Simplified Model for Penetration Depth Analysis of Extra-Large Concrete Caissons During Construction

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***Abstract*-Large concrete caissons are widely used in civil and ocean engineering. It is very important to keep caissons penetrate evenly and safely in the launching process. It is necessary to establish a suitable mechanical model to predict the penetration depth in the excavation. In this paper, a general multiple degree mechanical model is derived. The relationship between the penetration depth and the excavation amount are obtained. The model predicts the relationship between the penetration depth and the excavation. The numerical results obtained by the model are compared with those measured at the construction site. The results prove that they are in good agreement with the measured data, suggesting that the present model can be effectively applied for engineering predictions.**

Keywords*- *Large Extra Concrete Caissons; Simplified Mathematical Model; Penetration; Excavation; Runge-Kutta 4 Order Method

I. INTRODUCTION

The large extra concrete caisson is a structure with opened top and bottom with the closed side retaining wall. A caisson is sunk by self-weight and inner excavation to decrease friction. The leading edge of the caisson (blade) is sloped out at an angle to help sinking in a vertical manner. Normally it is adopted not only in the foundation of bridges and railways but also the foundation of drainage pump stations, oil storages and as well as buildings. A schematic diagram of a large extra concrete caisson is shown in Fig. 1. Some of the advantages of the extra caisson include adjustable final penetration depth, relatively small construction site needed, available inner space, no sheet pile supporting and high bearing capacity. With the development of construction technologies and machines, the number of cross-bays and cross-channels which separate the inner space of the caissons is increasing. Engineers are constructing large extra concrete caissons for deep water foundation [1, 2].

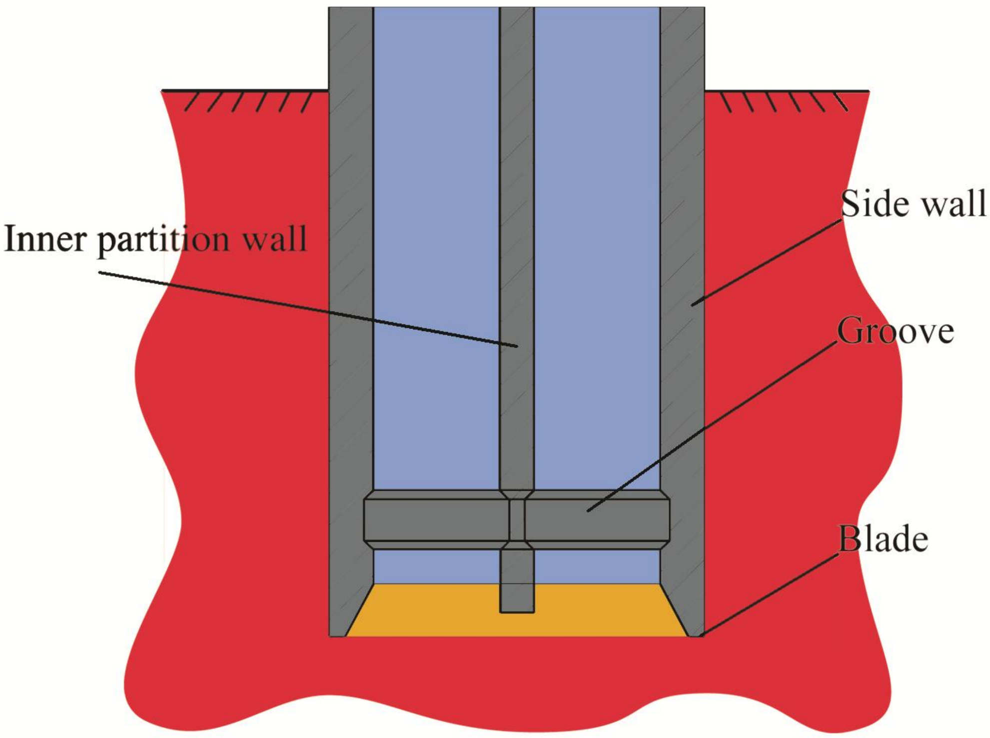

Fig. 1 Schematic diagram of a large extra concrete caisson

The complicate of geotechnical engineering together with uncertainties and time-variety existing in the construction of caissons makes it hard to control stabilizing penetrating, avoiding sink suddenly and ensuring final penetration depth on the site. These difficulties are attracted researchers and engineers to study the sinking process, especially in penetration speed, deflection and final penetration depth. Boothby and Johnstone [3] studied a deflection reduced method for steel caissons. Surendran [4] gave an innovative way to construct and launch heavy concrete caissons in a marine environment. However, the complexity of geotechnical engineering and dynamic variable system make it quite difficult to keep penetrating stably. In order to predict the penetration depth, it is necessary to establish a suitable analytical model to predict penetration depth for excavation. Some researchers [5, 6] gave some empirical formulas of bearing capacity analysis and side friction of foundations. There is still no full investigation of moving process in penetration. In this paper, a simplified model for large extra concrete caissons penetration is derived with multiple degrees of freedom. The relationship of penetration depth and excavation can be obtained through solving the model. The model has been applied to predict the extra caisson penetration. The numerical results obtained by the model are compared with those measured in the construction site of SB pump station of Shanghai (China) sewage treatment project. The results are in agreement with the measured data and thus the proposed model can be effectively applied for engineering predictions.

