Applying an Exact Solution of the Brutsaert and Nieber Baseflow Model for Watershed Yield Prediction:

A Case Study for the Spoon River at Seville, Illinois, USA

John Y. Ding

Principal, John Ding & Co., 134 Heatherside Drive, Toronto, Ontario, M1W 1T9, Canada

johnding_toronto@yahoo.com

***Abstract*-This paper focuses on the Brutsaert–Nieber (1977) drought flow or baseflow model, and its exact solution recently rediscovered by Ding (2013). The standard model, $-dQ/dt = aQ^b$, is well known for its log ($-dQ/dt$) versus log (Q) recession–slope plots. On such a plot, exponent *b* represents the slope of a regression line through the data point cloud, and coefficient or prefactor *a* the intercept at $Q = 1$.**

The exact solution method discovered a half–century ago by Ding (1966) transforms instead the flow rate *Q* by an inverse–fractional–power (IFP) into a new variable, $1/Q^{b-1}$. The transform solution is $1/Q^{b-1}(t) = 1/Q^{b-1}(0)+(b-1)at$, if $b \neq 1$. This converts a recession curve into a straight line. Since only two points, regardless of the distance, are required to define a transformed line, the linearized Brutsaert–Nieber model thus becomes temporal scale invariant (a third point in between is needed to falsify the given exponent *b* value).

A nonlinear groundwater storage–discharge relation was previously derived by integrating the recession hydrograph from time *t* to ∞. This, $Q = c^N S^N$, has two rescaled parameters *N* and *c*, both relate back to Brutsaert–Nieber parameters: $N = 1/(2-b)$, $c = (2-b)a$, and $Nc = a$.

Basin or aquifer characteristics, such as the lagtime or half–life, are analytically derivable from the transform solution and the inferred storage–discharge function. For storage-based lagtime, $t_{S/2} = (\log 2)/a = 0.693/a$, if $b = 1$; and $((2^{(b-1)/(2-b)} - 1)/((b-1)a))Q^{-(b-1)}(0)$, if $b \neq 1$.

Exponent *b* and prefactor *a* were recently calibrated by Ding (2013) for four recession events in the Spoon River at Seville, Illinois, USA, a very large watershed of 4237 km^2. Units of measurement are days for time *t*, mm/d for flow rate *Q*, and mm for storage *S*; exponent *b* is dimensionless, and prefactor *a* has the units of the flow rate *Q* and of time *t*. For water supply scenario analysis, three common types of the Ding transform are explored: the linear, and the reciprocals of the cube root and of square root (RoCR and RoSR) transforms. Their exponent *b* values are 1, 4/3 and 3/2, respectively. The mean calibrated (*b, a*) values are (1, 0.08), (1.33, 0.12) and (1.5, 0.15). These are used to construct or infer both the groundwater storage–discharge functions and the storage lagtimes for the Spoon River. The lagtimes (in days) are 8.66, $10.33/\sqrt[3]{Q(0)}$, and $13.33/\sqrt{Q(0)}$, respectively.

To summarize, the classical Brutsaert–Nieber groundwater flow recession model and an earlier Ding transform solution complement each other, the former having the latter as an exact solution. On an ungauged watershed, small or large, to set up the model for water yield prediction, this requires a minimum of three new flow measurements in the field during a rainless period, supplemented by concurrent evaporation pan measurements.

Keywords- Streamflow Recession Model; Drought Flow; Baseflow; Storage-discharge Function; Basin Lagtime; Water Yield

REFERENCES

[1] M. G. Kang, J. H. Lee, and K.W. Park, “Parameter regionalization of a Tank model for simulating runoffs from ungauged watersheds,” J. Korea Water Resour. As., vol. 46(5), pp. 519-530, (in Korean). 2013.

[2] A. L. Kay, D. A. Jones, S. M. Crooks, T.R. Kjeldsen, and C.F. Fung, “An investigation of site-similarity approaches to generalization of a rainfall-runoff model,” Hydrol. Earth Syst. Sci., vol. 11, pp. 500-515. 2007.

[3] C. Perrin, L. Oudin, V. Andreassian, C. Rojas-Serna, C. Michel, and T. Mathevet, “Impact of limited streamflow data on the efficiency and the parameters of rainfall-runoff models,” Hydrolog. Sci. J., vol. 52, pp. 131-151, 2007.

