New Homoclinic Solution for Davey-Stewartson Equation with Periodic Boundary

Zhengde Dai[1], Xiping Zeng[2], Murong Jiang[3], Chuanjian Wang[4]

School of Mathematics and Statistics, Yunnan University, Kunming, 650091, China

Department of Information and Computing Science, Guangxi Institute of Technology, Liuzhou, 545006, China

School of Information Science and Engineering, Yunnan University, Kunming, 650091, China

School of Science, Kunming University of Science and Technology, Kunming, 650093, China

[1]zhddai2004@yahoo.com.cn; [2]zengxiping@163.com; [3]jiangmr@ynu.edu.cn; [4]wcj20082002@yahoo.com.cn

Abstract- **A new kind of the homoclinic solution with oscillatory structure for Davey-Stewartson (DSI and DSII) equation with periodic boundary condition is constructed by using the Hirota's bilinear form and extended homoclinic test method, respectively. The mechanical feature of the solution is also investigated. Result shows the variety of the structure for the homoclinic solution of one integrable system with periodic boundary condition.**

Keywords- Homoclinic Solution; Breather; Davey-Stewartson Equation; Periodic Boundary

I. INTRODUCTION

Davey-Stewartson (DS) equation is written as [1]:

$$\begin{cases} iu_t = -u_{xx} - \frac{1}{\alpha_0^2}u_{yy} - \frac{2\varepsilon}{\alpha_0^2}|u|^2 u - \frac{2}{\alpha_0^2}uv \\ v_{yy} - \alpha_0^2 v_{xx} - 2\alpha_0^2\varepsilon(|u|^2)_{xx} = 0 \end{cases} \tag{1.1}$$

where $u: R_x \times R_y \times R_t^+ \to C, v: R_x \times R_y \times R_t^+ \to R$, ε and α_0 are constants. DS equation was derived by Davey et al. to model the evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseuille flow. The function $u(x,y,t)$ stands for the complex amplitude, and $v(x,y,t)$ describes the perturbation of the real velocity. DS equation is called the DSI as $\varepsilon = 1, \alpha_0 = \pm 1$ and DSII as $\varepsilon = 1, \alpha_0 = \pm i$. There are known results due to local well-posed, global existence and blow-up of some solutions, exact periodic soliton solutions, solitoff and dromion solutions [2-11]. Recently, homoclinic and heteroclinic tube solutions were obtained [12-15].

It is well known that the existence of homoclinic and heteroclinic orbits solutions is very important for studying the spatiotemporal chaos of partial differential equation. Many methods were developed for proving the existence of homoclinic orbits of perturbed soliton equation. As we know that the homoclinic solution is non-wave type solution, it is generally obtained using "Homoclinic test method" and can be expressed by function $F(\cos(p_1x + p_2y), \cosh(\gamma t + \delta))$ for (2+1)D system, where p_1, p_2, γ and δ are some parameters[12]. The non-wave type homoclinic solution has the stable and non-locally oscillatory structure with time evolution. It meanwhile satisfies periodic boundary condition and asymptotically tends to a fixed cycle as time tends to infinity.

In this work, we search for a new two-wave kind of homoclinic solution with locally oscillatory structure, homoclinic breather solution, for DSI and DSII equation using "extended homoclinic test method", and it can be expressed by function $F(\cos(p_1x + p_2y - \alpha t), \cosh(p_3x + p_4y - \beta t + \delta))$. The two-wave kind of homoclinic solution satisfies periodic boundary condition and asymptotically tends to a fixed cycle as time tends to infinity as well. We also exhibit locally structure of these solutions, respectively.

