

Multi-Aquifer Parameterization with Well Loss in Vertical Flows

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Abstract—Usually aquifer parameters are estimated through pumping/recharge test. Precondition for using those parameters in the numerical model is to ensure the synonymy of the hydro-geological condition between the field setup and the conceptual model. In many cases, it is erroneous. While numerical modelling in multi-aquifer considered vertical flow in general, it was reflected in the pumping solutions only to the extent of treating a leakage through the aquitard. As such well characterisation in multi aquifers separated by an aquiclude is illusive in the literature. As a result of which, interconnected aquifers were considered as isolated ones in the pumping/recharge tests. Therefore, estimated parameters were for a single aquifer, not for an aquifer connected to multi aquifers. In the present paper, the solution for a multi-aquifer system comprising of two interconnected aquifers separated by an aquiclude is presented. Vertical flow in the well bore with well loss due to friction has been considered. It is concluded that multi-aquifer parameters computed using present solution technique are more scientific and economical.

Keywords—Interconnected aquifers; Diffusivity Ratio; Well Bore Interaction; Equivalent Single Aquifer; the Friction Parameter

I. INTRODUCTION

Neuman and Witherspoon (1969) [21] provided the aquitard drawdown in FEM solution using Darcy's law for groundwater flow in the existing hydrostatic condition in a multi-aquifer system. They also encouraged the use of equivalent parameter values to resolve issues of vertical variation of hydraulic conductivity and specific storage of the aquitard. Herrera (1970) [16] realized the possibility of an analytical solution to describe the vertical flow in a homogeneous aquitard using Duhamel convolution theorem. Later on Hennart et al. (1981) [13] extended it to inhomogeneous multi-aquifer system using eigen function approaches. Using discrete kernel approach, Mishra et al. (1985) [27] analyzed the two-aquifer system. Analytical solutions developed for multi-aquifers in general catered more to evoke the aquifer diffusivity ratios (Sokol, 1963; Wikramaratna, 1984; Hemker, 1984; Hunt, 1985) [34], [38], [14], [17].

In analytical solutions, vertical flows in multi aquifers have been treated as leakage from the aquitards separating the two aquifers. Hemker (1999) [15] developed the uniform well-face drawdown solution including the effects of a finite diameter well, wellbore storage and thin skin. Bakker and Hemker (2002) [4] considered the vertical flow in the layer interfaces of the multi-aquifers. Yeh et al. (2003) [40] developed two layered aquifer model with skin effects. Based on the first order perturbation technique, problem of steady groundwater flow toward a pumping well in multi-aquifer system with anisotropy of the horizontal conductivity was solved by Meesters et al. (2004) [21]. Kim and Parizek (2004) [19] described the Rhode effect caused by (1) a slower head recovery of the un-pumping stress and (2) an amplification of the excessive extension in the lower part of the aquitard due to its relatively higher deformability. They did not found extension from the pumped aquifer into the adjacent aquitard and un-pumped aquifer due to the relatively lower hydraulic conductivity of the aquitard, Using numerical inversions of exact analytical solutions for Laplace transforms, Hunt and Scott (2007) [18] developed an approximate solution for flow to a well in an aquifer overlain by both an aquitard and a second aquifer containing a free surface. Sedghi et al. (2009) [33] developed Laplace domain solutions for three-dimensional groundwater flow to a well in confined and unconfined wedge-shaped aquifers.

Numerical well flow models considered radial and vertical groundwater flow near a well since seventies. Bennet et al. (1982) [5] devised for the calculation of three-dimensional groundwater flow. Sudicky et al. (1995) [35] applied finite elements to simulate well flow in heterogeneous aquifers with wellbore storage. Xiang (1995) [39] used a finite difference code to investigate the skin effect during flow meter tests in layered aquifers. Neville and Tonkin (2004) [20] reviewed four methods to represent multi-aquifer wells with the widely used code MODFLOW (McDonald and Harbaugh, 1988) [25] to demonstrate that results obtained with the Multi Aquifer Well (MAW1) package closely matched exact solutions of Sokol (1963) [34]. Bakker (2006) [3] presented an analytic element approach for the modeling of steady groundwater flow through multi-aquifer systems with aquifer and leaky layer properties. Craig and Gracie (2011) [11] applied extended finite element method to the problem of transient inter-aquifer leakage through passive wellbores. Precedence out of the literature, discussed above, does not guarantee the numerical model calibration of multi-aquifer system, like it does in the case of a single aquifer. The reason behind low reliability is the model parameters, which were estimated under different hydro-geological condition. Such inconsistency develops; firstly, due to difficulty in extending the slug or pump test solutions of single aquifer to multi-aquifer setup. Secondly, due to the fact that the complex analytical and numerical solution exclusively developed for multi-aquifers, could not be amicably adopted in the field tests. Hence in the multi aquifer tests, parameterisation actually dealt with isolated aquifers overlooking the interconnectivity factor.

Analytical solutions are available dealing with interconnectivity between various layers of multi-aquifer, mostly limited to aquifer-aquitard setups. Neuman and Witherspoon (1972) [22] put forward the idea of putting observation wells in the over and underlying aquitards for a pumping test in a multi-aquifer setup. Cheng and Morohunfola (1993) [8] presented an analytical solution of a radial flow to a pumping well at constant discharge located in a multilayered leaky aquifer system, to evaluate the drawdown at any given time and location in any layer. Miyake et al. (2008) [28] carried out multi-aquifer pumping tests, in multi-screen pumping well and multi-level piezometers in multi-layered confined aquifers to estimate parameters in aquifers, and also for low permeability layers between the aquifers, using the Cooper-Jacob method and a finite element groundwater model. Cihan et al. (2011) [9] developed analytical solutions accounting for the combined effect of diffuse and focused leakage to solve for pressure changes in a system of N aquifers with alternating leaky aquitards in response to fluid injection/extraction from any number of layers. Basic ingredient to the mathematical formulations of the above-stated solutions remained a minimum boundary condition of leakage face. In the absence of such boundary condition, solutions required numerical integration. Ruud and Kabala (1997) [32] and Zlotnik and Zurbuchen (2003) [41] showed the depth wise head loss distribution in borehole flow meter test, where parameter estimation required numerical techniques. Difficulty in adopting numerical solutions encouraged field testing in confined aquifers separated by aquiclude, to deal with each aquifer turn by turn, packing rest of the aquifers.