II. MECHANICS ANALYSIS

Fig. 2 shows the loads acting on the large extra concrete caisson: the gravity (G, in some cases, it may add additional weights), the friction between soil and the extra caisson wall (R_f), the resistance force on the blade (R_j) and the buoyancy force (N_w) for undrained penetration. It is difficult to get the exact lateral friction because of many uncertain influencing factors, such as roughness of the wall, adhesion and the friction angle between the wall and soil, the internal friction angle, the cohesive strength, the water content coefficient of soil, lateral soil pressure, gravity of soil, circumference of extra caissons and the instantaneous penetration depth etc. Furthermore, the lateral friction is affected by the penetration speed connecting with soil fluid property. Most tests of large extra concrete caissons indicated lateral friction increasing with penetration depth, and achieving to a constant at a certain depth (see Fig. 3) [7]. The other key load acting on the extra caisson is resistance on the blade. Its approximate analytical formula could be derived from ultimate ability theory using some simplifications [8].

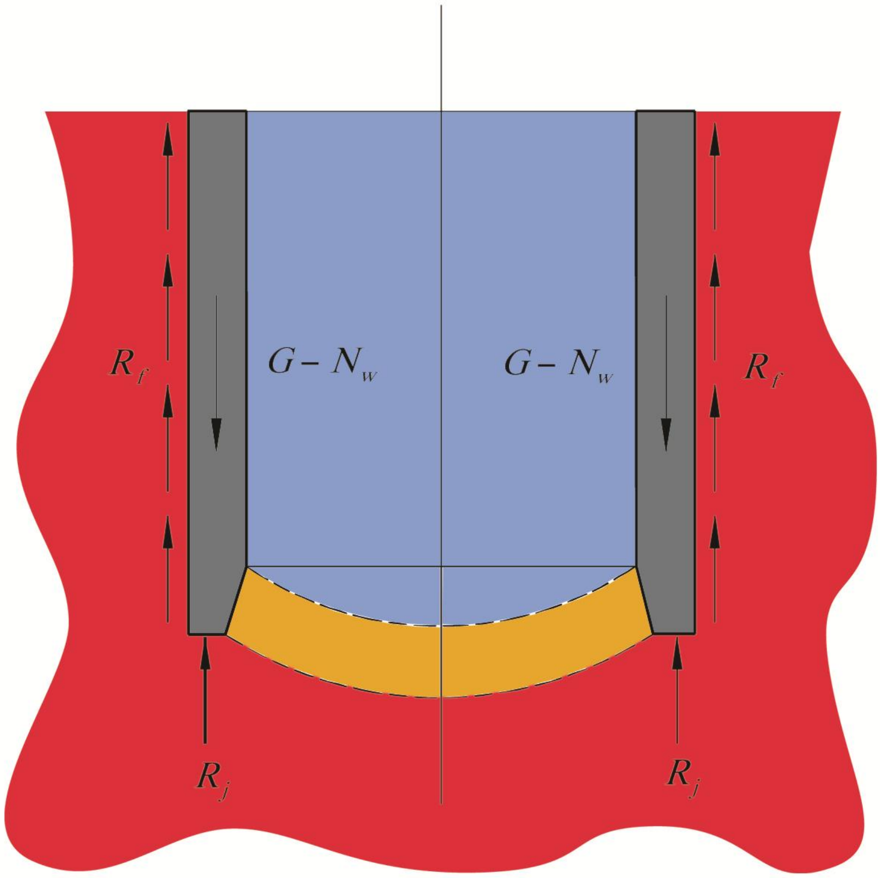

Fig. 2 Main loads distribution on the large extra concrete caisson

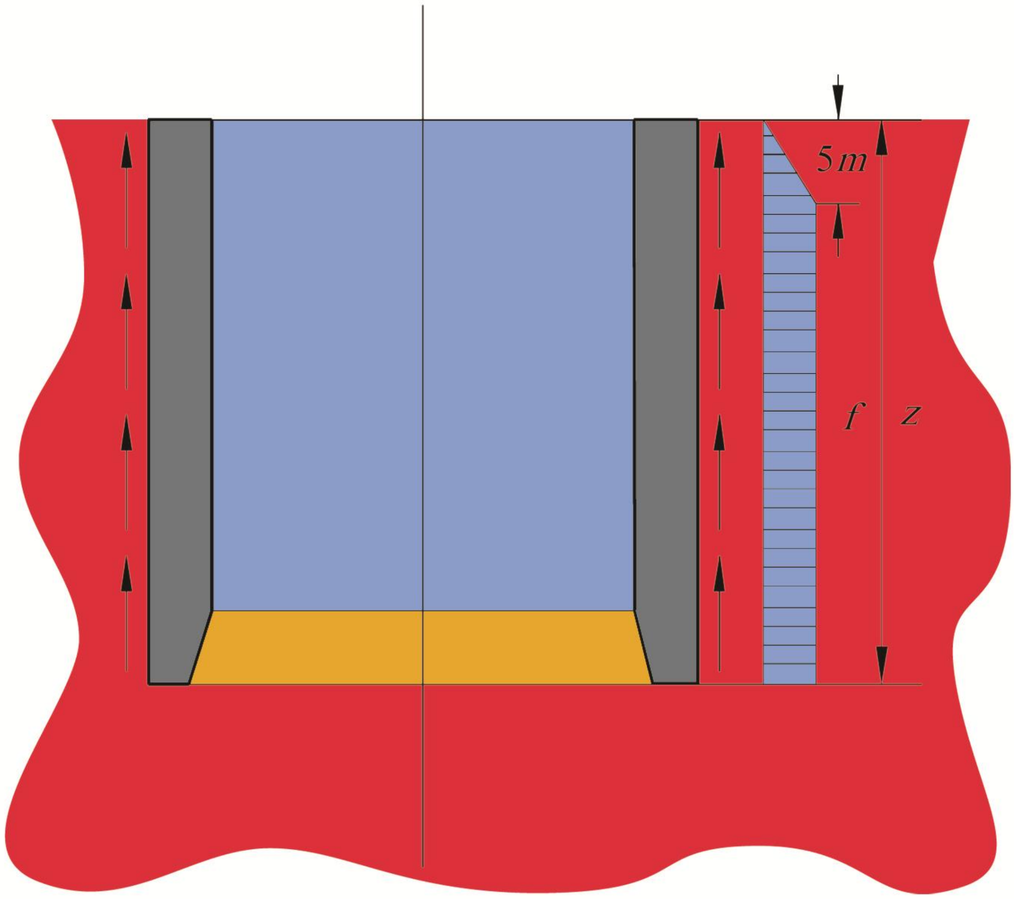

Fig. 3 Unit area lateral friction distribution along the wall

Large extra concrete caissons are space structures with large rigidity. Given the large size, the properties of soil in construction site are complicate. The penetration motion can be treated as a rigid motion of multiple freedom degrees in a three dimensional system, which can be resolved in six directions motion, especially in translation along x_0, y_0 and z_0 axis, and rotation around the axis x_0, y_0 and z_0 (see Fig. 4).