Consider DSI equation

$$\begin{cases} iu_t + u_{xx} + u_{yy} = -2|u|^2 u - 2uv \\ v_{xx} - v_{yy} = -2(|u|^2)_{xx} \end{cases} \tag{1.2}$$

with periodic boundary condition

$$u(x,y,t) = u(x+l_1, y+l_2, t);\ \ v(x,y,t) = v(x+l_1, y+l_2, t) \tag{1.3}$$

and DSII equation

$$\begin{cases} iU_t + U_{xx} - U_{yy} = 2|U|^2 U + 2UV \\ V_{xx} + V_{yy} = -2(|U|^2)_{xx} \end{cases} \tag{1.4}$$

with periodic boundary condition

$$U(x,y,t) = U(x+l_3, y+l_4, t);\ \ V(x,y,t) = V(x+l_3, y+l_4, t) \tag{1.5}$$

In this work, we analyse linear stability in neighbourhood of fixed cycle and then use the extended homoclinic test approach[16-18] to construct a new kind of the homoclinic solution different form homoclinic tube solution for DS equation[12, 13], some mechanical features are investigated and global structure of the solutions is exhibited.

II. HOMOCLINIC BREATHER SOLUTION FOR DSI EQUATION

It is easy to see that $(\frac{a}{\sqrt{2}}\exp(ia^2t), 0)$ is a fixed cycle of DSI equation [13]. We investigate the linear stability of fixed cycle by considering a small perturbation of the form

$$\begin{cases} u = \frac{a}{\sqrt{2}}\exp(ia^2t)(1 + q_\varepsilon(x,y,t)) \\ v = -\frac{\varphi_\varepsilon(x,y,t)}{2} \end{cases} \tag{2.1}$$

where $|q_\varepsilon(x,y,t)| << 1, |\varphi_\varepsilon(x,y,t)| << 1$. Substituting Eq. (2.1) into Eq. (1.1), we get the linearized equation as

$$\begin{cases} iq_{\varepsilon t} + q_{\varepsilon xx} + q_{\varepsilon yy} = -a^2 q_\varepsilon - a^2 q_\varepsilon^* + \varphi \\ \varphi_{\varepsilon xx} - \varphi_{\varepsilon yy} = 2a^2 q_{\varepsilon xx} + 2a^2 q_{\varepsilon xx}^* \end{cases} \tag{2.2}$$

where $*$ denote the conjugation. Assume that q_ε and φ_ε have the following forms

$$\begin{cases} q_\varepsilon = Ae^{i(\mu_n x+\overline{\mu}_n y)+\sigma_n t} + Be^{-i(\mu_n x+\overline{\mu}_n y)+\sigma_n t} \\ \varphi_\varepsilon = C(e^{i(\mu_n x+\overline{\mu}_n y)+\sigma_n t} + e^{-i(\mu_n x+\overline{\mu}_n y)+\sigma_n t}) \end{cases} \quad (2.3)$$

where A, B are complex constants and C is real, $\mu_n = pn = \frac{2\pi n}{l_1}$, $\overline{\mu}_n = p_1 n = \frac{2\pi n}{l_2}$ and σ_n is the growth rate of the n th mode.

Substitution of Eq. (2.3) into Eq. (2.2) leads to

$$\begin{cases} (i\sigma_n - \mu_n^2 - \overline{\mu}_n^2 + a^2)A = -a^2 B^* + c \\ (i\sigma_n - \mu_n^2 - \overline{\mu}_n^2 + a^2)B = -a^2 A^* + C \\ -c\mu_n^2 + C\overline{\mu}_n^2 = 2a^2 A\mu_n^2 + 2a^2 B^* \mu_n^2 \\ C\mu_n^2 + C\overline{\mu}_n^2 = 2a^2 B\mu_n^2 + 2a^2 A^* \mu_n^2 \end{cases} \quad (2.4)$$

Solving Eq. (2.4), we obtain

$$\sigma_n^2 = (\mu_n^2 + \overline{\mu}_n^2)(2a^2 - \mu_n^2 - \overline{\mu}_n^2) \quad (2.5)$$

Since $\sigma > 0$, we obtain

$$\mu_n^2 + \overline{\mu}_n^2 < 2a^2 \quad (2.6)$$

This shows that the fixed cycle is hyperbolic provided.

$$n^2 < \frac{2a^2}{p^2 + p_1^2} \quad (2.7)$$

Thus the number of unstable modes which determines the complexity of the homoclinic structure is given by the following the largest integer N with

$$0 < N < \frac{\sqrt{2}\,|a|}{\sqrt{p^2 + p_1^2}} \quad (2.8)$$

Now, by using extended homoclinic test approach [16], we construct the homoclinic breather solution of DSI equation.