Elsewhere, well bore model in Sandia Waste Isolation Flow and Transport (SWIFT) as detailed in Reeves et al. (1986) [31] included solution for frictional pressure drops in the wellbore as;

$$dp = \rho g \phi dz / g_c + \tau_f dz \quad (1)$$

Where τ_f is related to friction factor 'f' through $\tau_f = f \rho u^2 / 2r_T$, where $\tau_f r_T / 2$ is the shear stress in the well bore face in the direction of flow. For laminar flow, the friction factor is taken as $f = 64 / \text{Re}$; $\log_{10}(\text{Re}) > 3.3$; For the transition zone, $f = 10^x$; $3.3 \log_{10}(\text{Re}) > 3.6$, with the exponent given by the polynomial, $x = 260.67 - 228.62 \log_{10}(\text{Re}) + 66.307 \log_{10}(\text{Re})^2 + 6.3944 \log_{10}(\text{Re})^3$; For the turbulent flow region, the surface roughness fr of the wellbore becomes; $f = f(\text{Re}, fr)$. The second term on the right-hand side in equation (1), supplemented to the pressure difference between the two well skins open to successive aquifers. In the present paper, this term is included and treated analytically for the multi-aquifer setups to address the interaction between the aquifers via Darcian or non-Darcian vertical flow through the wellbore as highlighted by Chen and Jiao (1999) [7]. Well-loss characterization in the wellbore is studied using the approach of Duhamel Convolution theorem under hydrostatic conditions. Single aquifer solution of Majumdar et al. (2013) [23] has been extended to two aquifer systems separated by an aquiclude. Both steady and unsteady well water level conditions have been solved. Testing setup required for the use of the type curves generated in the present paper is different to that exist currently, which has lead to defer field validation till the theory becomes established, and subsequently commensurable methodology for field testing could be practised.

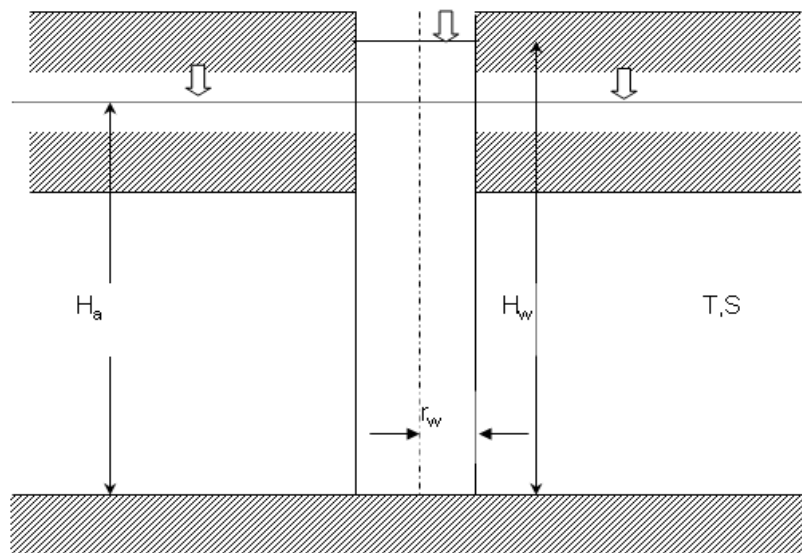


Fig. 1 Schematic diagram of the recharge well

II. SOLUTION FOR SINGLE AQUIFER

In Fig. 1, schematic cross-section of a fully penetrating recharge well in a single confined aquifer is shown with initial piezometric head in the aquifer at a height H_a from the bottom of the aquifer. Transmissivity and storage coefficient of the aquifer are T and S respectively. The well screen and unscreened portions have the same radius r_w . For augmenting recharge, water

well is filled to a height H_w , so that piezometric water level in the aquifer goes up whereas, water level in the well starts receding. In a particular time step n , the piezometric head in the aquifer at radial distance r from the recharge well is given by

$$H_a(r, n) = H_a(r, 0) + s(r, n) \quad (2)$$

Here $s(r, n)$ is the head rise in the aquifer at a radial distance r from the recharge well and given by (Morel-Seytoux and Daly, 1975) [29]

$$s(r, n) = \sum_{\gamma=1}^n Q_a(\gamma) \delta(r, \Delta t, n - \gamma + 1) \quad (3)$$

$\delta(r, \Delta t, n - \gamma + 1)$ is known as Discrete Kernel coefficient or Delta function. Other notations are listed at the end of the paper. Derivation of the Delta function ' δ ' is included as Appendix-A, using Theis (1935) [37] well function for the unsteady flow in a confined aquifer. With friction loss in the well bore, Duhamel's convolution equation is written as;

$$H_w(0) - H_a(0) = \frac{1}{\pi r_w^2} \sum_{\gamma=1}^n Q_a(\gamma) + \sum_{\gamma=1}^n Q_a(\gamma) \delta(r, \Delta t, n - \gamma + 1) + \left(\frac{Q_a(n)}{\pi r_w^2} \right)^2 \frac{1 + kl}{2g} \quad (4)$$

