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Dynamic Analysis of Simply Supported Functionally Graded Nanobeams Subjected to a Moving Force Based on the Nonlocal Euler-Bernoulli Elasticity Theory

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***Abstract*-Dynamic analysis of simply supported functionally graded nanobeams (FG nanobeams) subjected to a moving force based on the nonlocal Euler-Bernoulli elasticity theory has been investigated in this paper. It is generally believed that the material properties of the functionally graded nanobeams follow the power law change along its thickness. The model of the functionally graded nanobeams with the small deformation is based on the Euler-Bernoulli beam theory, and the governing equations of motion for the dynamic response of the nanobeams, including nonlocal effect, are derived from by the minimum total potential energy principle and the energy variational principle. The higher order partial differential equations have been reduced to a low order partial differential equation by using the Petrov-Galerkin method. The partial differential equations are solved by the employing Runge-Kutta numerical analysis method. Numerical results addressing the significance of the material distribution profile, velocity of the moving force, and nonlocal effect are discussed in detail. The results indicate that these parameters are decisive ones in analysing the dynamic response of the FG nanobeams.**

Keywords- FG Nanobeams; Moving Force; Nonlocal Elasticity Theory; Euler-Bernoulli Beam Theory

I. INTRODUCTION

In the fields of electronics, biological medicine and new energy, the dynamic analysis of functionally graded substances and engineering structures subjected to moving forces has received a lot of attention. Fryba collected a lot of results concerning a moving loads on rectangular plates and beams [1]. The general continuous Euler-Bernoulli beam subjected to a moving oscillator has been studied, and an analytical solution based on the surface elasticity theory has been derived by Wang [2]. A similar problem was considered by Henchi and Fafard, but with a convoy of moving loads and ignoring the effect of inertia on the load, employing the Wittrick and Williams algorithm [3]. Wang and Lin worked on stochastic vibrations of the Timoshenko beam applied to a stochastic moving load [4]. Takabatake discussed the dynamic analysis of a rectangular plate subjected to a moving mass in experiments. He also used the Galerkin approach to obtain a numerical solution when the thickness of the plate varied discontinuously [5].

In paper [6], Eringen proposed the nonlocal elasticity theory, which was widely used to predict the size-dependence of materials and structures. According to the typical method of the elastic theory, the stress at an arbitrary point depends only on the strain at the same point; however, in the nonlocal theory the stress at an arbitrary point depends on the strain at all points. When the structures are at a scale of nanosizes, the classical continuum theories fail to accurately predict the mechanical behavior of structures [7-24]. Simsek utilized the nonlocal Timoshenko beam theory to find the analytical solutions for bending and buckling of the FG nanobeams [25]. Eltaher used nonlocal beam theory to present the static stability of the FG nanobeams [26]. According to the nonlocal differential constitutive relations of Eringen, Reddy studied the static response of nanobeams [27]. By employing a meshless method, Roque explored the static response of Timoshenko nanobeams based on the Eringen's nonlocal theory [28]. Thai and Vo presented a nonlocal shear deformation beam theory for the solution of bending, buckling and vibration of the nanobeams [29, 30]. In addition, Civalek used the nonlocal elastic theory to analyze the bending of microtubules [31]. The paper presented a developed DQ method for obtaining accurate bending moments and displacements, and the results show that the nonlocal parameters lead to the increase of bending moment of MTs.

Along with the development of materials engineering, a kind of composite material that is composed of continuous or quasi-continuous materials has been extensively used. Its structure and properties are continuously changing in the thickness or length direction. The structure of the functionally graded materials has been widely researched by many pursuers. Rajabi and Kargarnovin studied the dynamic response of a simply supported FG Euler-Bernoulli beam subjected to a moving oscillator by using Hamilton's principle [32]. Reddy and Praveen provided exact solutions for the thermo-mechanical behavior of the functionally graded plates [33, 34]. Up to the present, a few studies have been carried out on functionally graded nanobeams on the basis of nonlocal theory. Babilio considered the dynamics of a simply supported beam under an axial time-dependent load [35]. The beam consisted of an axially functionally graded material, but not in the nanometer range. According to the nonlocal

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elasticity theory, Eltaher investigated free vibration of functionally graded Euler-Bernoulli nanobeams [36]. Also, Simsek has examined free vibration of axially functionally graded tapered nanorods in the frame of the nonlocal elasticity theory in [37].

Hence, based on the nonlocal Euler-Bernoulli beam theory, the dynamic of the FG nanobeam subjected to a moving force is discussed. The governing equations of motion for the dynamic response of the nanobeam, including nonlocal effect, are derived by the minimum total potential energy principle and the energy variational principle. The higher order partial differential equation has been reduced to a low order partial differential equation by using the Petrov-Galerkin method, which is different from the DQ method in [31]. The partial differential equations are solved by employing the Runge-Kutta numerical analysis method. Numerical results addressing the significance of the material distribution profile, the speed of the moving force, and the nonlocal effect are discussed in detail. The conclusions show that the nonlocal parameter and material parameter produce larger deflection of the FG nanobeam.