Make the transformation $u = \frac{a}{\sqrt{2}}\exp(ia^2 t)Q, v = -\frac{\varphi}{2}$ and substitute it into Eq. (1.1), we can get

$$\begin{cases} iQ_t + Q_{xx} + Q_{yy} = -a^2(|Q|^2 - 1)Q + Q\varphi \\ \varphi_{xx} - \varphi_{yy} = 2a^2(|Q|^2)_{xx} \end{cases} \quad (2.9)$$

where $Q = Q(x, y, t)$ is a complex function, and φ is a real.

By the dependent variable transformation

$$Q = \frac{G}{F}, \qquad \varphi = -4(\ln F)_{xx} \quad (2.10)$$

where G is a complex and F a real. Then, Eq. (2.9) can be converted into the form

$$\begin{cases} iG_t F - iF_t G + G_{xx}F - 2G_x F_x + GF_{xx} + G_{yy}F \\ \qquad - 2G_y F_y + GF_{yy} - (a^2 + B)GF = 0, \\ 2(F_{yy}F - F_y^2 - F_{xx}F + F_x^2) - BF^2 - a^2 GG^* = 0 \end{cases} \quad (2.11)$$

where B is an integral constant and $*$ denotes the complex conjugation.

By means of the extended homoclinic test approach [16,17], we take the test function as follows:

$$\begin{aligned} G &= e^{-p(x+\frac{y}{2}+\alpha t)} + a_1\cos(p_1(x+2y-\alpha t)) + a_2 e^{p(x+\frac{y}{2}+\alpha t)} \\ F &= e^{-p(x+\frac{y}{2}+\alpha t)} + a_3\cos(p_1(x+2y-\alpha t)) + a_4 e^{p(x+\frac{y}{2}+\alpha t)} \end{aligned} \quad (2.12)$$

where all of a_3, a_4, p, p_1, α are real, and a_1, a_2 are complex. Substituting Eq. (2.12) into Eq. (2.11), equating the coefficients of all powers of $e^{jp(x+\frac{y}{2}+\alpha t)}\cos(p_1(x+2y-\alpha t))$, $e^{jp(x+\frac{y}{2}+\alpha t)}\sin(p_1(x+2y-\alpha t))$ and $e^{\pm 2(p(x+\frac{y}{2}+\alpha t))}$ $(j=-1,0,1)$ to zero, we can obtain a set of algebraic equations for p, p_1, α, a_k $k = 1,2,3,4$ with

$$\begin{aligned} &B = -a^2 \\ &(4pp_1 - ip_1\alpha)a_1 + (4pp_1 + ip_1\alpha)a_3 = 0 \\ &(-ip_1\alpha - 4pp_1)a_4 a_1 + (ip_1\alpha - 4pp_1)a_3 a_2 = 0 \\ &(ip\alpha + \tfrac{5}{4}p^2 - 5p_1^2)a_1 + (\tfrac{5}{4}p^2 - 5p_1^2 - ip\alpha)a_3 = 0 \\ &(\tfrac{5}{4}p^2 - ip\alpha - 5p_1^2)a_4 a_1 + (ip\alpha + \tfrac{5}{4}p^2 - 5p_1^2)a_3 a_2 = 0 \\ &-10a_1 a_3 p_1^2 + (5p^2 + 2ip\alpha)a_2 + (5p^2 - 2ip\alpha)a_4 = 0 \\ &a^2(a_3^2 - a_1 a_1^*) = 0 \\ &a^2(a_4^2 - a_2 a_2^*) = 0 \\ &(2a^2 - \tfrac{3}{2}p^2 - 6p_1^2)a_3 - a^2(a_1 + a_1^*) = 0 \\ &(2a^2 - \tfrac{3}{2}p^2 - 6p_1^2)a_3 a_4 - a^2(a_1 a_2^* + a_1^* a_2) = 0 \\ &(2a^2 - 6p^2)a_4 - 6a_3^2 p_1^2 - a^2(a_2 + a_2^*) = 0 \end{aligned}$$