In equation 3, k is the friction parameter (f/D), where f is the friction factor in Darcy-Weisbach equation for head loss due to friction h_f in the recharge well given by the equation 5.

$$h_f = \frac{flv^2}{2gD} \quad (5)$$

Value of friction factor is available in Moody's diagram (Featherstone and Nalluri, 1982) [21] for known Reynolds number. Development of the single aquifer solution with head loss due to friction could be followed elsewhere (Majumdar et al., 2013) [23], where the solution was validated with the theoretical solution of Cooper et al. (1967) [10] and field data of Hansol injection well near Ahmedabad, Gujarat, India. Majumdar and Mishra (2007) [24] have documented the type curves, which require only well water level observations. In the present article, single aquifer solution has been extended to two aquifer setups separated by an aquiclude, to discuss good characterization in multi aquifers.

III. EXTENSION TO MULTI-AQUIFER SETUP

For the multi-aquifer setups, simultaneous equations were formulated under hydrostatic condition using Bernoulli's theorem. A two-layered confined aquifer with an initial head of $H_a(0)$ is shown in Fig. 2. Aquifers have transmissivity T_1 in the first aquifer and T_2 in the second, and the respective storativity values are S_1 and S_2 . Radius of the well for both screened and unscreened portion is r_w . The aquifers are recharged by a fully penetrating well up to the height of $H_w(0)$. Let the quantity of water moved to first, and second aquifers are Q_{a1} and Q_{a2} respectively. Q_w is the quantity of water moved out of the well. It is required to derive expressions to compute Q_{a1} and Q_{a2} for a constant head well condition and variable head conditions.

A. Constant Water Level in the Well

Applying Bernoulli's equation between well water level and elevation head at point 2 (at aquifer 1 top level) of a multi-aquifer system in Fig. 2, using the notations there and k as friction factor per unit diameter of recharge well,

$$H_w(0) = kl_1 \frac{V^2}{2g} + \frac{V^2}{2g} + \sum_{\gamma=1}^n Q_{a1}(\gamma) \delta_1(r, \Delta t, n - \gamma + 1) + h_2 + h_1 \quad (6a)$$

$$H_w(0) = [1 + kl_1] \left[\frac{Q_{a1}(n) + Q_{a2}(n)}{\pi r_w^2} \right]^2 \frac{1}{2g} + \sum_{\gamma=1}^n Q_{a1}(\gamma) \delta_1(r, \Delta t, n - \gamma + 1) + h_2 + h_1 \quad (6b)$$

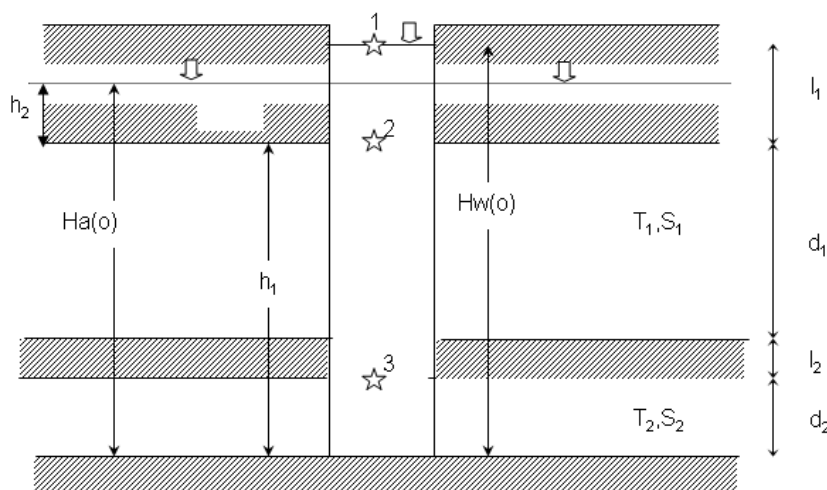


Fig. 2 Schematic diagram of recharge well in multi-aquifer setup

Then applying same between point 3 (at aquifer 2 top level) and 2,

$$h_1 + \frac{1}{2g} \left[\frac{\{Q_{a1}(n) + Q_{a2}(n)\}}{\pi r_w^2} \right]^2 + \sum_{\gamma=1}^n Q_{a1}(\gamma) \delta_1(r, \Delta t, n - \gamma + 1) + h_2 =$$

$$\int_0^{d_1} \frac{k}{2g} \left[\frac{\left\{ Q_{a1}(n) + \left(1 - \frac{x}{d_1}\right) Q_{a2}(n) \right\}}{\pi r_w^2} \right]^2 dx + \frac{(1 + k l_2)}{2g} \left[\frac{Q_{a2}(n)}{\pi r_w^2} \right]^2 +$$

$$\sum_{\gamma=1}^n Q_{a2}(\gamma) \delta_2(r, \Delta t, n - \gamma + 1) + d_2 + l_2 + d_1 + h_2 \quad (7)$$

Where, δ_1 and δ_2 are the Discrete Kernel coefficient or Delta function of aquifer 1 and 2 respectively. Other notations are listed at the end of the paper.