II. MATHEMATICAL FORMULATION

A. *Functionally Graded Materials*

Consider a FG nanobeam with length L, thickness h and width b, meaning that the cross-section is rectangular. The coordinate system x, y, z is introduced in the central axis of the beam, while the x-axis is taken along the length of the beam, the y-axis in the width direction and the z-axis is taken along the height direction. The beam is subjected to moving force q with speed v. The most left of the beam is set to the start of the coordinate system, as shown in Fig. 1.

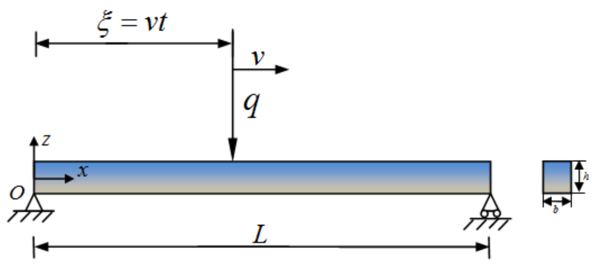

Fig. 1 A FG simple supported beam model for dynamic displacement

Assume that the functionally graded nanobeam is composed of two different kinds of materials, and properties of this hybrid material of the functionally graded nanobeam (e.g., Young's modulus E, Poisson's ratio ν, shear modulus μ and density ρ) vary continuously in the thickness direction pursuant to power law distribution. Based on the properties of the mixed material, the mixed material properties can be written as [25]

$$P = P_1V_1 + P_2V_2\,, \tag{1}$$

where P_1, P_2 are the two material properties, respectively, and V_1 and V_2 are the volume fractions of the first and the second material related by

$$V_1 + V_2 = 1\,. \tag{2}$$

The mixed material properties of the functionally graded nanobeam can be expressed in the form of power-law. The volume fraction of the second material can be expressed as

$$V_2(\mathrm{z}) = \left(\frac{z}{h} + \frac{1}{2}\right)^k, \tag{3}$$

where k is not negative (power-law index is a non-negative constant) and dictates the material properties change along the thickness of the beam. The Young's modulus and the density of the functionally graded nanobeam can be given, respectively, as

$$E(\mathrm{z}) = (E_2 - E_1)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_1\,, \tag{4}$$

$$\rho(\mathrm{z}) = (\rho_2 - \rho_1)\left(\frac{z}{h} + \frac{1}{2}\right)^k + \rho_1\,. \tag{5}$$

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Clearly, $E = E_2, \nu = \nu_2$ when $z = +h/2$, and $E = E_1, \nu = \nu_1$ when $z = -h/2$. Fig. 2 shows $V_2(z)$ along the thickness.

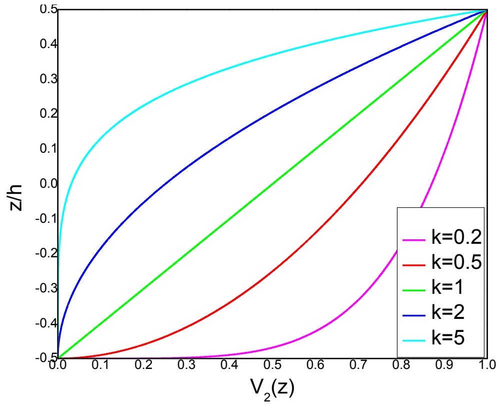

Fig. 2 The change of the volume fraction V_2 along the thickness of the functionally graded nanobeam

B. *Euler-Bernoulli Beam Theory*

The Euler-Bernoulli beam theory is based on the assumption that plane sections perpendicular to the axis of the beam before deformation remain (1) plane, (2) rigid, and (3) perpendicular to the (deformed) axis after deformation. Based on these assumptions, the components of displacement u, v, and w along the x, y, z axis which depend on the x and z coordinates and time t can be written

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_0(x,t)}{\partial x}, \tag{6}$$

$$v(x,z,t) = 0, \tag{7}$$

$$w(x,z,t) = w_0(x,t), \tag{8}$$

where u_0, w_0 is the axial and transverse displacement of any point on the mid-plane of the beam, respectively. Under the premise of considering the small deformation and Hooke's law, the normal stress of the beam can be expressed as

$$\varepsilon_{xx}(x,z,t) = \varepsilon_{xx}^0 - z\kappa^0, \varepsilon_{xx}^0 = \frac{\partial u_0}{\partial x}, \kappa^0 = \frac{\partial^2 w_0}{\partial x^2}. \tag{9}$$

In order to apply the principle of minimum total potential energy, the following expressions of energy must be used. The strain energy of the beam is given as

$$U = \int_{t_1}^{t_2}\int_V \sigma_{ij}\varepsilon_{ij}\, dVdt \tag{10}$$

The velocity vector representation of any point on the beam is

$$\vec{v} = \frac{\partial}{\partial t}\left(u_0(x,t) - z\frac{\partial w_0(x,t)}{\partial x}\right)\vec{i} + \frac{\partial}{\partial t}w_0(x,t)\,\vec{j}. \tag{11}$$

