 - k_3 - k_4)\varphi_1\varphi_2 - 2(k_3 - k_4)\varphi_1\varphi_3 - 2k_4\varphi_1\varphi_4 -$$
$$- 2(k_3 - k_4)\varphi_2\varphi_3 - 2k_4\varphi_2\varphi_4 - 2k_4\varphi_3\varphi_4\} + ..., \qquad (3)$$

where the points denote the members of the containing φ_1 and φ_2 raised to powers higher than second.

Let us introduce the notations

$$c_{11} = k_1 + k_2 + k_3 + k_4 - (m_1n_1 + m_2 + m_3 + m_4)gl_1,$$
$$c_{22} = k_2 + k_3 + k_4 - (m_2n_2 + m_3 + m_4)gl_2,$$
$$c_{33} = k_3 + k_4 - (m_3n_3 + m_4)gl_3, c_{44} = k_4 - m_4gl_4n_4,$$
$$c_{12} = c_{21} = -k_2 + k_3 + k_4,$$
$$c_{13} = c_{31} = c_{23} = c_{32} = -k_3 + k_4,$$
$$c_{14} = c_{41} = c_{24} = c_{42} = c_{34} = c_{43} = -k_4. \qquad (4)$$

Then

$$P = (1/2)(c_{11}\varphi_1^2 + c_{22}\varphi_2^2 + c_{33}\varphi_3^2 + c_{44}\varphi_4^2 +$$
$$+ 2c_{12}\varphi_1\varphi_2 + 2c_{13}\varphi_1\varphi_3 + 2c_{23}\varphi_2\varphi_3 + 2c_{14}\varphi_1\varphi_4 +$$
$$+ 2c_{24}\varphi_2\varphi_4 + 2c_{34}\varphi_3\varphi_4) + .. \qquad (5)$$

Sylvester criterion in this case is of the form

$$\Delta_1 = c_{11} > 0,$$

$$\Delta_2 = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} > 0,$$

$$\Delta_3 = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix} > 0, \qquad (6)$$

$$\Delta_4 = \begin{vmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{vmatrix} > 0.$$

Therefore, a mathematical model describing the state of the balance of a human is created. Let us analyze this model numerically.

III. NUMERICAL STUDY

The solution of the received system of inequalities in analytical is quite lengthy. It is impossible to construct a four-dimensional space area (k_1, k_2, k_3, k_4) corresponding to the solution of these inequalities. Therefore let us build three-dimensional areas at one of the fixed arguments in the system of computer mathematics Mathematica [3] (Fig. 2).
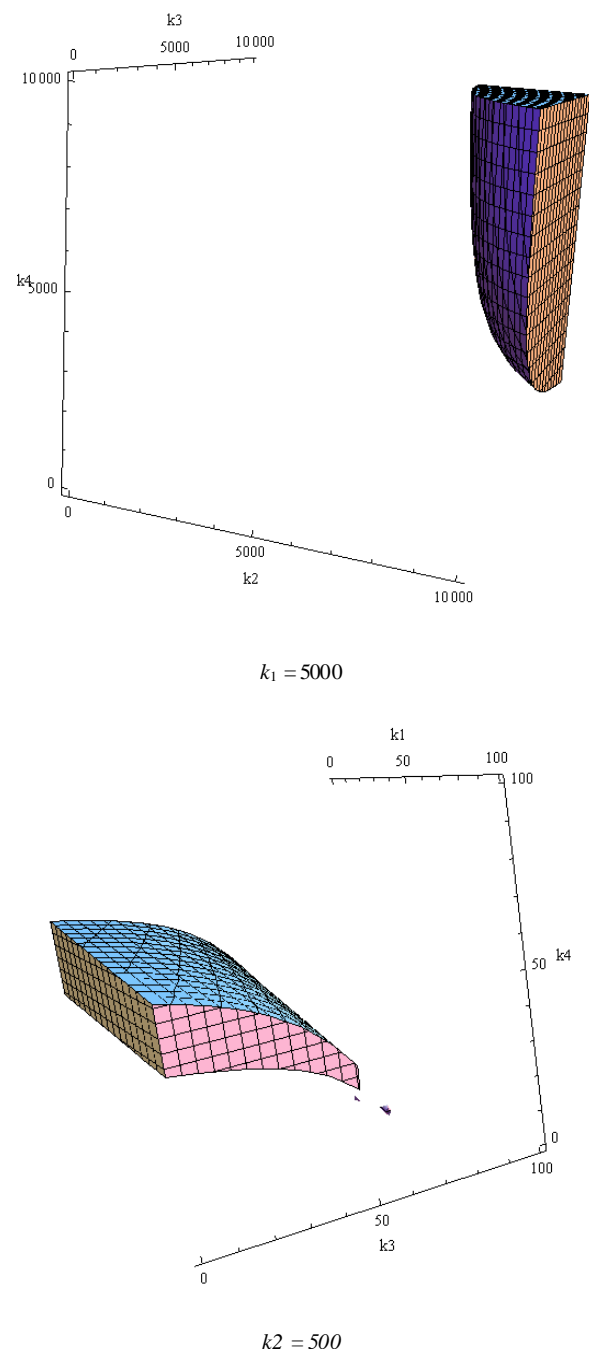
$k_1 = 5000$