Solving these equations, we obtain the relations between the parameters as

$$\begin{aligned} &B = -a^2, \quad p_1^2 = \frac{21p^2}{20}, \quad p^2 = \frac{320a^2 - 39\alpha^2}{624} \\ &a_1 = \frac{(i\alpha + 4p)a_3}{i\alpha - 4p}, \quad a_2 = \frac{(i\alpha + 4p)^2 a_4}{(i\alpha - 4p)^2}, \quad a_3^2 = \frac{4(21\alpha^2 - 80p^2)a_4}{21(\alpha^2 + 16p^2)} \end{aligned} \quad (2.13)$$

From $p^2 \ge 0$ and $a_3^2 \ge 0$ in Eq. (2.13), we have $\frac{800a^2}{507} < \alpha^2 < \frac{320a^2}{39}$. Substituting Eq. (2.13) into Eq. (2.12) and then Eq. (2.10), taking $a_4 > 0$, we obtain the solution for DSI equation as

$$\begin{cases} u = \dfrac{a}{\sqrt{2}} e^{i(\theta + a^2 t)} \dfrac{2\cosh(\xi + \gamma + i\theta) + \frac{a_3}{\sqrt{a_4}}\cos(\eta)}{2\cosh(\xi + \gamma) + \frac{a_3}{\sqrt{a_4}}\cos(\eta)} \\ v = \dfrac{2H(\xi, \gamma, \eta)}{(a_3\cos(\eta) + 2\sqrt{a_4}\cosh(\xi + \gamma))^2} \end{cases} \quad (2.14)$$

where

$$\begin{aligned} H(\xi, \gamma, \eta) &= 2a_3\sqrt{a_4}(p^2 - p_1^2)\cos(\eta)\cosh(\xi + \gamma) \\ &\quad + 4a_3 pp_1\sqrt{a_4}\sin(\eta)\sinh(\xi + \gamma) + 4a_4 p^2 - a_3^2 p_1^2 \end{aligned}$$

$$\xi = p(x + \tfrac{y}{2} + \alpha t), \quad \eta = p_1(x + 2y - \alpha t), \quad \gamma = \ln\sqrt{a_4}$$

and $e^{i\theta} = \frac{i\alpha + 4p}{i\alpha - 4p}$, p, p_1, α, a_3, a_4 are given by Eq. (2.13). Note that if $(u(x,y,t), v(x,y,t))$ is the solution of DSI equation, then $(u(x,-y,t), v(x,-y,t))$ is the solution as well. So, we also obtain the solution of DSI equation as

$$\begin{cases} u_1 = \dfrac{a}{\sqrt{2}} e^{i(\theta + a^2 t)} \dfrac{2\cosh(\xi_1 + \gamma + i\theta) + \frac{a_3}{\sqrt{a_4}}\cos(\eta_1)}{2\cosh(\xi_1 + \gamma) + \frac{a_3}{\sqrt{a_4}}\cos(\eta_1)} \\ v_1 = \dfrac{2H_1(\xi_1, \gamma, \eta_1)}{(a_3\cos(\eta_1) + 2\sqrt{a_4}\cosh(\xi_1 + \gamma))^2} \end{cases} \tag{2.15}$$

where

$$H_1(\xi, \gamma, \eta) = 2a_3\sqrt{a_4}(p^2 - p_1^2)\cos(\eta_1)\cosh(\xi_1 + \gamma)$$
$$+ 4a_3 p p_1 \sqrt{a_4}\sin(\eta_1)\sinh(\xi_1 + \gamma) + 4a_4 p^2 - a_3^2 p_1^2$$
$$\xi_1 = p(x - \tfrac{y}{2} + \alpha t), \quad \eta_1 = p_1(x - 2y - \alpha t), \quad \gamma = \ln\sqrt{a_4}$$

We suitably choose p, p_1 such that all wave numbers n of x and y satisfy Eq. (2.7), then Eq. (2.15) is the homoclinic solution of DSI equation. In addition, the solution given by Eq. (2.15) satisfies periodic boundary condition. In fact, if we take $l_1 = 2\pi/3p_1, l_2 = 4\pi/3p_1$, then we have

$$u_1(x, y, t) = u_1(x + l_1, y + l_2, t), \quad v_1(x, y, t) = v_1(x + l_1, y + l_2, t)$$

