Twelve-Link Exoskeleton with Biological Endoskeleton Properties

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***Abstract*-A model of a novel twelve-link exoskeleton, which allowed body torsion in space, was proposed. The equations of motion in regards to deformability of the exoskeleton links, i.e. biological properties of the endoskeleton, were composed based on theoretical mechanics. A matrix form of the differential equations of motion was used. Generalizations for the matrix elements were also obtained for the first time. The obtained results allowed for an approximation of exoskeletal motions in regards to the endoskeleton, and calculations of separate links as well as of the whole system were carried out, which had a practical value for approximating exoskeletal motions to human bodies.**

Keywords*- *Matrix; Nonlinear Differential Equations; A Single-Support Phase; A Human Locomotorium; Exoskeleton; Endoskeleton

I. INTRODUCTION

The first exoskeletons were developed in the 1960s [1] by the American company General Electric. It was a hydraulic design called Hardiman that weighed about 700 kg, rendering the exoskeleton slow and low capacity. However, the interest in exoskeletons increased. In addition to military robots, a course on robotics and cybernetical war demanded new equipment for living units. Perspective developer teams in the USA were given allocations of 20 million dollars each year, with multiple teams being encouraged to participate. The result of the competition was a rapid and tangible breakthrough in the development of exoskeletons, and within ten years scientists had presented many different designs of exoskeletons. The Defense Advanced Research Projects Agency (DARPA) is responsible for developing new technologies for use in the United States armed forces, and continues to finance the development of exoskeletons [2].

As a result of the combined design of Raytheon and Sarcos, the XOS exoskeleton was created. A model unveiled in 2008 had the disadvantages of high power consumption and limited range of motion. Several ensuing releases managed to reduce energy consumption and increase motor potential. However, even the latest model of the exoskeleton has failed to meet all the requirements of DARPA [3], falling short of real autonomy due to high power consumption when driving, which will quite possibly limit the XOS to utility work.

Designers from the University of California have gone in a different direction [1]. They identified three main functions of a soldier: shoot, run, and carry, and developed an exoskeleton for the latter two functions to strengthen the legs and back of a soldier. In 2004 the development of the HULC exoskeleton began with the Lockheed Martin firm [4]. The exoskeleton allows a person to carry loads up to 90 kg and run at speeds of 16 km/h, working autonomously. A lithium battery charge lasts for 72 hours of active use. The eLEGS exoskeleton, produced by Berkeley Bionics (Berkeley, CA), is based on the HULC exoskeleton and was designed for people with disabilities [5].

Japanese engineers and scientists are actively working on exoskeletal designs. A hybrid assistive limb model called HAL-5 from the Japanese Corporation Cyberdyne has been in production since 2005, and has foot and hand strengtheners aimed to help rescue services and people with disabilities. It can work autonomously and costs about 60 thousand dollars [6].

A support structure developed in Norway called Atlas [7] supports a human skeleton in a certain position, which helps shield against injury during strenuous physical activity. There is an electromotor in the hinge joint of the exoskeleton, and sensors detect pressure at any given time. The exoskeleton allows the user to move loads up to 100 kg and the battery is charged with an electricity supply network.

An experimental apparatus created at the University of California in 2009 has actuators controlled by the wearer's neural signals. In previous exoskeleton models, a tracking servo measured the force (or weak movement) of the arms and legs of the user, multiplied it by a constant, and implemented it [8]. The UC model uses non-invasive surface electromyography to link to the musculoskeletal system using sensors to collect the currents and connect to the muscle commanders. This simultaneous functioning of human and exoskeleton is achieved if the actuators change the links simultaneously with the start of the movement of live limbs.

Models of the human muscular system for musculoskeletal systems have also been reported in the works of foreign authors.

A simulation of human body parts is provided in the work of Johnson and Spalding [9]. The development of mechanisms based on soft models is continuously active. A soft robot octopus model based on formable limbs was developed in Italy [10] for their OCTOPUS project. They are trying to simulate the muscles so that they could change the direction of the action, the length and stiffness.

Kinematics of a foot for a bipedal robot that is capable of moving up stairs has been studied by Chinese scholars. The simulation results of a dynamic robot model with elastic links have also been discussed [11].

A large number of different robot models were presented in the Hong Kong ICRA conference in 2014 [12].

The common drawback of all models of exoskeletons is their rigidity. The small amount of freedom in areas of movable joints is not enough; it creates constraint and unnatural movements. If the number of degrees of freedom is increased, there is great difficulty in controlling movements and large amounts of energy are required among other complications. As can be seen with the proposed materials, engineering, based on empirical data, is primarily involved. Judging by the available data, existing exoskeletons consume a lot of energy and are short-lived. This means that the fundamental mathematical development of human walking and the mechanical and kinematic models of the musculoskeletal system are not being used sufficiently. If the scientific and technical works are analyzed, it can be seen that the development of exoskeletons and anthropomorphic robots is still in the initial stages. Mechanisms have been built without taking into account the properties of human tissues, structure of bones, joints and muscles, wear and destruction that defines the inadequacy of human movement, and high energy consumption. It is essential to thoroughly examine the structure and functions of the musculoskeletal system of a human, low coefficients of friction in the joints, and mechanisms of recuperation in order to make exoskeletons more comfortable for to wearer. The motion of the exoskeleton and anthropomorphic robots have thus far failed to adequately imitate the human body.

II. MODEL LINK OF VARIABLE LENGTH

Model links, which can change their lengths, are a basic component of this study. The basic concepts used will be introduced and the behavior of the link explained. A mathematical model of a walking anthropomorphic mechanism with rigid links was considered in earlier studies of biomechanics of human movement [13-16]. However, erroneous calculations resulted in shock accelerations that were too large while setting the foot on a support. With each step on a hard surface, the foot experienced an impact load equal to 18-20 g acceleration, a significant portion of which was absorbed due to the structure of the foot; about 6-7 g passed through the shank [17]. Forces arising when putting the legs on a support in a rigid model would lead to the destruction of the musculoskeletal system of a human, which is why a model of links that minimizes shock loads on the wearer’s musculoskeletal system is necessary.

Numerical and analytical solutions of the differential equations of motion for rigid model links are easier in comparison to formable models. However, the speed of propagation of perturbations in a rigid model is infinitely large. Therefore, when walking, shock in the human body would affect all elements simultaneously, including the brain, if all sectors were absolutely rigid. But in practice this does not occur. Consequently, an approach to the act of human walking using rigid models is not realistic. Therefore, the body should be considered as an elastic system, and the deformability of the links of the musculoskeletal system of a person should be considered.

Nearly all the impact energy when setting a foot on a support is dissipated in the lower limb. In this paper a macroscopic limb was considered, and not just the joint was assumed to deform, but the entire rod. A model of a rod of variable length was considered with a possibility of application in the joints of the control points through which it was possible to maintain a vertical position and also to navigate.

Practically, these control points can be implemented in the form of electric motors with angle sensors, angular velocity, and a control unit with feedback. The implementation itself is the optimal control problem; it is not the purpose of this study.

Consider a rod of variable length, as presented in Fig. 1, where φ is the degree of rotation, C_{l_0} is the initial position of the center of gravity, C_l is the current position of the center of gravity, initial length of unstressed rod l_0, current length of rod while movement l, then the change in length of the rod can be represented in the following form:

$$\Delta l = l_0 - l. \quad (1)$$

When $\Delta l > 0$, there is a compression of the rod, and when $\Delta l < 0$ there is stretching.

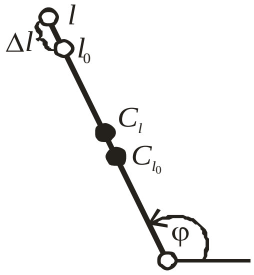


Fig. 1 Changing of the length of the rod during its movement

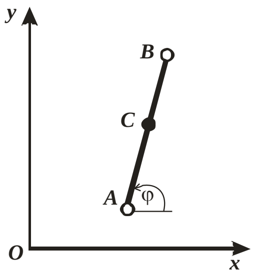


Fig. 2 Plane-parallel motion of uncontrolled chain of variable length

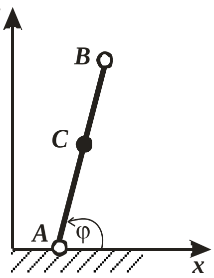


Fig. 3 Planar motion of a link of variable length with one fixed point

While simulation of the musculoskeletal system of the person changing the length of the link is related to the deformations of the bone or joint, it is also the result of the combined action of these factors and the offsets in the joint by the synovial fluid and the non-ideality of the form of the joint. The most effective way to describe the net change of length of the element under the influence of various factors is to take the link of variable length as a model like a function of time. All the characteristics, such as angle, length, moment of inertia, and position of the center of link mass are functions of time: $\varphi = \varphi(t)$, $l = l(t)$, $\Delta l = \Delta l(t)$, $I = I(t)$, $C_l = C_l(t)$.

Changing the length of the link does not control the system. It can be physically implemented in a real model of the exoskeleton, as shown in the model view of the telescopic link.

A. *Plane-parallel Motion of Uncontrolled Chain of Variable Length*

Consider the plane motion of a significant link AB (Fig. 2), which is a rod whose length is a function of time: l = l(t). In this model for the implementation of variability of the length of the rod of the mechanical system for the numerical calculation used in the experimental data, the distance between the markers drawn on the axis of rotation of the joints were determined by the results of video of a human walking. In each frame the distance between the markers was different. To study this movement, the stationary right-hand Cartesian coordinate system xyz with origin at point O was introduced, assuming that the center of mass motion occurs in the plane of XOY.