The kinetic energy of the nanobeam, T, at any instant can be stated as

$$T = \frac{1}{2}\int_{t_1}^{t_2}\int_0^L\int_A \rho(z)\cdot(\vec{v}\cdot\vec{v})\, dAdxdt \tag{12}$$

According to the Dirac's delta function, the unknown direct loads on the FG nanobeam can be written as

$$f_c(x,t) = q\delta(x - vt), \tag{13}$$

where q, δ and v are the moving force, the Dirac's delta function, and the speed of the moving force, respectively. Then, the work done by the moving force can be obtained by

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$$W = \int_{t_1}^{t_2}\int_0^L q\delta(x-vt)\cdot w_0(x,t)\,dxdt\,. \tag{14}$$

III. THE GOVERNING EQUATIONS

Based on the principle of minimum total potential energy, the first-order variation of the total potential energy must be zero.

$$\delta\prod = \delta(T-U+W) = 0\,, \tag{15}$$

where $\prod$ is the total energy. For convenience, the varition is performed by parts. The first variation of the strain energy is

$$\delta U = \int_{t_1}^{t_2}\left[\int_0^L\left(-\frac{\partial N}{\partial x}\right)\delta u_0 dx + \int_0^L\left(-\frac{\partial^2 M}{\partial x^2}\right)\delta w_0 dx\right]dt\,, \tag{16}$$

where U is the strain energy. N and M are the axial normal force and the bending moment at the beam's cross section respectively, and they are defined as

$$N = \int_A \sigma_{xx} dA, M = \int_A z\sigma_{xx} dA\,. \tag{17}$$

The first variation of the kinetic energy of the nanobeam can be expressed by

$$\begin{aligned}\delta T &= \frac{1}{2}\int_{t_1}^{t_2}\int_0^L\int_A \rho(\mathrm{z})\delta(\bar{v}\cdot\bar{v})\,dAdxdt\\ &= A_{11}\int_0^L\int_{t_1}^{t_2}\left(-\frac{\partial^2 u_0}{\partial t^2}\right)\delta u_0 dtdx - B_{11}\int_0^L\int_{t_1}^{t_2}\left(\frac{\partial^3 u_0}{\partial t^2\partial x}\right)\delta w_0 dtdx + B_{11}\int_0^L\int_{t_1}^{t_2}\left(\frac{\partial^3 w_0}{\partial x\partial t^2}\right)\delta u_0 dtdx\\ &\quad + D_{11}\int_0^L\int_{t_1}^{t_2}\left(\frac{\partial^4 w_0}{\partial x^2\partial t^2}\right)\delta w_0 dtdx + A_{11}\int_0^L\int_{t_1}^{t_2}\left(-\frac{\partial^2 w_0}{\partial t^2}\right)\delta w_0 dtdx\end{aligned} \tag{18}$$

where the constants A_{11}, B_{11}, D_{11} are

$$(A_{11}, B_{11}, D_{11}) = \int_A \rho(z)(1, z, z^2)dA\,. \tag{19}$$

The first-order variation of the work done by the moving force can be obtained by

$$\delta W = \int_{t_1}^{t_2}\int_0^L q\delta(x-vt)\,\delta w_0(x,t)\,dxdt\,. \tag{20}$$

Substituting Eqs. (16), (18) and (20) into Eq. (15), integrating by parts and setting the coefficient δu_0 and δw_0 to zero lead to the following governing equations:

$$\frac{\partial N}{\partial x} - A_{11}\frac{\partial^2 u_0}{\partial t^2} + B_{11}\frac{\partial^3 w_0}{\partial x\partial t^2} = 0\,, \tag{21a}$$

$$\frac{\partial^2 M}{\partial x^2} - B_{11}\frac{\partial^3 u_0}{\partial t^2\partial x} + D_{11}\frac{\partial^4 w_0}{\partial x^2\partial t^2} - A_{11}\frac{\partial^2 w_0}{\partial t^2} + q\delta(x-vt) = 0\,. \tag{21b}$$

For simply supported FG nanobeams subjected to a moving force, the boundary conditions at the edges of the nanobeam are obtained by

$$\begin{aligned} &u_0 = 0, w_0 = 0 \quad \text{at} \quad x = 0,\\ &w_0 = 0, N = 0 \quad \text{at} \quad x = L. \end{aligned} \tag{22}$$

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IV. NONLOCAL EULER-BERNOULLI BEAM