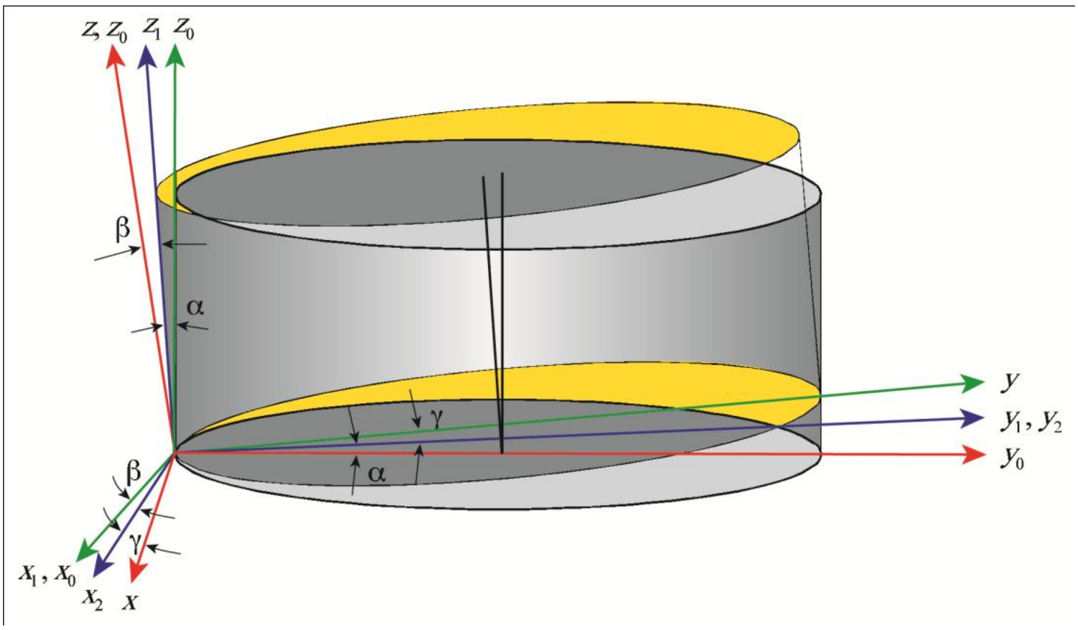

Fig. 4 Defection of the extra caisson during the penetration

Although the penetration speed of the large extra concrete caisson is slow, its instantaneous velocity cutting into soil is large; in addition, because its gravity is large, the variation of instantaneous kinetic energy penetrating in the soil is large. Moreover, the moment of inertia of the extra caisson is large considering its size, so the variation of the kinetic energy is large when the extra caisson deflects slight. The simplified model can be derived from instantaneous force analysis of the extra caisson motion in a three dimensional system.

Generally, the extra caisson moves by translation and by rotation. It can be considered as a six degrees of freedom. The reference coordinate can be supposed to be $Ox_0y_0z_0$, and conjoined coordinate is $Oxyz$ that fixed connect to a certain point O. Let the Cardan angles α, β and γ be the generalized coordinates (Fig. 5). The kinetic energy for rotation can be expressed as:

$$
\begin{aligned}
T_1 &= \frac{1}{2}\begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}\begin{bmatrix}I_0\end{bmatrix}\begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}^T \\
&= \frac{1}{2}\left[I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 - 2I_{yz}\omega_y\omega_z - 2I_{zx}\omega_z\omega_x - 2I_{xy}\omega_x\omega_y\right]
\end{aligned} \tag{1}
$$

where ω is the extra caisson angular velocity and I is the moment of inertia.