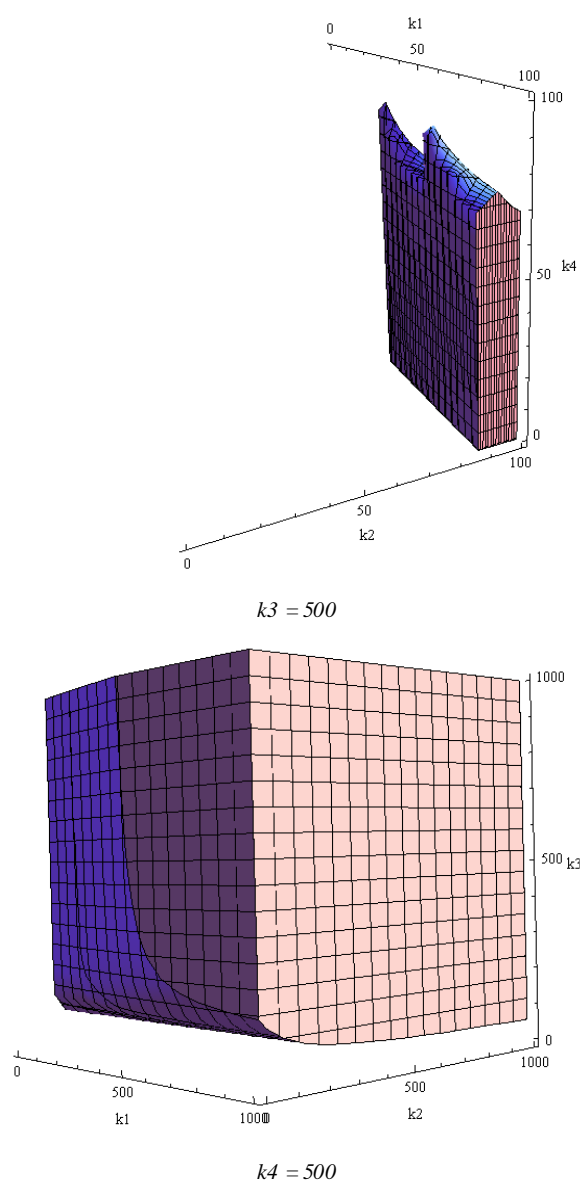$k2 = 500$

$k3 = 500$

$k4 = 500$

Fig. 2 Stability areas at one of the fixed arguments

It is also possible to build the areas of stability at two fixed arguments. As twelve of such areas are received, let us take two of them as an illustration (Fig. 3).