Following the nonlocal elasticity theory [6], the nonlocal stress tensor σ at point x can be read as

$$\sigma = \int_{V} \alpha\left(\left|x' - x\right|, \tau\right) T\left(x'\right) \mathrm{d}\, x' , \tag{23}$$

$$T\left(x\right) = C\left(x\right) : \varepsilon\left(x\right), \tag{24}$$

where $T\left(x'\right)$ is the classical macroscopic stress tensor at point x', $\alpha\left(\left|x'-x\right|,\tau\right)$ is the nonlocal modulus or attenuation function specifying the nonlocal effects at the reference point x produced by local strain at the source x', $C\left(x\right)$ is the fourth-order elasticity tensor, $\varepsilon(x)$ is the strain tensor, τ is the material parameter which is defined as $\tau = e_0 a/l$ where e_0 is a material constant, a is an internal characteristics length, such as lattice parameter or granular distance, and l is an external characteristic length, such as crack length or wavelength. In order to solve the integral constitutive Eq. (23), a simplified differential equation is obtained in terms of the nonlocal constitutive formulation

$$\left(1 - \tau^2 l^2 \nabla^2\right) \sigma = T, \tau = e_0 a/l , \tag{25}$$

where ∇^2 is the Laplace operator. Because the beam is a two-dimensional structure, the nonlocal response can be ignored in the thickness direction. Hence, the nonlocal constitutive relation can be expressed as

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(\mathrm{z}) \varepsilon_{xx} , \tag{26}$$

where $\mu = (\mathrm{e_0 a})^2$, E is the elasticity modulus, σ_{xx} is the axial normal stress and ε_{xx} is the axial strain. When the nonlocal parameter is zero ($\mu = 0$), we can derive the classical theory's constitutive relation. By using Eqs. (9), (17) and (26), the force-strain and the moment-strain relations of the nonlocal Euler-Bernoulli beam theory can be taken as [25]

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{22} \frac{\partial u_0}{\partial x} - B_{22} \frac{\partial^2 w_0}{\partial x^2} , \tag{27a}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{22} \frac{\partial u_0}{\partial x} - D_{22} \frac{\partial^2 w_0}{\partial x^2} , \tag{27b}$$

where the constants A_{22}, B_{22}, D_{22} are defined:

$$(A_{22}, B_{22}, D_{22}) = \int_A E(z)(1, z, z^2) dA.$$

Substituting Eqs. (21a) and (21b) into Eqs. (27a) and (27b), respectively, the nonlocal displacement type control equations can be expressed as

$$N = \mu\left(A_{11} \frac{\partial^3 u_0}{\partial t^2 \partial x} - B_{11} \frac{\partial^4 w_0}{\partial x^2 \partial t^2}\right) + A_{22} \frac{\partial u_0}{\partial x} - B_{22} \frac{\partial^2 w_0}{\partial x^2} , \tag{28a}$$

$$M = \mu\left[B_{11} \frac{\partial^3 u_0}{\partial t^2 \partial x} - D_{11} \frac{\partial^4 w_0}{\partial x^2 \partial t^2} + A_{11} \frac{\partial^2 w_0}{\partial t^2} - q\delta(x - vt)\right] + B_{22} \frac{\partial u_0}{\partial x} - D_{22} \frac{\partial^2 w_0}{\partial x^2} . \tag{28b}$$

Then, the nonlocal displacement type governing equations can be written as

$$\mu\left(A_{11} \frac{\partial^4 u_0}{\partial t^2 \partial x^2} - B_{11} \frac{\partial^5 w_0}{\partial x^3 \partial t^2}\right) + A_{22} \frac{\partial^2 u_0}{\partial x^2} - B_{22} \frac{\partial^3 w_0}{\partial x^3} - A_{11} \frac{\partial^2 u_0}{\partial t^2} + B_{11} \frac{\partial^3 w_0}{\partial x \partial t^2} = 0 , \tag{29a}$$

$$\mu\left[B_{11} \frac{\partial^5 u_0}{\partial t^2 \partial x^3} - D_{11} \frac{\partial^6 w_0}{\partial x^4 \partial t^2} + A_{11} \frac{\partial^4 w_0}{\partial t^2 \partial x^2} - q\delta''(x - vt)\right] + B_{22} \frac{\partial^3 u_0}{\partial x^3} - D_{22} \frac{\partial^4 w_0}{\partial x^4} \tag{29b}$$

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$$-B_{11}\frac{\partial^3 u_0}{\partial t^2\partial x}+D_{11}\frac{\partial^4 w_0}{\partial x^2\partial t^2}-A_{11}\frac{\partial^2 w_0}{\partial t^2}+q\delta(x-vt)=0$$

V. NUMERICAL RESULTS

A. Static Analysis

An exact numerical solution has been obtained based on the Petrov-Galerkin method by solving the governing equations of the system in Eqs. (29) and the boundary conditions in Eq. (22). In order to solve the equations more easily, the axial and transverse displacements u_0 and w_0 are expressed in a series form in order to meet the boundary conditions. In addition, we assume zero initial conditions, which means that the nanobeam is at the time $t=0$ when the moving force is on the left side of the nanobeam,

$$w_0(x,t)=\sum_{i=1}^{N}w_i^o(t)\sin\frac{i\pi}{L}x,$$
$$u_0(x,t)=\sum_{j=1}^{N}u_j^o(t)\left(1-\cos\frac{j\pi}{L}x\right), \tag{30}$$

where $w_i^o(t)$ and $u_j^o(t)$ are undetermined functions of time, which are needed to determine in the process of solving the governing equations.