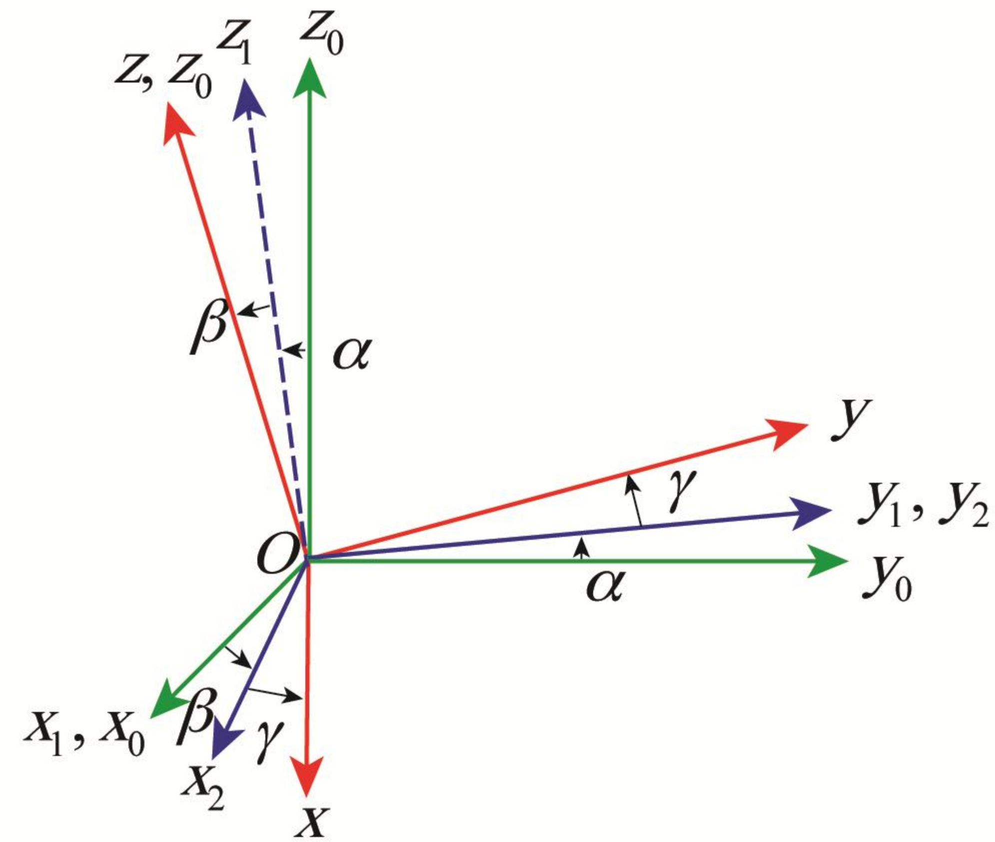

Fig. 5 The angles of Cartesian Coordinate

The angular velocity can be expressed in the Cardan coordinate system (Fig. 5) as:

$$\omega_x = \dot{\alpha}\cos\beta,\ \omega_y = \dot{\beta},\ \omega_z = \dot{\gamma} + \dot{\alpha}\sin\beta \tag{2}$$

Substituting Eq. (2) into Eq. (1), the kinetic energy of rotation can be written as

$$
\begin{aligned}
T_1 = \frac{1}{2}\Big[& I_x\dot{\alpha}^2\cos^2\beta + I_y\dot{\beta}^2 + I_z(\dot{\gamma} + \dot{\alpha}\sin\beta)^2 \\
& - 2I_{yz}(\dot{\gamma} + \dot{\alpha}\sin\beta)\dot{\beta} - 2I_{zx}(\dot{\gamma} + \dot{\alpha}\sin\beta)\dot{\alpha}\cos\beta - 2I_{xy}\dot{\alpha}\dot{\beta}\cos\beta\Big]
\end{aligned} \tag{3}
$$

The kinetic energy of translation is determined as

$$T_2 = \frac{1}{2}m\dot{x}_0^2 + \frac{1}{2}m\dot{y}_0^2 + \frac{1}{2}m\dot{z}_0^2 \tag{4}$$

where m is mass of the extra caisson, $\dot{x}_0$, $\dot{y}_0$ and $\dot{z}_0$ are the components of the translation velocity.

Defining the original position of the extra caisson to be zero potential level, the potential of the system is $V = -mgz_0$. The Euler-Lagrange equation is:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} = -\frac{\partial V}{\partial q_k} + Q_k \tag{5}$$

where q_k is a generalized displacement, Q_k is a generalized force, and $T = T_1 + T_2$. Since lateral friction of the extra caisson wall is too large to rotate around to z_0 axis, γ angle can be neglected. In addition, the translation displacements along the x_0, y_0 can be neglected because the soil lateral pressure are very large. Furthermore, β is small, it's value is about $\tan\beta < 0.01$, thus, one can obtained the simplified formation of $\sin\beta \approx \beta$ and $\cos\beta \approx 1$. Thus, according to Eq. (5), the motion equations of the large extra concrete caisson can be expressed as:

$$m\ddot{z}_0 = Q_{z_0} \tag{6}$$

$$(I_x - 2I_{zx}\beta)\ddot{\alpha} - I_{yz}\dot{\beta}^2 - 2I_{zx}\dot{\alpha}\dot{\beta} - (I_{yz} + I_{xy}\beta)\ddot{\beta} = Q_{M_\alpha} \tag{7}$$

$$-(I_{yz}\beta + I_{xy})\ddot{\alpha} + (I_x - I_z)\dot{\alpha}^2\beta + I_{zx}\dot{\alpha}^2 - I_{xy}\dot{\alpha}\dot{\beta}\beta + I_y\ddot{\beta} = Q_{M_\beta} \tag{8}$$

where Q_{z_0} are generalized forces corresponding to z_0, Q_{M_α}, Q_{M_β} are generalized forces corresponding to α, β respectively, I_x, I_y, I_z, I_{xy}, I_{yz} and I_{zx} are moments of inertia. The generalized forces Q_{M_α} and Q_{M_β} are given by:

$$Q_{M_\alpha} = R_j l_\alpha - (G - N_w)\left(\frac{1}{2}d_1 - \frac{1}{2}H\sin\alpha\right) - \frac{1}{2}R_f d_1 \tag{9}$$

$$Q_{M_\beta} = R_j l_\beta - \frac{1}{2}(G - N_w)H\sin\beta \tag{10}$$

where R_j is the resultant resistance on the blade, H is the height of the large extra concrete caisson, l_α and l_β are the distances from the action point of resultant resistance on the blade to the point O when the extra caisson rotates with α and β angles respectively, d_1 is the outer diameter of the large extra concrete caisson.

The generalized force Q_{z_0} in Eq. (6) can be calculated as

$$Q_{z_0} = G - R_f - R_j - N_w \tag{11}$$

The side frictions of the wall are calculated [6] using the following equations:

$$R_f(h) = \frac{1}{10}U_1 fh^2 + \frac{1}{10}U_2 f(h - h_e)^2 \quad \text{when } h \le 5(m) \tag{12}$$

$$R_f(h) = U_1 fh - 2.5U_1 f + \frac{1}{10}U_2 f(h - h_e)^2 \quad \text{when } h > 5(m) \tag{13}$$

where h_e is the excavation depth in the extra caisson, U_1 is outer perimeter of the extra caisson, U_2 is inner perimeter of the extra caisson, f is the lateral side friction force per area.

