Valuation of a Typical Nigerian Crude Oil Reserve Using Real Option Analysis

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***Abstract*-Real options analysis is a valuable tool in asset valuation; however, its application in evaluating oil and gas reserves is yet to be widely accepted, though many investigators have presented its potential advantages. This paper demonstrates the applicability and importance of real options analysis in the valuation of a typical Nigerian crude oil reserve. The cost data utilized represent real data for a typical Nigerian oil field.**

An improved version of the Black and Scholes (BS) model was developed by eliminating the assumption of a constant volatility. Crude oil price data from 1987 to 2012 were analysed with the volatilities of the return on crude oil price computed, and a time series model was developed which replicates this pattern of volatility. This was achieved by the use of a GARCH (General Autoregressive Conditional Heteroskedacity) model. The project was evaluated using the traditional NPV valuation method, the original BS model and the BS model with non-constant volatility. The results obtained using these methods were then compared. The real options valuation method provided the most accurate and reliable estimate of crude oil reserves using the BS model, which incorporates option values for various levels of volatility.

Keywords- Real Options; Valuation; Volatility; Net Present Value; Uncertainty; Crude Oil Reserves

I. INTRODUCTION

A. Background

The activities of the oil and gas industry are flooded with myriads of uncertainties and huge capital investments, right from the initial lease acquisition phase to abandonment. During each of these phases, an oil and/or gas company is confronted with a number of decisions, and accuracy in making these decisions is one of the major factors that determine the profitability, survivability and growth of such firms. Therefore, a firm faces pressure to accurately quantify and incorporate the risks and uncertainties that evolve during each of these phases in order to properly value its assets.

It has been recently discovered that the most popular decision criterion, the Discounted Cash Flow (DCF) which is expressed as Net Present Value (NPV) amidst other traditional methods, is not particularly suitable for the valuation of oil and gas assets or projects because it neglects managerial flexibility and does not appropriately incorporate uncertainties and risks. These deficiencies in the DCF method necessitate the development of better valuation methods, which properly account for uncertainty in the valuation of a developed oil reserve. In order to solve this problem, this paper considers the following questions:

1 What are the factors to consider when valuing an undeveloped and a developed reserve?

2 Which factor constitutes the primary source of uncertainty in the two types of reserves?

3 How are the terms of the appropriate valuation model determined?

4 How is uncertainty incorporated into asset valuation?

B. Objectives

The objectives of this study can be summarized as follows:

1 Compute the Net Present Value of X project, which represents a developed field; the discount rate used is calculated using Weighted Average Cost of Capital (WACC).

2 Compute the value of the developed reserve using the Black-Scholes model, assuming a constant volatility.

3 Compute annual volatilities for crude oil prices using monthly prices from September 1987 to September 2012, and develop a model for volatility based on the computed annual volatilities.

4 Incorporate the volatility model into the existing BS model.

5 Compute the value of the X oil reserve using the modified BS model, and compare the three models.

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II. LITERATURE REVIEW

The theory of real options uses tools developed to price financial derivatives in order to price investment opportunities. A derivative is a financial contract with a value based on the value of an underlying asset. The underlying assets may be stocks, stock indices, future contracts, interest rates, etc.

The Black–Scholes or Black–Scholes-Merton model [1], a mathematical model of a financial market containing certain derivative investment instruments, was developed for the pricing of financial options. The model has been previously applied to valuation of hydrocarbon reserves, and assumes a stochastic process for the movement of the underlying stock prices. A stochastic process is a variable that evolves over time in a way that is random, at least in part. The Black and Scholes method assumes that the price process of the underlying stock can be described by an appropriate Ito's process, namely the Geometric Brownian Motion. The model was developed on the basis of the non-arbitrage argument, which implies that any option written on underlying stocks can be replicated perfectly by an acceptable trading strategy applied to a portfolio of the underlying stocks and the risk-free asset.

An early study by Dixit and Pyndick [2] suggested that price follows a stochastic process which can either be explained by the Geometric Brownian Motion (GBM) or by a Mean Reverting process. Dixit and Pindyck [2] also provided an extensive review of the various applications of this theory in monopolistic and competitive industries. Schluyer's survey indicated that DCF techniques, particularly NPV and Internal rate of return (IRR), have become state-of-the-art in evaluating projects within the petroleum industry [3]. The models are sometimes accompanied by Sensitivity analysis (SA), Monte Carlo simulation (MCS), and/or decision tree analysis (DTA) to support the decision making process related to investment in or rejection of a project. SA offers the shortcoming that each scenario remains fixed on a single future outcome and investment plan. There is usually no clear way to reconcile, aggregate or choose between scenarios. The disadvantage of MCS lies in the fact it lays thousands of possible paths for the uncertain variables. Thus it is extremely difficult to deal with decision opportunities that arise before the final date in a simulation model. In addition, it is often difficult to interpret the results of a simulation analysis because simulation models use subjective discount rates and do not incorporate market information. DTA also offers a disadvantage of reliance on subjective assessments of probabilities, subjective discount rates and preferences regarding the objective.

Zettl supported the insight that a real options approach allows the incorporation of management flexibility [4]. In contrast to traditional DCF analysis, real options equip management with opportunities to change the course of projects as time passes and more information is known. Vivian [5] developed a model for pricing an option to invest in an oil and gas project according to a binomial model and the backward induction methodology. Yao [6] compared four approaches for the valuation of real options: the Leuhrman approach, [7, 8] Marketed Asset Disclaimer (MAD) approach, the Smith approach [9] and the Luenberger approach [10]. John et al. [11] suggested the use of real options technique (ROT) for the valuation of smart wells rather than the use of NPV, due to the uncertain timing and impact of a Smart Well® completion. Operators intuitively understand the benefits of the technology, but are unable to compute a realistic value with conventional NPV valuation techniques. A real options technique was employed in the valuation of these smart wells by applying the concept of Flexibility Real Options.