$k2 = 500, k3 = 50$
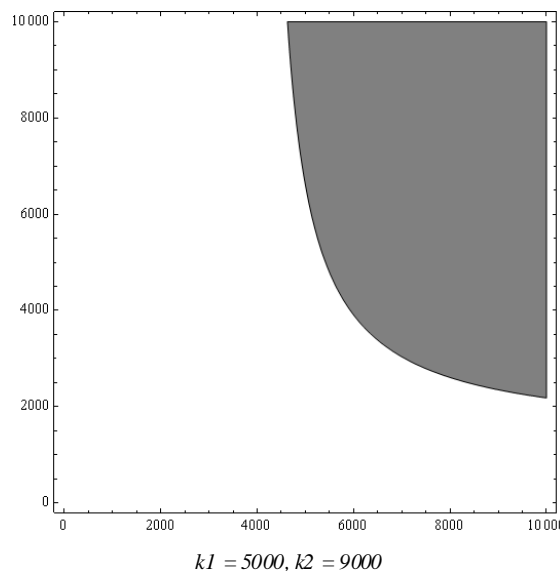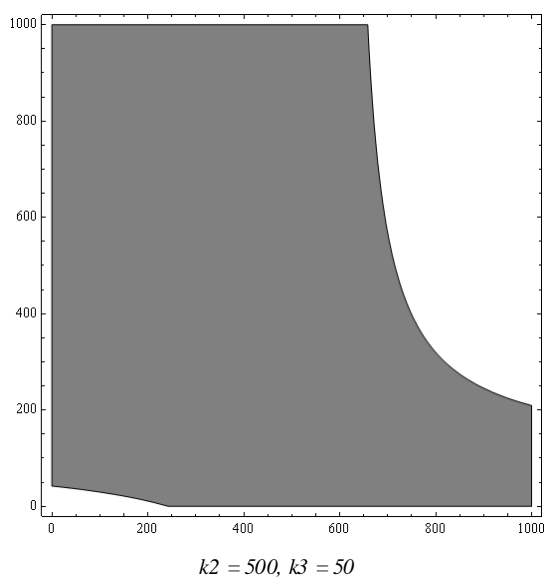

$k1 = 5000, k2 = 9000$

Fig. 3 Stability areas at two fixed arguments

Thus, the areas of stability for a four-link model are estimated.

IV. PRACTICAL EXAMPLE: STABILITY OF AN EXOSKELETON WITH THE MAN INSIDE

Let us consider the stability of the exoskeleton model, taking into account the mass of the shifting leg, hand and head (Fig. 4). Let us also take into account the own mass of the exoskeleton itself. Suppose that the mass of each link of the exoskeleton is 30% of the mass of the appropriate link of a man. In Fig. 4, in addition to the cases discussed above, the following notations are introduced: the shifting leg with the same mass as the supporting leg is slung in point C; in the point two hands are attached, with the mass m_5; in the point E is the head, with the mass m_6.
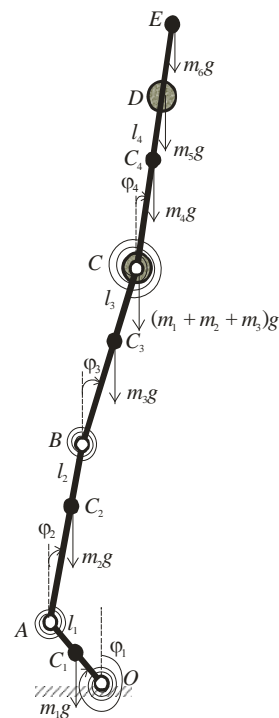

Fig. 4 The model of the supporting leg of the exoskeleton in the form of four-link rod-spring system

In this case, the potential energy P of the system is found according to the formula:

$$P_1 = k_1\varphi_1^2/2 + k_2(\varphi_2 - \varphi_1)^2/2 + k_3(\varphi_3 - \varphi_2 - \varphi_1)^2/2 +$$

$$+ k_4(\varphi_4 - \varphi_3 - \varphi_2 - \varphi_1)^2/2,$$

$$P_2 = - m_1 g l_1 n_1 (1 - \cos\varphi_1) - m_2 g [l_1(1 - \cos\varphi_1) +$$

$$+ l_2 n_2(1 - \cos\varphi_2)] - m_3 g [l_1(1 - \cos\varphi_1) +$$

$$+ l_2(1 - \cos\varphi_2) + l_3 n_3(1 - \cos\varphi_3)] -$$

$$- (m_1 + m_2 + m_3)g[l_1(1 - \cos\varphi_1) + l_2(1 - \cos\varphi_2) + \quad (7)$$

$$+ l_3(1 - \cos\varphi_3)] - m_4 g[l_1(1 - \cos\varphi_1) + l_2(1 - \cos\varphi_2) +$$

$$+ l_3(1 - \cos\varphi_3) + l_4 n_4(1 - \cos\varphi_4)] -$$

$$- m_5 g[l_1(1 - \cos\varphi_1) + l_2(1 - \cos\varphi_2) + l_3(1 - \cos\varphi_3)$$