Assuming that the center of mass of the link is at point C, its position was set as a ratio of the length from the start (pole) of the corresponding branch to the center of mass to the entire length of the link, via the multiplier n $(0 < n < 1)$, $n = const$. This was because positions of the centers of mass of the limbs for a person were determined empirically and defined as the relationship of one part of the level to another; additionally, it allows the changing of the position of the center of mass during movement by changing the length of the link. The weight of the link was denoted as m, the axial moment of inertia passing through the center of mass of link I = I(t) was variable due to changes in the length of the link. The coordinates of the center of mass of the mechanism were determined from geometrical relations.

$$x_C = x(t) + l(t)n\cos\varphi(t), \qquad y_C = y(t) + l(t)n\sin\varphi(t). \tag{2}$$

In further records, the argument (t) for shortening the size of formulas is omitted.

Differentiating expression (2) with respect to time, the velocity components are obtained in the following form:

$$Vx_C = \dot{x} - \dot{\varphi} nl\sin\varphi + n\dot{l}\cos\varphi, \qquad Vy_C = \dot{y} + \dot{\varphi} nl\cos\varphi + n\dot{l}\sin\varphi. \tag{3}$$

By comparing the expressions for the projections of the velocities with the expression given for the velocity distribution in a solid body, the following is obtained [18-22]:

$$\vec{v}_i = \vec{v}_0 + \vec{\omega} \times \vec{r}_i' . \tag{4}$$

Analyzing data of expression, it was concluded that the first terms in expressions (3) correspond to the first term of expression (4) – it is the velocity of the pole, and describes the translational motion of the link. The second summand describes the rotational motion of the link, which also demonstrates that there is a correspondence between formulas (3) and (4). The summands in expression (3) are related to changes in the length of the link and describe the displacement of the center of mass. If a rigid link is considered, i.e., the link length is a constant value, then the derivative = 0 and formula (3) moves into formula (4), the formula for the link of variable length transforms into a rigid formula link.

Furthermore, according to the method described in the work of [23], the square of the velocity of the center of mass can be found. After simplifying and grouping, the following is obtained:

$$V_C^2 = Vx_C^2 + Vy_C^2 = \dot{x}^2 + \dot{y}^2 + 2\dot{\varphi} nl(\dot{y}\cos\varphi - \dot{x}\sin\varphi) + l^2n^2\dot{\varphi}^2 + \dot{l}^2 n^2 + 2\dot{l} n(\dot{x}\cos\varphi + \dot{y}\sin\varphi) \tag{5}$$

Then, similar to the work of [23], the kinetic energy of the link can be written as the following:

$$T = \frac{1}{2}\,[m(\dot{x}^2 + \dot{y}^2 + \dot{l}^2 n^2) + 2m\dot{\varphi}\,ln(\dot{y}\cos\varphi - \dot{x}\sin\varphi) + J\dot{\varphi}^2 + 2m\dot{l}\,n(\dot{x}\cos\varphi + \dot{y}\sin\varphi)], \tag{6}$$

where: $l = l(t)$ и $J = J(t)$ is the variable value. The moment of inertia of the rod relative to the pole (point A) is defined by the theorem of Steiner in the form of $J = I + ml^2n^2$, as the moment of inertia of rod I is defined relative to the axis passing through the center of mass. In addition, this method of specifying the moment of inertia brings the model to the biological prototype – link of musculoskeletal system of the person, the distribution of mass, which generally speaking, is not symmetrical. Thus, the model approaches a biological prototype.

The first term in Eq. (6) is the velocity of the pole, the second and the last terms account for the portable motion of the center of mass relative to the pole, and the third ingredient is the rotational movement relative to the pole.

In fact, comparing the expression for the kinetic energy (6) with expressions (1.2) and (1.3) given in [19], it can be noted that the structure of the first three summands coincide with the method of reference angles and notation, the last term in the first parenthesis and the last term in the kinetic energy associated with the changes in the length of the link. If the link is non-formable, then $\dot{l} = 0$ and the kinetic energy coincide with the kinetic energy for a rigid link.

Potential energy can be written as:

$$\Pi = mg(y + ln\sin\varphi). \tag{7}$$

Lagrange function:

$$L = T - \Pi. \tag{8}$$

The Lagrange equations of the second kind include the following:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \quad (i = 1, \ldots, 4) \tag{9}$$

Omitting intermediate calculations related to the prescription of the Lagrange function and calculation of derivatives, the equations of motion are presented as the following:

$$m\ddot{x} - \ddot{\varphi}\,mln\sin\varphi - \dot{\varphi}^2\,mln\cos\varphi - 2\dot{l}\dot{\varphi}\,mn\sin\varphi + mn\cos\varphi\,\ddot{l} = 0, \tag{10}$$

$$m\ddot{y} + \ddot{\varphi}\,mln\cos\varphi - \dot{\varphi}^2\,mln\sin\varphi + 2\dot{l}\dot{\varphi}\,mn\cos\varphi + mn\sin\varphi\,\ddot{l} + mg = 0, \tag{11}$$

$$-\ddot{x}\,lnm\sin\varphi + \ddot{y}\,lnm\cos\varphi + \ddot{\varphi}\,(I_1 + ml^2n^2) + glnm\cos\varphi + 2ln^2m\,\dot{l}\dot{\varphi} + \dot{l}\dot{\varphi} = 0, \tag{12}$$

$$\ddot{x}\,mn\cos\varphi + \ddot{y}\,mn\sin\varphi - lmn^2\dot{\varphi}^2 + gmn\sin\varphi + mn^2\ddot{l} = 0. \tag{13}$$

If a homogeneous element with uniform distribution of mass in a particular case is considered, then the center of mass is at the geometric middle of the link, i.e. the multiplier $n = ½$. The moment of inertia of the link relative to the center of mass $I = \frac{1}{12}ml^2$, then $J = \frac{1}{12}ml^2 + ml^2\left(\frac{1}{2}\right)^2 = \frac{1}{3}ml^2$. By substituting these values in Eqs. (9)-(12), the equations of motion of a homogeneous rod can be obtained.

B. Planar Motion of a Link of Variable Length with One Fixed Point

Considering a system consisting of one tangible link AB, fixed at point A, Fig. 3 illustrates the link schematically and the corresponding notations are introduced. All assumptions and notations are consistent with the previous case of a free link. The differences are that the coordinates of point A (pole) are constant, i.e., x = const, y = const.

The position of the rod in this case is uniquely determined by the angle $\varphi = \varphi(t)$ and the rod length $l = l(t)$. The considered system has two degrees of freedom, and the center of mass of the link is at point C.

Differences in the preparation of the equations will be in the fact that derivatives are $\dot{x} = 0$, $\dot{y} = 0$.

The coordinates of the center of mass of the mechanism are as follow:

$$x_C = x + l(t)n\cos\varphi(t), \qquad y_C = y + l(t)n\sin\varphi(t). \tag{14}$$

In further recordings, to reduce the size of formulas, argument (t) is omitted.

Differentiating expression (4) with respect to time (14), the velocity components can be obtained in the following forms:

$$V_{x_C} = -\dot{\varphi} nl\sin\varphi + n\dot{l}\cos\varphi, \qquad V_{y_C} = \dot{\varphi} nl\cos\varphi + n\dot{l}\sin\varphi. \tag{15}$$

The kinetic energy of the link can be written as:

$$T = \frac{1}{2}(m\dot{l}^2 n^2 + J\dot{\varphi}^2) \tag{16}$$

The meanings of all terms in Eq. (16) remain the same.

Expression (7) for the potential energy in this case is not changed.

There is a system of differential equations of motion of the considered terminal using the Lagrange equations of the second kind, as presented by the following:

$$\ddot{\varphi}(I + ml^2n^2) + gmln\cos\varphi + 2mln^2\dot{l}\dot{\varphi} + \dot{l}\dot{\varphi} = 0 \tag{17}$$

$$-lmn^2\dot{\varphi}^2 + gmn\sin\varphi + \ddot{l}mn^2 = 0 \tag{18}$$

The transition to the equations of oscillations of a homogeneous rod is implemented similarly to the free rod, namely replacing the moment of inertia in the first summand on $J = \frac{1}{3}ml^2$.

If the equations of oscillations are considered with a constant of a long line, then the second Eq. (18) of the resulting system will not exist; only the first two components will remain in the first equation. This takes a form similar to the equations of oscillations of mathematical pendulum. Differences will be in the selection of the counting direction angles and inertial characteristics. Assuming it is a uniform rod, the equation then becomes the following:

$$\frac{1}{3}ml^2\ddot{\varphi} + gml\frac{1}{2}\cos\varphi = 0. \tag{19}$$

If other forces excluding gravity do not act on the link, then the following is true:

$$\ddot{\varphi} + \frac{3}{2}\frac{g}{l}\cos\varphi = 0. \tag{20}$$

Introducing the notation $\omega = \sqrt{\frac{3}{2}\frac{g}{l}}$, there is:

$$\ddot{\varphi} + \omega^2\cos\varphi = 0. \tag{21}$$

Eq. (21) represents the equation of harmonic oscillations of mathematical pendulum. If the angle is calculated from the vertical and not the horizontal, then sinφ would stand in the place of cosφ. In that case, the equation would have its usual form. Thus, a general scientific principle of consistency holds.