By substituting Eq. (30) into Eqs. (29), the following Eqs. are obtained by employing the orthogonalization method.

$$\sum_{i=1}^{N}\left(\mu B_{11}\frac{i^3\pi^3}{L^3}+B_{11}\frac{i\pi}{L}\right)\delta_{ij}w_i^{o\prime\prime}+\sum_{i=1}^{N}\left(\mu A_{11}\frac{i^2\pi^2}{L^2}+A_{11}\right)\delta_{ij}u_i^{o\prime\prime}+\sum_{i=1}^{N}B_{22}\frac{i^3\pi^3}{L^3}\delta_{ij}w_i^o+\sum_{i=1}^{N}A_{22}\frac{i^2\pi^2}{L^2}\delta_{ij}u_i^o=0 \tag{31a}$$

$$\sum_{i=1}^{N}\left(-\mu D_{11}\frac{i^4\pi^4}{L^4}-\mu A_{11}\frac{i^2\pi^2}{L^2}-D_{11}\frac{i^2\pi^2}{L^2}-A_{11}\right)\delta_{ij}w_i^{o\prime\prime}+\sum_{i=1}^{N}\left(-\mu B_{11}\frac{i^3\pi^3}{L^3}-B_{11}\frac{i\pi}{L}\right)\delta_{ij}u_i^{o\prime\prime}-\sum_{i=1}^{N}\left(B_{22}\frac{i^3\pi^3}{L^3}\delta_{ij}\right)\delta_{ij}u_i^{o\prime\prime}$$
$$-\sum_{i=1}^{N}\left(D_{22}\frac{i^2\pi^2}{L^2}\delta_{ij}\right)\delta_{ij}w_i^{o\prime\prime}=\frac{2q}{L}\sin\left(\frac{k\pi}{L}vt\right)\left(\mu\frac{k^2\pi^2}{L^2}+1\right) \tag{31b}$$

We can obtain the following matrix equations by introducing the generalized variables, which can be set to $\{\nabla\}=\left[w_1^o\cdots w_N^o u_1^o\cdots u_N^o\right]^T$.

$$[\mathrm{A}]\left\{\frac{d^2\nabla}{dt^2}\right\}+[\mathrm{B}]\{\nabla\}=\{\mathrm{F(t)}\}, \tag{32}$$

where the size of column vector $\{\mathrm{F(t)}\}$ is $(2N\times1)$ and the size of matrices $[\mathrm{A}]$ and $[\mathrm{B}]$ is $(2N)\times(2N)$. In Appendix A, the contents of the matrix $[\mathrm{A}],[\mathrm{B}]$ and $\{\mathrm{F(t)}\}$ are given concretely.

Next, the second-order differential equations must be reduced to the ordinary differential equations of the first order system. According to the new definition of the generalized variable $\{\Delta\}$, the above Eq. (32) can be turned into

$$[\mathrm{K}]\left\{\frac{d\Delta}{dt}\right\}=[\mathrm{M}]\{\Delta\}+\{\mathrm{C}\}, \tag{33}$$

where $\{\Delta\}=\left[\{\nabla\}^T\left\{\frac{d\nabla}{dt}\right\}^T\right]^T$ is a space column vector. The size of matrices $[\mathrm{K}]=\begin{bmatrix}\mathrm{I} & [0]\\ [0] & [\mathrm{A}]\end{bmatrix}$ and $[\mathrm{M}]=\begin{bmatrix}[0] & \mathrm{I}\\ -[\mathrm{B}] & [0]\end{bmatrix}$ is $(4N)\times(4N)$, and the size of vector $\{C\}$ is $(4N\times1)$. The matrix equations given in Eq. (33) are solved by applying the Runge-Kutta method. A MATLAB code can be programmed to obtain the numerical results. For details, see Appendix B.

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B. Numerical Results and Discussion

The FG material of the nanobeam is made up of Steel (304) and Alumina (Al_2O_3). The entire upper surface of the FG beam is Alumina, and the entire lower surface of the FG beam is stainless Steel. Table 1 clearly shows the properties of the nanobeams [26]:

TABLE 1 MATERIAL PROPERTIES OF FG NANOBEAM CONSTITUENTS

L/h	Properties	Unit	Steel	Alumina (Al_2O_3)
20	E	GPa	193	395
20	ρ	kg/m^3	7930	3970

Fig. 3 states clearly that for different values of constant k, the maximum dimensionless deflection of the functionally graded nanobeam varies with the change of dimensionless time. These images can be drawn, as with the red line in the picture, $k = 0$, which means the entire nanobeam becomes an Alumina nanobeam and the deflection of the nanobeam is also the smallest. When the value of the constant k is at its maximum, the entire nanobeam becomes a Steel nanobeam and the deflection of the nanobeam is also the largest. Obviously, the increase of speed of the moving force leads to the decrease of deflection of the functionally graded nanobeam due to the fact that with an increase of speed, the moving force traverses the functionally graded nanobeam more quickly. Therefore, compared with the situation of the small velocity through the beam, the time of the moving force stimulating the beam is relatively small. That is to say, when the force moves at a faster velocity, the FG nanobeam does not have enough time to respond to the force.