$$+ l_4 n_5(1 - \cos\varphi_4)] - m_6 g[l_1(1 - \cos\varphi_1) +$$

$$+ l_2(1 - \cos\varphi_2) + l_3(1 - \cos\varphi_3) + l_4(1 - \cos\varphi_4)].$$

Expanding the squares and using extantion in a McLaurin series, after grouping we receive

$$P = (1/2)\{[k_1 + k_2 + k_3 + k_4 - (m_1 n_1 + m_1 + 2m_2 +$$

$$+ 2m_3 + m_4 + m_5 + m_6)gl_1]\varphi_1^2 + (k_2 + k_3 + k_4 - (m_2 n_2 +$$

$$+ m_2 + 2m_3 + m_4 + m_5 + m_6)gl_2)\varphi_2^2 + (k_3 + k_4 -$$

$$- (m_3 n_3 + m_2 + m_3 + m_4 + m_5 + m_6)gl_3)\varphi_3^2 + \quad (8)$$

$$+ (k_4 - (m_4 n_4 + m_5 n_5 + m_6)gl_4)\varphi_4^2 -$$

$$- 2(k_2 - k_3 - k_4)\varphi_1\varphi_2 - 2(k_3 - k_4)\varphi_1\varphi_3 - 2k_4\varphi_1\varphi_4 -$$

$$- 2(k_3 - k_4)\varphi_2\varphi_3 - 2k_4\varphi_2\varphi_4 - 2k_4\varphi_3\varphi_4\} + ...,$$

where the points denote the members of the containing φ_1 and φ_2 raised to powers higher than second.

Let us introduce the notations

$$c_{11} = k_1 + k_2 + k_3 + k_4 -$$

$$- (m_1 n_1 + m_1 + 2m_2 + 2m_3 + m_4 + m_5 + m_6)gl_1,$$

$$c_{22} = k_2 + k_3 + k_4 -$$

$$- (m_2 n_2 + m_2 + 2m_3 + m_4 + m_5 + m_6)gl_2,$$

$$c_{33} = k_3 + k_4 - (m_3 n_3 + m_2 + m_3 + m_4 + m_5 + m_6)gl_3, \quad (9)$$

$$c_{44} = k_4 - (m_4 n_4 + m_5 n_5 + m_6)gl_4,$$

$$c_{12} = c_{21} = - k_2 + k_3 + k_4,$$

$$c_{13} = c_{31} = c_{23} = c_{32} = - k_3 + k_4,$$

$$c_{14} = c_{41} = c_{24} = c_{42} = c_{34} = c_{43} = - k_4.$$

Then the potential energy goes over to (5), and Sylvestre criterion has a form (6).

