# Consumers' Activities for Brand Selection and Their Matrices Structure in the Case of Jewelry/Accessory Purchasing 

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#### Abstract

Consumers often buy higher ranked brand after they are bored using current brand goods. This may be analysed utilizing matrix. Suppose past purchasing data are set input and current purchasing data are set output, then transition matrix is identified using past and current data. If all brand selections are composed by the upper shifts, then the transition matrix becomes an upper triangular matrix.

Utilizing jewelry/accessory purchasing history record of online shopping (especially we focus on pierced earrings, ring, bracelet/bangle) over three years, above structure is investigated and confirmed. Comparison with our previous research is also executed. Some interesting results are obtained.


Keywords- Brand Selection; Matrix Structure; Brand Position; Jewelry; Accessory

## I. INTRODUCTION

It is often observed that consumers select upper class brand when they buy next time. Suppose that the former buying data and the current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then the transition matrix becomes upper triangular matrix under the supposition that the former buying variables are set input and the current buying variables are set output. If the top brand were selected from lower brand in jumping way, the corresponding part in upper triangular matrix would be 0 . These are verified in numerical examples with simple models. There may be also the case that customers select lower brand to seek suitable price when they have chosen higher brand. Then it may compose items of lower triangular matrix.

If the transition matrix is identified, S-step forecasting can be executed. Unless planner for products does not notice its brand position whether it is higher or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables making effective marketing plan and/or establishing new brand.

Quantitative analysis concerning brand selection has been executed by Yamanaka ${ }^{[5]}$, Takahashi et al. ${ }^{[4]}$. Yamanaka ${ }^{[5]}$ examined purchasing process by Markov Transition Probability with the input of advertising expense. Takahashi et al. ${ }^{[4]}$ made analysis by the Brand Selection Probability model using logistics distribution.

In this paper, matrix structure is analysed for the case
brand selection is executed for upper class and for lower class, utilizing jewelry/accessory purchasing history record of on-line shopping over three years. Comparison with our previous research ${ }^{[7]}$ is also executed. Some interesting results are obtained. Such research cannot be found as long as searched. Hereinafter, matrix structure is clarified for the selection of brand in Section 2. Block matrix structure is analysed when brands are handled in group in Section 3. Purchase history investigation of jewelry/accessory on-line shopping is examined and its numerical calculation is executed in Section 4. Application of this method is extended in Section 5, which is followed by the remarks of Section 6.

## II. BRAND SELECTION AND ITS MATRIX STRUCTURE

## A. Upper Shift of Brand Selection

It is often observed that consumers select upper class brand when they buy next time.

Now, suppose that $x$ is the most upper class brand, $y$ is the second upper brand, and $z$ is the lowest brand. Consumer's behaviour of selecting brand would be $z \rightarrow y$, $y \rightarrow x, z \rightarrow x$ etc. $x \rightarrow z$ might be few.

Suppose that $x$ is the current buying variable, and $x_{b}$ is the previous buying variable. Shift to $x$ is executed from $x_{b}, y_{b}$, or $z_{b}$.

Therefore, $x$ is stated in the following equation.

$$
x=a_{11} x_{b}+a_{12} y_{b}+a_{13} z_{b}
$$

Similarly,

$$
y=a_{22} y_{b}+a_{23} z_{b}
$$

and

$$
z=a_{33} z_{b}
$$

These are re-written as follows.

$$
\left(\begin{array}{c}
x  \tag{1}\\
y \\
z
\end{array}\right)=\left(\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
0 & a_{22} & a_{23} \\
0 & 0 & a_{33}
\end{array}\right)\left(\begin{array}{c}
x_{b} \\
y_{b} \\
z_{b}
\end{array}\right)
$$

Set

$$
\begin{gathered}
\mathbf{X}=\left(\begin{array}{l}
x \\
y \\
z
\end{array}\right) \\
\mathbf{A}=\left(\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
0 & a_{22} & a_{23} \\
0 & 0 & a_{33}
\end{array}\right) \\
\mathbf{X}_{\mathbf{b}}=\left(\begin{array}{l}
x_{b} \\
y_{b} \\
z_{b}
\end{array}\right)
\end{gathered}
$$

then, $\mathbf{X}$ is represented as follows.

$$
\begin{equation*}
\mathbf{X}=\mathbf{A} \mathbf{X}_{\mathbf{b}} \tag{2}
\end{equation*}
$$

Here,

$$
\mathbf{X} \in \mathbf{R}^{3}, \mathbf{A} \in \mathbf{R}^{3 \times 3}, \mathbf{X}_{\mathbf{b}} \in \mathbf{R}^{3}
$$