Eq. (2.15) (Resp. Eq. (2.14)) is a breather kind of homoclinic solution. It is a new kind of homoclinic solution different from homoclinic tube solution obtained in [13]. It has a homoclinic structure. Indeed, we have

$$(u_1, v_1) \to (\frac{a}{\sqrt{2}}\exp(i(a^2 t + 2\theta)), 0) \quad \text{as} \quad t \to +\infty$$
$$(u_1, v_1) \to (\frac{a}{\sqrt{2}}\exp(i(a^2 t)), 0) \quad \text{as} \quad t \to -\infty$$

where 2θ is a phase shift, $(a\exp(ia^2 t), 0)$ a fixed circle of DSI [12]. Note that Eq. (2.15) contains not only a periodic wave $\cos(p_1(x - 2y - \alpha t))$, so its amplitude periodically oscillates with the evolution of time (the breather effect), but also a solitary wave $\dfrac{1}{\cosh(p(x - \frac{y}{2} + \alpha t) + \gamma)}$ which shows that the interaction between a solitary wave and a periodic wave with the same velocity α and the opposite propagation direction can form a new family of homoclinic solution. This is a new phenomenon of evolution of a three-dimensional disturbance in the nonlinear regime of plane Poiseeuille flow (Ref. Fig. 1 and 2).

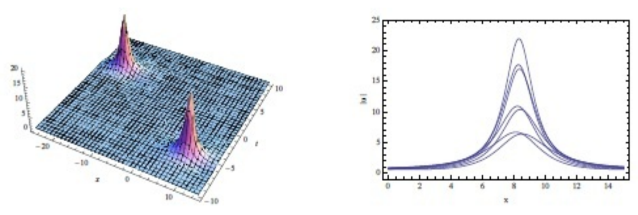

Fig. 1 Behaviour of $|u_1|$ in DSI and the homoclinic breather variation in $x - |u_1|$ plane

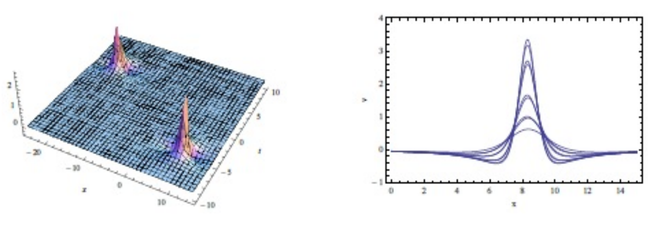

Fig. 2 Behaviour of v_1 in DS1 and the homoclinic breather variation in $x - v_1$ plane

III. HOMOCLINIC BREATHER SOLUTION FOR DSII EQUATION

As we know that $(a\exp(-2i|a|^2 t), 0)$ is a hyperbolic fixed cycle of DSII equation when the period of y is larger than the period of x [12]. Similar to the argument in [12], we can analyse the linear stability of fixed cycle $(a\exp(-2i|a|^2 t), 0)$. By similar process of dealing with of Eq. (1.1), we take the transformation

$$U = \frac{G}{F}, \quad V = A - 2(\ln F)_{xx} \tag{3.1}$$

Eq. (1.3) can be converted into the bilinear form

$$\begin{cases} (iD_t + D_x^2 - D_y^2)G \cdot F = (\lambda + 2A)G \cdot F \\ (D_x^2 + D_y^2 + \lambda)F \cdot F = 2GG^* \end{cases} \tag{3.2}$$

where A is a constant, G is a complex function, F is a real. Now, we take the following ansatz:

$$G = ae^{-i2|a|^2 t}[e^{-p_1(\sqrt{2}x + y - \alpha t)} + b_1\cos(p_2(x - \sqrt{2}y + \alpha t))$$
$$+ b_2 e^{p_1(\sqrt{2}x + y - \alpha t)}]$$
$$F = e^{-p_1(\sqrt{2}x + y - \alpha t)} + b_3\cos(p_2(x - \sqrt{y} + \alpha t))$$
$$+ b_4 e^{p_1(\sqrt{2}x + y - \alpha t)} \tag{3.3}$$