Equations 1 and 2 are simplified in the following form,

$$bx^2 + by^2 + 2bxy + \delta_1(r, \Delta t, 1)x - a = 0 \quad (8)$$

$$vx^2 + wy^2 + sxy - \delta_1(r, \Delta t, 1)x + \delta_2(r, \Delta t, 1)y - c = 0 \quad (9)$$

where,

$$x = Q_{a1}(n) \quad (10)$$

$$y = Q_{a2}(n) \quad (11)$$

$$a = H_w(o) - h_2 - h_1 - \sum_{\gamma=1}^{n-1} Q_{a1}(\gamma) \delta_1(n - \gamma + 1) \quad (12)$$

$$b = \frac{(1 + k l_1)}{2g \pi^2 r_w^4} \quad (13)$$

$$c = \sum_{\gamma=1}^{n-1} Q_{a1}(\gamma) \delta_1(n - \gamma + 1) - \sum_{\gamma=1}^{n-1} Q_{a2}(\gamma) \delta_2(n - \gamma + 1) \quad (14)$$

$$r = \frac{1}{2g\pi^2 r_w^4} \quad (15)$$

$$u = \frac{kd_1}{2g\pi^2 r_w^4} \quad (16)$$

$$e = \frac{(1 + kl_2)}{2g\pi^2 r_w^4} \quad (17)$$

$$v = (u + r) \quad (18)$$

$$s = \left(\frac{3}{2}u + 2r \right) \quad (19)$$

$$w = \left(\frac{1}{3}u + r + e \right) \quad (20)$$

For solving equations (6) and (7) Newton-Raphson method was employed.

B. Variable Water Level in the Well

The following three equations were written for the solution of three unknowns Q_{a1} , Q_{a2} and Q_w for varying water level in the well, where Q_w is the change in well storage.

$$Q_{a1}(n) + Q_{a2}(n) = Q_w(n) \quad (21)$$

$$H_w(o) - \frac{1}{\pi r_w^2} \sum_{\gamma=1}^n Q_{a1}(\gamma) - \frac{1}{\pi r_w^2} \sum_{\gamma=1}^n Q_{a2}(\gamma) = H_a(o) + \sum_{\gamma=1}^n Q_{a1}(\gamma) \delta_1(r, \Delta t, n - \gamma + 1) + \left[\frac{Q_{a1}(n) + Q_{a2}(n)}{\pi r_w^2} \right]^2 \frac{1}{2g} + \frac{kl_1 \left[\frac{Q_{a1}(n) + Q_{a2}(n)}{\pi r_w^2} \right]^2}{2g} \quad (22)$$

and

$$H_a(o) + \sum_{\gamma=1}^n Q_{a1}(\gamma) \delta_1(r, \Delta t, n - \gamma + 1) + \left[\frac{Q_{a1}(n) + Q_{a2}(n)}{\pi r_w^2} \right]^2 \frac{1}{2g} = H_a(o) + \left[\frac{Q_{a2}(n)}{\pi r_w^2} \right]^2 \frac{1}{2g} + \frac{kl_2 \left[\frac{Q_{a2}(n)}{\pi r_w^2} \right]^2}{2g} + \int_0^{d_1} \frac{k}{2g} \left[\frac{Q_{a1}(n) + \left(1 - \frac{x}{d_1}\right) Q_{a2}(n)}{\pi r_w^2} \right] dx + \sum_{\gamma=1}^n Q_{a2}(\gamma) \delta_2(r, \Delta t, n - \gamma + 1) \quad (23)$$

Further simplification reduced equation 22 and 23 similar to equations 4 and 5 respectively, except the current value of $H_w(o)$ required to be modified every time step. Again Newton-Raphson method was applied for the solution of the set of equations to evaluate the values of Q_{a1} and Q_{a2} .

IV. RESULTS OF ANALYSIS

For the schematic single aquifer configuration of Fig. 3, illustrations depicting computation of well heads and recharge rates are shown in Fig. 4 and Fig. 5 respectively. Values were computed using equation 4, with discrete kernel of Theis (1935) [37] well function. Influence of the friction parameter is well evident with well loss dissipating the well water column in Fig. 4 and non-dimensional recharge rates in Fig. 5 much slowly. The range of friction parameters used in the analyses covered Darcian and non-Darcian flow regimes in a well bore. The single aquifer formulation when extended to the two aquifer setup, the friction parameter generated a difference between the piezometric head of successive aquifers, causing an interaction between

the aquifers through well bore. Interaction through the interfaces between aquifer and aquiclude was not considered. Such demonstrative analysis was carried out, comparing the recharge well water levels in the multi-aquifer with those for an equivalent single aquifer. A hypothetical case of two-aquifer system, each 5 m thick, separated by a 3 m thick aquiclude was considered. Confining layer of the top most aquiclude was having a thickness of 5 m. Diameter of the well was 1.0 m. Initial water level depth was 14 m and the recharge well was poured up to the depth of 16 m. Storage coefficient was kept constant as 0.01 and the friction factor per unit diameter of the well was considered as zero and 1.0 respectively for respective sets of analyses.