A is an upper triangular matrix.
To examine this, generating following data, which are all consisted by upper brand shift data,

$$
\begin{array}{rlll}
\mathbf{X}^{\mathbf{i}} & =\left(\begin{array}{l}
1 \\
0 \\
0
\end{array}\right) & \left(\begin{array}{l}
1 \\
0 \\
0
\end{array}\right) & \ldots \\
\mathbf{X}_{\mathbf{b}}^{\mathbf{i}} & =\left(\begin{array}{l}
0 \\
1 \\
0
\end{array}\right) & \left(\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right) & \ldots  \tag{4}\\
i & =1 & 2 & \ldots
\end{array}\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right)
$$

parameter can be estimated using least square method.
Suppose

$$
\begin{equation*}
\mathbf{X}^{i}=\mathbf{A} \mathbf{X}_{\mathbf{b}}^{i}+\boldsymbol{\varepsilon}^{i} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
J=\sum_{i=1}^{N} \boldsymbol{\varepsilon}^{i T} \boldsymbol{\varepsilon}^{i} \rightarrow \text { Min } \tag{6}
\end{equation*}
$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$
\begin{equation*}
\hat{\mathbf{A}}=\left(\sum_{i=1}^{N} \mathbf{X}^{i} \mathbf{X}_{\mathbf{b}}^{i T}\right)\left(\sum_{i=1}^{N} \mathbf{X}_{\mathbf{b}}^{i} \mathbf{X}_{\mathbf{b}}^{i T}\right)^{-1} \tag{7}
\end{equation*}
$$

In the data group of upper shift brand, an estimated value $\hat{\mathbf{A}}$ should be an upper triangular matrix.

If following data that have lower shift brand are added only a few in Equations (3) and (4),

$$
\begin{aligned}
& \mathbf{X}^{i}=\left(\begin{array}{l}
0 \\
1 \\
0
\end{array}\right) \\
& \mathbf{X}_{\mathbf{b}}^{i}=\left(\begin{array}{l}
1 \\
0 \\
0
\end{array}\right)
\end{aligned}
$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part of triangle.

## B. Sorting Brand Ranking by Re-Arranging Row

In a general data, variables may not be in order as $x, y, z$. In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in the upper triangular matrix, and the small value parts are gathered in the lower triangular matrix.

## A

$\left(\begin{array}{l}x \\ y \\ z\end{array}\right)\left(\begin{array}{ccc}\bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc\end{array}\right)$
Shifting row $\left(\begin{array}{l}z \\ x \\ y\end{array}\right)\left(\begin{array}{lll}\varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc\end{array}\right)$

## C. In the Case That Brand Selection Shifts in Jump

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the middle class brand.

We suppose $v, w, x, y, z$ brands (suppose they are laid from upper position to lower position as $v>w>x>y>z)$.

In the above case, selection shifts would be:

$$
\begin{aligned}
& v \leftarrow z \\
& v \leftarrow y
\end{aligned}
$$

Suppose they do not shift to $y, x, w$ from $z$, to $x, w$ from $y$, and to $w$ from $x$, then Matrix structure would be as follows.

$$
\left(\begin{array}{c}
v  \tag{9}\\
w \\
x \\
y \\
z
\end{array}\right)=\left(\begin{array}{ccccc}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
0 & a_{22} & 0 & 0 & 0 \\
0 & 0 & a_{33} & 0 & 0 \\
0 & 0 & 0 & a_{44} & 0 \\
0 & 0 & 0 & 0 & a_{55}
\end{array}\right)\left(\begin{array}{c}
v_{b} \\
w_{b} \\
x_{b} \\
y_{b} \\
z_{b}
\end{array}\right)
$$

## III. BLOCK MATRIX STRUCTURE IN BRAND GROUPS

Next, we examine the case in brand groups. Matrices are composed by Block Matrix.

## A. Brand Shift Group - in the Case of Two Groups

Suppose brand selection shifts from Corolla class to Mark II class in car. In this case, it does not matter which company's car they choose. Thus, selection of cars is executed in a group and brand shift is considered to be done from group to group. Suppose brand groups at time $n$ are as follows.
$\mathbf{X}$ consists of $p$ varieties of goods, and $\mathbf{Y}$ consists of $q$ varieties of goods.