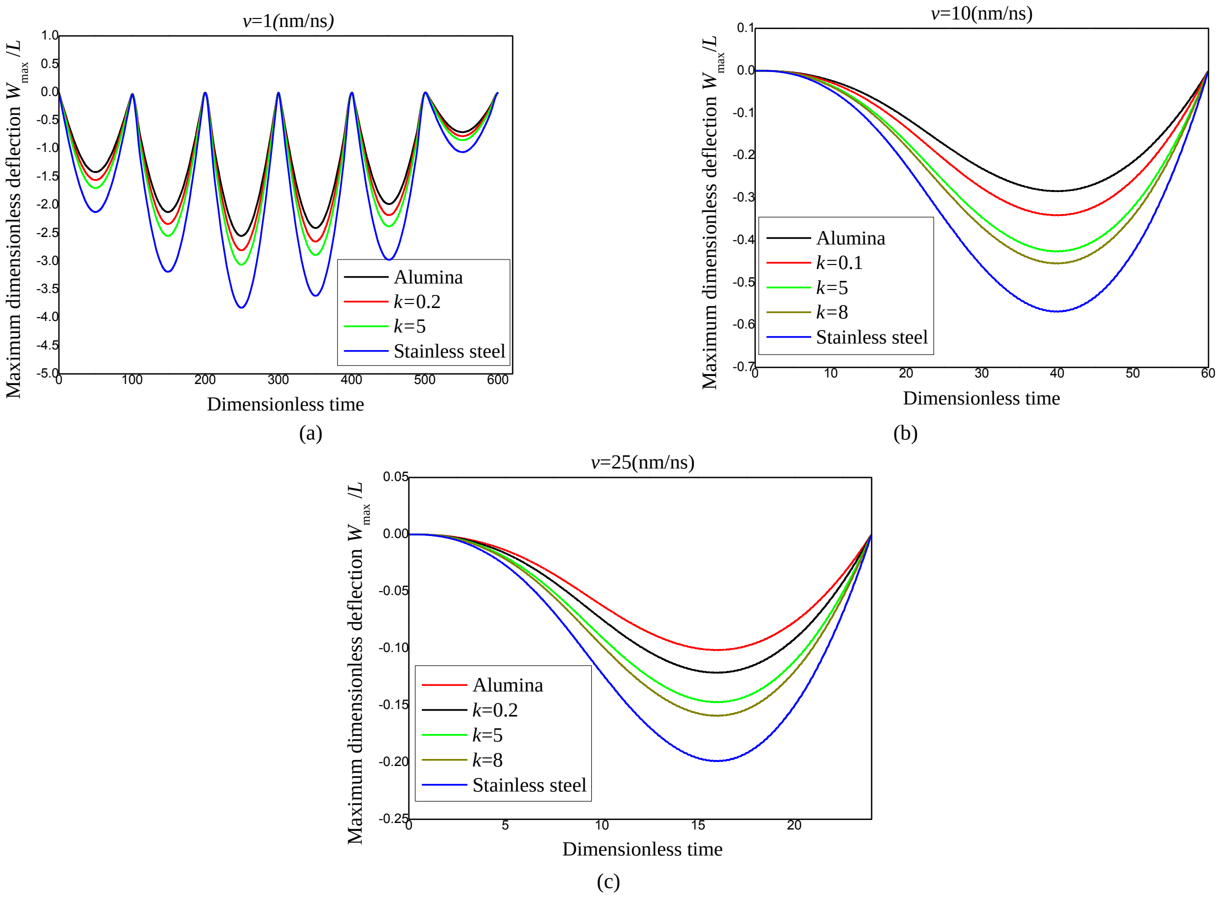

Fig. 3 The relationship between maximum dimensionless deflection and the dimensionless time of the FG nanobeam for $L/h = 20$, $\mu = 1.0E-12$.

(a) v=1nm/ns , (b) v = 10nm/ns , (c) v = 25nm/ns

Fig. 4 shows the normalized deflections of the FG nanobeam with respect to the dimensionless axial coordinate x/L, or variable k at different values, $L/\mathrm{h} = 20$, $v = 1$, 10, 25nm/ns when the maximum deflection of the nanobeam reaches the maximum value. We can draw from these data that the maximum deflection in the position of the FG beam will change with the increase of the speed of the moving force.