[4] Z. Xie, F. Yuan, Q. Duan, J. Zheng, M. Liang, and F. Chen, “Regional parameter estimation of the VIC land surface model: Methodology and application to river basins in China,” J. Hydromet., vol. 8(3), pp. 447-468, 2007.

[5] K. H. Son, M.H. Lee, and D. H. Bae, “Runoff analysis and assessment using land surface model on East Area,” J. Korea Water Resour. As., vol. 45(2), pp. 165-178, (in Korean). 2012.

[6] B. H. Cho, “A study on regionalization of long-term runoff model parameters,” M.S. Thesis, Sejong Univ., Republic of Korea, p. 86, (in Korean). 2004.

DOI: 10.5963/JWRHE0402005

[7] A. R. Young, “Stream flow simulation within UK ungauged catchments using a daily rainfall-runoff model,” J. Hydrol., vol. 320, pp. 155-172, 2006.

[8] S. H. Lee and S.U. Kang, “A parameter regionalization study of a modified Tank model using characteristic factors of watersheds,” KSCE J. Civ. Eng., vol. 27(4B), pp. 379-385, (in Korean). 2007.

[9] J. Wang, Y. Hong, L. Li, J.J. Gourley, S.I. Khan, K.K. Yilmaz, R.F. Adler, F.S. Policeli, S. Habib, D. Irwin, A.S. Limaye, T. Korme, and L. Okello, “The coupled routing and excess storage (CREST) distributed hydrological model,” Hydrolog. Sci. J., vol. 56(1), pp. 84-98, 2011.

[10] G. J. Huffman, R. F. Adler, D.T. Bolvin, G. Gu, E.J. Nelkin, K.P. Bowman, Y. Hong, E.F. Stocker, and D.B. Wolff, “The TRMM Multisatellite Precipitation Analysis (TMPA): Quasi-global, multiyear, combined-sensor precipitation estimates at fine scales,” J. Hydrometeo., vol. 8, pp. 38-55, 2007.

[11] J. E. Janowiak, R. J. Joyce, and Y. Yarosh, “A real-time global half-hourly pixel-resolution infrared dataset and its applications,” Bull. Am. Meteorol. Soc., vol. 82, pp. 205-217, 2001.

[12] Z. S. Haddad, E. A. Smith, C. D. Kummerow, T. Iguchi, M.R. Farrar, S.L. Durden, M. Alves, and W.S. Olson, “The TRMM “day-1” radar/radiometer combined rain-profiling algorithm,” J. Meteorol. Soc. Jpn., vol. 75, pp. 799-809, 1997.

[13] B. Rudolf, “Management and analysis of precipitation data on a routing basis,” Proc. intern. Symp. on precipitation and evaporation, vol. 1, pp. 69-76, 1993.

[14] P. Xie and P. A. Arkin, “Gauge-based monthly analysis of global land precipitation from 1971 to 1994,” J. Geophys. Res., vol. 101, pp. 19023-19034, 1996.

[15] F. Su, Y. Hong, and D. P. Lettenmaier, “Evaluation of TRMM multi-satellite precipitation analysis (TMPA) and its utility in hydrologic prediction in La Plata basin,” J. Hydrometeorol., vol. 9(4), pp. 622-640, 2008.

[16] S.I. Khan, Y. Hong, J. Wang, K. K. Yilmaz, J. J. Gourley, R.F. Adler, G.R. Brakenridge, F. Policelli, S. Habib, and D. Irwin, “Satellite remote sensing and hydrologic modeling for flood inundation mapping in Lake Victoria basin: Implications for hydrologic prediction in ungauged basins,” IEEE Trans. Geosci. Remote Sens., vol. 49(1), pp. 85-95, 2011.

[17] J. Singh, H. V. Knapp, G. Arnold, and M. Demissie, “Hydrologic modeling of the Iroquois River watershed using HSPF and SWAT,” J. Am. Water Resour. As., vol. 41(2), pp. 361-375, 2005.

[18] D. N. Moriasi, J. G. Arnold, M.W. Van Liew, R. L. Bingner, R. D. Harmel, and T. L. Veith, “Model evaluation guidelines for systematic quantification of accuracy in watershed simulations,” Trans. of the ASABE, vol. 50(3), pp. 885-900, 2007.

DOI: 10.5963/JWRHE0402005