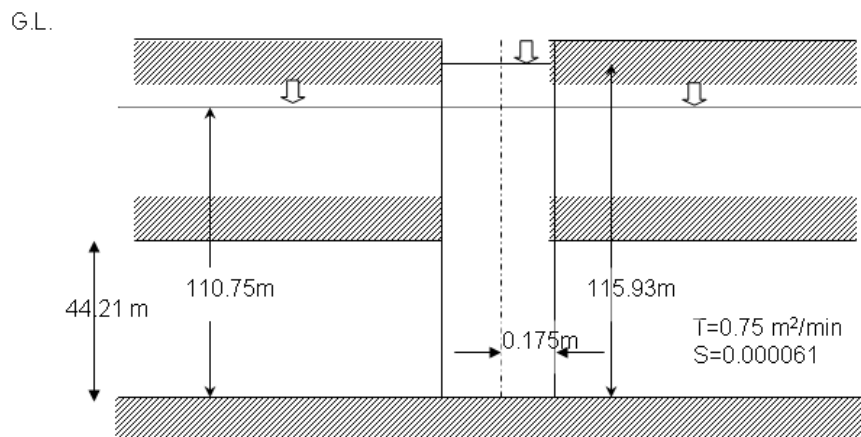


Fig. 3 Conceptual aquifer configurations for the illustrative example

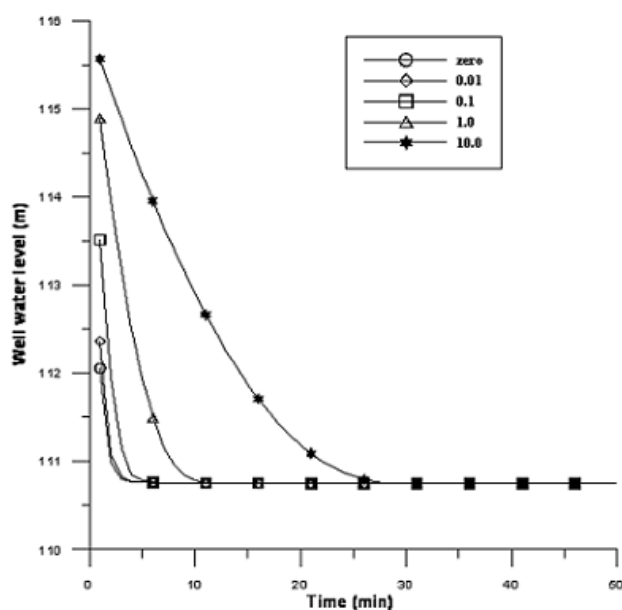


Fig. 4 Change in well water level with friction parameter

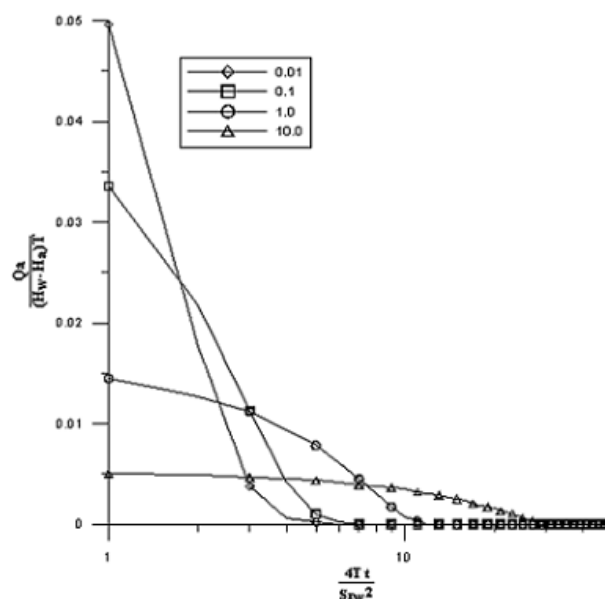


Fig. 5 Non-dimensional recharge rates for different friction parameter

For the equivalent single aquifer, T and S values were obtained by summing the T and S values of the individual aquifers of the multi-aquifer system, because T , S and thickness in each of the aquifers of multi-aquifer system were equal. Recharge well water column and static water level in the multi-aquifer case and the equivalent single aquifer case were maintained at the same level. Consequently, the aquiclude thickness between the two aquifers of multi-aquifer system was accounted in the equivalent single aquifer thickness. In Fig. 6, results of multi-aquifer solution in case of no friction loss show a good match between the head decline in the well for single and two-aquifer cases. In the case of friction loss ($k=1$) the water level in the well declined differently for equivalent single and two-aquifer system as shown in Fig. 7, indicating nonlinear behavior arising due to well loss and subsequent interaction between the aquifers through well bore. This behavior could be Darcian or non-Darcian, depend upon the range of k and well radius used in the analysis. Outcome of this analysis is of paramount practical importance, as during the analysis it was found that existing concept of equivalent single aquifer did not apply, when head loss during recharge test was accounted. Therefore, present parameterisation techniques in multi aquifer are questionable. Incorporation of such parameters for numerical model calibration is an added time-consuming effort due to the increase in the number of trials. Seldom calibrated parameters could be the field tested values.

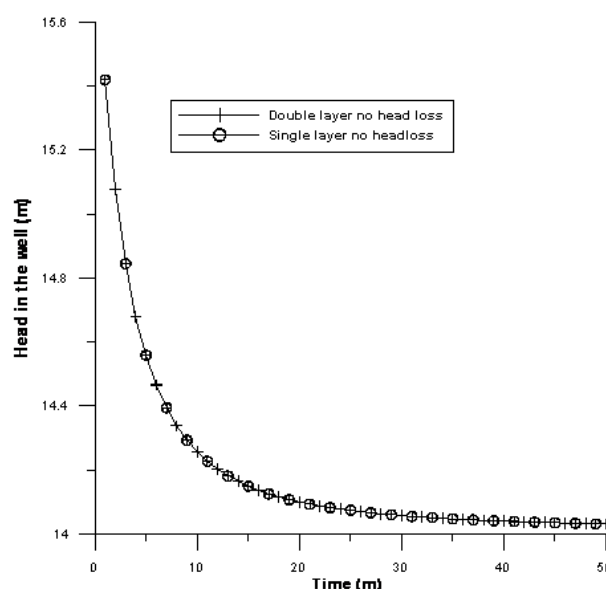


Fig. 6 Comparison of recharge well water levels for single and multi aquifer without friction loss