III. METHODOLOGY

The Black and Scholes (BS) model for real options valuation has been discovered to have many weaknesses, as stated in the following assumptions. Constant volatility for return on stock prices (in the case of the oil and gas industry, the volatility is related to the price of crude oil/gas, volumetric estimates, costs, inflation, and any other variable that can constitute uncertainty and risk) is one major assumption that weakens the model. The BS model was built upon the European style option, in which a European option gives its owner the right to purchase or sell an underlying asset for a given price (exercise price, or strike price) on the expiration date. This is applicable to oil and gas leases because the expiration date is related to the time of the relinquishment of the lease.

The assumptions of the BS model include:

1 Risk-free and constant interest rate.

2 Constant volatility.

3 No arbitrage opportunity.

4 Fixed strike price.

5 Stock pays no dividends and commissions during the life of the option.

6 European exercise styles are employed.

7 Known and constant interest rates.

8 Returns are log-normally distributed.

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The Geometric Brownian Motion for return on stocks is given by:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dz(t)dt \tag{1}$$

where S (t) = Stock price at time t, μ is the expected return on the stock, σ is the volatility of the return on the stock, and dz (t) represents a stochastic process.

A. Development of the Valuation Model

The solution to the Black-Scholes Partial differential equation (PDE) is given by:

$$V(P,t) = Pe^{-\delta t}N(d_a) - Ce^{-r(T-t)}N(d_b) \tag{2}$$

where

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-z^2}\,dz,\ d_{at} = \frac{\ln\left(\frac{P}{C}\right)+\left(r+\frac{\sigma}{2}\right)(T-t)}{\sigma\sqrt{T}},\ d_{bt} = d_{at} - \sigma\sqrt{T-t},$$

P = price of the crude oil at time t = 0, C = cost of development at time t = 0, r = risk-free interest rate, δ = risk-adjusted discount rate (risk-free interest rate + risk premium), and σ = volatility of crude oil price.

Thus far, σ has been assumed to be constant. If a period of 1 year is considered with T number of years, it is observed from Table 1A and Table 1B (representing annualized volatility) that volatility is never constant. Volatility changes daily but, in order to reduce data volume, monthly prices of typical Nigerian crude oil were captured. The value of the volatility of crude oil price impacts the value of any project. In the succeeding sections, the model for estimating annualized volatility is obtained, and the time until expiration T is split into yearly intervals (i.e., t =1, 2, 3, 4... T-1, T).

The value at time t =1 year is given as:

$$V_1 = Pe^{-\delta}N(d_{a1})\text{-}Ce^{-r(T-1)}N(d_{b1}) \tag{3}$$

and

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-z^2}\,dz, \qquad d_{a1} = \frac{\ln\left(\frac{P}{C}\right) + \left(r + \frac{{\sigma_1}^2}{2}\right)(T-1)}{\sigma_1\sqrt{T}}, \qquad d_{b1} = d_{a1} - \sigma_1\sqrt{T-1}$$

where σ_1 = annualized volatility for the first year period.

In year 2:

$$V_2 = Pe^{-2\delta}N(d_{a2})Ce^{-r(T-2)}N(d_{b2}) \tag{4}$$

and

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-z^2}\,dz, \qquad d_{a2} = \frac{\ln\left(\frac{P}{C}\right) + \left(r + \frac{{\sigma_2}^2}{2}\right)(T-2)}{\sigma_2\sqrt{T}}, \qquad d_{b2} = d_{a2} - \sigma_2\sqrt{T-2}$$

where σ_2 = annualized volatility for the second year, etc.

B. Model for Crude Oil Price Volatility

To generate a model for the volatility of crude oil prices, annualized volatilities were obtained from monthly prices of crude oil from September 1987 to September 2012. Volatility was computed by calculating the standard deviation of the natural logarithm of the returns for a 12-month period.

1) Calculation of Crude Oil Volatility

If P_t is the price/bbl at period t, then the return on the price $\boldsymbol{r_t}$ is defined as the ratio of the prices at period t and t-1:

$$r_t = \frac{P_t}{P_{t-1}} \tag{5}$$

To approximate continuously compounded returns, the natural logarithm of $\boldsymbol{r_t}$ was used in the computation. Denoting the mean of returns over n time periods by m:

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$$m = \frac{1}{n}\sum_{t=1}^{n} \ln \mathrm{r_t} \tag{6}$$

The variance can be computed using a standard likelihood variance estimator:

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n} (\ln \mathrm{r_t} - \mathrm{m})^2 \tag{7}$$

The volatility is given by:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n} (\ln \mathrm{r_t} - \mathrm{m})^2} \tag{8}$$

The volatility is adjusted to provide the annualized value. In this case, monthly oil price data are used. Therefore, annualized volatility is given by:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n} (\ln \mathrm{r_t} - \mathrm{m})^2 * \ 12} \tag{9}$$

The annualized volatilities were generated for September 1988 to September 2012 using Microsoft Excel. The results obtained are presented in Tables 1A and 1B. A time-series model was employed to model the annualized volatilities. The GARCH (general autoregressive conditional heteroskedasticity) model for the variance is given by GARCH (1, 1). The basic principle of the GARCH model assumes that the shocks of an asset are serially uncorrelated but dependent, which can be described by a simple quadratic function of the lagged values.