The resultant resistance on the blade has been derived by [8]. The equation can be expressed as:

$$R_j = \frac{K_m K_{mp}}{2n} \sum_{i=1}^{n} \left\{ c_i N_{ci} + \frac{1}{2}\gamma_i b N_{\gamma i} + N_{qi} \left[\frac{k_0 \gamma_i \tan\delta}{b \tan(45^0 + \varphi_i/2) e^{\pi \tan\varphi_i}} h_{1i}^2 + \gamma h_{1i} \right] \right\} U_{si} b$$
$$+ \frac{K_m K_{mp}}{2n} \sum_{i=1}^{n} \left\{ c_{n_i} N_{cn_i} + \frac{1}{2}\gamma_{n_i} b N_{\gamma n_i} + N_{qn_i} \left[\frac{k_0 \gamma_{n_i} \tan\delta}{b \tan(45^0 + \varphi_{n_i}/2) e^{\pi \tan\varphi_{n_i}}} h_{1n_i}^2 + \gamma h_{1n_i} \right] \right\} U_{sn_i} b \tag{14}$$

where $h_{i1} = h - h_{ei}$; $h_{1n_i} = h - h_{en_i}$ ($i = 1 \sim n$, n is total number of chambers in the wall, please refer to Fig. 6), h is the penetrated depth, b is the tread width of blade, c is cohesion for unit soil, h_{ei} and h_{en_i} are the excavation depth of the i th chamber and the n_i th chamber respectively, U is the mean perimeter of the large extra concrete caisson, U_{si} and U_{sn_i} are mean perimeters of the i th and the n_i th chamber matching with the inner wall of the large extra concrete caisson respectively, $N_q = \frac{(1+\sin\varphi)e^{\pi\tan\varphi}}{1-\sin\varphi\sin(2\eta+\varphi)}$, $N_c = \left[\frac{(1+\sin\varphi)e^{\pi\tan\varphi}}{1-\sin\varphi\sin(2\eta+\varphi)} - 1\right]\text{ctan}\,\varphi = (N_q - 1)\text{ctan}\,\varphi$, $N_\gamma = (N_q - 1)\tan 1.4\varphi$, φ is the internal friction angle of soil, γ_i and γ_{n_i} are the gravity of soil at i th and n_i th chamber respectively, k_0 is the static lateral press coefficient of the soil, K_{mp} is the influence coefficient of the geometrical cross-section and the resistance on the blade, K_m is the soil turbulence correction factor.

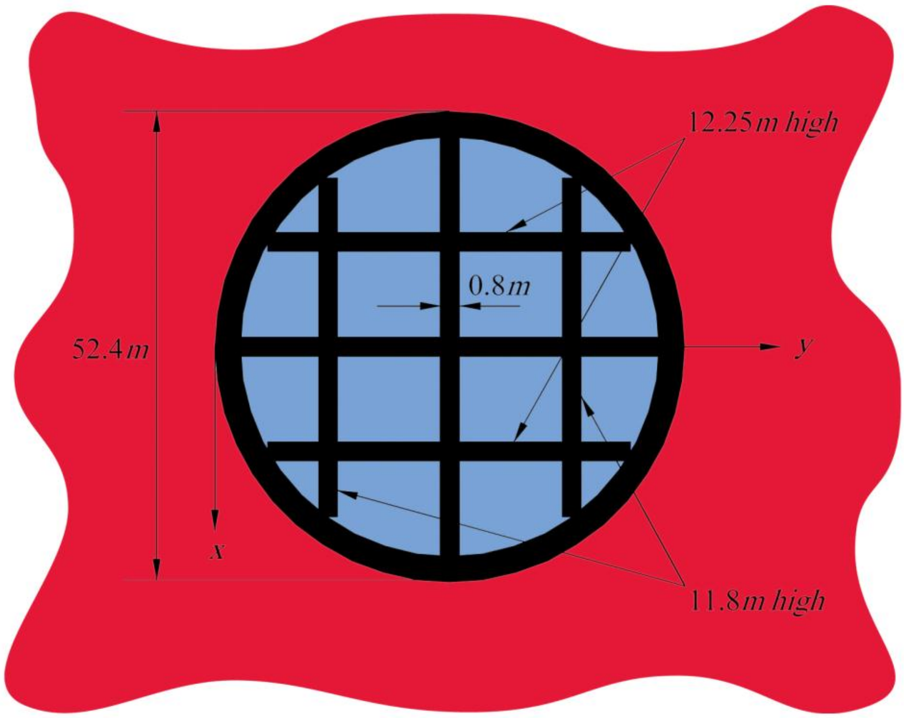

Fig. 6 Layout chart of the extra caisson in the site

In Eq. (9) and Eq. (10), the Q_{M_α} and Q_{M_β} can be calculated as

$$Q_{M_\alpha} = R_j l_\alpha - (G - N_w)\left(\frac{1}{2}d_1 - \frac{1}{2}H\alpha\right) - \frac{1}{2}R_f d_1 \tag{15}$$

$$Q_{M_\beta} = R_j l_\beta - \frac{1}{2}(G - N_w)H\beta \tag{16}$$

Substituting Eqs. (11), (15-16) into Eqs. (6-8), then

$$m\ddot{h} = G - R_f - R_j - N_w \tag{17}$$

$$\left(I_x - 2I_{zx}\beta\right)\ddot{\alpha} - I_{yz}\dot{\beta}^2 - 2I_{zx}\dot{\alpha}\dot{\beta} - \frac{G - N_w}{2}H\alpha - \left(I_{yz} + I_{xy}\beta\right)\ddot{\beta}$$
$$= R_j l_\alpha - \frac{G - N_w}{2}d_1 - \frac{1}{2}R_f d_1 \tag{18}$$

$$-\left(I_{yz}\beta + I_{xy}\right)\ddot{\alpha} + \left(I_x - I_z\right)\dot{\alpha}^2\beta + I_{zx}\dot{\alpha}^2 - I_{xy}\dot{\alpha}\dot{\beta}\beta + \frac{1}{2}\left(G - N_w\right)H\beta + I_y\ddot{\beta} = R_j l_\beta \tag{19}$$