As such, it can be concluded that the approach associated with the approximation of the deformability of the element of the human musculoskeletal system during walking as a model link changing its length depending on time is adequate to the existing models of theoretical mechanics.

III. MODEL DESCRIPTION

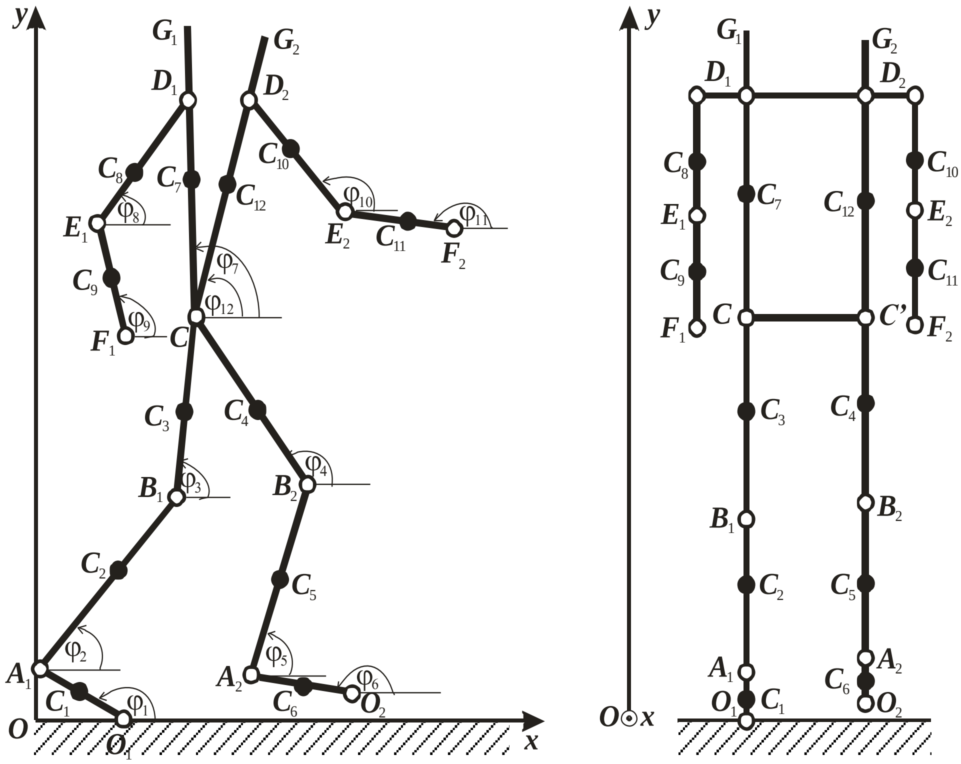
The exoskeleton model had two three-link ponderable legs, two two-link ponderable arms, and two ponderable bodies. Fig. 4 illustrates the mechanism schematically and introduces the notations. For the names of the exoskeleton elements, corresponding terms of human anatomy were used [20-24]. With the aim of acquisition of a model that reproduced real systems more adequately, the formability of the links was taken into account, allowing to consider energy recuperation in the process of genuine walking. For the first time, a more realistic behavior of model exoskeleton links of variable length are proposed. The given study not only accounted for bone deformations, but also for deformations related to the presence of muscular tissue, tendons, and joints, and these deformations were higher then the bone deformations. In a general case, any of these components are nonlinear when under load, but if the deformations are distributed along the whole link, these average deformations, at a first approximation, may be considered as linear.

To investigate the in-plane motion of the exoskeleton in a single-support phase, a right-handed system of coordinates *xyz* with the origin at *O* and the *xy* plane is introduced, on which the motion of mass center takes place. Let $O_1A_1 = l_1$, $A_1B_1 = l_2$, $B_1C = l_3$, $B_2C = l_4$, $A_2B = l_5$, $O_2A_2 = l_6$, $CG_1 = l_7$, $D_1E_1 = l_8$, $E_1F_1 = l_9$, $D_2E_2 = l_{10}$, $E_2F_2 = l_{11}$, $CG_2 = l_{12}$ be the lengths of the biomechanical system links. All the links are elastic and the lengths of rods are the functions of time: $l_i = l_i(t)$ (i = 1, ..., 12). The position in the single-support phase is unequivocally defined by angles φ_i and by the lengths of rods l_i (i = 1, ..., 12), and that is why the system under investigation has twenty-four degrees of freedom. The torques developed in an i-th joint (i = 1, ..., 12) are denoted by M_i.

The mass centers are at the following points: C_1 – the supporting leg foot, C_2 – the supporting leg shank, C_3 – the supporting leg thigh, C_4 – the working leg thigh, C_5 – the working leg shank, C_6 – the working leg foot, C_7, C_{12} – the bodies, C_8, C_{10} – the shoulders, C_9, C_{11} – the forearms. Their positions were set as ratios of the length from the beginning of the corresponding link up to the mass center to the entire length of the link, expressed in terms of the multipliers n_i, (i = 1, …, 12), ($0 < n_i < 1$). This method is more preferable as the positions of the mass center of the extremities of a human body are defined empirically and are set in percentage terms of one part of the link to the other [25].

It also allows taking into account the changes of the mass center position in the process of motion in terms of certain link deformations. Consequently, they must also change in the exoskeleton to provide comfortable movement of the person inside.

The masses: m_1, m_6; m_2, m_5; m_3, m_4; m_8, m_{10}; m_9, m_{11}; m_7, m_{12} are the masses of the foot, shank, thigh, shoulders, forearms and bodies, respectively. The moments of inertia: I_1, I_6; I_2, I_5; I_3, I_4; I_8, I_{10}; I_9, I_{11}; I_7, I_{12} are the moments of inertia of the foot, shank, thigh, shoulders, forearms and bodies, respectively. The right and the left extremities are considered different.



a) *b)*

Fig. 4 Model of a human exoskeleton for body rotation a) profile view; b) face

There are no active external forces; the motion takes place affected only by internal forces and external reactions. The connection at point O_1 is realized as an ideal joint and is two-sided or hold-in (Fig. 5).

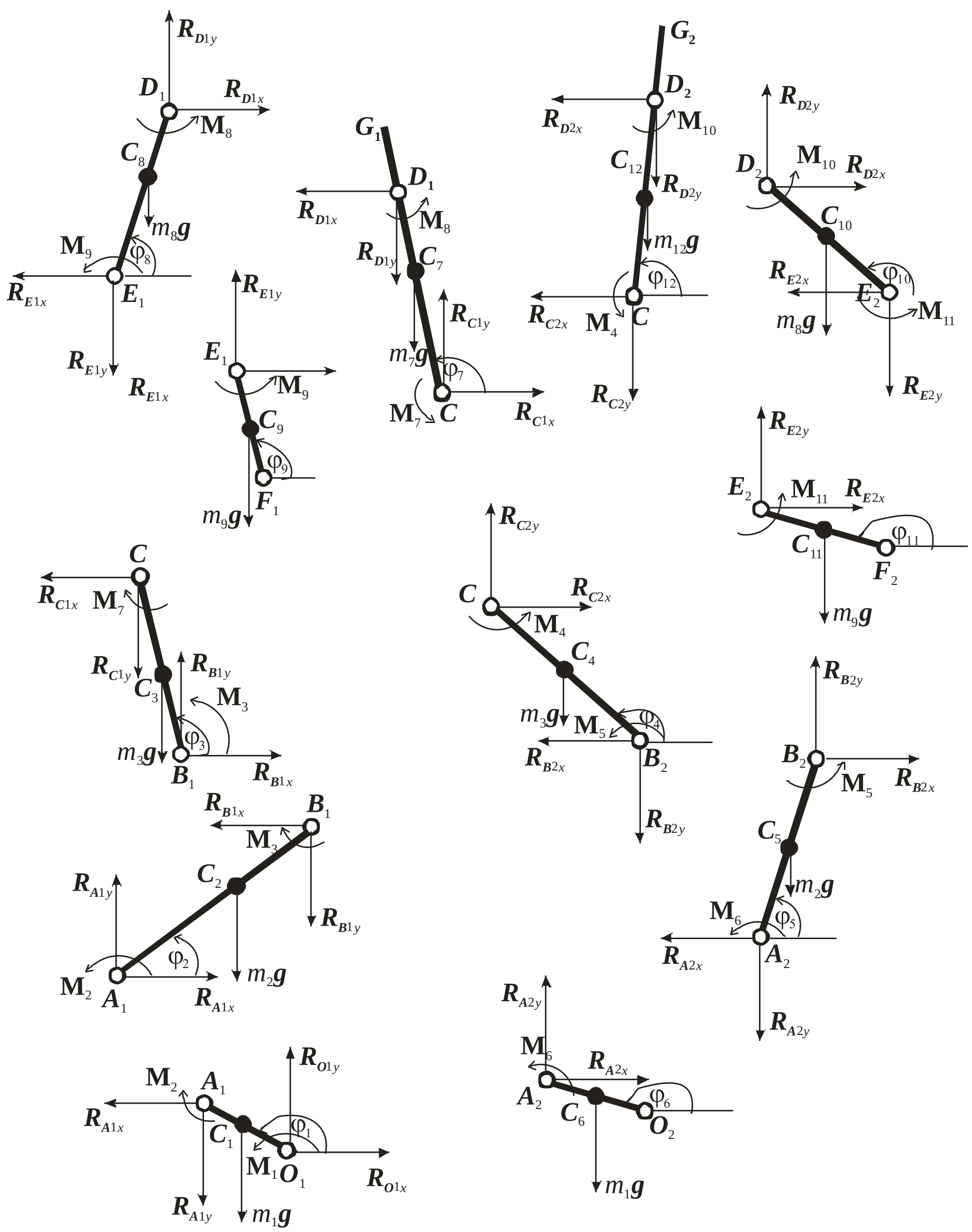


Fig. 5 The synthesis of the 12-link exoskeleton of separate links

Suppose the working leg is free and no external forces are applied to it. Composing an expression for elementary work δA of all the forces applied to the system, the following generalized forces are obtained:

$$Q_1 = -\, g l_1 (m + m_2 + m_3 + m_1 n_1)\cos\varphi_1 + M_1 - M_2,$$

$$Q_2 = -\, g l_2 (m + m_3 + m_2 n_2)\cos\varphi_2 + M_2 - M_3,$$

$$Q_3 = -\, g l_3 (m + m_3 n_3)\cos\varphi_3 - M_3 + M_4 + M_7,$$

$$Q_4 = m_n g l_4 \cos\varphi_4 + M_4 - M_5,$$

$$Q_5 = m_g g l_5 \cos\varphi_5 + M_5 - M_6,$$

$$Q_6 = m_s g l_6 \cos\varphi_6 + M_6,$$

$$Q_7 = -\, g l_7 m_k \cos\varphi_7 + M_3 + M_7,$$

$$Q_8 = g l_8 m_r \cos\varphi_8 + M_8 - M_9,$$

$$Q_9 = g l_9 m_p \cos\varphi_9 + M_9,$$

$$\begin{gathered}
Q_{10} = gl_{10}m_r\cos\varphi_{10} + M_{10} - M_{11}, \\
Q_{11} = gl_{11}m_p\cos\varphi_{11} + M_{11}, \\
Q_{12} = -gl_{12}m_k\cos\varphi_{12} + M_4 + M_7, \\
Q_{13} = -k_1\Delta l_1 - g(m + m_2 + m_3 + m_1n_1)\sin\varphi_1. \\
Q_{14} = -k_2\Delta l_2 - g(m + m_3 + m_2n_2)\sin\varphi_2, \\
Q_{15} = -k_3\Delta l_3 - g(m + m_3n_3)\sin\varphi_3, \\
Q_{16} = -k_4\Delta l_4 + m_ng\sin\varphi_4, \\
Q_{17} = -k_5\Delta l_5 + m_gg\sin\varphi_5, \\
Q_{18} = -k_6\Delta l_6 + m_sg\sin\varphi_6, \\
Q_{19} = -k_7\Delta l_7 - gm_k\cos\varphi_7, \\
Q_{20} = -k_8\Delta l_8 + gm_r\sin\varphi_8, \\
Q_{21} = -k_9\Delta l_9 + gm_p\sin\varphi_9, \\
Q_{22} = -k_{10}\Delta l_{10} + gm_r\sin\varphi_{10}, \\
Q_{23} = -k_{11}\Delta l_{11} + gm_p\sin\varphi_{11}, \\
Q_{24} = -k_{12}\Delta l_{12} - gm_k\cos\varphi_{12},
\end{gathered} \tag{22}$$

where Δl_i is the changing of the length of the link, determined according to formula (1); $m = m_1 + m_2 + m_3 + m_7 + m_8 + m_9 + m_{10} + m_{11}$; $m_k = m_7n_7 + (m_8 + m_9 + m_{10} + m_{11})n_{7R}$; $m_r = m_8n_8 - m_8 - m_9$, $m_p = m_9(n_9 - 1)$ is one hand, the expression for Q with indexes 8, 9, 20, 21; $m_r = m_{10}n_{10} - m_{10} - m_{11}$, $m_p = m_{11}(n_{11} - 1$ is the other hand, expression for Q with indexes 10, 11, 22, 23; $m_n = m_1 + m_2 + m_3 - m_3n_3$, $m_g = m_1 + m_2 - m_2n_2$, $m_s = m_1(n_1 - 1)$ is the bearing leg, expression for Q with indexes 1, 2, 3, 12, 14, 15; $m_n = m_6 + m_5 + m_4 - m_4n_4$, $m_g = m_6 + m_5 - m_5n_5$, $m_s = m_6(n_6 - 1)$ is the portable leg, expression for Q with indexes 4, 5, 6, 16, 17, 18.

Substituting (22) into the right-hand member (9), the system of equations of motion anthropomorphic 20-link mechanism is obtained describing the change of the angular coordinates of links and fluctuations of the lengths of the links of the mechanism.

IV. COMPILATION OF EQUATIONS

For the model of the 12-link exoskeleton, a possibility of the body rotation due to the differences of the tilting angles φ_7 и φ_{12} is provided. There are two parallel splines linked together by the rods in the hip and shoulder joints, and the motion of these joints may be considered as they are in parallel planes and the linkage provides motion synchronization. Control synchronization provides motion stability of the exoskeleton with a man inside.

Having used the Langrangian description and the software package «Mathematica»[1] [26, 27] a system of differential equations was obtained, simulating the motions of the twelve-link exoskeleton with deformable structure elements.

The equations of the exoskeleton element motions in a single-support phase are the system of nonlinear differential equations of angle variables vector, which can be given in the form of a matrix, as in the following:

$$A(q,l)\,\ddot{q} + B(q,l)\,\dot{q}^2 + gC(q)l + 2D(q,l)(\dot{l}\;\dot{q}) + E(q,l)\,\ddot{l} + \dot{I}\dot{q} = F(q,l). \tag{23}$$

The equation for the length change is in the following form:

$$G(q,l)\,\ddot{q} + H(q,l)\,\dot{q}^2 + gK(q) + 2L(q,l)(\dot{l}\;\dot{q}) + P(q,l)\,\ddot{l} = S(k,l), \tag{24}$$

where q denotes the generalized coordinates of the mass center $q = (\varphi_1, \ldots, \varphi_n)^T$; l is the generalized coordinates associated with the link deformations $l = (l_1, \ldots, l_n)^T$; $A(q,l)$, $G(q,l)$ are the matrixes that take into account inertia characteristics; $B(q,l)$, $H(q,l)$ are the matrixes that take into account viscosity; $C(q)$, $K(q)$ are the matrixes that take into account viscosity; $D(q,l)$, $E(q,l)$, $L(q,l)$, $P(q,l)$ are the matrixes that take into account the link deformations; $F(q,l)$ is the column matrix of the generalized forces, i.e. control moments; $S(E,l)$ is the column matrix associated with the elastic characteristics of the links material; $\ddot{q}$ is the matrix of the generalized acceleration; $\dot{q}^2$ is the matrix of the generalized velocities; $(\dot{l}\;\dot{q}) = (\dot{l}_1\,\dot{q}_1, \ldots, \dot{l}_n\,\dot{q}_n)^T$ is the matrix composed of products $\dot{l}\;\dot{q}$ at equal indexes, matrix of inertia $\dot{I} = (\dot{I}_1, \ldots, \dot{I}_n)$.

---

[1] A licensed version of this program was funded by a grant from the President of the Russian Federation, the number MK-2524.2008.1

As an example, the motion equation for link 1 which is the foot of the supporting leg, can be written as:

$$
\begin{aligned}
&(I_1 + l_1^2(m + m_2 + m_3 + m_1n_1^2))\ddot{\varphi}_1 + l_1l_2(m + m_3 + m_2n_2)\cos(\varphi_1 - \varphi_2)\ddot{\varphi}_2 + l_1l_3(m + m_3n_3)\cos(\varphi_1 - \varphi_3)\ddot{\varphi}_3 - \\
&l_1l_4m_n\cos(\varphi_1 - \varphi_4)\ddot{\varphi}_4 - l_1l_5m_g\cos(\varphi_1 - \varphi_5)\ddot{\varphi}_5 + l_1l_5m_s\cos(\varphi_1 - \varphi_6)\ddot{\varphi}_6 + l_1l_7m_k\cos(\varphi_1 - \varphi_7)\ddot{\varphi}_7 + l_1l_{12}m_k\cos(\varphi_1 - \varphi_{12}) \\
&\ddot{\varphi}_{12} + l_1l_8m_r\cos(\varphi_1 - \varphi_8)\ddot{\varphi}_8 + l_1l_9m_p\cos(\varphi_1 - \varphi_9)\ddot{\varphi}_9 + l_1l_{10}m_r\cos(\varphi_1 - \varphi_{10})\ddot{\varphi}_{10} + l_1l_{11}m_p\cos(\varphi_1 - \varphi_{11})\ddot{\varphi}_{11} + l_1l_2(m + \\
&m_3 + m_2n_2)\sin(\varphi_1 - \varphi_2)\dot{\varphi}_2^2 + l_1l_3(m + m_3n_3)\sin(\varphi_1 - \varphi_3)\dot{\varphi}_3^2 - l_1l_4m_n\sin(\varphi_1 - \varphi_4)\dot{\varphi}_4^2 - l_1l_5m_g\sin(\varphi_1 - \varphi_5)\dot{\varphi}_5^2 + \\
&l_1l_6m_s\sin(\varphi_1 - \varphi_6)\dot{\varphi}_6^2 + l_1l_7m_k\sin(\varphi_1 - \varphi_7)\dot{\varphi}_7^2 + l_1l_{12}m_k\sin(\varphi_1 - \varphi_{12})\dot{\varphi}_{12}^2 + l_1l_8m_r\sin(\varphi_1 - \varphi_8)\dot{\varphi}_8^2 + l_1l_9m_p\sin(\varphi_1 - \varphi_9) \\
&\dot{\varphi}_9^2 + l_1l_{10}m_r\sin(\varphi_1 - \varphi_{10})\dot{\varphi}_{10}^2 + l_1l_{11}m_p\sin(\varphi_1 - \varphi_{11})\dot{\varphi}_{11}^2 + gl_1(m + m_2 + m_3 + m_1n_1)\cos\varphi_1 + 2l_1(m + m_2 + m_3 + m_1n_1^2) \\
&\dot{l}_1\dot{\varphi}_1 + 2l_1(m + m_3 + m_2n_2)\cos(\varphi_1 - \varphi_2)\dot{l}_2\dot{\varphi}_2 + 2l_1(m + m_3n_3)\cos(\varphi_1 - \varphi_3)\dot{l}_3\dot{\varphi}_3 - 2l_1m_n\cos(\varphi_1 - \varphi_4)\dot{l}_4\dot{\varphi}_4 - \\
&2l_1m_g\cos(\varphi_1 - \varphi_5)\dot{l}_5\dot{\varphi}_5 - 2l_1m_s\cos(\varphi_1 - \varphi_6)\dot{l}_6\dot{\varphi}_6 + 2l_1m_k\cos(\varphi_1 - \varphi_7)\dot{l}_7\dot{\varphi}_7 + 2l_1m_k\cos(\varphi_1 - \varphi_{12})\dot{l}_{12}\dot{\varphi}_{12} + \\
&2l_1m_r\cos(\varphi_1 - \varphi_8)\dot{l}_8\dot{\varphi}_8 + 2l_1m_p\cos(\varphi_1 - \varphi_9)\dot{l}_9\dot{\varphi}_9 + 2l_1m_r\cos(\varphi_1 - \varphi_{10})\dot{l}_{10}\dot{\varphi}_{10} + 2l_1m_p\cos(\varphi_1 - \varphi_{11})\dot{l}_{11}\dot{\varphi}_{11} - l_1(m \\
&+ m_3 + m_2n_2)\sin(\varphi_1 - \varphi_2)\ddot{l}_2 - l_1(m + m_3n_3)\sin(\varphi_1 - \varphi_3)\ddot{l}_3 + l_1m_n\sin(\varphi_1 - \varphi_4)\ddot{l}_4 + l_1m_g\sin(\varphi_1 - \varphi_5)\ddot{l}_5 - l_1m_s\sin(\varphi_1 \\
&- \varphi_6)\ddot{l}_6 - l_1m_k\sin(\varphi_1 - \varphi_7)\ddot{l}_7 - l_1m_k\sin(\varphi_1 - \varphi_{12})\ddot{l}_{12} - l_1m_r\sin(\varphi_1 - \varphi_8)\ddot{l}_8 - l_1m_p\sin(\varphi_1 - \varphi_9)\ddot{l}_9 - l_1m_r\sin(\varphi_1 - \\
&\varphi_{10})\ddot{l}_{10} - l_1m_p\sin(\varphi_1 - \varphi_{11})\ddot{l}_{11} + i_1\dot{\varphi}_1 = M_1 - M_2,
\end{aligned}
\tag{25}
$$

For link 2, the equation of motion of the lower leg supporting leg has the form:

$$
\begin{aligned}
&l_1l_2(m + m_3 + m_2n_2)\cos(\varphi_1 - \varphi_2)\ddot{\varphi}_1 + (I_2 + l_2^2(m + m_3 + m_2n_2^2))\ddot{\varphi}_2 + l_2l_3(m + m_3n_3)\cos(\varphi_2 - \varphi_3)\ddot{\varphi}_3 - l_2l_4m_n\cos(\varphi_2 \\
&- \varphi_4)\ddot{\varphi}_4 - l_2l_5m_g\cos(\varphi_2 - \varphi_5)\ddot{\varphi}_5 + l_2l_6m_s\cos(\varphi_2 - \varphi_6)\ddot{\varphi}_6 + l_2l_7m_k\cos(\varphi_2 - \varphi_7)\ddot{\varphi}_7 + l_2l_{12}m_k\cos(\varphi_2 - \varphi_{12})\ddot{\varphi}_{12} + \\
&l_2l_8m_r\cos(\varphi_2 - \varphi_8)\ddot{\varphi}_8 + l_2l_9m_p\cos(\varphi_2 - \varphi_9)\ddot{\varphi}_9 + l_2l_{10}m_r\cos(\varphi_2 - \varphi_{10})\ddot{\varphi}_{10} + l_2l_{11}m_p\cos(\varphi_2 - \varphi_{11})\ddot{\varphi}_{11} - l_1l_2(m + m_3 + \\
&m_2n_2)\sin(\varphi_1 - \varphi_2)\dot{\varphi}_1^2 + l_2l_3(m + m_3n_3)\sin(\varphi_2 - \varphi_3)\dot{\varphi}_3^2 - l_2l_4m_n\sin(\varphi_2 - \varphi_4)\dot{\varphi}_4^2 - l_2l_5m_g\sin(\varphi_2 - \varphi_5)\dot{\varphi}_5^2 + l_2l_6m_s\sin(\varphi_2 \\
&- \varphi_6)\dot{\varphi}_6^2 + l_2l_7m_k\sin(\varphi_2 - \varphi_7)\dot{\varphi}_7^2 + l_2l_{12}m_k\sin(\varphi_2 - \varphi_{12})\dot{\varphi}_{12}^2 + l_2l_8m_r\sin(\varphi_2 - \varphi_8)\dot{\varphi}_8^2 + l_2l_9m_p\sin(\varphi_2 - \varphi_9)\dot{\varphi}_9^2 + \\
&l_2l_{10}m_r\sin(\varphi_2 - \varphi_{10})\dot{\varphi}_{10}^2 + l_2l_{11}m_p\sin(\varphi_2 - \varphi_{11})\dot{\varphi}_{11}^2 + gl_2(m + m_3 + m_2n_2)\cos\varphi_2 + 2l_2(m + m_3 + m_2n_2)\cos(\varphi_1 - \varphi_2)\dot{l}_1\dot{\varphi}_1 \\
&+ 2l_2(m + m_3 + m_2n_2^2)\dot{l}_2\dot{\varphi}_2 + 2l_2(m + m_3n_3)\cos(\varphi_2 - \varphi_3)\dot{l}_3\dot{\varphi}_3 - 2l_2m_n\cos(\varphi_2 - \varphi_4)\dot{l}_4\dot{\varphi}_4 - 2l_2m_g\cos(\varphi_2 - \varphi_5)\dot{l}_5 \\
&\dot{\varphi}_5 + 2l_2m_s\cos(\varphi_2 - \varphi_6)\dot{l}_6\dot{\varphi}_6 + 2l_2m_k\cos(\varphi_2 - \varphi_7)\dot{l}_7\dot{\varphi}_7 + 2l_2m_k\cos(\varphi_2 - \varphi_{12})\dot{l}_{12}\dot{\varphi}_{12} + 2l_2m_r\cos(\varphi_2 - \varphi_8)\dot{l}_8\dot{\varphi}_8 + \\
&2l_2m_p\cos(\varphi_2 - \varphi_9)\dot{l}_9\dot{\varphi}_9 + 2l_2m_r\cos(\varphi_2 - \varphi_{10})\dot{l}_{10}\dot{\varphi}_{10} + 2l_2m_p\cos(\varphi_2 - \varphi_{11})\dot{l}_{11}\dot{\varphi}_{11} + l_2(m + m_3 + m_2n_2)\sin(\varphi_1 - \varphi_2)\ddot{l}_1 \\
&- l_2(m + m_3n_3)\sin(\varphi_2 - \varphi_3)\ddot{l}_3 + l_2m_n\sin(\varphi_2 - \varphi_4)\ddot{l}_4 + l_2m_g\sin(\varphi_2 - \varphi_5)\ddot{l}_5 - l_2m_s\sin(\varphi_2 - \varphi_6)\ddot{l}_6 - l_2m_k\sin(\varphi_2 - \varphi_7) \\
&\ddot{l}_7 - l_2m_k\sin(\varphi_2 - \varphi_{12})\ddot{l}_{12} - l_2m_r\sin(\varphi_2 - \varphi_8)\ddot{l}_8 - l_2m_p\sin(\varphi_2 - \varphi_9)\ddot{l}_9 - l_2m_r\sin(\varphi_2 - \varphi_{10})\ddot{l}_{10} - l_2m_p\sin(\varphi_2 - \varphi_{11})\ddot{l}_{11} \\
&+ i_2\dot{\varphi}_2 = M_2 - M_3,
\end{aligned}
\tag{26}
$$