The GARCH option price is not preference-neutral, and depends on the unit risk premium (λ) as well as the two GARCH (1, 1) process parameters ((α_1 ,β_1). In general, the GARCH option price does not seem overly sensitive to these parameters; however, deep-out-of the-money and short maturity options are an exception [12]. The variance persistence parameter, $\gamma = \alpha_1 + \beta_1$, has a material bearing on the magnitude of the Black-Scholes model bias. The risk preference parameter alternatively determines the "leverage effect" and can be important in determining the direction of the Black-Scholes model bias. Consequently, a time varying risk premium may help explain a general underpricing or overpricing of traded options.

If ε_t represents the shocks of an asset, then the GARCH (*p*, *q*) model assumes:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{10}$$

where α_i and β_j are non-negative constants and ω is a strictly positive constant. The log return series is given by: $\varepsilon_t = \log\left(x_t / x_{t-1}\right)$, while σ_t^2 represents the time varying variance.

Using the GARCH (1, 1) model:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \qquad t = 1, 2, ..., \tag{11}$$

where $\omega > 0$; $\alpha > 0$; $\beta \geq 0$; and $\alpha + \beta < 1$ for the full series and each of the subseries of the return for the nine indices. The unconditional variance is measured by $\omega / (1 - \alpha - \beta)$ while the level of persistence is measured by $(\alpha + \beta)$; the closer to unity, the more persistent is the volatility of return. The half-life volatility, a measure of the average time it takes the persistence to reduce by one-half is obtained by $\ln(0.5)/\ln(\alpha + \beta)$. As $\alpha + \beta$ approaches 1, the half-life of the volatility increases. The unconditional standard deviation of the return series is measured by $\overline{\sigma} = \sqrt{\omega/(1-\alpha-\beta)}$ [13].

C. Valuing X Oil Reserve

The time until relinquishment of X oil field, T, is 16 years from present. Three models were used: NPV, a BS model with constant volatility and a BS model with variable annualized volatility computed from the volatility model.

1) Net Present Value (NPV)

The NPV for this project is obtained by:

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$$\mathrm{NPV} = \sum_{t=0}^{T} \frac{(\mathrm{NCF})_t}{(1+r)^t} \tag{12}$$

Royalty was placed at 15% (for onshore fields), and tax has been neglected. Cost of capital = r = 5% (assumed to be the risk-free rate). Internal rate of return (IRR) was also obtained for the project.

2) Black and Scholes with Constant Crude Oil Price Volatility

According to the BS model:

$$V(P,t) = Pe^{-\delta t}N(d_{at}) - Ce^{-r(T-t)}N(d_{bt}) \tag{13}$$

where

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-z^2}\,dz, \qquad d_{at} = \frac{\ln\left(\frac{P}{C}\right) + \left(r + \frac{\sigma^2}{2}\right)T - t}{\sigma\sqrt{T}}, \qquad d_{bt} = d_{at} - \sigma\sqrt{T-t}$$

N (d) is the cumulative standard normal distribution function (computed by the Excel function NORMSDIST), and represents the probability that a normally distributed variable with a mean of zero and a standard deviation of one would have a value less than d. N (d) was obtained using the *Normal (z)* function in MS Excel, and substituted into the above equation to obtain the value of the X developed reserve on a one-year interval. The volatility was assumed to be constant, and the value of the most recent annualized volatility (September 2012) was assumed for each time period. The crude oil price used at time zero was that of September 2012, which was equal to $113.37.

The term **C** is the total cost per barrel, and was estimated by computing the total cost of facilities, development, and operation per year (Table 2) and dividing the obtained result by the total estimated number of barrels of the annual crude oil produced.

3) Black and Scholes with Varying Annualized Volatilities

As explained in section B1, the obtained annualized volatility model was used to estimate the annualized volatilities for the subsequent months (from October 2012 to October 2038); the annualized volatilities were then averaged over a 12-month period to obtain a single volatility for each year. The time until relinquishment of X oil field is 16 years; hence, the average annualized volatilities were computed for years 1 through 16.

IV. RESULTS AND DISCUSSION

Fig. 1 shows the distribution of the annualized volatilities of return on crude oil price with respect to time in months. Shocks in volatilities were noted between September 1990 and July 1991, as well as between September 2008 and October 2009. The results depicted that a time series model was necessary to fit this pattern, rather than simply averaging the annualized volatilities and using a single value.