For round extra concrete caissons, x, y and z are principles axis of inertia, so Eq. (18) and Eq. (19) can be simplified as:

$$I_x\ddot{\alpha} - \frac{1}{2}\left(G - N_w\right)H\alpha - R_j l_\alpha + \frac{G - N_w}{2}d_1 + \frac{1}{2}R_f d_1 = 0 \tag{20}$$

$$\left(I_x - I_z\right)\dot{\alpha}^2\beta + I_y\ddot{\beta} - R_j l_\beta + \frac{1}{2}\left(G - N_w\right)H\beta = 0 \tag{21}$$

where $I_x = I_y = \frac{1}{48}m\left[3\cdot\left(5d_1^2 + d_2^2\right) + 16H^2\right]$, $I_z = \frac{1}{8}m\left(3d_1^2 + d_2^2\right)$, d_1 and d_2 are the outer diameter and the inner diameter of the extra caisson.

To calculate l_α and l_β, it is assumed that there are $2r$ chambers along the wall considering symmetry excavation, the resultant resistance on the blade of wall can be written as sum of $\sum_{i=1}^{r} R_{ji}$ and $\sum_{i=1}^{r} R_{jr_i}$, in terms of Eq. (14), R_{ji} and R_{jr_i} are given by

$$R_{ji} = K_m K_{mp}\left\{c_i N_{ci} + \frac{1}{2}\gamma_i b N_{\gamma i} + N_{qi}\left(\frac{k_0\gamma_i \tan\delta}{b\tan\left(45^0 + \varphi_i/2\right)e^{\pi\tan\varphi_i}}h_{1i}^2 + \gamma h_{1i}\right)\right\}U_{si}b \tag{22}$$

$$R_{jr_i} = K_m K_{mp}\left\{c_{n_i} N_{cr_i} + \frac{1}{2}\gamma_{r_i} b N_{\gamma r_i} + N_{qr_i}\left[\frac{k_0\gamma_{r_i} \tan\delta}{b\tan\left(45^0 + \varphi_{r_i}/2\right)e^{\pi\tan\varphi_{r_i}}}h_{1r_i}^2 + \gamma h_{1r_i}\right]\right\}U_{sr_i}b \tag{23}$$

The distances (l_α and l_β) between the point of resultant force of resistance on each chamber to the point of rotation can be measured, resultant moment of resistance of each chamber to the rotation point can be calculated by

$$M_O = \sum_{i=1}^{r} R_{ji}l_i + \sum_{i=1}^{r} R_{jr_i}l_{r_i} \tag{24}$$

The moment can also be calculated by the total resistance (R_j) on the blade of each chamber multiplying the distance between the action points of total resistance to the point of rotation (l_α or l_β). $M_O = R_j l_\alpha$ or $M_O = R_j l_\beta$. Then, l_α and l_β can be obtained as

$$l_\alpha = \frac{1}{R_j}\left(\sum_{i=1}^{r} R_{ji}l_i + \sum_{i=1}^{r} R_{jr_i}l_{r_i}\right) \tag{25}$$

$$l_\beta = \frac{1}{R_j}\left(\sum_{i=1}^{r} R_{ji}l_i + \sum_{i=1}^{r} R_{jr_i}l_{r_i}\right) \tag{26}$$

Eqs. (17-19) are the total motion equations of the large extra concrete caisson, among these, Eq. (17) is the translation motion equation and Eqs. (18) and (19) are the rotation motion equations. Penetration depth and declination angle can be solved for from these differential equations.

Substituting Eqs. (12-14) into Eq. (17), the simplified model of penetration depth and excavation can be obtained and this model is the focus of this paper. To simplify the calculations, the difference of physical parameters of each chamber can be ignored since it has little influence on the whole penetration. Then, the simplified model of the relationship between penetration depth and excavation is given by:

when $h < 5\,(m)$

$$\ddot{h} = -\left[\frac{K_m K_{mp} k_1 b U_2}{m} + \frac{f}{10m}\left(U_1 + U_2\right)\right]h^2 + \left[\frac{2K_m K_{mp} k_1 b}{m}\bar{h}_e \bar{U}_s - \frac{K_m K_{mp} b\gamma U_2}{m} N_q + \frac{f U_2 h_e}{5m}\right]h$$

$$- \frac{f U_2}{10m} h_e^2 - \frac{K_n K_{mp} b}{m}\sum_{i=1}^{n} k_1\left(h_{ei}^2 U_{si} + h_{en_i}^2 U_{sn_i}\right) + \frac{K_m K_{mp} b\gamma}{m} N_q \cdot \bar{h}_e \bar{U}_s \tag{28}$$

$$- \frac{K_m K_{mp}}{m}\left(cN_c + \frac{1}{2}\gamma b N_\gamma\right)Ub + g - \frac{N_w}{m}$$

when $h > 5\,(m)$

$$\ddot{h} = -\left[\frac{K_m K_{mp} k_1 U_2 b}{m} + \frac{f U_2}{10m}\right]h^2 + \left[\frac{2K_m K_{mp} k_1 b}{m}\bar{h}_e \bar{U}_s + \frac{f U_2 h_e}{5m} - \frac{f U_1}{m}\right]h - \frac{f U_2}{10m} h_e^2$$