where $a, p, p_1, \alpha, b_3, b_4$ are real, b_1, b_2 are complex. Substituting Eq. (3.3) into Eq. (3.2), we get

$$\lambda = 2a^2, \quad A = -a^2, \quad p_1^2 = \frac{64a^2 + 3(1 - 2\sqrt{2})\alpha^2}{96(2\sqrt{2} - 1)}$$
$$p_2^2 = (4\sqrt{2} - 1)p_1^2, \quad b_1 = \frac{i\alpha + 4\sqrt{2}p_1}{i\alpha - 4\sqrt{2}p_1}b_3$$
$$b_2 = \left(\frac{i\alpha + 4\sqrt{2}p_1}{i\alpha - 4\sqrt{2}p_1}\right)^2 b_4, \quad b_3^2 = \frac{4((4\sqrt{2} - 1)\alpha^2 + 32p_1^2)}{(4\sqrt{2} - 1)(\alpha^2 + 32p_1^2)}b_4 \tag{3.4}$$

where $\alpha^2 < \frac{64a^2}{3(2\sqrt{2} - 1)}$. Substituting Eq. (3.4) into Eq. (3.1), we obtain the following solution for DSII equation:

$$\begin{cases} U = ae^{i(\theta - 2|a|^2 t)} \dfrac{2\cosh(R_1(x, y, t) + i\theta) + \frac{b_3}{\sqrt{b_4}}\cos(R_2(x, y, t))}{2\cosh(R_1(x, y, t)) + \frac{b_3}{\sqrt{b_4}}\cos(R_2(x, y, t))} \\ V = -\dfrac{2M}{b_4(2\cosh(R_1(x, y, t)) + \frac{b_3}{\sqrt{b_4}}\cos(R_2(x, y, t)))^2} \end{cases} \tag{3.5}$$

where

$$R_1(x, y, t) = p_1(\sqrt{2}x + y - \alpha t) + \ln\sqrt{b_4}$$
$$R_2(x, y, t) = p_2(x - \sqrt{2}y + \alpha t)$$

and

$$M = (8b_4 - (4\sqrt{2} - 1)b_3^2)p_1^2$$
$$+ 2p_1^2 b_3\sqrt{b_4}\{(3 - 4\sqrt{2})\cos(R_2(x, y, t))\cosh(R_1(x, y, t))$$
$$- \sqrt{32\sqrt{2} - 8}\sin(R_2(x, y, t))\sinh(R_1(x, y, t))\}$$

$p_1, p_2, \alpha, b_1, b_2, b_3, b_4$ are given by Eq. (3.4) and $e^{i\theta} = \frac{i\alpha + 4\sqrt{2}p_1}{i\alpha - 4\sqrt{2}p_1}$.

Note that if $(U(x, y, t), V(x, y, t))$ is the solution of DSII equation, then $(U(x, -y, t), V(x, -y, t))$ is the solution as well. So, we obtain the solution of DSII equation

$$\begin{cases} U_1 = ae^{i(\theta - 2|a|^2 t)} \dfrac{2\cosh(R_3(x,y,t)+i\theta) + \frac{b_3}{\sqrt{b_4}}\cos(R_4(x,y,t))}{2\cosh(R_3(x,y,t)) + \frac{b_3}{\sqrt{b_4}}\cos(R_4(x,y,t))} \\ V_1 = -\dfrac{2M_1}{b_4(2\cosh(R_{31}(x,y,t)) + \frac{b_3}{\sqrt{b_4}}\cos(R_4(x,y,t)))^2} \end{cases} \quad (3.6)$$

where

$$R_3(x,y,t) = p_1(\sqrt{2}x - y - \alpha t) + \ln\sqrt{b_4}$$
$$R_4(x,y,t) = p_2(x + \sqrt{2}y + \alpha t)$$

and

$$\begin{aligned} M_1 &= (8b_4 - (4\sqrt{2}-1)b_3^2)p_1^2 \\ &+ 2p_1^2 b_3\sqrt{b_4}\{(3-4\sqrt{2})\cos(R_4(x,y,t))\cosh(R_3(x,y,t)) \\ &- \sqrt{32\sqrt{2}-8}\sin(R_4(x,y,t))\sinh(R_3(x,y,t))\} \end{aligned}$$