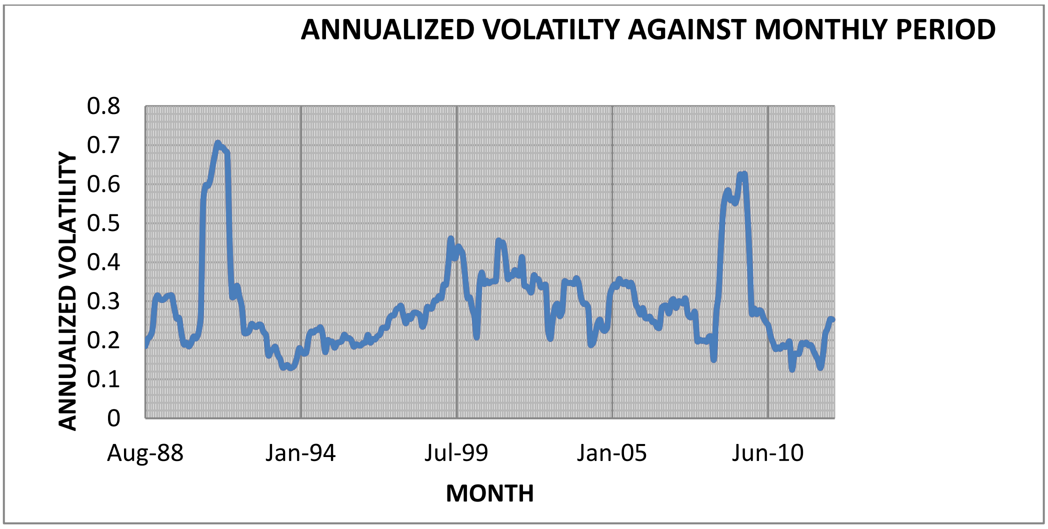

Fig. 1 Annualized volatility against monthly period

A. Result from NPV Model

Recoverable reserves for X oil field were estimated to be 22,967,387 MMbbls. A production profile was generated according to decline curve analysis. The economic production limit was estimated at 806 bbls/day. The valuation of X oil

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project using the NPV valuation method resulted in an NPV of $413.80MM using a risk-free rate of 5%. An additional risk factor of 3% (i.e., discount rate = 8%) was incorporated, resulting in an NPV of $231.44MM, while an additional risk value of 4% (i.e., discount rate = 9%) provided an NPV of $175.69MM. Any attempt to increase the risk factor above 7% (discount rate = 12%) yielded a negative NPV. Fig. 2 shows the results of NPV against discount rate. An internal rate of return (IRR) of approximately 12.69% was obtained. The project was not viable above this discount rate.

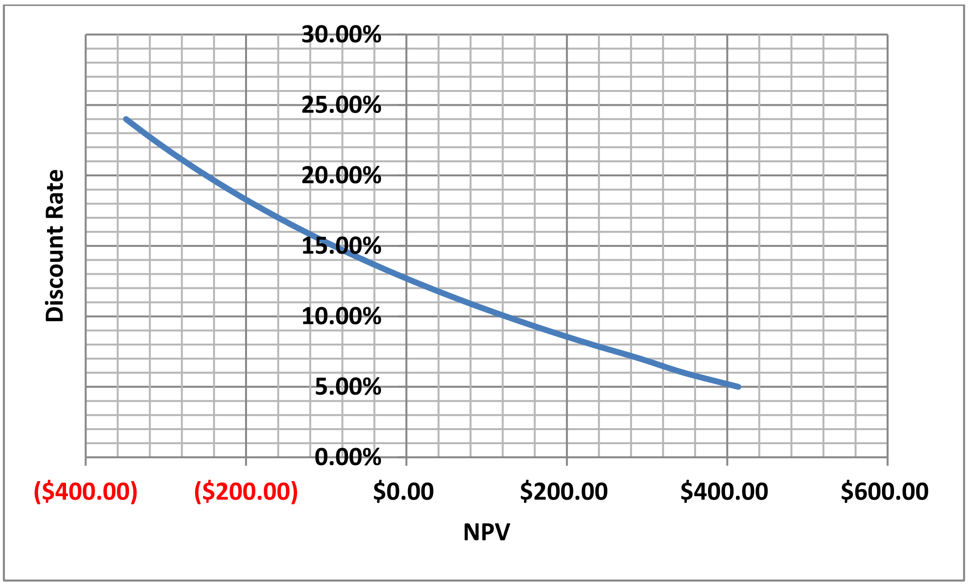

Fig. 2 Discount Rate against NPV

B. Results from Black and Scholes Model with Constant Volatility

The results obtained using the Black and Scholes model with constant volatilities are indicated in Tables 3A and 3B. The BS model is expressed by:

$$\boldsymbol{V(P,t) = Pe^{-\delta t}N(d_{at}) - Ce^{-r(T-t)}N(d_{bt})}$$

where C = development and operating cost per barrel ($/bbl), estimated to be $58.41/bbl based on the data provided; P = price per bbl at time t=0, assumed to be the price as of September 2012 (i.e., $113.37/bbl); r =risk free rate of return = 5%; δ = risk-free rate + risk premium (the risk premium for this investment was 13.5%, therefore δ = (5.0+13.5) % = 18.5%); T = time until lease relinquishment = 16years.

The values of $N(d_a)$ and $N(d_b)$ were computed from a normal distribution.

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-z^2}\,dz, d_{at} = \frac{\ln\left(\frac{P}{C}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T}}, d_{bt} = d_{at} - \sigma\sqrt{T-t}$$

The annualized volatility used was that of September 2012, which was equal to 0.253.The value obtained was $441.97MM for X oil project, which depicts a 7% increase over the value obtained using the NPV model at identical r=5% ($413.80MM). This was expected because in the BS model, the introduction of a volatility term removes the assumption that the crude price remains constant at $113.37/bbl throughout the production period; there is either a probability of increase or decrease. However, the idea of constant volatility implies that the variation in oil prices follows a uniform pattern; which is one limiting problem for assuming a constant volatility.