$$
\begin{gather*}
\mathbf{X}_{\mathbf{n}}=\left(\begin{array}{c}
x_{1}^{n} \\
x_{2}^{n} \\
\vdots \\
x_{p}^{n}
\end{array}\right) \\
\mathbf{Y}_{\mathbf{n}}=\left(\begin{array}{c}
y_{1}^{n} \\
y_{2}^{n} \\
\vdots \\
y_{q}^{n}
\end{array}\right) \\
\binom{\mathbf{X}_{\mathbf{n}}}{\mathbf{Y}_{\mathbf{n}}}=\left(\begin{array}{cc}
\mathbf{A}_{11}, & \mathbf{A}_{\mathbf{1 2}} \\
\mathbf{0}, & \mathbf{A}_{22}
\end{array}\right)\binom{\mathbf{X}_{\mathbf{n}-1}}{\mathbf{Y}_{\mathbf{n}-1}} \tag{10}
\end{gather*}
$$

Here,

$$
\begin{gathered}
\mathbf{X}_{\mathbf{n}} \in \mathbf{R}^{p}(n=1,2, \cdots), \quad \mathbf{Y}_{\mathbf{n}} \in \mathbf{R}^{q}(n=1,2, \cdots), \\
\mathbf{A}_{\mathbf{1 1}} \in \mathbf{R}^{p \times p}, \quad \mathbf{A}_{\mathbf{1 2}} \in \mathbf{R}^{p \times q}, \quad \mathbf{A}_{22} \in \mathbf{R}^{p \times q}
\end{gathered}
$$

## B. Brand Shift Group -in the Case of Three Groups

Suppose brand selection is executed in the same group or to the upper group, and also suppose that brand position is $x>y>z$ ( $x$ is upper position). Then the brand selection transition matrix would be expressed as:

$$
\left(\begin{array}{l}
\mathbf{X}_{\mathrm{n}}  \tag{11}\\
\mathbf{Y}_{\mathrm{n}} \\
\mathbf{Z}_{\mathrm{n}}
\end{array}\right)=\left(\begin{array}{rrr}
\mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\
\mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\
\mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}
\end{array}\right)\left(\begin{array}{l}
\mathbf{X}_{\mathrm{n}-1} \\
\mathbf{Y}_{\mathrm{n}-1} \\
\mathbf{Z}_{\mathrm{n}-1}
\end{array}\right)
$$

Where

$$
\begin{aligned}
& \mathbf{X}_{\mathbf{n}}=\left(\begin{array}{c}
x_{1}^{n} \\
x_{2}^{n} \\
\vdots \\
x_{p}^{n}
\end{array}\right) \\
& \mathbf{Y}_{\mathbf{n}}=\left(\begin{array}{c}
y_{1}^{n} \\
y_{2}^{n} \\
\vdots \\
y_{q}^{n}
\end{array}\right) \\
& \mathbf{Z}_{\mathbf{n}}=\left(\begin{array}{c}
z_{1}^{n} \\
z_{2}^{n} \\
\vdots \\
z_{r}^{n}
\end{array}\right)
\end{aligned}
$$

Here,

$$
\begin{gathered}
\mathbf{X}_{\mathrm{n}} \in \mathbf{R}^{p}(n=1,2, \cdots), \quad \mathbf{Y}_{\mathrm{n}} \in \mathbf{R}^{q}(n=1,2, \cdots), \\
\mathbf{Z}_{\mathrm{n}} \in \mathbf{R}^{r}(n=1,2, \cdots), \quad \mathbf{A}_{11} \in R^{p \times p}, \quad \mathbf{A}_{12} \in R^{p \times q} \\
\mathbf{A}_{13} \in R^{p \times r}, \mathbf{A}_{22} \in R^{q \times q}, \mathbf{A}_{23} \in R^{q \times r}, \mathbf{A}_{33} \in R^{r \times r}
\end{gathered}
$$

These are re-stated as:

$$
\begin{equation*}
\mathbf{W}_{\mathbf{n}}=\mathbf{A} \mathbf{W}_{\mathbf{n}-1} \tag{12}
\end{equation*}
$$

Where

$$
\begin{gathered}
\mathbf{W}_{\mathrm{n}}=\left(\begin{array}{l}
\mathbf{X}_{\mathrm{n}} \\
\mathbf{Y}_{\mathrm{n}} \\
\mathbf{Z}_{\mathrm{n}}
\end{array}\right) \\
\mathbf{A}=\left(\begin{array}{rr}
\mathbf{A}_{11}, & \mathbf{A}_{12}, \\
\mathbf{\mathbf { A } _ { 1 3 }} \\
\mathbf{0}, & \mathbf{A}_{22}, \\
\mathbf{0}, & \mathbf{A}_{23}, \\
\mathbf{0}, & \mathbf{A}_{33}
\end{array}\right) \\
\mathbf{W}_{\mathrm{n}-1}=\left(\begin{array}{c}
\mathbf{X}_{\mathrm{n}-1} \\
\mathbf{Y}_{\mathrm{n}-1} \\
\mathbf{Z}_{\mathrm{n}-1}
\end{array}\right)
\end{gathered}
$$

## IV. PURCHASE HISTORY INVESTIGATION AND NUMERICAL <br> <br> CALCULATION

 <br> <br> CALCULATION}Jewelry/Accessory purchase history investigation is executed.