The solution given by Eq. (3.6) is a homoclinic breather solution different from the homoclinic tubes solution obtained in [12]. It satisfies the periodic boundary condition. In fact, if we take $l_3 = 2\pi/3p_2, l_4 = 2\sqrt{2}\pi/3p_2$, then

$$U_1(x,y,t) = U_1(x+l_3, y+l_4, t), \quad V_1(x,y,t) = V_1(x+l_3, y+l_4, t)$$

It is obvious that Eq. (3.6) is a two-wave kind of homoclinic solution, which has similar structure to Eq. (2.15). But the periodic boundary for DSII is different from DSI. Specially, the oscillation of two-wave for DSII is stronger (Ref. Fig. 3 and 4).

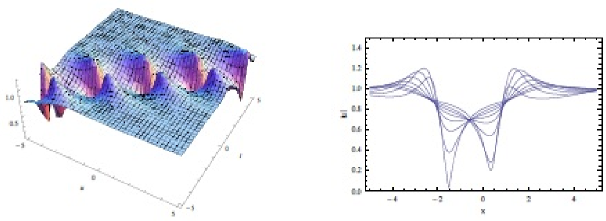

Fig. 3 Behaviour of $|U_1|$ in DSII and the homoclinic breather variation in $x-|U_1|$ plane

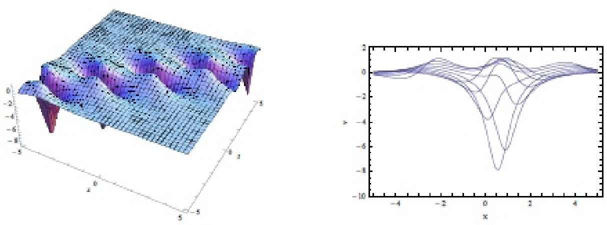

Fig. 4 Behaviour of V_1 in DSII and the homoclinic breather variation in $x-V_1$ plane

IV. CONCLUSIONS

Based on the Hirota bilinear form, by applying the extended homoclinic test method to DSI and DSII equations, we obtain a new kind of homoclinic solutions of two-wave type with locally oscillatory structure. We also investigate and exhibit the different homoclinic structures of solutions. These results show the complexity and variety of dynamical behavior for DS system. Following these ideas in this work, the problem needed to be further studied is whether the other types of nonlinear evolutions have this kind of homoclinic solutions or not.

ACKNOWLEDGMENT

This work was supported by Chinese Natural Science Foundation Grant No. 11061028 and No. 11161055.

REFERENCES

[1] A. Davey and K. Stewartson, "On three-dimensional packets of surface waves," *Proc. R. Soc. Lond.* Ser. A, 338, pp. 101 – 110, 1974.

[2] V. A. Arkadiev, A. K. Pogrebkov and M. C. Polivanov, "Inverse scattering transform method and solitons for Davey-Stewartson II equation," *Physica D*, 36, pp. 189 – 196, 1989.

[3] M. Boiti, J. P. Leon, L. Martina and F. Pempinelli, "Scattering of localized solitons in the plane," *Phys. Lett. A.*, 132, pp. 432 – 439, 1988.

[4] M. Boiti, J. P. Leon and F. Pempinelli, "Multidimensional solitons and their spectral transforms," *J. Math. Phys.*, 31, pp. 2612 – 2618, 1990.

[5] K. W. Chow, "A calss of doubly periodic waves for nonlinear evolution equations," *Wave Motion*, 35, pp. 71 – 90, 2002.

[6] A. S. Fokas and P. M. Santina, "Coherent structures in multidimensions," *Phys. Rew. Lett.*, 63, pp. 1329 – 1333, 1989.

[7] C. R. Girson, "Resonant behavior in the Davey-Stewartson equations," *Phy. Lett. A*, 161, pp. 432 – 428, 1992.

[8] J. Hietarinta and R. Hirota, Multidromion solutions to the Davey-Stewartson equation," *Phys. Lett. A*, 145, pp. 237 – 243, 1990.