$$+ \frac{K_m K_{mp} b}{m}\sum_{i=1}^{n} k_1\left(h_{ei}^2 U_{si} - h_{en_i}^2 U_{sn_i}\right) + \frac{K_m K_{mp} b\gamma}{m} N_q \cdot \bar{h}_e \bar{U}_s \tag{28}$$

$$- \frac{K_m K_{mp}}{m}\left(cN_c + \frac{1}{2}\gamma b N_\gamma\right)Ub + \frac{2.5 U_1 f}{m} + g - \frac{N_w}{m}$$

where $k_1 = \dfrac{k_0 \gamma \tan\delta}{b\tan\left(45^0 + \varphi/2\right)e^{\pi\tan\varphi}}$, $\bar{h}_e \bar{U}_s = \sum_{i=1}^{n}\left(h_{ei} U_{si} + h_{en_i} U_{sn_i}\right)$, $U_2 = \sum_{i=1}^{n}\left(U_{si} + U_{sn_i}\right)$, $k_0 = 0.95 - \sin\varphi$.

Eqs. (27) and (28) are the simplified model of the relationship between penetration depth and excavation in construction. Solving these two equations gives the penetration depth for a certain excavation. In the following engineering application, the forth order Runge-Kutta method [9] is used to numerically solve the nonlinear differential Eqs. (27-28) they are programed using Matlab.

III. ENGINEERING APPLICATION

To verify the accuracy of the simplified mechanical model, the model is applied in SB pump station, a largest round extra concrete caisson of Shanghai (China) sewage treatment projects phase II (Fig. 6). The outside perimeter is 52.4m, and the caisson is 15.9 m high. There are two cross walls in the vertical position through the center of the structure with height of 12m, and width of 0.8m. The other four inside walls are 0.8m width and about 5m high, and inner space of the caisson was consist of 16 chambers. The construction site is located at an ancient river area (Suzhou River, China) with complex soil layers distribution. The soil material properties are measured at the site and given in Table 1. Drained excavation was adopted. Duration of excavation is 35 days, the final penetration depth of the caisson is 16.9m, and the total amount of excavated earth is about 25000m^3. Unfortunately, in this project construction site, it may happen drift sand under a certain dynamic water pressure at the gray clay silt layer. Furthermore, there is an artesian aquifer below 15.91m to 20.09m embedment, this may induce sharp penetration suddenly and large settlement. Thus penetration depth analysis is extremely important.

TABLE 1 GEOLOGIC SURVEY REPORT AT THE SITE

Layer Number	Soil Name	Buried elevation (m)	gravity γ (kN/m^3)	Shear strength index	
				Φ	c(kPa)
1	Plain fill	3.89~1.93			
2_1	Brown yellow silty clay	3.29~1.59	19.4	22	23
2_2	Grey yellow silty clay	2.64~0.79	17.9	17	17

3_1	Grey mud silty clay	1.50~-3.41	18	19	13
3_2	Grey clay silt	0.01~-2.81	18.4	28	8
4	Grey mud clay	-2.41~-16.10	16.9	15	12
5_1	Grey clay silt and sandy silt	-14.71~-18.89	18.5	28	10
5_2	Grey silty caly	-16.73~-21.63	17.7	18	11
5_3	Grey clay silt and sandy silt	-19.93~-23.51	18		
5_4	Grey silty clay	-20.73~39.07	17.7	21	14

In practical construction, to satisfy the penetration quality, the penetration is separated into three phases according to the site situation. These phases are: the initial penetration phase, the general penetration phase, and the final penetration phase. In the initial penetration phase, the extra concrete caisson penetration depth is less than 6.0m. During this phase, the center of gravity is high and the extra caisson stability penetration is bad. The penetration and deflection should be measured in every two hours. The soil inside the extra caisson should be excavated symmetrically. The extra caisson position should be kept vertically. In the general penetration phase, the penetration depth is between the 6.0 to 12.0m. The deflection can be corrected when the level error is more than 0.1m. The soil in 2m above the extra caisson blade cannot be excavated. The soil outside the extra caisson should be filled back to keep the resident force. The penetration depth of final penetration phase is the last 3.0 m. The velocity of excavation should be reduced. The deflection need to be corrected less than 0.05 m.

The penetration coefficient [7] is generally used to check the penetration condition. It is defined as:

$$K_s = \frac{G - N_w}{R_j + R_f} \tag{29}$$

where K_s is the penetration coefficient, and is initially set to be zero since the bottom is open. During the calculation, the penetration coefficient is changed at each time step. The penetration coefficient at time t is used to predict the displacement and the quantity of excavation at the next time $t + \Delta t$.

Fig. 7 shows the relationship between the predicted extra caisson penetration depth and the quantity of excavation. The figure shows that the extra caisson has an initial penetration depth. During the initial penetration phase, the caisson had a relative large penetration depth with less excavation. This is because the friction force is small and the penetration coefficient is large in this phase. The penetration acceleration and the penetration velocity are also large. In the general and final penetration phases, the penetration tends to a stable state. From Fig. 8, it can be seen that the tangent of the curve at these phases which represent penetration speed are smaller than these of the initial phase. Figs. 7 and 8 show that the predicted values are in agreement with the corresponding measured values at the site.