First of all, the framework of jewelry/accessory purchasing via on-line shopping is as follows.

- On-line shop: Ciao! / Happy gift

Host site: http://www.happy-gift.jp/
Branch site: http://www.rakuten.co.jp/ciao/
http://store.shopping.yahoo.co.jp/b-ciao/index.html
Managed by Cherish Co. Ltd.

- Customers: all over Japan (Every Prefecture)
- Data gathering period: April 2008 - May 2011
- Order number: 4411 (limited to the order number which has repeated order)
- Main residents of customers
- Tokyo $11.9 \%$
- Kanagawa $8.7 \%$
- Osaka $6.0 \%$
- Aichi 5.8\%
- Chiba $5.7 \%$
- Saitama 5.4\%

The share of Tokyo capital area consists of $31.7 \%$.

- Sales goods:

Necklace / Pendant
Pierced earrings
Ring
Bracelet / Bangle
Brooch
Necktie Pin
Miscellaneous (Package/Ribbon etc.)

- Classification of goods by price

| Rank | Price(Yen) |
| :---: | :---: |
| Necklace / Pendant |  |
| N6 | 40001~ |
| N5 | $\sim 40000$ |
| N4 | $\sim 30000$ |
| N3 | $\sim 20000$ |
| N2 | $\sim 15000$ |
| N1 | $\sim 10000$ |
| Pierced earrings |  |
| P6 | $24001 \sim$ |
| P5 | $\sim 24000$ |
| P4 | $\sim 16000$ |
| P3 | $\sim 10000$ |
| P2 | $\sim 6000$ |
| P1 | $\sim 2000$ |
| Ring |  |
| R6 | 40001~ |
| R5 | $\sim 40000$ |
| R4 | ~30000 |
| R3 | $\sim 20000$ |
| R2 | $\sim 15000$ |
| R1 | $\sim 10000$ |
| Bracelet / Bungle |  |
| B6 | 40001~ |
| B5 | $\sim 40000$ |
| B4 | $\sim 35000$ |
| B3 | $\sim 30000$ |
| B2 | $\sim 15000$ |
| B1 | $\sim 10000$ |

We have made an analysis about Necklace/ Pendant in another paper. Therefore we pick up Pierced earrings, Ring and Bracelet / Bungle in this paper.

The total number of shifts in one genre is as follows.

| Necklace / Pendant | 2181 |
| :---: | :---: |
| Pierced earrings | 567 |
| Ring | 1205 |
| Bracelet / Bungle | 458 |

The shift status is as follows.
Case1 Pierced earrings
(1) Number of shift from P1 to P1: 166
(2) Number of shift from P1 to P2:8
(3) Number of shift from P1 to P3:1
(4) Number of shift from P1 to P4:3
(5) Number of shift from P1 to P5:5
(6) Number of shift from P1 to P6:0
(7) Number of shift from P2 to P1:9
(8) Number of shift from P2 to P2: 184
(9) Number of shift from P2 to P3:21
(10) Number of shift from P2 to P4:6

11 Number of shift from P2 to P5:0
12 Number of shift from P2 to P6:0
13 Number of shift from P3 to P1:4
14 Number of shift from P3 to P2 : 12
15 Number of shift from P3 to P3:42
16 Number of shift from P3 to P4:13
17 Number of shift from P3 to P5:0
18 Number of shift from P3 to P6:0
19 Number of shift from P4 to P1 : 5
20 Number of shift from P4 to P2 : 2
21 Number of shift from P4 to P3: 12
22 Number of shift from P4 to P4:55
23 Number of shift from P4 to P5 : 1
24 Number of shift from P4 to P6 : 1
25 Number of shift from P5 to P1 : 2
26 Number of shift from P5 to P2 : 1
27 Number of shift from P5 to P3:2
28 Number of shift from P5 to P4:2
29 Number of shift from P5 to P5:6
30 Number of shift from P5 to P6 : 0
31 Number of shift from P6 to P1:0
32 Number of shift from P6 to P2 : 0
33 Number of shift from P6 to P3:0
21 Number of shift from P6 to P4:2
22 Number of shift from P6 to P5:0
23 Number of shift from P6 to P6:1
Total: 567
Case2 Ring
(1) Number of shift from R1 to R1:332
(2) Number of shift from R1 to R2:55
(3) Number of shift from R1 to R3:21
(4) Number of shift from R1 to R4: 1
(5) Number of shift from R1 to R5: 1
(6) Number of shift from R1 to R6:2
(7) Number of shift from R2 to R1: 61
(8) Number of shift from R2 to R2 : 503
(9) Number of shift from R2 to R3: 28
(10) Number of shift from R2 to R4:0