The solution of the received system of inequalities in analytical is quite lengthy. It is impossible to construct a four-dimensional space area (k_1, k_2, k_3, k_4) corresponding to the solution of these inequalities. Therefore let us build the three-dimensional areas at one of the fixed arguments using the system of computer mathematics Mathematica (Fig. 5).
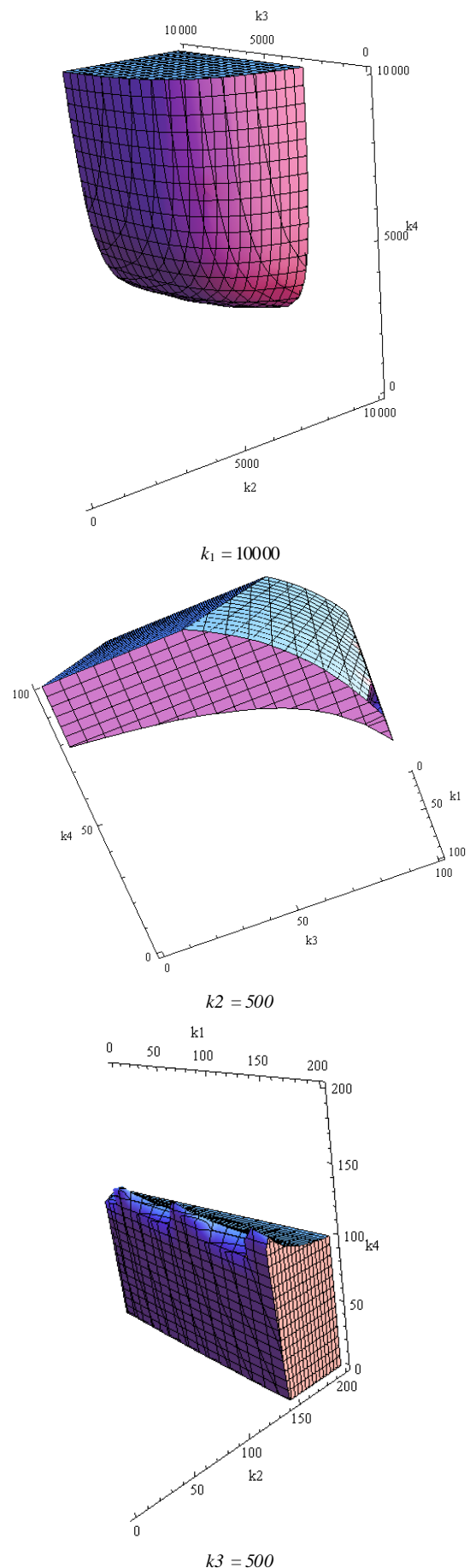
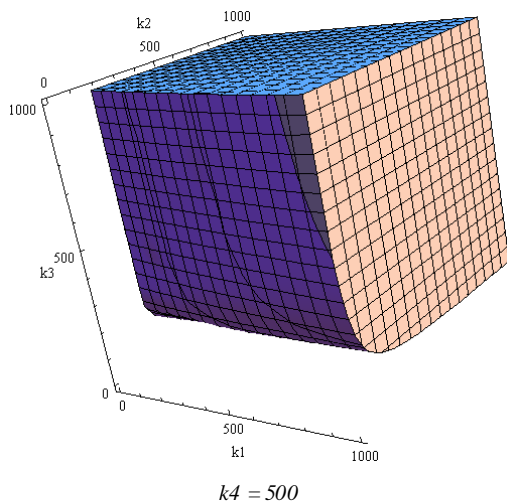$k_1 = 10000$

$k2 = 500$

$k3 = 500$

$k4 = 500$

Fig. 5 Stability areas at one of the fixed arguments

It is also possible to build the areas of stability at two fixed arguments. As twelve of such areas are received, let us take two of them as an illustration (Fig. 6).
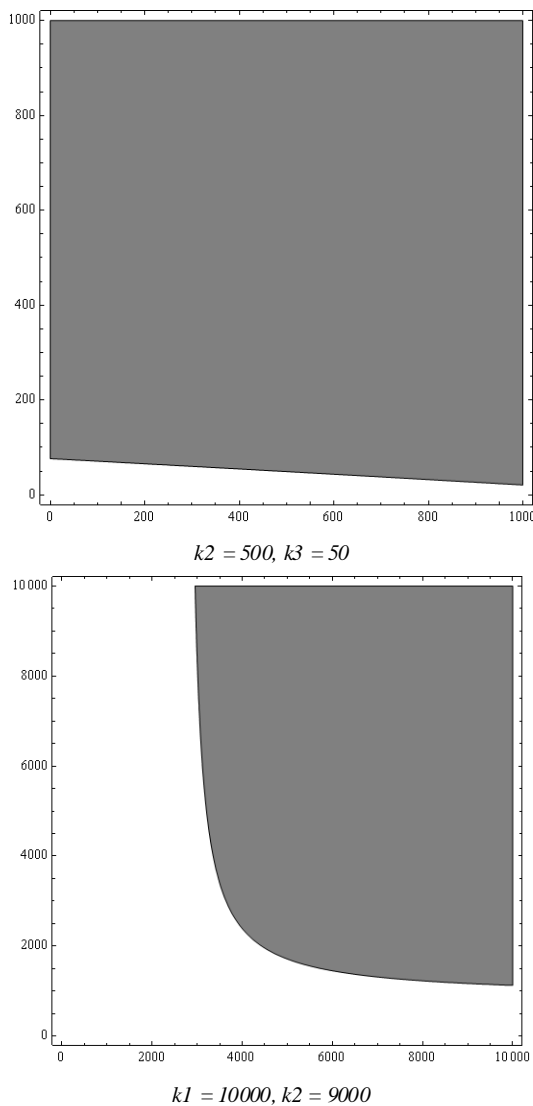

$k2 = 500, k3 = 50$

$k1 = 10000, k2 = 9000$

Fig. 6 Stability areas at two fixed arguments

Thus, the areas of stability for a four-link model are estimated.