C. Result Obtained Using Varying Annualized Volatilities

In the Black and Scholes model with varying annualized volatilities, the volatilities were modeled using GARCH (1, 1). The results are presented as follows:

Dependent Variable: Y

Method: ML - ARCH (Marquardt) - Normal distribution

Date: 30/10/12 Time: 22:11

Sample (adjusted): 1987M03 2012M01

Included observations: 299 after adjustments

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Estimation settings: tol = 0.00010, derivs = accurate numeric (linear)

Initial Values: C(2) = 0.26213, C(3) = 0.00491, C(4) = 0.15000, C(5) = 0.60000

Convergence achieved after 18 iterations

Variance backcast: ON

Y=C(2)*Y(-1)

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

From the analysis, the following models were obtained:

$$r = 0.2093357651 r_t + \varepsilon_t \quad (13)$$

$$(0.0012)$$

$$\sigma_t^2 = 0.001195 + 0.220871\,\varepsilon_{t-1}^2 + 0.638259\,\sigma_{t-1}^2 \quad (14)$$

$$(0.1031) \qquad (0.0001) \qquad (0.0000)$$

The results of the GARCH (1, 1) model are displayed in Table 4. The results obtained using the BS model with non-constant volatilities are also presented in Tables 5A and 5B. The value obtained for X oil project using the volatility model was $945.13MM. This is greater than a 100% increase over the value obtained from the BS model with constant volatility. Although this may seem exaggerated, it does significantly indicate the impact of the variable volatility of crude oil prices on the valuation of an oil project.

TABLE 1A ANNUALIZED VOLATILITIES (SEPTEMBER 1988 TO AUGUST 2000)

Month	σ_a	Month	σ_a	Month	σ_a	Month	σ_a
Sep-88	0.1849393	Sep-91	0.310894	Sep-94	0.226973	Sep-97	0.26252
Oct-88	0.2032513	Oct-91	0.313356	Oct-94	0.233127	Oct-97	0.243505
Nov-88	0.2096929	Nov-91	0.340034	Nov-94	0.211631	Nov-97	0.262143
Dec-88	0.2278819	Dec-91	0.311482	Dec-94	0.169606	Dec-97	0.254785
Jan-89	0.2973902	Jan-92	0.286146	Jan-95	0.200002	Jan-98	0.268816
Feb-89	0.3151501	Feb-92	0.219279	Feb-95	0.195645	Feb-98	0.271182
Mar-89	0.3059346	Mar-92	0.218266	Mar-95	0.196305	Mar-98	0.269039
Apr-89	0.3031139	Apr-92	0.22344	Apr-95	0.181036	Apr-98	0.263283
May-89	0.3062689	May-92	0.241699	May-95	0.19281	May-98	0.234918
Jun-89	0.3129314	Jun-92	0.237053	Jun-95	0.19516	Jun-98	0.24902
Jul-89	0.3145254	Jul-92	0.233858	Jul-95	0.198187	Jul-98	0.285061
Aug-89	0.3144243	Aug-92	0.240103	Aug-95	0.21436	Aug-98	0.282048
Sep-89	0.2829713	Sep-92	0.238566	Sep-95	0.205621	Sep-98	0.281996
Oct-89	0.2554079	Oct-92	0.220834	Oct-95	0.205644	Oct-98	0.301538
Nov-89	0.2572324	Nov-92	0.213379	Nov-95	0.199685	Nov-98	0.302187
Dec-89	0.2171145	Dec-92	0.161725	Dec-95	0.183729	Dec-98	0.313429
Jan-90	0.1893579	Jan-93	0.17185	Jan-96	0.189737	Jan-99	0.308451
Feb-90	0.1942119	Feb-93	0.177683	Feb-96	0.187167	Feb-99	0.342675
Mar-90	0.1843251	Mar-93	0.183286	Mar-96	0.186764	Mar-99	0.342453
Apr-90	0.1927605	Apr-93	0.162334	Apr-96	0.192987	Apr-99	0.39746
May-90	0.209585	May-93	0.151333	May-96	0.194952	May-99	0.461131
Jun-90	0.2047212	Jun-93	0.130404	Jun-96	0.213894	Jun-99	0.411833
Jul-90	0.2183939	Jul-93	0.132267	Jul-96	0.193936	Jul-99	0.411
Aug-90	0.2637227	Aug-93	0.136581	Aug-96	0.203161	Aug-99	0.439519
Sep-90	0.5554473	Sep-93	0.128907	Sep-96	0.202116	Sep-99	0.432902
Oct-90	0.5972631	Oct-93	0.130304	Oct-96	0.210674	Oct-99	0.420205
Nov-90	0.5957537	Nov-93	0.135828	Nov-96	0.214706	Nov-99	0.365527

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Dec-90	0.6136056	Dec-93	0.155155	Dec-96	0.231023	Dec-99	0.308142
Jan-91	0.6489449	Jan-94	0.17972	Jan-97	0.232288	Jan-00	0.309879
Feb-91	0.6805739	Feb-94	0.171372	Feb-97	0.232772	Feb-00	0.280222
Mar-91	0.7060456	Mar-94	0.165735	Mar-97	0.255431	Mar-00	0.261044
Apr-91	0.6951256	Apr-94	0.169023	Apr-97	0.263532	Apr-00	0.208829
May-91	0.6948339	May-94	0.206857	May-97	0.264407	May-00	0.335064
Jun-91	0.6863183	Jun-94	0.222201	Jun-97	0.279179	Jun-00	0.373491
Jul-91	0.6788441	Jul-94	0.219681	Jul-97	0.282573	Jul-00	0.344808
Aug-91	0.4401706	Aug-94	0.226177	Aug-97	0.288087	Aug-00	0.352449

TABLE 1B ANNUALIZED VOLATILITIES (SEPTEMBER 2000 - SEPTEMBER 2012)