11 Number of shift from R2 to R5 : 1
12 Number of shift from R2 to R6: 6
13 Number of shift from R3 to R1:13
14 Number of shift from R3 to R2:38
15 Number of shift from R3 to R3:86
16 Number of shift from R3 to R4:0

17 Number of shift from R3 to R5:0
18 Number of shift from R3 to R6: 1
19 Number of shift from R4 to R1:0
20 Number of shift from R4 to R2:0
21 Number of shift from R4 to R3: 0
22 Number of shift from R4 to R4:1
23 Number of shift from R4 to R5: 0
24 Number of shift from R4 to R6:0
25 Number of shift from R5 to R1: 1
26 Number of shift from R5 to R2: 1
27 Number of shift from R5 to R3: 0
28 Number of shift from R5 to R4:0
29 Number of shift from R5 to R5 : 5
30 Number of shift from R5 to R6: 1
31 Number of shift from R6 to R1:3
32 Number of shift from R6 to R2: 9
33 Number of shift from R6 to R3: 0
34 Number of shift from R6 to R4:0
35 Number of shift from R6 to R5: 1
21 Number of shift from R6 to R6:34
Total: 1205
Case3 Bracelet / Bungle
(1) Number of shift from B1 to B1: 139
(2) Number of shift from B1 to B2: 22
(3) Number of shift from B1 to B3:9
(4) Number of shift from B1 to B4:0
(5) Number of shift from B1 to B5:0
(6) Number of shift from B1 to B6:0
(7) Number of shift from B2 to B1:20
(8) Number of shift from B2 to B2: 182
(9) Number of shift from B2 to B3: 10
(10) Number of shift from B2 to B4:0

11 Number of shift from B2 to B5:0
12 Number of shift from B2 to B6:0
13 Number of shift from B3 to B1:4
14 Number of shift from B3 to B2 : 6
15 Number of shift from B3 to B3: 65
16 Number of shift from B3 to B4:0
17 Number of shift from B3 to B5:0
18 Number of shift from B3 to B6:0
19 Number of shift from B4 to B1:0
20 Number of shift from B4 to B2: 1
21 Number of shift from B4 to B3: 0
22 Number of shift from B4 to B4:0
23 Number of shift from B4 to B5:0
24 Number of shift from B4 to B6 : 0
25 Number of shift from B5 to B1:0
26 Number of shift from B5 to B2 : 0
27 Number of shift from B5 to B3:0
28 Number of shift from B5 to B4:0
29 Number of shift from B5 to B5 : 0

30 Number of shift from B5 to B6:0
31 Number of shift from B6 to B1:0
32 Number of shift from B6 to B2:0
33 Number of shift from B6 to B3:0
34 Number of shift from B6 to B4:0
35 Number of shift from B6 to B5:0
36 Number of shift from B6 to B6:0
Total: 458
The shift to P1 to P3 in $\mathbf{X}_{\mathbf{b}}, \mathbf{X}$, for example, is expressed as follows when one event arises.

$$
\mathbf{X}=\left(\begin{array}{l}
0 \\
0 \\
0 \\
1 \\
0 \\
0
\end{array}\right), \mathbf{X}_{\mathbf{b}}=\left(\begin{array}{l}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{array}\right)
$$

Substituting these to Equation (7), we obtain following equations. We state and analyse them by each case.

Case1 Pierced earrings

$$
\begin{align*}
\hat{\mathbf{A}} & =\left(\begin{array}{cccccc}
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 6 & 1 & 0 & 0 & 5 \\
2 & 2 & 55 & 13 & 6 & 3 \\
0 & 2 & 12 & 42 & 21 & 1 \\
0 & 1 & 3 & 12 & 184 & 8 \\
0 & 2 & 5 & 4 & 9 & 166
\end{array}\right)\left(\begin{array}{cccccc}
3 & 0 & 0 & 0 & 0 & 0 \\
0 & 13 & 0 & 0 & 0 & 0 \\
0 & 0 & 77 & 0 & 0 & 0 \\
0 & 0 & 0 & 81 & 0 & 0 \\
0 & 0 & 0 & 0 & 220 & 0 \\
0 & 0 & 0 & 0 & 0 & 183
\end{array}\right)^{-1} \\
& =\left(\begin{array}{cccccc}
\frac{1}{3} & 0 & \frac{1}{77} & 0 & 0 & 0 \\
0 & \frac{6}{13} & \frac{1}{77} & 0 & 0 & \frac{5}{183} \\
\frac{2}{3} & \frac{2}{13} & \frac{5}{7} & \frac{13}{81} & \frac{3}{110} & \frac{1}{61} \\
0 & \frac{2}{13} & \frac{12}{77} & \frac{14}{27} & \frac{21}{220} & \frac{1}{183} \\
0 & \frac{1}{13} & \frac{3}{77} & \frac{4}{27} & \frac{46}{55} & \frac{8}{183} \\
0 & \frac{2}{13} & \frac{5}{77} & \frac{4}{81} & \frac{9}{220} & \frac{166}{183}
\end{array}\right) \tag{13}
\end{align*}
$$