[9] J. Satsuma and M. J. Ablowitz, "Two-dimensional lumps in nonlinear dispersive system," *J. Math. Phys.*, 20, pp. 1496 – 1502, 1979.

[10] M. Tajiri and T. Arai, "Periodic soliton solutions to the Davey-Stewartson equation," *Proc. of Institute of NAS of Ukraine*, 30, pp. 210 – 219, 2002.

[11] Y. Watanabe and M. Tajiri, "Periodic soliton resonance: solutions to the Davey-Stewartson equation," *J. Phys. Soc. Jpn.*, 67, 705 – 710, 1998.

[12] Z. Dai and J. Huang, Homoclinic tubes for the Davey-Stewartson II equation with periodic boundary condition," *Chin. J. Phys.*, 43: 2, pp. 349 – 356, 2005.

[13] J. Huang and Z. Dai, "Homoclinic solution for Davey-Stewartson I equation," *Chaos, Soliton and Fractals*, 35, pp. 996 – 1002, 2008.

[14] Z. Dai, A. Zhu, S. Li and D. Li, "Heteroclinic tubes for Davey-Stewartson I equation," *Chin. J. Phys.*, 45: 1, pp. 62 – 71, 2007.

[15] Z. Dai and S. Li, "Homoclinic and heteroclinic flows in blobal attractor for Davey-Stewartson II equation," *Acta. Math, Appl. Sinica*, 24: 4, pp. 599 – 612, 2008.

[16] Z. Dai, J. Liu, X. Zeng and Z. Liu, "Periodic kink-wave and kinky periodic-wave solutions for the Jimbo-miwa equation," *Phys. Lett. A*, 372, pp. 5984 – 5986.

[17] Z. Dai, Z. Li, Z. Liu and D. Li, "Exact homoclinic wave and soliton solutions for the 2D Ginzburg-Landau equation," *Phys. Lett. A.*, 372, pp. 3010 – 3014, 2008.

[18] Z. dai, Z. Liu and D. Li, "Exact periodic solitary-wave solution for KdV equation," *Chin. Phys. Lett.*, 25, pp. 1523 – 1534, 2008.

[19] S. A. EI-Wakil, M. A. Abdou and E. M. Abulwafa, "The three-wave method and its applications," *Nonlinear Science Letters A*, 1, pp. 373 – 378, 2010.

[20] S. A. EI-Wakil, E. M. Abulwafa and M. A. Abdou, "New exact traveling wave solutions of (2+1)-dimensional Burgers equation via three wave method," *Nonlinear Science Letters A*, 1, pp. 391 – 396, 2010.

Zhengde Dai was born in Jiangsu, China, on July, 1945. He received the B.S. degree in mathematics in 1968 from Nanjing University and the M.S. degree in mathematics in 1981 from Yunnan University, China. He got the professor fellowship in 1992. Now, he is the professor in the Department of Mathematics, Yunnan University. His research interests include partial differential equation, nonlinear evolution equation, nonlinear science and infinite dimensional dynamical system. He has published two monographs and more than 130 papers in ISI-listed journals.

Xiping Zeng was born in Guangxi, China, on May, 1966. She received the B.S. degree in mathematics in 1988 from Guangxi Normal University and the M.S. degree in mathematics in 2000 from Fudan University, China. Now she is the assistant professor in the Department of Information and Computing Science, Guangxi University of Technology. Her research interests include partial differential equation and differential dynamical system.

Murong Jiang was born in Hunan, China, on June, 1963. She received the B.S. degree in mathematics in 1984 and the M.S. degree in mathematics in 1989 from Yunnan University, China. She received the Ph.D. degree in mathematics in 1999 from the Institute of Applied Physics and Computational Mathematics of Beijing, China. Now she is the professor in the Department of Computer Science and Engineering, Yunnan University. Her research interests include partial differential equation and its application in parallel image processing.

Chuanjian Wang was born in Yunnan, China, on September, 1984. He received the B.S. degree in mathematics in 2007 from Chuxiong Normal University and the M.S. degree in mathematics in 2010 from Yunnan University, China. Now he is the teaching assistant in the School of Science, Kunming University of Science and Technology. His research interests include partial differential equation and integrable system.