Month	σ_a	Month	σ_a	Month	σ_a	Month	σ_a
Sep-00	0.3463082	Sep-03	0.344543	Sep-06	0.232116	Sep-09	0.625481
Oct-00	0.3500738	Oct-03	0.359252	Oct-06	0.282798	Oct-09	0.541743
Nov-00	0.3514965	Nov-03	0.343449	Nov-06	0.288523	Nov-09	0.41526
Dec-00	0.3524705	Dec-03	0.308357	Dec-06	0.287849	Dec-09	0.268011
Jan-01	0.4546542	Jan-04	0.293717	Jan-07	0.268861	Jan-10	0.281516
Feb-01	0.4431099	Feb-04	0.293725	Feb-07	0.299659	Feb-10	0.266948
Mar-01	0.4497358	Mar-04	0.283075	Mar-07	0.305082	Mar-10	0.27667
Apr-01	0.4133563	Apr-04	0.188571	Apr-07	0.28293	Apr-10	0.274209
May-01	0.3575103	May-04	0.193112	May-07	0.299153	May-10	0.256911
Jun-01	0.3665446	Jun-04	0.218931	Jun-07	0.298647	Jun-10	0.245997
Jul-01	0.3651493	Jul-04	0.244219	Jul-07	0.294593	Jul-10	0.23732
Aug-01	0.379885	Aug-04	0.252641	Aug-07	0.306901	Aug-10	0.20927
Sep-01	0.3681821	Sep-04	0.228323	Sep-07	0.267187	Sep-10	0.193225
Oct-01	0.3666822	Oct-04	0.224262	Oct-07	0.258841	Oct-10	0.177625
Nov-01	0.4130569	Nov-04	0.234949	Nov-07	0.26189	Nov-10	0.181951
Dec-01	0.340692	Dec-04	0.31137	Dec-07	0.272332	Dec-10	0.178162
Jan-02	0.3377565	Jan-05	0.333383	Jan-08	0.197546	Jan-11	0.187699
Feb-02	0.3313801	Feb-05	0.343168	Feb-08	0.201418	Feb-11	0.182865
Mar-02	0.3233866	Mar-05	0.336667	Mar-08	0.198454	Mar-11	0.186501
Apr-02	0.3659566	Apr-05	0.356845	Apr-08	0.19959	Apr-11	0.196119
May-02	0.356495	May-05	0.348035	May-08	0.196524	May-11	0.124914
Jun-02	0.3559871	Jun-05	0.344216	Jun-08	0.20992	Jun-11	0.164283
Jul-02	0.3361462	Jul-05	0.349621	Jul-08	0.208721	Jul-11	0.165753
Aug-02	0.3402647	Aug-05	0.338354	Aug-08	0.1505	Aug-11	0.165978
Sep-02	0.3418383	Sep-05	0.347579	Sep-08	0.270524	Sep-11	0.192365
Oct-02	0.2273322	Oct-05	0.330288	Oct-08	0.320976	Oct-11	0.190033
Nov-02	0.2037099	Nov-05	0.293393	Nov-08	0.446723	Nov-11	0.19413
Dec-02	0.2542857	Dec-05	0.281359	Dec-08	0.540018	Dec-11	0.186726
Jan-03	0.2867473	Jan-06	0.265996	Jan-09	0.572875	Jan-12	0.187334
Feb-03	0.2931332	Feb-06	0.282569	Feb-09	0.584373	Feb-12	0.172978
Mar-03	0.2618023	Mar-06	0.257948	Mar-09	0.559248	Mar-12	0.161083
Apr-03	0.2748985	Apr-06	0.256124	Apr-09	0.563455	Apr-12	0.147931
May-03	0.3508985	May-06	0.26043	May-09	0.550717	May-12	0.129163

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Jun-03	0.3471597	Jun-06	0.247276	Jun-09	0.573493	Jun-12	0.161147
Jul-03	0.347054	Jul-06	0.247322	Jul-09	0.62417	Jul-12	0.215452
Aug-03	0.3468375	Aug-06	0.233809	Aug-09	0.613218	Aug-12	0.230948
						Sep-12	0.253685

TABLE 2 EVALUATION OF PROFITABILITY OF X OIL PROJECT USING NPV

YEAR	**Crude oil Production BOPD**	**Annual Production Forecast BBLS/yr**	**Total Development Cost $MM**	**Operating Cost (Fixed + Variable) $MM**	**Gross Revenue $MM**	**NCF $MM**	**NPV $MM**	**Discount Rate**
2011		-	1,272.72			-1,272.72	**$413.80**	**5.00%**
2012	12,000	4,008,425		12.03	454.44	374.24	**$345.89**	**6.00%**
2013	10,023	3,348,118		10.04	379.58	312.60	**$291.33**	**7.00%**
2014	8,372	2,796,583		8.39	317.05	261.10	**$231.44**	**8.00%**
2015	6,993	2,335,903		7.01	264.82	218.09	**$175.69**	**9.00%**
2016	5,841	1,951,110		5.85	221.20	182.16	**$123.69**	**10.00%**
2017	4,879	1,629,704		4.89	184.76	152.16	**$75.11**	**11.00%**
2018	4,075	1,361,243		4.08	154.32	127.09	**$29.63**	**12.00%**
2019	3,404	1,137,006		3.41	128.90	106.16	**($13.02)**	**13.00%**
2020	2,843	949,707		2.85	107.67	88.67	**($53.07)**	**14.00%**
2021	2,375	793,262		2.38	89.93	74.06	**($90.76)**	**15.00%**
2022	1,984	662,588		1.99	75.12	61.86	**($126.26)**	**16.00%**
2023	1,657	553,440		1.66	62.74	51.67	**($159.76)**	**17.00%**
2024	1,384	462,272		1.39	52.41	43.16	**($191.42)**	**18.00%**
2025	1,156	386,122		1.16	43.77	36.05	**($221.37)**	**19.00%**
2026	966	322,516		0.97	36.56	30.11	**($249.75)**	**20.00%**
2027	806	269,388		0.81	30.54	25.15	**($276.67)**	**21.00%**
							($302.24)	**22.00%**
							($326.56)	**23.00%**
	Np	22,967,387					**($349.70)**	**24.00%**
							ROR	***12.69%***