Total number of upper shift from P1: 17
Total number of lower shift from P1 : -
Total number of upper shift from P2:27
Total number of lower shift from P2:9
Total number of upper shift from P3:13
Total number of lower shift from P3:16
Total number of upper shift from P4:2
Total number of lower shift from P4:20
Total number of upper shift from P5:0
Total number of lower shift from P5:7
Total number of upper shift from P6 : -
Total number of lower shift from P6 : 2

We can observe that the upper shift from P1 and P2 is dominant and on the contrary, the lower shift from P4 and P5 is dominant.

This implies that customers buy rather cheap goods at first for the trial, and after confirming the quality, they make the upper shift in selecting brands.

After reaching higher brands, they buy cheaper goods and that leads to a lower shift in brand selection.

Hearing from customers, we can also find that she buys pierced earrings for herself and confirms the quality. After that, she makes gift by selecting upper brand. Sometimes she buys lower brand goods for herself after making gift.

That scene can often be seen and the result shows its sequence well. When the shop owner introduces new brand goods, he/she has to determine the price. If it is not reasonable, customers do not select their brand position as the shop owner assumes. These are confirmed by the brand shift transition, which forces the shop owner to re-consider the price and brand position of the new brand goods.

Case2 Ring

$$
\begin{align*}
\hat{\mathbf{A}} & =\left(\begin{array}{cccccc}
34 & 1 & 0 & 1 & 6 & 2 \\
1 & 5 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 86 & 28 & 21 \\
9 & 1 & 0 & 38 & 503 & 55 \\
3 & 1 & 0 & 13 & 61 & 332
\end{array}\right)\left(\begin{array}{cccccc}
47 & 0 & 0 & 0 & 0 & 0 \\
0 & 8 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 138 & 0 & 0 \\
0 & 0 & 0 & 0 & 599 & 0 \\
0 & 0 & 0 & 0 & 0 & 412
\end{array}\right)^{-1} \\
& =\left(\begin{array}{cccccc}
\frac{34}{47} & \frac{1}{8} & 0 & \frac{1}{138} & \frac{6}{599} & \frac{1}{206} \\
\frac{1}{47} & \frac{5}{8} & 0 & 0 & \frac{1}{599} & \frac{1}{412} \\
0 & 0 & 1 & 0 & 0 & \frac{1}{412} \\
0 & 0 & 0 & \frac{43}{69} & \frac{28}{559} & \frac{21}{412} \\
\frac{9}{47} & \frac{1}{8} & 0 & \frac{19}{69} & \frac{503}{559} & \frac{55}{412} \\
\frac{3}{47} & \frac{1}{8} & 0 & \frac{13}{138} & \frac{61}{599} & \frac{83}{103}
\end{array}\right) \tag{14}
\end{align*}
$$

Total number of upper shift from R1: 80
Total number of lower shift from R 1 : -
Total number of upper shift from R 2:35
Total number of lower shift from R $2: 61$
Total number of upper shift from R 3:1
Total number of lower shift from R 3 : 51
Total number of upper shift from R $4: 0$
Total number of lower shift from R 4:0
Total number of upper shift from R $5: 1$
Total number of lower shift from R $5: 2$
Total number of upper shift from R 6 :-
Total number of lower shift from R 6:13
We can observe that the upper shift from R1 is dominant and on the contrary, the lower shift from R2, R3 and R6 is dominant.