For link 3, the equation of motion of the hip of the supportive leg becomes the following:

$$
\begin{aligned}
&l_1l_3(m + m_3n_3)\cos(\varphi_1 - \varphi_3)\ddot{\varphi}_1 + l_2l_3(m + m_3n_3)\cos(\varphi_2 - \varphi_3)\ddot{\varphi}_2 + (I_3 + l_3^2(m + m_3n_3^2))\ddot{\varphi}_3 - l_3l_4m_n\cos(\varphi_3 - \varphi_4)\ddot{\varphi}_4 - \\
&l_5l_3m_g\cos(\varphi_3 - \varphi_5)\ddot{\varphi}_5 + l_6l_3m_s\cos(\varphi_3 - \varphi_6)\ddot{\varphi}_6 + l_3l_7m_k\cos(\varphi_3 - \varphi_7)\ddot{\varphi}_7 + l_3l_{12}m_k\cos(\varphi_3 - \varphi_{12})\ddot{\varphi}_{12} + l_3l_8m_r\cos(\varphi_3 - \varphi_8) \\
&\ddot{\varphi}_8 + l_3l_9m_p\cos(\varphi_3 - \varphi_9)\ddot{\varphi}_9 + l_3l_{10}m_r\cos(\varphi_3 - \varphi_{10})\ddot{\varphi}_{10} + l_3l_{11}m_p\cos(\varphi_3 - \varphi_{11})\ddot{\varphi}_{11} - l_1l_3(m + m_3n_3)\sin(\varphi_1 - \varphi_3)\dot{\varphi}_1^2 - \\
&l_2l_3(m + m_3n_3)\sin(\varphi_2 - \varphi_3)\dot{\varphi}_2^2 - l_3l_4m_n\sin(\varphi_3 - \varphi_4)\dot{\varphi}_4^2 - l_5l_3m_g\sin(\varphi_3 - \varphi_5)\dot{\varphi}_5^2 + l_6l_3m_s\sin(\varphi_3 - \varphi_6)\dot{\varphi}_6^2 + l_3l_7m_k\sin(\varphi_3 \\
&- \varphi_7)\dot{\varphi}_7^2 + l_3l_{12}m_k\sin(\varphi_3 - \varphi_{12})\dot{\varphi}_{12}^2 + l_3l_8m_r\sin(\varphi_3 - \varphi_8)\dot{\varphi}_8^2 + l_3l_9m_p\sin(\varphi_3 - \varphi_9)\dot{\varphi}_9^2 + l_3l_{10}m_r\sin(\varphi_3 - \varphi_{10})\dot{\varphi}_{10}^2 + \\
&l_3l_{11}m_p\sin(\varphi_3 - \varphi_{11})\dot{\varphi}_{11}^2 + gl_3(m + m_3n_3)\cos\varphi_3 + 2l_3(m + m_3n_3)\cos(\varphi_1 - \varphi_3)\dot{l}_1\dot{\varphi}_1 + 2l_3(m + m_3n_3)\cos(\varphi_2 - \varphi_3)\dot{l}_2\dot{\varphi}_2 \\
&+ 2l_3(m + m_3n_3^2)\dot{l}_3\dot{\varphi}_3 - 2l_3m_n\cos(\varphi_3 - \varphi_4)\dot{l}_4\dot{\varphi}_4 - 2l_3m_g\cos(\varphi_3 - \varphi_5)\dot{l}_5\dot{\varphi}_5 + 2l_3m_s\cos(\varphi_3 - \varphi_6)\dot{l}_6\dot{\varphi}_6 +
\end{aligned}
\tag{27}
$$

$$2l_3m_k\cos(\varphi_3-\varphi_7)\dot{l}_7\dot{\varphi}_7 + 2l_3m_k\cos(\varphi_3-\varphi_{12})\dot{l}_{12}\dot{\varphi}_{12} + 2l_3m_r\cos(\varphi_3-\varphi_8)\dot{l}_8\dot{\varphi}_8 + 2l_3m_p\cos(\varphi_3-\varphi_9)\dot{l}_9\dot{\varphi}_9 +$$
$$2l_3m_r\cos(\varphi_3-\varphi_{10})\dot{l}_{10}\dot{\varphi}_{10} + 2l_3m_p\cos(\varphi_3-\varphi_{11})\dot{l}_{11}\dot{\varphi}_{11} + l_3(m+m_3n_3)\sin(\varphi_1-\varphi_3)\ddot{l}_1 + l_3(m+m_3n_3)\sin(\varphi_2-\varphi_3)\ddot{l}_2 +$$
$$l_3m_n\sin(\varphi_3-\varphi_4)\ddot{l}_4 + l_3m_g\sin(\varphi_3-\varphi_5)\ddot{l}_5 - l_3m_s\sin(\varphi_3-\varphi_6)\ddot{l}_6 - l_3m_k\sin(\varphi_3-\varphi_7)\ddot{l}_7 - l_3m_k\sin(\varphi_3-\varphi_{12})\ddot{l}_{12} -$$
$$l_3m_r\sin(\varphi_3-\varphi_8)\ddot{l}_8 - l_3m_p\sin(\varphi_3-\varphi_9)\ddot{l}_9 - l_3m_r\sin(\varphi_3-\varphi_{10})\ddot{l}_{10} - l_3m_p\sin(\varphi_3-\varphi_{11})\ddot{l}_{11} + i_3\dot{\varphi}_3 = -M_3 + M_4 + M_7$$
$$+ M_{12},$$

where: $m = m_1 + m_2 + m_3 + 2m_7 + 2m_8 + 2m_9$, $m_r = m_8n_8 - m_8 - m_9$, $m_p = m_9(n_9 - 1)$, $m_n = m_1 + m_2 + m_3 - m_3n_3$, $m_g = m_1 + m_2 - m_2n_2$, $m_s = m_1(n_1 - 1)$, $m_k = m_7n_7 + (m_8 + m_9)n_{7R}$.

Here is an equation for a single frame with a number 7; however, for a case of 12 in the equation, the index numbers 7, 8, 9, 12, 10, and 11 must be changed.

$$l_1l_7m_k\cos(\varphi_1-\varphi_7)\ddot{\varphi}_1 + l_2l_7m_k\cos(\varphi_2-\varphi_7)\ddot{\varphi}_2 + l_3l_7m_k\cos(\varphi_3-\varphi_7)\ddot{\varphi}_3 + (I_7 + l_7^2(m_7n_7^2 + (m_8+m_9)n_{7R}^2))\ddot{\varphi}_7 +$$
$$l_7l_8n_{7R}m_r\cos(\varphi_7-\varphi_8)\ddot{\varphi}_8 + l_7l_9n_{7R}m_p\cos(\varphi_7-\varphi_9)\ddot{\varphi}_9 - l_1l_7m_k\sin(\varphi_1-\varphi_7)\dot{\varphi}_1^2 - l_2l_7m_k\sin(\varphi_2-\varphi_7)\dot{\varphi}_2^2 + l_3l_7m_k\sin(\varphi_3-$$
$$\varphi_7)\dot{\varphi}_3^2 + l_7l_8n_{7R}m_r\sin(\varphi_7-\varphi_8)\dot{\varphi}_8^2 + l_7l_9n_{7R}m_p\sin(\varphi_7-\varphi_9)\dot{\varphi}_9^2 + gl_7m_k\cos\varphi_7 + 2l_7m_k\cos(\varphi_1-\varphi_7)\dot{l}_1\dot{\varphi}_1 + 2l_7m_k\cos(\varphi_2 \tag{28}$$
$$-\varphi_7)\dot{l}_2\dot{\varphi}_2 + 2l_7m_k\cos(\varphi_3-\varphi_7)\dot{l}_3\dot{\varphi}_3 + 2l_7(m_7n_7^2 + (m_8+m_9)n_{7R}^2)\dot{l}_7\dot{\varphi}_7 + 2l_7n_{7R}m_r\cos(\varphi_7-\varphi_8)\dot{l}_8\dot{\varphi}_8 +$$
$$2l_7n_{7R}m_p\cos(\varphi_7-\varphi_9)\dot{l}_9\dot{\varphi}_9 + l_7m_k\sin(\varphi_1-\varphi_7)\ddot{l}_1 + l_7m_k\sin(\varphi_2-\varphi_7)\ddot{l}_2 + l_7m_k\sin(\varphi_3-\varphi_7)\ddot{l}_3 + l_7n_{7R}m_r\sin(\varphi_7-\varphi_8)$$
$$\ddot{l}_8 + l_7n_{7R}m_p\sin(\varphi_7-\varphi_9)\ddot{l}_9 + i_7\dot{\varphi}_7 = M_3 + M_4,$$

where $m_r = m_8n_8 - m_8 - m_9$, $m_p = m_9(n_9 - 1)$, $m_k = m_7n_7 + (m_8 + m_9)n_{7R}$.

Thus, the equations describe the change of the angular variables and the lengths of the rods of a 12-link exoskeleton with elements of variable length. The equation of motion of the mechanism in the single-support phase is a system of 24 nonlinear differential equations.