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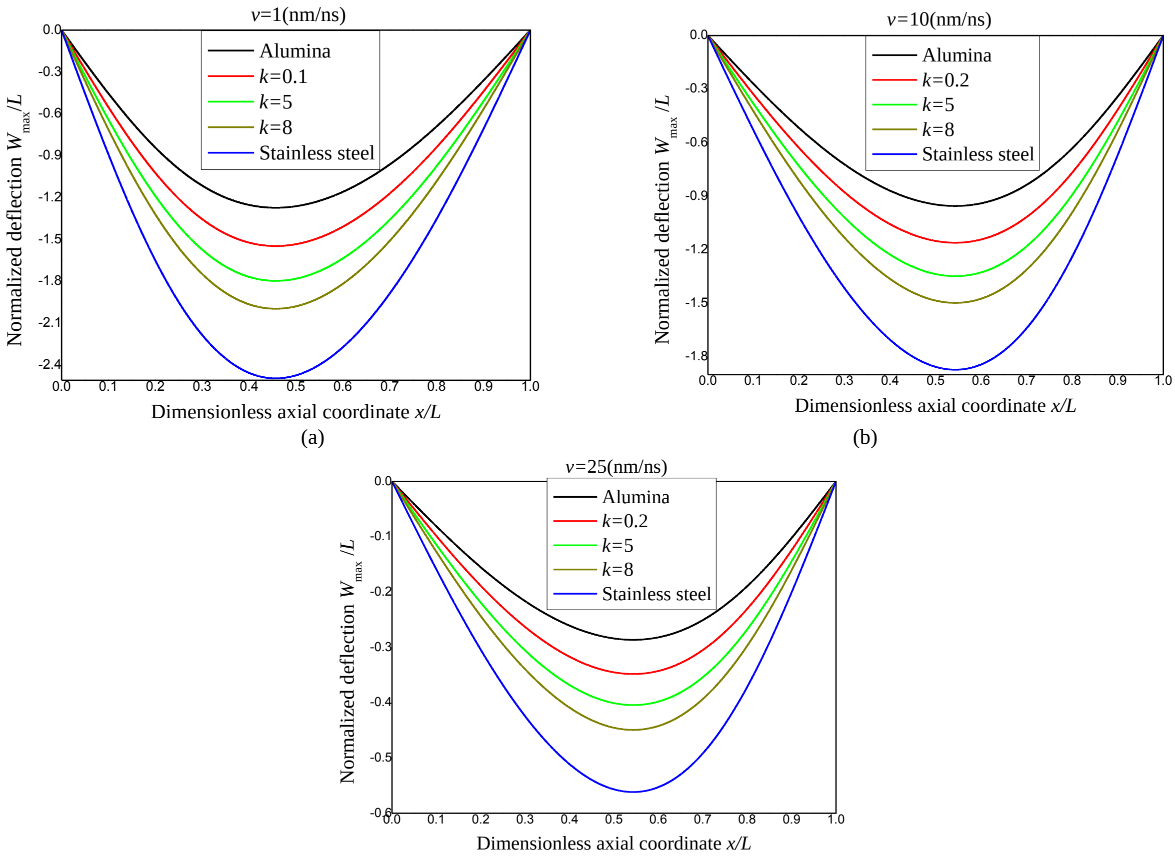

(c)

Fig. 4 The deflection at different locations to the dimensionless axial coordinate x/L, for $L/h = 20$, $\mu = 1.0E-12$.

(a) $v = 1$nm/ns, (b) $v = 10$nm/ns, (c) $v = 25$nm/ns

Fig. 5 presents the consequence of the material distribution parameter k on the maximum dimensionless deflections for various values of the nonlocal constant μ of a simply supported FG nanobeam. For a given nonlocal constant μ, the maximum dimensionless deflection increases with the increase of the material distribution parameter k. What's more, with a certain material distribution parameter k, the maximum dimensionless deflection becomes larger as the nonlocal parameter μ increases. With the change of the material constant from 0 to 10, the composition of the material moves from unmixed isotropic alumina (Al_2O_3) to almost pure stainless steel. We conclude that the FG nanobeam has become soft under the conditions of nonlocal theory.

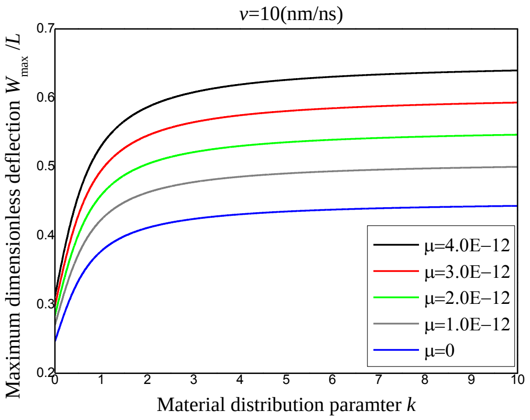

Fig. 5 The relationship between maximum dimensionless deflections and material distribution parameter of a simply supported FG nanobeam

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VI. CONCLUSIONS

The dynamic analysis of simply supported functionally graded nanobeams subjected to a moving force based on the nonlocal Euler-Bernoulli elasticity theory has been researched in this paper. The governing equations of the system of the nanobeams are derived by applying the energy variational principle. In this text, the effects of nonlocal effect, aspect ratio and various material compositions on the responses of the functionally graded nanobeam are also discussed. Numerical results indicate that the nonlocal effects have an important position in the dynamic responses of the FG nanobeam. According to the nonlocal theory, the maximum deflection of the nanobeam is much larger than that of the classical theory. Therefore, the small scale effects should not be negligible considered in the analysis of mechanical behaviour of nanostructures. It is shown that the power-law exponent index produces an effect on the responses of functionally graded nanobeams, and the responses can be dominated by selecting proper numerical values of the power-law index. The conclusion of this paper is guidance for research and manufacture of functional graded nanobeam structures.