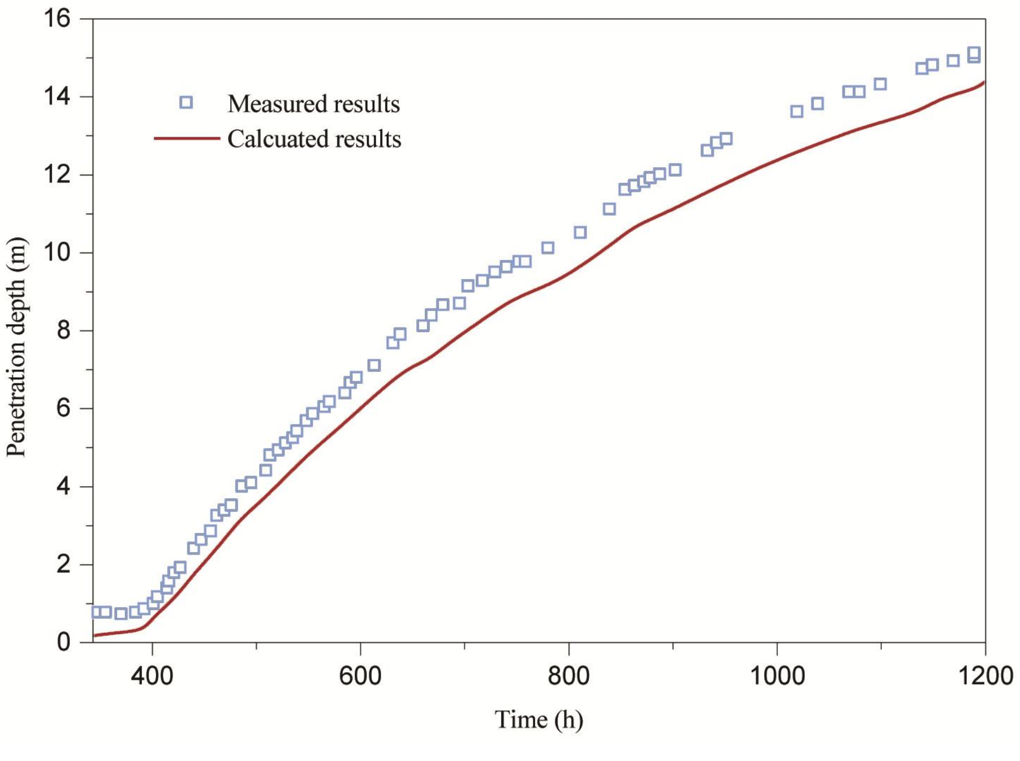

Fig. 7 Time history of the extra caisson penetration depth

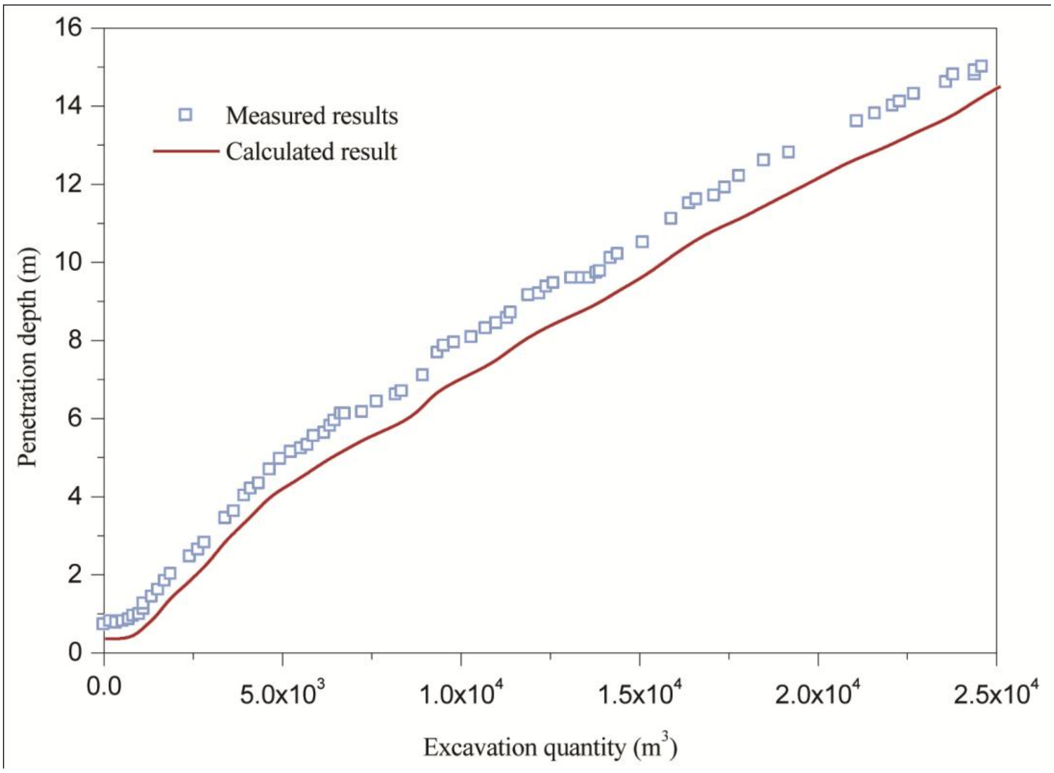

Fig. 8 Penetration depth varying with the quantity of excavation

IV. CONCLUSIONS

Large extra concrete caisson penetration is a complex multiple degree problem. In this paper, the prediction equations of the extra caisson penetration are derived by the application of Euler-Lagrange equations with assumptions of conservative systems. Equations are solved by Runge-Kutta Fourth order Method. The relative traction and moment are derived. The numerical program to solve the differential equations is written by Matlab. The predicted relationship between the penetration depth and the quantity of excavation is completely consistent with the measurements at the construction site. The real time penetration coefficient is adopted to monitor the evolution of the relationship between the penetration depth and the quantity of excavation in time. The simplified model can be used to supervise the extra caisson penetration and keep the extra caisson from sudden penetration. The present model provides a convenient and efficient method for engineering applications.

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