TABLE 3A COMPUTATION OF RESULTS FROM BLACK AND SCHOLES MODEL WITH CONSTANT VOLATILITY

Year	**t**	**σ**	**d_{at}**	**d_{bt}**	**$N(d_{at})$**	**$N(d_{bt})$**	**$P*(e^{-\delta t})*N(d_{at})$**	**C , $/bbl**
2012	1	0.25369	1.87786	0.89534	0.96980	0.81470	91.37678	58.41
2013	2	0.25369	1.79688	0.84768	0.96382	0.80169	75.47554	58.41
2014	3	0.25369	1.71589	0.80122	0.95691	0.78850	62.27812	58.41
2015	4	0.25369	1.63491	0.75612	0.94897	0.77521	51.32995	58.41
2016	5	0.25369	1.55392	0.71255	0.93990	0.76194	42.25293	58.41
2017	6	0.25369	1.47294	0.67072	0.92962	0.74880	34.73242	58.41
2018	7	0.25369	1.39196	0.63090	0.91803	0.73595	28.50655	58.41
2019	8	0.25369	1.31097	0.59344	0.90507	0.72356	23.35731	58.41
2020	9	0.25369	1.22999	0.55880	0.89065	0.71185	19.10313	58.41

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2021	10	0.25369	1.14900	0.52760	0.87472	0.70111	15.59279	58.41
2022	11	0.25369	1.06802	0.50076	0.85724	0.69173	12.70028	58.41
2023	12	0.25369	0.98703	0.47966	0.83819	0.68427	10.32061	58.41
2024	13	0.25369	0.90605	0.46666	0.81755	0.67963	8.36627	58.41
2025	14	0.25369	0.82507	0.46630	0.79533	0.67950	6.76432	58.41
2026	15	0.25369	0.74408	0.49040	0.77159	0.68807	5.45401	58.41
2027	16	0.25369	0.66310	0.66310	0.74637	0.74637	4.38468	58.41

TABLE 3B RESULTS FROM BLACK AND SCHOLES MODEL WITH CONSTANT VOLATILITY

T-t, yr	$e^{-r(T-t)}$	$Ce^{-r(T-t)}$	$N(d_{at})*Ce^{-r(T-t)}$	**Value ($/bbl)**	**Annual Prod, bbl**	**Vt ($MM) at r=5%**
15	0.47237	27.59293	26.75962017	64.61716	4008424.856	259.01
14	0.49659	29.00765	27.95822218	47.51732	3348117.877	159.09
13	0.52205	30.49490	29.18085177	33.09726	2796583.127	92.56
12	0.54881	32.05841	30.42233729	20.90762	2335902.579	48.84
11	0.57695	33.70208	31.67654291	10.57639	1951109.841	20.64
10	0.60653	35.43002	32.93632965	1.79609	1629703.93	2.93
9	0.63763	37.24656	34.19353638	-5.68698	1361243.146	-7.74
8	0.67032	39.15623	35.43898465	-12.08168	1137005.85	-13.74
7	0.70469	41.16381	36.66251133	-17.55938	949707.1169	-16.68
6	0.74082	43.27432	37.85303254	-22.26025	793262.0643	-17.66
5	0.77880	45.49305	38.99864123	-26.29836	662588.1722	-17.42
4	0.81873	47.82552	40.08674047	-29.76613	553440.1626	-16.47
3	0.86071	50.27759	41.10421249	-32.73794	462272.0817	-15.13
2	0.90484	52.85538	42.0376229	-35.27330	386122.0994	-13.62
1	0.95123	55.56533	42.87345727	-37.41945	322516.2876	-12.07
0	1.00000	58.41423	43.59838617	-39.21370	269388.2477	-10.56
				Estimated Reserves	**22967387.44**	**441.97**

TABLE 4 RESULTS OBTAINED BY THE GARCH(1, 1) MODEL

	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	**0.209336**	0.064789	3.231059	***0.0012***
		Variance Equation		
C	**0.001195**	0.000733	1.629989	***0.1031***
RESID(-1)^2	**0.220871**	0.058029	3.806220	***0.0001***
GARCH(-1)	**0.638259**	0.103163	6.186922	***0.0000***
R-squared	0.061777	Mean dependent var		0.006001
Adjusted R-squared	0.052235	S.D. dependent var		0.090034
S.E. of regression	0.087651	Akaike info criterion		-2.089103
Sum squared resid	2.266391	Schwarz criterion		-2.039598
Log likelihood	316.3209	Durbin-Watson stat		1.876055