Comparing the dependences obtained earlier for the four-link mechanism (Figs. 2 and 3) and the dependences obtained for the model of exoskeleton, taking into account the shifting leg, hands, head and own mass (Figs. 5 and 6), it can be seen that there are no fundamental changes in the forms of dependencies, but the areas of stability have shifted to higher deflection rates of the springs.

Thus, the mathematical model describing the stability of exoskeleton with the man inside in the phase standing on one leg is obtained. The areas of stability are determined.

V. GENERALIZATION AN *N*-LINKS MODEL

Let us sum up the results for an arbitrary multi-links model with n elements.

As all the parameters, characterizing mechanical system is included into the c coefficients, and the pattern of their generalization is clear, let us introduce the notations:

$$c_{tt} = \sum_{i=t}^{n} k_i - gl_t \left(m_t n_t - \sum_{i=t+1}^{n} m_i \right) \quad (t = 1, \ldots n),$$

$$c_{ts} = c_{st} = -k_s + \sum_{i=s+1}^{n} k_i \quad (t = 1, \ldots n, s = 2, \ldots n, t \neq s) \qquad (10)$$

Then

$$P = \frac{1}{2} \sum_{s,t=1}^{n} c_{st} \varphi_t \varphi_s + \ldots$$

$$(t = 1, \ldots n, s = 1, \ldots n). \qquad (11)$$

Sylvester criterion in this case is of the form

$$\Delta_1 = c_{11} > 0,$$

$$\Delta_2 = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} > 0,$$

$$\Delta_3 = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix} > 0, \qquad (12)$$

$$\ldots,$$

The solution of the received system of inequalities in analytical is quite lengthy. It is impossible to construct a n-dimensional space area $(k_1, k_{2,;\ldots} k_n)$ corresponding to the solution of these inequalities. It is only possible to build three-dimensional and two-dimensional areas at several fixed arguments using the system of computer mathematics Mathematica, similar to the cases discussed above.

Thus, the generalisation of the criterion of stability for an arbitrary multi-links mechanical system is obtained.

VI. CONCLUSIONS

The problem of studying and modeling of dynamic and static capabilities of human musculoskeletal system and creation of an exoskeleton is very complicated, not only technically, but also theoretically, due to the versatility of its possible solutions. Much depend on the selection of scientific and technical design of the mechanism. But with any design a theoretical solution should have a complex character. This task can be done by collaboration of practitioners and theoreticians.

The study proposes a variant of a complex theoretical solution of exoskeleton and anthropomorphic robots creation

based on mathematical modeling of the human musculoskeletal system.

The main results of the study are:

- In this work, a mathematical model for describing static stability of a man when standing on one foot is created;

- The possibility of its practical application when creating an exoskeleton of a man is shown. The obtained stability zones are fairly large. Therefore, different combinations of elastic elements in a joint are possible;

- The proposed model is sensitive to the parameters changes as reflected in changes of stability zones of the models of a man and of exoskeleton with a man inside. Consequently, this mechanical model is adequate to the real biological prototype.

Need for exoskeletons helping people to move while people are aging increases. The developed model will assist in calculating an optimal design of exoskeleton for a human musculoskeletal system.

Such synthetic bio-mechanical and man-like anthropomorphic system in the form of an exoskeleton can be used in practice when creating human-machine systems for military purposes and emergency situations, as well as for helping people with limited capabilities and those who need rehabilitation.

REFERENCES

[1] A.V. Borisov Methods of determination of the length of human extremeties and links with inertial characteristics calculations. //Proceedings of the international theoretical and practical conference of scientists of MADI (STU), MAA, LNAU January 5-6 2004. Volume 4. Pedagogy and methodology. Moscow-Lugansk. Published By MADI (STU), MAA, LNAU, Smolensk "Inga". 2004. pp. 77-90.

[2] D.R. Merkin Introduction to the theory of the movement stability. -M.: Nauka, 1971. -312 p.

[3] A.V.Chigarev, A.I. Kravchuk, A.S. Kravchuk Basis Mathematica 6.0. System Challenges and solutions. -Minsk: Tehnoprint, 2008. – 136 p.

[4] A.V. Chigarev Stochastic and regular dynamics of heterogeneous environments. -Minsk: UE "Tehnoprint", 2000. – 426 p.