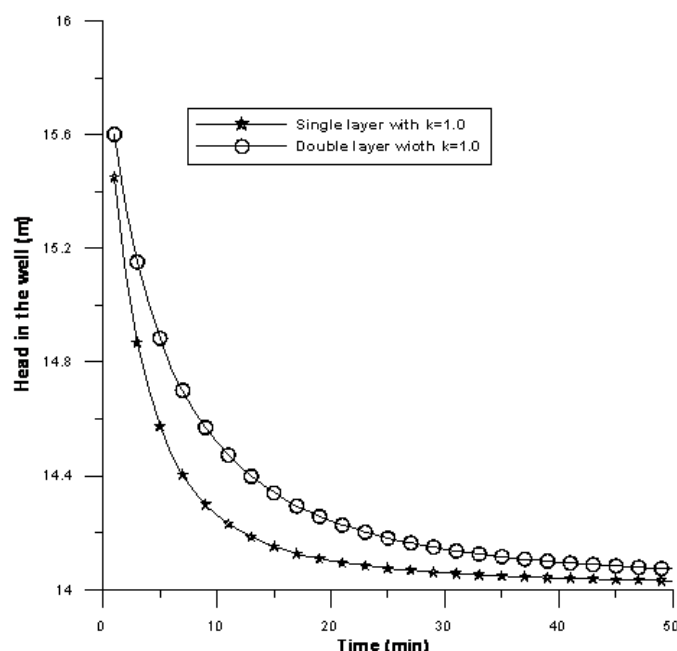


Fig. 7 Comparison of recharge well water levels for single and multi aquifer with friction loss

To study the effect of diffusivity between the two aquifers, different ratios ranging from 0.2 to 5.0, were tried out. Recharge rate variations to both the aquifers under constant and variable head conditions are shown in Figs. 8 to 10 for different diffusivity ratios. Recharge to the individual aquifer increases with the increase in the diffusivity of that particular aquifer. Accordingly the recharge to the remaining aquifer decreases. However, there is no indication that recharge ratio between individual aquifer follows the diffusivity proportions. In a multi-aquifer setup, type curves such as Fig. 8 to Fig. 10 could make it convenient to ascertain the diffusivity ratios between the two aquifers. Consequently, if diffusivity of one aquifer is known using Majumdar and Mishra (2007) [24], parameterization for the other aquifers becomes hand calculation. When applied to model calibration, lesser trials would be required due to synonymy in the hydro-geological conditions of the testing setup and the conceptual model. Further, field testing could be carried out at much cheaper financial investments, as packers in the remaining layers are no more required.

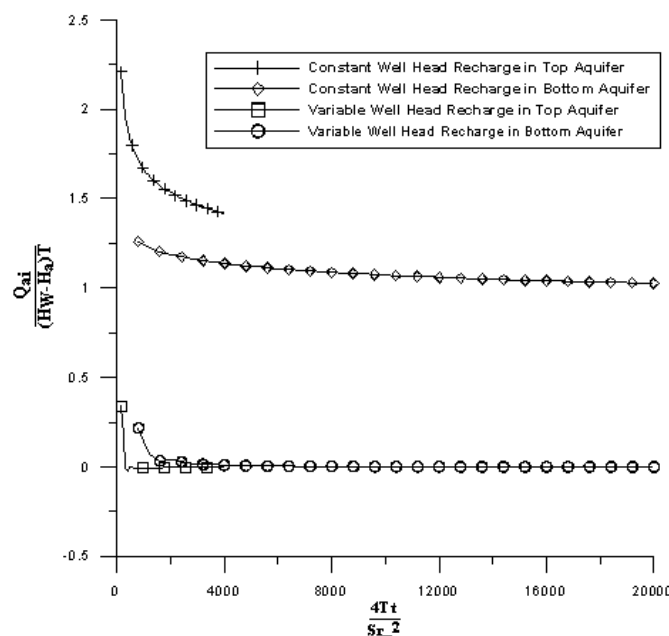


Fig. 8 Recharge rates for aquifer diffusivity ratio (top/bottom) as 0.2

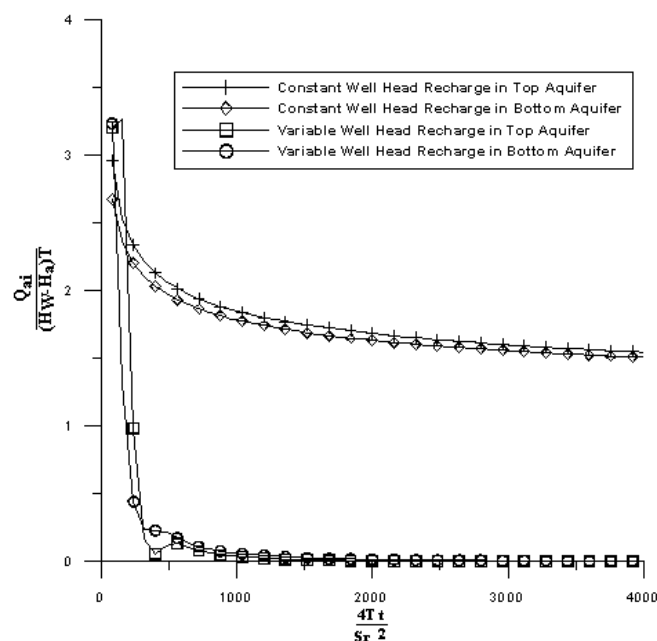


Fig. 9. Recharge rates for aquifer diffusivity ratio (top/bottom) as 1.0.