The general solution of the equations of motion depends on 48 arbitrary constants. To identify the motion uniquely, initial conditions must be identified. For angular coordinates, formula (29) can be used; for changes of the lengths of the links, use formula (30).

$$\begin{gathered}
\varphi_1\big|_{t=0} = \varphi_{1_0};\varphi_2\big|_{t=0} = \varphi_{2_0};\varphi_3\big|_{t=0} = \varphi_{3_0};\varphi_4\big|_{t=0} = \varphi_{4_0};\varphi_5\big|_{t=0} = \varphi_{5_0};\varphi_6\big|_{t=0} = \varphi_{6_0};\\
\varphi_7\big|_{t=0} = \varphi_{7_0};\varphi_8\big|_{t=0} = \varphi_{8_0};\varphi_9\big|_{t=0} = \varphi_{9_0};\varphi_{10}\big|_{t=0} = \varphi_{10_0};\varphi_{11}\big|_{t=0} = \varphi_{11_0};\varphi_{12}\big|_{t=0} = \varphi_{12_0};\\
\dot{\varphi}_1\big|_{t=0} = \dot{\varphi}_{1_0};\dot{\varphi}_2\big|_{t=0} = \dot{\varphi}_{2_0};\dot{\varphi}_3\big|_{t=0} = \dot{\varphi}_{3_0};\dot{\varphi}_4\big|_{t=0} = \dot{\varphi}_{4_0};\dot{\varphi}_5\big|_{t=0} = \dot{\varphi}_{5_0};\dot{\varphi}_6\big|_{t=0} = \dot{\varphi}_{6_0};\\
\dot{\varphi}_7\big|_{t=0} = \dot{\varphi}_{7_0};\dot{\varphi}_8\big|_{t=0} = \dot{\varphi}_{8_0};\dot{\varphi}_9\big|_{t=0} = \dot{\varphi}_{9_0};\dot{\varphi}_{10}\big|_{t=0} = \dot{\varphi}_{10_0};\dot{\varphi}_{11}\big|_{t=0} = \dot{\varphi}_{11_0};\dot{\varphi}_{12}\big|_{t=0} = \dot{\varphi}_{12_0};
\end{gathered} \tag{29}$$

$$\begin{gathered}
l_1\big|_{t=0} = l_{1_0};\ l_2\big|_{t=0} = l_{2_0};\ l_3\big|_{t=0} = l_{3_0};\ l_4\big|_{t=0} = l_{4_0};\ l_5\big|_{t=0} = l_{5_0};\ l_6\big|_{t=0} = l_{6_0};\\
l_7\big|_{t=0} = l_{7_0};\ l_8\big|_{t=0} = l_{8_0};\ l_9\big|_{t=0} = l_{9_0};\ l_{10}\big|_{t=0} = l_{10_0};\ l_{11}\big|_{t=0} = l_{11_0};\ l_{12}\big|_{t=0} = l_{12_0};\\
\dot{l}_1\big|_{t=0} = \dot{l}_{1_0};\ \dot{l}_2\big|_{t=0} = \dot{l}_{2_0};\ \dot{l}_3\big|_{t=0} = \dot{l}_{3_0};\ \dot{l}_4\big|_{t=0} = \dot{l}_{4_0};\ \dot{l}_5\big|_{t=0} = \dot{l}_{5_0};\ \dot{l}_6\big|_{t=0} = \dot{l}_{6_0};\\
\dot{l}_7\big|_{t=0} = \dot{l}_{7_0};\ \dot{l}_8\big|_{t=0} = \dot{l}_{8_0};\ \dot{l}_9\big|_{t=0} = \dot{l}_{9_0};\ \dot{l}_{10}\big|_{t=0} = \dot{l}_{10_0};\ \dot{l}_{11}\big|_{t=0} = \dot{l}_{11_0};\ \dot{l}_{12}\big|_{t=0} = \dot{l}_{12_0};
\end{gathered} \tag{30}$$

In a similar way, the equations for all angular coordinates of the twelve exoskeleton links and for each link deformation can be composed, but they are not given here.

The matrixes for a certain n-linked system obtained by induction can be generalized. Matrixes A и D are symmetrical, and that is why it is sufficient to enter only the diagonal and off-diagonal components for them, i.e., if i is a row number, j is the column number, then $i,j = 1,2, \ldots, n$, while $j \geq i$, the rest of the below-diagonal elements are equal to the corresponding symmetrical in respect to the main diagonal of the off-diagonal elements. The value can be set to $n = 12$.

For matrix A:

$$a_{ij} = \delta_{ij} I_i + l_i l_j \left(m_j n_j \tilde{\delta} n_i + \sum_{k=i+1}^{n} m_k \right) \cos(\varphi_i - \varphi_j), \tag{31}$$

where δ_{ij} - Kronecker symbol: $\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$, symbol: $\tilde{\delta} n_i = \begin{cases} n_j, i = j \\ 1, i \neq j \end{cases}$.

For matrix D:

$$d_{ij} = l_i \left(m_j n_j \tilde{\delta} n_i + \sum_{k=i+1}^{n} m_k \right) \cos(\varphi_i - \varphi_j). \tag{32}$$

For matrix *P*:

$$p_{ij} = \left(m_j n_j \tilde{\delta} n_i + \sum_{k=i+1}^{n} m_k \right) \cos(\varphi_i - \varphi_j). \tag{33}$$

Matrixes *B* and *E* are antisymmetric. Then the elements on the main diagonal are equal to zero and the below-diagonal elements are equal to the corresponding off-diagonal components, taken with the opposite signs. Thus, it is enough to define only the off-diagonal elements. If i is the line number and j is the column number, then $i, j = 1, 2, \ldots, n$. In this case $j > i$.

For matrix *B*:

$$b_{ij} = l_i l_j \left(m_j n_j + \sum_{k=i+1}^{n} m_k \right) \sin(\varphi_i - \varphi_j). \tag{34}$$

For matrix *E*:

$$e_{ij} = - l_i \left(m_j n_j + \sum_{k=i+1}^{n} m_k \right) \sin(\varphi_i - \varphi_j). \tag{35}$$

For matrix *L*:

$$l_{ij} = \left(m_j n_j + \sum_{k=i+1}^{n} m_k \right) \sin(\varphi_i - \varphi_j), \tag{36}$$

Matrixes *C* and *K* are diagonal, i.e. it is enough to consider the elements, which have the indexes equal to $j = i$, at $i, j = 1, 2, \ldots, n$; the rest of the elements are equal to zero. It is enough to use any index.

For matrix *C*:

$$c_{ij} = \left(m_j n_j + \sum_{k=j+1}^{n} m_k \right) \cos(\varphi_j). \tag{37}$$

For matrix *K*:

$$k_{ij} = \left(m_j n_j + \sum_{k=j+1}^{n} m_k \right) \sin(\varphi_j). \tag{38}$$

The column matrix of the generalized forces *F* can be written as follows:

$$f_i = M_i - M_{i+1}, \tag{39}$$

where $i = 1, 2, \ldots, n$.

The column matrix containing the information of the elastic properties of material *S* is:

$$s_i = - k_i \Delta l_i, \tag{40}$$

where $i = 1, 2, \ldots, n$.

Comment 1: In all the cases where the index value exceeds n, the value corresponding to the magnitude is set to zero.

Comment 2: The summation over repeated indices is not performed in the given formulas.

Comment 3: The matrixes for the beam system with the ramification of an exoskeleton are governed by generalizations, obtained for nonramified systems like the considered model; however, it is necessary to consider the change of the sign for the opposite after passing through the ramification point.

This means that the matrix form of the motion equation is universal and may be used for exoskeleton movements with any number of links. The matrix structures will be the same; only their dimensionality, mass quantities, and link lengths for every element of the matrix changes.

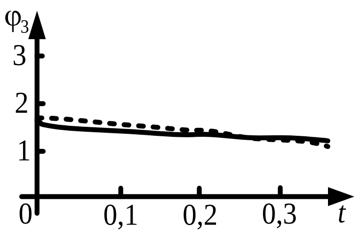
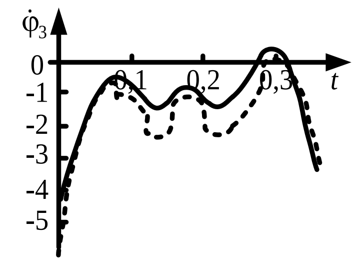
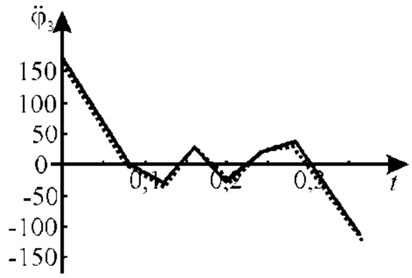
Thus, generalizations for the motion equations of a beam system with deformed links were obtained for the first time.

V. NUMERICAL SOLUTION OF SYSTEM OF DIFFERENTIAL EQUATIONS OF MOTION

In this study [20] deformations arising in the links of the biomechanical system during walking were experimentally determined. The inverse problem of dynamics was solved in that the forces between the movable deformable core elements of the biomechanical system were determined by the known kinematic characteristics [19]. To check the adequacy of the proposed mathematical model of the exoskeleton, the direct problem of dynamics needs to be solved. The control points were put in the equations of motion of the exoskeleton with links of variable lengths, and the dependence of the angles, velocities, and accelerations were found.

A mathematical model of the motion of an exoskeleton is a system of nonlinear differential equations of the second order. The resulting system of differential equations is large and non-linear. Therefore, its numerical solution is difficult. Of all known universal mathematical packages, only SCM Mathematica successfully handled the numerical solution of the system of differential equations in a reasonable time. The Runge-Kutt method was used for the solution, which was selected for the numerical solution of differential equations because it is a tested, proven method that has a small margin of error, is sustainable, and ensures convergence to a single solution of the Cauchy problem. The numerical values of the linear and inertial characteristics of the model were considered to be equal to the corresponding specific parameters of an experimentally simulated person [20]. Initial conditions were set at the beginning of the step.

The calculation was carried out for one step. Fig. 6 illustrates the dependence of the angular displacement, velocity, and acceleration for one of the links of the exoskeleton (the hip), which was the result of the solution of the system of differential equations of motion (denoted by solid lines in the graphs) and experimental data for comparison [20] (denoted by dotted lines in the graphs).