This characteristic is common in each brand genre.
Case3 Bracelet/ Bangle

$$
\begin{align*}
\hat{\mathbf{A}} & =\left(\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 65 & 10 & 9 \\
0 & 0 & 1 & 6 & 182 & 22 \\
0 & 0 & 0 & 4 & 20 & 139
\end{array}\right)\left(\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 75 & 0 & 0 \\
0 & 0 & 0 & 0 & 212 & 0 \\
0 & 0 & 0 & 0 & 0 & 170
\end{array}\right)^{-1} \\
& =\left(\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{13}{15} & \frac{5}{106} & \frac{9}{170} \\
0 & 0 & 1 & \frac{2}{25} & \frac{91}{106} & \frac{11}{85} \\
0 & 0 & 0 & \frac{4}{75} & \frac{5}{53} & \frac{139}{170}
\end{array}\right) \tag{15}
\end{align*}
$$

If the determinant of the matrix is 0 , we cannot calculate the inverse matrix. But we have made an analysis when the new brand came into the market, where there was no before data and could not calculate inverse matrix.

Setting very small event $\varepsilon$ and making $\varepsilon \rightarrow 0$, we found that we could calculate such situation by setting diagonal 0 data to $1^{[6]}$.

Total number of upper shift from B1:31
Total number of lower shift from B 1 : -
Total number of upper shift from B 2:10
Total number of lower shift from B 2:20
Total number of upper shift from B $3: 0$
Total number of lower shift from B 3:10
Total number of upper shift from B $4: 0$
Total number of lower shift from B $4: 1$
Total number of upper shift from B 5:0
Total number of lower shift from B $5: 0$
Total number of upper shift from B 6 : -
Total number of lower shift from B 6:0
We can observe that the upper shift from B1 is dominant and on the contrary, the lower shift from B2 and B3 is dominant.

## V. APPLICATION OF THIS METHOD

There may arise following case. Consumers and producers do not recognize the brand position clearly. But analysis of consumers' behaviour let them know their brand position in the market. In such a case, strategic marketing guidance to select brand would be introduced.

Setting in order the brand position of various goods and taking the suitable marketing policy, enhancement of sales would be enabled.

## VI．REMARKS

We have made an analysis before ${ }^{[7]}$ ．Hereinafter comparison with the previous research is executed，mainly focusing on Pierced earrings，Ring and Bracelet／Bungle．

|  | Previous <br> Research | Research in this <br> Paper |
| :---: | :---: | :---: |
| Data gathering period | October 2006 <br> －May 2009 <br> （32 months） | April 2008－May <br> 2011 <br> （38 months） |
| Order Number |  | 3438 |
| Total <br> number <br> of <br> Shits <br> in <br> one <br> genre | Necklace／ <br> Pendant | Pierced <br> earrings |
|  | Ring | 117 |
| Bracelet／ <br> Bungle | 1611 |  |

We can observe that the repeated purchasing has increased for several times compared with the former research period．

In particular，Bracelet／Bungle has increased tremendously compared with other genres．It is because shop owner has introduced new products of leathered pair Bracelet into the market and they made a great hit．

Now，we compare the contents of the upper shift and the lower shift in each genre．

## A．Pierced Earrings

〈 Previous Research 〉

|  | P1 | P2 | P3 | P4 |
| :---: | :---: | :---: | :---: | :---: |
| Upper shift | - | 0 | 2 | 2 |
| Same Rank <br> movement | 2 | 1 | 3 | 14 |
| Lower shift | 4 | 3 | 4 | 1 |
| Summary | 6 | 4 | 9 | 17 |
| Share（\％） | 5.1 | 3.4 | 7.7 | 14.5 |
|  | P5 | P6 | Summary | Share（\％） |
| Upper shift | 4 | 6 | 14 | 12 |
| Same Rank <br> movement | 38 | 29 | 87 | 74.4 |
| Lower shift | 4 | - | 16 | 13.7 |
| Summary | 46 | 35 | 117 | - |
| Share（\％） | 39.3 | 29.9 | - | - |

〈 Research in this paper 〉

|  | P1 | P2 | P3 | P4 |
| :---: | :---: | :---: | :---: | :---: |
| Upper shift | - | 0 | 2 | 13 |
| Same Rank <br> movement | 1 | 6 | 55 | 42 |
| Lower shift | 2 | 7 | 20 | 16 |
| Summary | 3 | 13 | 77 | 71 |
| Share（\％） | 0.5 | 2.3 | 13.6 | 12.5 |
|  | P5 | P6 | Summary | Share（\％） |
| Upper shift | 27 | 17 | 59 | 10.4 |
| Same Rank <br> movement | 184 | 166 | 454 | 80 |
| Lower shift | 9 | - | 54 | 9.5 |
| Summary | 220 | 183 | 567 |  |
| Share（\％） | 38.8 | 32.3 | - | - |

Compared with the former one，the share of P3 has increased greatly，while P1 made a big decrease．Consumers tends to have bought rather＂upper middle＂class items．