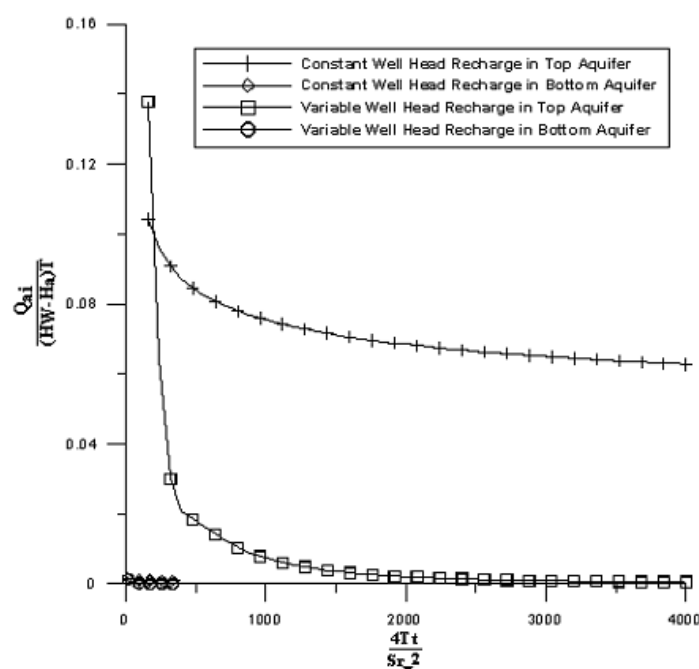


Fig. 10 Recharge rates for aquifer diffusivity ratio (top/bottom) as 5.0

Similarly, piezometric heads in the recharge well for constant and variable head conditions are shown in Fig. 11. Under the variable head condition, water level in the well goes down with a steeper rate initially and subsequently gains a flatter trend with time. It depends upon the ratio of the diffusivity of the two aquifers and the friction parameter. In Fig. 11, with the increase in the ratio of diffusivities for the top and bottom aquifer, water level in the recharge well is improving. It is attributed to the effect of the decreasing total diffusivity value of the two aquifers and likely to differ in case of an overall increase in the total diffusivity value. These exercises accounted head loss due to velocity of flow and friction in the recharge well, which caused unequal piezometric heads in the individual aquifers at the well face. Unique feature of Duhamel's convolution theorem allowed coupling of momentum with mass balance in the recharge well. Present formulation estimates unsteady recharge to the individual aquifers under current wellhead condition. Flexibility in transforming the head into a flux and vice-versa at the well face makes it possible to generate pressure and recharge solutions simultaneously but successively. Algorithm-wise, present exact solution satisfies the conservation of mass and momentum in an identical manner with which an approximate solution derives boundary conditions in the numerical methods. Therefore, present solution technique could be termed as semi-analytical rather than strictly analytical. Although conceptually the results were verified, however, it could not be compared

with the field observation, mainly due to the absence of reconcilable testing methodology. Present accurate solutions are simple to adopt for parameterization of multi-aquifer setups with much cheaper experimental setup, not warranting the use of packers anymore.

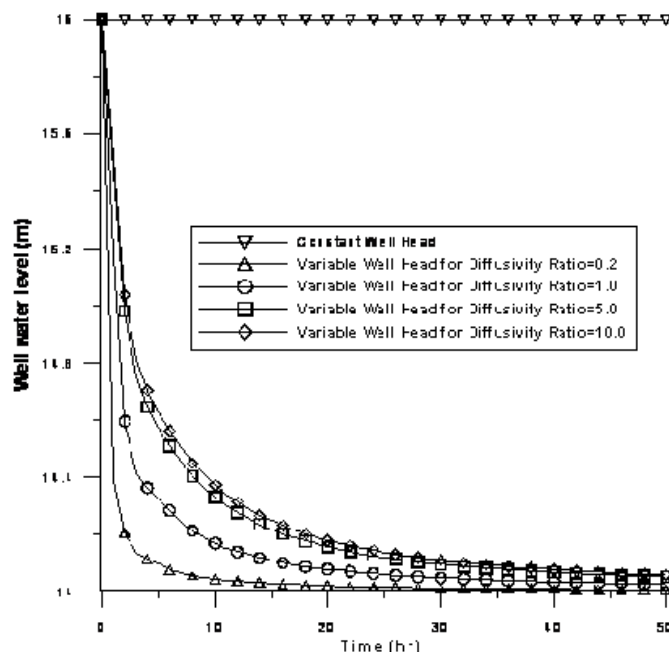


Fig. 11 Recharge well water levels for different aquifer diffusivity ratios

V. CONCLUSIONS

In the present paper, the semi-analytical solution for a single aquifer is extended to the two-layered aquifer system for both constant and variable wellhead conditions. With the incorporation of head loss due to friction in the recharge well, aquifer-to-aquifer interaction through well bore was formulated and established in an exact solution in a way, similar to that of numerical models. Solution technique is flexible and can be extended to the systems comprising of any number of aquifers separated by suitable numbers of aquicludes. Parameter estimation through present solution will ensure precision along-with time and cost effectiveness. More importantly, estimated parameters have a scope of reconciling with the field conditions, when used for the model development, using numerical methods. Model calibration is expected to be much smoother.

Notations

H_a = Height of piezometric level above datum (L)

H_w = Height of well water level above datum (L)

d_1, d_2 = Aquifer thickness (L)

l_1, l_2 = Aquiclude thickness (L)

r_w = Well radius (L)

r_c = Well casing radius (L)

r = Radial coordinate (L)

D = Well bore diameter (L)

h_f = Head loss due to friction (L)

S = Storativity (ratio)

T = Transmissivity (L^2/T)

Q_a = Rate of recharge at well face (L^3/T)

Q_r = Rate of injection (L^3/T)

Q_w = Change in well storage (L^3/T)

v = Darcy flux (L^3/T)

δ = Discrete Kernel ($L/(L^3/T)$)

f = Friction factor (non-dimensional)

g = Acceleration due to gravity ($L/T/T$)

R_e = Reynold's number (non-dimensional)

ν = Coefficient of viscosity (L^2/T)

k = Dimensional friction coefficient ($/L$)

t = Time (T)

Δt = Time step size (T)

n = Time step count (number)

γ = Time step count (number)

l = Distance between well water level and top of the aquifer (L)

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APPENDIX - A

Derivation for Discrete Kernel, $\delta(m)$ in Duhamel's Convolution Theorem (After Patel and Mishra, 1983) [30].