The thigh of the supporting leg		Angular displacement, (rad)
The thigh of the supporting leg		Angular velocity, (rad/s)
The thigh of the supporting leg		Angular acceleration (rad/s^2)

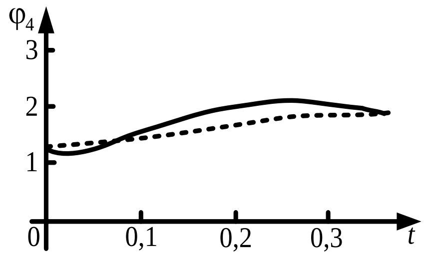
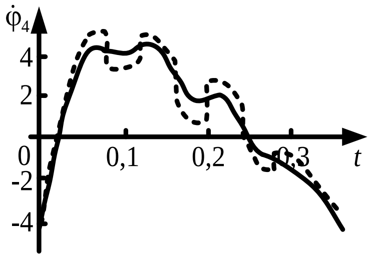
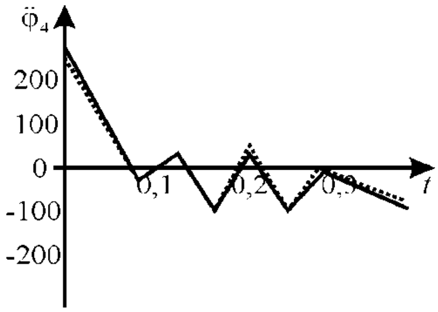
The hip of the portable leg		Angular displacement, (rad)
The hip of the portable leg		Angular velocity, (rad/s)
The hip of the portable leg		Angular acceleration (rad/s^2)

Fig. 6 Dependence of angular displacement (rad), speed (rad/s), and acceleration (rad/S2) (from left to right in rows) solve the system of equations with deformable links (*the result of solving the system of differential equations of motion = the solid lines in the graphs; the experimental data = the dotted lines on the graphs)*

The calculations were performed for the model with rigid links and links of variable lengths. The results of calculations of the ratio of maximum values of kinematic and dynamic characteristics of the model with rigid links are shown to the same characteristics of the model with links of variable lengths in Table 1. The calculations were not conducted for housing or units with numbers 7 and 12 due to their complex anatomical structure, flexibility, and mobility.

TABLE 1 RESULTS OF CALCULATIONS OF THE RATIO OF MAXIMUM VALUES OF KINEMATIC AND DYNAMIC CHARACTERISTICS OF THE MOTION OF THE MODEL WITH RIGID LINKS AND THE LINKS OF VARIABLE LENGTHS

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_8	φ_9	φ_{10}	φ_{11}
ω_T/ω_D	1,29	1,24	1,14	1,03	1,00	1,39	2,33	2,14	3,09	1,20
$\varepsilon_T/\varepsilon_D$	1,94	2,33	1,40	1,88	1,87	1,25	1,79	1,09	1,78	1,25
M_T/M_D	3,50	2,20	1,16	1,08	1,00	1,13	1,09	1,00	1,00	1,00

The table below shows that the highest value of change in the length of the links was to the legs and body. For portable feet and hands, accounting for changes in the lengths of the links had a negligible effect. This disagreement was especially pronounced in the control points. Accounting for the changes in the lengths of the links significantly reduced the requirements to the control points of the supporting legs and body, localized in the area of the large joints.

A numerical evaluation of the main characteristics of the changes of the lengths of the links is shown in Table 2 below.

TABLE 2 RESULTS OF CALCULATIONS OF THE MAXIMUM ABSOLUTE VALUE OF THE CHANGE IN LENGTH OF THE LINKS, ELONGATION, AND THEIR FIRST AND SECOND DERIVATIVES WITH RESPECT TO TIME FOR EACH LINK

	l_1	l_2	l_3	l_4	l_5	l_6	l_8	l_9	l_{10}	l_{11}
Δl, m	0,016	0,015	0,024	0,013	0,011	0,009	0,036	0,036	0,044	0,023
$\Delta l/l_0$, %	6,84	3,91	5,09	2,61	2,88	3,84	14,70	15,01	17,89	9,50

$\dot{l}$, m/s	0,40	0,42	0,38	0,55	0,81	0,17	0,33	0,49	0,59	0,61
$\ddot{l}$, m/s^2	11,88	14,38	15,63	15,63	18,75	22,50	26,25	30,63	34,38	16,88

As the table shows, a significant acceleration arose from the changes of the lengths of the links. This lead to reduced shock loads on the musculoskeletal system of the user. Therefore, accounting for variability in the lengths of the links discussed in the work was necessary to create an adequate model.

VI. CONCLUSIONS

A dynamic model of an exoskeleton with twelve links of variable lengths was created for the first time. A system of nonlinear differential equations for modeling movements of a twelve-linked mechanical system was compiled and recorded. These results can be used to describe the motion of an exoskeleton or anthropomorphic robot of similar structure. The direct problem of dynamics of movement of the exoskeleton, with a rigid and length-changing links for given actions was formulated and solved, and the numerical solution held true using experimental efforts. The success of the simulations gives rise to the possibility of their application as a bipedal walking exoskeleton with links of variable lengths as a method to restore lost motor abilities. It will be possible to measure pulses with appropriate developments in technology, indicating the direction of a person's actions and synthesizing the movements of the exoskeleton. Checking adequacy of the developed exoskeleton model showed its potential to restore and enhance the user's motor abilities. The experimentations also showed the possibility of using information obtained from the wearer to control the movement of the exoskeleton.

For the first time, a comparative analysis of a model with rigid and variable-length links showed that accounting for changes in the lengths of the links is necessary; despite the fact that the changes themselves are small, their derivatives are significant. Shock loads on the musculoskeletal system and requirements to control actions were shown to be reduced using links with changeable lengths. The results of the study give a better understanding of functioning mechanisms of the body's musculoskeletal system and can help create a model of exoskeleton with properties similar to a human.

REFERENCES

[1] http://vzvod.ucoz.ru/publ/ehkzoskelet_dospekhi_budushhego/1-1-0-29.

[2] http://www.venture-news.ru/tech_and_design/6398-raytheon-xos-2-ekzoskelet-vtorogo-pokoleniya.html.

[3] http://www.3dnews.ru/news/bolee_podrobnii_obzor_prototipa_ekzoskeleta_xos_i_istoriya_ego_sozdaniya/.

[4] http://neotech.3dn.ru/publ/mekhanika_i_ehlektronika/ehkzoskelet_hulc_i_sarcos/12-1-0-55.

[5] http://www.dislife.ru/flow/theme/10412/.

[6] http://www.popmech.ru/blogs/post/2356-nachalis-probnyie-postavki-kiberkostyuma-ekzoskeleta-hal-5/.

[7] http://www.belych.ru/index.php/-atlas.

[8] http://www.membrana.ru/particle/13444.

[9] Johnson G.R., "Modelling the muscles of the scapula morphometric and coordinate date and functional implications," *J. Biomech*, vol. 29, no. 8, pp. 1039-1051, 1996.

[10] http://www.octopusproject.eu/.

[11] www.microbot.ru.

[12] http://edurobots.ru/konferenciya-icra-2014-slon-gepard-riba-i-drugie-roboti/.

[13] Borisov A.V., "Modeling of the human musculoskeletal system and use the results to develop a model for anthropomorphic robot," Monograph, Publishing House, p. 212, 2009 (in Russian).

[14] Zaciorskij V. M., Aruin A. S., and Seluyanov V. N., "Biomechanics of the locomotor apparatus of man," *Physical culture and sport*, p. 143, 1981.

[15] http://www.it-med.ru/library/p/ploskostopie.htm.

[16] Lurie A. I., "Analytical mechanics," p. 824, 1961.

[17] Formalskii, A. M., "Moving anthropomorphic mechanisms," M.: Nauka, p. 368, 1982.

[18] V. I. Gulyaev and T. V. Zavrazhina, "Dynamics of robotic manipulator with elastic links," *Applied Mechanics*, vol. 37, no. 11, pp. 130-140, 2001.

[19] U. I. Nyashin and V. A. Lohov, "Biomechanics Theory," *PSTU Publ.*, p. 210, 2007.

[20] M. R. Pitkin, "Theory of construction and design practice of anthropomorphic lower extremity prostheses: Technical Science Doctoral Dissertation," Saint Petersburg, p. 205, 2006.

[21] U. I. Fantalov, "Methods of computation and design of biped walking robot with translational kinematic pairs in legs joints. Author's abstract of Technical Science Candidate's dissertation," Moscow, p. 23, 2009.

[22] A. V. Chigarev and A. V. Borisov, "Simulation of controlled motion of bipedal anthropomorphic mechanism," *Russian Journal of Biomechanics*, vol. 15, no. 1(51), pp. 74-88, 2011.

[23] Bernstein N. A., "Selected works on biomechanics and Cybernetics," SportAcademPress, p. 296, 2001.

[24] Borisov A. V., "Modeling the dynamics of the compound-chain mechanism: dis. ... candidate of technical Sciences," p. 147, 2005 (in

Russian).

[25] A. V. Chigarev, G. I. Mikhasev, and A. V. Borisov, "Biomechanics: tutorial," Minsk: Grevtsov Publishings, p. 284 (in Russian) 2010.

[26] V. P. Dyakonov, "Mathematica 5.1/5.2/6 in mathematical and technical-scientific computations," Moscow: SOLON-PRESS, p. 744, 2010 (in Russian).

[27] A. V. Chigarev, A. V. Borisov, "On the use SCM "MATHEMATICA" as a software environment for anthropomorphic robot motion simulation," Theoretical and applied mechanics. International technical-scientific journal, Minsk, BNTU, iss. 26, pp. 35-44, 2011 (in Russian).

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