[5] A.V. Borisov Elastic analysis of multilayered thick-walled spheres under external load. // Mechanika Nr. 4(84). Technologija Kaunas 2010. P. 28-32.

[6] Bražėnas, A., Vaičiulis, D. Determination of stresses and strains in two-layer mechanically inhomogeneous pipe subjected to internal pressure at elastic plastic loading. - Mechanika. - Kaunas: Technologija, 2009, Nr.6 (80), p.12-17.

[7] A.V.Chigarev, A.V. Borisov Calculation of the sizes of irreciprocal demolition of two ellipsoids of a software to the limit of a contact. // Mechanika 2010. Proceedings of the 15th international conference. 8, 9 April 2010 Kaunas University of Technology, Lithuania. – Technologija Kaunas. 2010. P. 116-121.

[8] A.V.Chigarev, A.V. Borisov Problems of Strength at Loading Multilayer Bones of the Person. // Mechanika 2009. Proceedings of the 14th international conference. April 2-3, 2009 Kaunas University of Technology, Lithuania. – Technologija Kaunas. 2009. P. 76-79.

[9] Ghannad, M., Zamani Nejad, M., Rahimi, G. H. Elastic solution of axisymmetric thick truncated conical shells based on first-order shear deformation theory. - Mechanika. - Kaunas: Technologija, 2009, Nr.5(79), p.13-20.

[10] Matlin, M., Kazankina, E., Kazankin, V. Mechanics of initial dot contact. - Mechanika. - Kaunas: Technologija, 2009, Nr.2 (76), p.20-23.

[11] Židonis, I. Method for a direct calculation of stress-strain state parameters at normal right-angled sections of structural members given curvilinear stress diagrams. - Mechanika. - Kaunas: Technologija, 2009, Nr.3 (77), p.27-33.

[12] Zilinskaite, A., Ziliukas, A. General Deformation Flow Theory. - Mechanika. - Kaunas: Technologija, 2008, Nr.2 (70), p.11-15.

Anatoliy Chigarev was born on December 2, 1941. He received PhD in 1971, and the place of thesis presentation is the city of Voronezh, Voronezh State University. He received Doctor of physics and mathematics in 1986, and the place of thesis presentation is the city of Novosibirsk, the Siberian Branch of the Academy of Sciences, and he has been served as Professor in the Institute of Theoretical and Applied Mechanics.

He was the head of the Department of Theoretical Mechanics of the Belarusian National Technical University. His research interests include: analytical methods and computer simulations in mechanics of deformable solids, biomechanics. His publications include: A.V. Chigarev Stochastic and regular dynamics of heterogeneous environments. -Minsk: UE "Tehnoprint", 2000. – 426 p.; A.V.Chigarev, A.I. Kravchuk, A.S. Kravchuk Basis Mathematica 6.0. System Challenges and solutions. -Minsk: Tehnoprint, 2008. – 136 p.; V.V. Mikhasev,. A.V Chigarev, A.V. Borisov. Biomechanics: tutorial. -Minsk Grevcov Publishings, 2010. – 284 p.

He is a now a member of the "German Society of Applied Mathematics and Mechanics ", the New York Academy, the Academy of Information Technologies, and the Mining Academy. He is the author of over 250 scientific works, which include twenty monographs and ten text books.

Andrei Borisov was born on October 2, 1977. He received PhD in 2005, and the place of thesis presentation is the city of Tula, Tula State University. He was a Senior Lecturer and an assistant Professor of the of Higher Mathematics department of the Smolensk branch of National Research University "MPEI" (Technical University). His research interests include: theoretical mechanics, mechanics of deformable solids, biomechanics, and anthropomorphic robots. His publications include: Borisov A.V. modeling of the human musculoskeletal system and use the results to develop a model for anthropomorphic robot. Monograph. M.: Publishing House, 2009. – 212 p.; A.V. Borisov, A.V. Chigarev. Biomechanics of human walking. Tutorial. – M.: Publishing House, 2009. -200 p.; A.V. Chigarev, A.V. Borisov Modeling managed traffic Biped anthropomorphic mechanism. Russian Journal of biomechanics. 2011 t. 15, no. 1 (51). C. 74-88.

He is the winner of the contest of young scientists of the Smolensk region for the year 2010, the winner of the Grant of the President of the Russian Federation for young scholars for the year 2007-2008, and author of over 150 scientific works, which include one monograph, one textbook and five tutorials.