TABLE 5A RESULTS USING BLACK AND SCHOLES WITH NON-CONSTANT VOLATILITY

Year	**t**	**σ**	d_{at}	d_{bt}	$N(d_{at})$	$N(d_{bt})$	$P*(e^{-\delta t})*N(d_{at})$	**C , $/bbl**
2012	1	0.25369	1.87786	0.89534	0.96980	0.81470	91.37678	58.41
2013	2	0.20588	1.87341	1.10309	0.96949	0.86501	75.91966	58.41
2014	3	0.16957	1.89697	1.28560	0.97108	0.90071	63.20066	58.41
2015	4	0.14085	1.93936	1.45145	0.97377	0.92667	52.67167	58.41
2016	5	0.11849	1.98650	1.59352	0.97651	0.94448	43.89882	58.41

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2017	6	0.10132	2.02346	1.70306	0.97849	0.95572	36.55833	58.41
2018	7	0.08839	2.03534	1.77018	0.97909	0.96165	30.40256	58.41
2019	8	0.07886	2.01009	1.78705	0.97779	0.96304	25.23409	58.41
2020	9	0.07200	1.94139	1.75089	0.97389	0.96002	20.88861	58.41
2021	10	0.06719	1.82978	1.66521	0.96636	0.95206	17.22628	58.41
2022	11	0.06388	1.68135	1.53850	0.95365	0.93804	14.12860	58.41
2023	12	0.06166	1.50479	1.38146	0.93381	0.91643	11.49803	58.41
2024	13	0.06019	1.30869	1.20443	0.90468	0.88579	9.25795	58.41
2025	14	0.05923	1.10001	1.01625	0.86434	0.84524	7.35119	58.41
2026	15	0.05860	0.88373	0.82513	0.81158	0.79535	5.73669	58.41
2027	16	0.05820	0.66310	0.66310	0.74637	0.74637	4.38468	58.41

TABLE 5B RESULTS USING BLACK AND SCHOLES WITH NON-CONSTANT VOLATILITY

T-t, yr	$e^{-r(T-t)}$	$Ce^{-r(T-t)}$	$N(d_{at})*Ce^{-r(T-t)}$	Value ($/bbl)	Annual Prod, bbl	Vt ($MM) at r=5%
15	0.06235	3.64210	3.532105108	87.84467	4008424.856	352.12
14	0.07502	4.38224	4.248552287	71.22699	3348117.877	238.48
13	0.09027	5.27279	5.120322387	57.15779	2796583.127	159.85
12	0.10861	6.34432	6.177912079	45.15204	2335902.579	105.47
11	0.13068	7.63360	7.454292409	34.79864	1951109.841	67.90
10	0.15724	9.18489	8.987293223	25.74513	1629703.93	41.96
9	0.18919	11.05143	10.82035858	17.68619	1361243.146	24.08
8	0.22764	13.29728	13.00193658	10.35537	1137005.85	11.77
7	0.27390	15.99953	15.58185145	3.52128	949707.1169	3.34
6	0.32956	19.25093	18.60330136	-3.01051	793262.0643	-2.39
5	0.39653	23.16308	22.08952652	-9.38925	662588.1722	-6.22
4	0.47711	27.87024	26.02553533	-15.70492	553440.1626	-8.69
3	0.57407	33.53399	30.33751796	-21.97125	462272.0817	-10.16
2	0.69073	40.34871	34.87482893	-28.11050	386122.0994	-10.85
1	0.83110	48.54831	39.40076524	-33.94676	322516.2876	-10.95
0	1.00000	58.41423	43.59838617	-39.21370	269388.2477	-10.56
				Np	**22967387.44**	**945.13**

D. Comparison of Three Models

As shown in Table 6, the real options valuation method yielded higher values for the oil reserve. The volatile nature of crude oil prices indicates that using a fixed value for crude oil price in the computation of a reserve value would be unrealistic; this is one of the weaknesses of the NPV approach. In the BS model with constant volatility, a higher value was observed for the reserve because this model incorporated the uncertainty of crude oil prices. The BS model with varying volatility presented a much higher value for the reserve because the aspect of constant volatility was eliminated. Using the GARCH (1, 1) model in e-View software, a model for volatilities was obtained and incorporated into the BS model to obtain the value of the reserve.

TABLE 6 COMPARISON OF RESULTS

MODEL	VALUE OF X OIL RESERVE ($MM)
NPV @ 5%	413.80
BS with constant volatility	441.97
BS with varying volatility	945.13

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V. CONCLUSION AND RECOMMENDATION

From the results obtained, X oil project (which would have been discarded if a discount rate greater than or equal to 13% had been considered appropriate to account for uncertainty) is very viable according to the real options valuation models.

Real options models generally present a better method to incorporate uncertainties, rather than indiscriminately choosing a discount rate. The data input in the real options model are observed data from the crude oil market, which helps to predict the value of a reserve from the trends observed in the data.

With the BS model, it was also observed that assuming a constant volatility underestimated the value of the reserve because crude oil prices are highly volatile. It was therefore necessary to account for changes in volatility by discretizing the total time of the lease and estimating the annualized volatility corresponding to each time period. Inputting each annualized volatility in the BS model thus obtained a more realistic value. Whether the change follows an increasing or decreasing trend, changes in volatility can be assumed with a great deal of confidence.

It is therefore highly recommended that real options models be employed in the valuation of oil and gas reserves in the oil and gas industry. Real options models prevent the unnecessary abandonment of projects that would have been viable but are determined to be otherwise due to inability to determine an appropriate discount rate in the NPV approach. Real options models can also prevent inadvertent initiation of projects whose values might have been exaggerated by the same reason.

Another advantage that real option models provide is the ability to determine the value of the option of delaying, expanding or abandoning a project. Although these areas were not explained in the methodology, real option models are capable of accurately quantifying these options based on market information and from the observation of other empirical data endemic to the oil industry.

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