The basic differential equation for axially symmetric, radial, unsteady flow groundwater flow in a homogeneous, isotropic, confined aquifer of uniform thickness is given by

$$\frac{\partial^2 s}{\partial t^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t}.$$

where S is the storage coefficient and T is the transmissivity.

For initial condition $s(r, 0) = 0$ (complete horizontal water table) and boundary condition $s(\infty, t) = 0$, solution to the above PDE, when a unit impulse quantity is withdrawn from the aquifer is given by (Carslaw and Jaeger, 1959) [6]

$$s(r, t) = \frac{e^{-\frac{r^2}{4\beta t}}}{4\pi T t}; \text{ where } \beta = \frac{T}{S}$$

This is a unit impulse kernel $K(t)$. Therefore, draw down during time t for a variable pumping rate Q_p at time τ can be written in the form

$$s(r, t) = \int_0^t Q_p(\tau) K(t - \tau) d\tau$$

$$s(r, t) = \int_0^t \frac{Q_p(\tau)}{4\pi T} \frac{e^{-\frac{r^2}{4\beta(t-\tau)}}}{(t-\tau)} d\tau$$

Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each time step but varies from step to step, the drawdown at the end of time step n , if $t = n\Delta t$, can be written as (Morel-Seytoux, 1975) [29]

$$\begin{aligned} s(r, t) &= \int_0^{\Delta t} \frac{Q_p(\tau)}{4\pi T} \frac{e^{-\frac{r^2}{4\beta(t-\tau)}}}{(t-\tau)} d\tau + \int_{\Delta t}^{2\Delta t} \frac{Q_p(\tau)}{4\pi T} \frac{e^{-\frac{r^2}{4\beta(t-\tau)}}}{(t-\tau)} d\tau + \dots + \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} \frac{Q_p(\tau)}{4\pi T} \frac{e^{-\frac{r^2}{4\beta(t-\tau)}}}{(t-\tau)} d\tau + \dots \\ &\quad + \int_{(n-1)\Delta t}^{n\Delta t} \frac{Q_p(\tau)}{4\pi T} \frac{e^{-\frac{r^2}{4\beta(t-\tau)}}}{(t-\tau)} d\tau \\ &= \sum_{\gamma=1}^n \int_{(\gamma-1)\Delta t}^{\gamma\Delta t} \frac{Q_p(\tau)}{4\pi T} \frac{e^{-\frac{r^2}{4\beta(n\Delta t-\tau)}}}{(n\Delta t-\tau)} d\tau \end{aligned}$$

$$\text{Let } \frac{r^2}{4\beta(n\Delta t-\tau)} = u$$

$$\text{So that } \tau = n\Delta t - \frac{r^2}{4\beta(u)} \text{ and } d\tau = \frac{r^2}{4\beta u^2} du$$

$$\begin{aligned} \text{Therefore } s(r, t) &= \sum_{\gamma=1}^n \int_{\frac{r^2}{4\beta(n\Delta t-(\gamma-1)\Delta t)}}^{\frac{r^2}{4\beta(n\Delta t-\gamma\Delta t)}} \frac{Q_p(\tau)}{4\pi T} \frac{e^{-u}}{\frac{r^2}{4\beta u}} \left(\frac{r^2}{4\beta u^2} du \right) \\ &= \sum_{\gamma=1}^n \int_{\frac{r^2}{4\beta(n-\gamma+1)\Delta t}}^{\frac{r^2}{4\beta(n-\gamma)\Delta t}} \frac{Q_p(\gamma)}{4\pi T} \frac{e^{-u}}{u} du \\ &= \sum_{\gamma=1}^n \int_{\frac{r^2}{4\beta(n-\gamma+1)\Delta t}}^{\infty} \frac{Q_p(\gamma)}{4\pi T} \frac{e^{-u}}{u} du - \int_{\frac{r^2}{4\beta(n-\gamma)\Delta t}}^{\infty} \frac{Q_p(\gamma)}{4\pi T} \frac{e^{-u}}{u} du \\ &= \sum_{\gamma=1}^n \frac{Q_p(\gamma)}{4\pi T} \left[E\left(\frac{r^2}{4\beta(n-\gamma+1)\Delta t}\right) - E\left(\frac{r^2}{4\beta(n-\gamma)\Delta t}\right) \right] \\ s(r, n\Delta t) &= \sum_{\gamma=1}^n Q_p(\gamma) \delta(r, n-\gamma+1) \end{aligned}$$

where the discrete kernel coefficient $\delta(r, n-\gamma+1)$ is defined as

$$\delta(r, n-\gamma+1) = \frac{1}{4\pi T} \left[E_1\left(\frac{r^2}{4\beta(n-\gamma+1)\Delta t}\right) - E_1\left(\frac{r^2}{4\beta(n-\gamma)\Delta t}\right) \right]$$

where $E_1(\cdot)$ is an exponential integral defined as $E_1(X) = \int_X^\infty (e^{-u}/u) du$ (Abramowitz and Stegun, 1970) [1]. The discrete kernel coefficient $\delta(r, n - \gamma + 1)$ is the drawdown at the end of the n^{th} time period at a distance r from the pumping well in response to withdrawal of a unit quantity of water from the aquifer storage during the first time period. A unit time period may be 0.1 day, 1 day, or 1 week. Mishra et al. (1985) [27] found that with any time step size the error in the solution decreases with time and with minimum number of $t/10$ time steps the response of the aquifers at time t can be calculated with reasonable accuracy.