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Appendix A

$$[A]=\begin{bmatrix}[A_1]_{N\times N} & [A_2]_{N\times N}\\ [A_5]_{N\times N} & [A_6]_{N\times N}\end{bmatrix}\qquad [B]=\begin{bmatrix}[A_3]_{N\times N} & [A_4]_{N\times N}\\ -[A_7]_{N\times N} & -[A_8]_{N\times N}\end{bmatrix}\qquad \{F(t)\}=\begin{bmatrix}[0]_{N\times 1}\\ \{f_k(t)\}_{N\times 1}\end{bmatrix}$$

$$A_1(i,j)=\left(\mu B_{11}\frac{i^3\pi^3}{L^3}+B_{11}\frac{i\pi}{L}\right)\delta_{ij},$$

$$A_2(i,j)=\left(\mu A_{11}\frac{i^2\pi^2}{L^2}+A_{11}\right)\delta_{ij},$$

$$A_3(i,j)=B_{22}\frac{i^3\pi^3}{L^3}\delta_{ij},$$

$$A_4(i,j)=A_{22}\frac{i^2\pi^2}{L^2}\delta_{ij},$$

$$A_5(i,j)=\left(-\mu D_{11}\frac{i^4\pi^4}{L^4}-\mu A_{11}\frac{i^2\pi^2}{L^2}-D_{11}\frac{i^2\pi^2}{L^2}-A_{11}\right)\delta_{ij},$$

$$A_6(i,j)=\left(-\mu B_{11}\frac{i^3\pi^3}{L^3}-B_{11}\frac{i\pi}{L}\right)\delta_{ij},$$

$$A_7(i,j)=D_{22}\frac{i^2\pi^2}{L^2}\delta_{ij},$$

$$A_8(i,j)=B_{22}\frac{i^3\pi^3}{L^3}\delta_{ij},$$

$$\{f_k(t)\}=-\frac{2q}{L}\sin\left(\frac{k\pi}{L}vt\right)\left(\mu\frac{k^2\pi^2}{L^2}+1\right)$$

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Appendix B

Runge-Kutta methods are a family of implicit and explicit iterative methods used in temporal discretization for the approximate solutions of ordinary differential equations. This algorithm's principle is more complex because of its high precision and the measures for the error suppression. The algorithm is based on the mathematical support. One member of the family of Runge-Kutta methods is often referred to as "RK4", "classical Runge-Kutta method". Let an initial value problem be specified as follows:

$$\dot{y} = f\left(t, y\right)$$
$$y\left(t_0\right) = y_0$$

Here, y is an unknown function of time t which we would like to approximate; we are told that $\dot{y}$, the rate at which y changes, is a function of t and of y itself. At the initial time t_0 the corresponding y-value is y_0. The function f and the data t_0, y_0 are given.

Now, pick a step-size RK4 and define

$$y\left(n+1\right) = y\left(n\right) + h*\left(K_1 + 2*K_2 + 2*K_3 + K_4\right)/6$$
$$t\left(n+1\right) = t\left(n\right) + h$$

for $n = 0,1,2,3\cdots$, using

$$K_1 = f\left(t_n, y_n\right)$$
$$K_2 = f\left(t\left(n\right)+h, y\left(n\right)+h*K_1/2\right)$$
$$K_3 = f\left(t\left(n\right)+h/2, y\left(n\right)+h*K_2/2\right)$$
$$K_4 = f\left(t\left(n\right)+h, y\left(n\right)+h*K_3\right)$$

where $y\left(n+1\right)$ is the RK4 approximation of $y\left(t_n+1\right)$, and the next value $y\left(n+1\right)$ is determined by the present value $y\left(n\right)$ plus the weighted average of four increments, where each increment is the product of the size of the interval, h, and an estimated slope specified by function f on the right-hand side of the differential equation.

K_1 is the increment based on the slope at the beginning of the interval, using y;

K_2 is the increment based on the slope at the midpoint of the interval, using $y+h*K_1/2$;

K_3 is again the increment based on the slope at the midpoint, but now using $y+h*K_2/2$;

K_4 is the increment based on the slope at the end of the interval, using $y+h*K_3$.

In averaging the four increments, greater weight is given to the increments at the midpoint. If f is independent of y so that the differential equation is equivalent to a simple integral, then RK4 is Simpson's rule.

The RK4 method is a fourth-order method, meaning that the local truncation error is on the order of $O\left(h^5\right)$, while the total accumulated error is order $O\left(h^4\right)$.

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