## B．Ring

〈 Previous Research 〉

|  | R1 | R2 | R3 | R4 |
| :---: | :---: | :---: | :---: | :---: |
| Upper shift | - | 1 | 1 | 0 |
| Same Rank <br> movement | 1 | 6 | 3 | 4 |
| Lower shift | 5 | 7 | 3 | 2 |
| Summary | 6 | 14 | 7 | 6 |
| Share（\％） | 3.7 | 8.7 | 4.3 | 3.7 |
|  | R5 | R6 | Summary | Share（\％） |
| Upper shift | 1 | 4 | 7 | 4.3 |
| Same Rank <br> movement | 64 | 41 | 119 | 73.9 |
| Lower shift | 18 | - | 35 | 21.7 |
| Summary | 83 | 45 | 161 |  |
| Share（\％） | 51.6 | 28 | - | - |

〈 Research in this paper〉

|  | R1 | R2 | R3 | R4 |
| :--- | :--- | :--- | :--- | :--- |
| Upper shift | - | 1 | 0 | 1 |
| Same Rank <br> movement | 34 | 5 | 1 | 86 |
| Lower shift | 13 | 2 | 0 | 51 |
| Summary | 47 | 8 | 1 | 138 |
| Share（\％） | 3.9 | 0.7 | 0.1 | 11.5 |
| - | R5 | R6 | Summary | Share（\％） |
| Upper shift | 35 | 80 | 117 | 9.7 |
| Same Rank <br> movement | 503 | 332 | 961 | 79.8 |
| Lower shift | 61 | - | 127 | 10.5 |
| Summary | 599 | 412 | 1205 |  |
| Share（\％） | 49.7 | 34.2 | - |  |

Compared with the former one，the share of upper shift has increased greatly．The number of the upper shift of R5 and R6 is a big one，therefore it effected well．As for each rank，the share of R4 and R6 has increased greatly，while R2 and R3 made a big decrease．

## C．Bracelet／Bungle

〈 Previous Research 〉

|  | B1 | B2 | B3 | B4 |
| :---: | :---: | :---: | :---: | :---: |
| Upper shift | - | 0 | 0 | 0 |
| Same Rank <br> movement | 0 | 0 | 0 | 1 |
| Lower shift | 1 | 0 | 0 | 0 |
| Summary | 1 | 0 | 0 | 1 |
| Share（\％） | 4.5 | 0 | 0 | 4.5 |


|  | B5 | B6 | Summary | Share（\％） |
| :---: | :---: | :---: | :---: | :---: |
| Upper shift | 0 | 1 | 1 | 4.5 |
| Same Rank <br> movement | 6 | 12 | 19 | 86.4 |
| Lower shift | 1 | - | 2 | 9.1 |
| Summary | 7 | 13 | 22 |  |
| Share（\％） | 31.8 | 59.1 |  |  |

〈 Research in this paper 〉

|  | B1 | B2 | B3 | B4 |
| :---: | :---: | :---: | :---: | :---: |
| Upper shift | 0 | 0 | 0 | 0 |
| Same Rank <br> movement | 0 | 0 | 0 | 65 |
| Lower shift | 0 | 0 | 1 | 10 |
| Summary | 0 | 0 | 1 | 75 |
| Share (\%) | 0 | 0 | 0.2 | 16.4 |
|  | B5 | B6 | Summary | Share (\%) |
| Upper shift | 10 | 31 | 41 | 9 |
| Same Rank <br> movement | 182 | 139 | 386 | 84.3 |
| Lower shift | 20 | - | 31 | 6.8 |
| Summary | 212 | 170 | 458 | - |
| Share (\%) | 46.3 | 37.1 | - | - |

Compared with the former one, the total number of sales increased greatly. We can observe an impact of a great hit of leathered pair Bracelet. We can clearly see the increase of the share of upper shift. As for each rank, the share of B4 and B5 has increased greatly, while B6 made a big decrease.

Rather middle class rank increased the share on the whole for each of them.

The movement from Necklace / Pendant to these genres may be considered.

The shift among genres is the next step research to be investigated.

## VII. CONCLUSION

It is often observed that consumers select the upper class brand when they buy the next time.

Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition matrix becomes upper triangular matrix under the supposition that the former buying variables are set input and the current buying variables are set output.

There may also be the case that customers select lower brand to seek suitable price when they have chosen higher brand. Then it may compose items of lower triangular matrix. Utilizing jewelry/accessory purchasing history record of on-line shopping (especially we focus on pierced
earrings, ring, bracelet/bangle) over three years, above structure was investigated and confirmed. Comparison with our previous research was also executed. Some interesting results are obtained.

Various fields should be examined hereafter.

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