# Modeling and Parameter Estimation of Roll Stability of a Single-Rotor Boat Model 

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#### Abstract

This paper presents experiments for determining the rotational stiffness coefficient $\boldsymbol{k}_{\boldsymbol{\theta}}$ for water, and the restoring-force coefficient $\gamma_{\theta}$ of the gyroscopic moment. These parameters are based on non-autonomous basic equations derived from the autonomous roll-stability equation for a floating body, the wind force moment, and the gyroscopic moment of the Flettner rotor dynamics for a single-rotor boat model with various wind speeds from both the port and starboard sides in a small pool. The tilt angle of the rotor boat was measured with 3D accelerometers, and the restoration angle was computed. The nonlinearity of the stiffness coefficient was identified, and in a simulation, accurate results for roll stability were obtained.


Keywords- Eco-ship; Flettner Rotor; Rolling Stability; Gyro

## I. INTRODUCTION

Following recent nuclear accidents, there has been an increase in the perceived importance of reducing total energy consumption and increasing the use of renewable energy. This can be achieved with the use of eco-friendly cars and eco-ships, which reduce greenhouse gas emissions by using renewable energy sources.

Here, eco-ships [1-2] are considered, which are ships that utilize renewable power sources. In particular, we consider rotor ships, a type of ship that uses the Magnus effect for propulsion through the drag and lift forces applied to some rotors. The first rotor ship, the Baden Baden, was designed and constructed by Anton Flettner in Germany in 1924 [3-4].

We have been investigating the characteristics of the rotor propulsion system using land vehicles [5-6], and thus far, we have evaluated and proposed the MRT (Main Rotor Trip similar to Main Fuel Trip) and RUNBACK interlock systems, which operate on a thermal power plant [7].

Another team at the same laboratory is researching the design of wing sails, with the aim of improving safety and efficiency, for the Wind Challenger project currently being conducted in Japan [8].

For the Wind Challenger project, we considered the hull design that would best lower the center of gravity, since this is often a problem with wind-powered eco-ships. We developed a new standard, and we confirmed that this maximizes the wind receiving area. It was assumed that it can perform at a maximum inclination angle of $2^{\circ}$ in a strong wind [9].

Recently, there have been many studies that consider boat roll stabilization via active fin stabilizers [10] or by adding conventional passive fin stabilizers [11]. These may be of particular help for the rotor boat, because the primary objective of the rotor is not to stabilize the rolling of the boat but to reduce the energy needed to drive the boat by assisting the propulsion. However, the rotor can be designed to act as an active stabilizer.

The object of this paper is to examine the rolling stability of a rotor boat for various rotation speeds of the rotor, after accounting for the wind force, based on the overall shape of the boat. We consider decreasing the sensitivity of the rolling stability to the wind power using a low-height rotor, and also increasing the sensitivity using a small boat with a shape that inclines easily and can be examined in a small pool. Clearly, a very wide vessel that does not incline easily is not suitable for a stability experimental use, but is suitable for actual rotor ships like the E-ship1 with 4 rotors.

## II. EXPERIMENTAL DEVICES AND RESULTS

In this chapter, we present the experimental devices and results. First, we consider the inclination of a boat due to wind and the restoration of the inclination by rolling.

## A. Inclination of Boat due to Wind and Restoration of the Inclination by Rolling

One small rotor, with an aspect ratio of 1 , was set up in a radio-controlled boat (NEW27DA, 1/16), and a 3D accelerometer (Microstone Co., Ltd, MVP-RF8-GC-500) [12] was set up in the bow and on the top board of the frame for fixing the rotor (Fig. 1). The boat was then set in a small, round pool with a diameter of 1.7 m . Then, the influence of the reflection of the wave cannot be disregarded. Two strings were horizontally supported at a low height in the vicinity of the roll rotation axis;
they were placed so that they would not obstruct the roll of the rotor boat and would not change the boat's angle relative to the wind (although it is not necessary to consider the angle of attack, as in the case of wings). The major objective of these strings was to avoid greatly biasing the support line from a central line and to prevent the boat from being capsized due to the wind and the lift. We used a factory fan (TRUSCO Co., Ltd, TFZ4S) to produce three wind velocities: weak ( $3.7 \mathrm{~m} / \mathrm{s}$ ), moderate ( 4.6 $\mathrm{m} / \mathrm{s}$ ), and strong ( $5.6 \mathrm{~m} / \mathrm{s}$ ).


Fig. 1 Rotor boat and wrapped 3D accelerometer on ceiling of rotor
The rotor rotational speed was varied using a DC/DC converter (Velleman CARS2000) [13] and a 12 V battery (TAKEGAWA MF12V), using five different voltages ( $6.0 \mathrm{~V}, 4.5 \mathrm{~V}, 3.0 \mathrm{~V}, 1.5 \mathrm{~V}$, and 0 V ). The relation between the DC/DC converter scale and the actual voltage was approximately linear, even though the voltage of the battery changed somewhat during the experiment, and the rotational speed was saturated above approximately 6 V .

## B. Measurement of Inclination and Restoration Angles

The inclination angle relative to the x -axis, that is, the rolling angle, can be computed from experimental data as follows:

$$
\begin{equation*}
\theta=\pi / 2-\cos ^{-1}\left(A_{x} /\left(g \times \cos \left(\tan ^{-1}\left(A_{y} / A_{x}\right)\right)\right)\right) \tag{1}
\end{equation*}
$$

where $g$ is the gravitational acceleration, and $A_{x}$ and $A_{y}$ are the $x$-axial and $y$-axial acceleration as measured by a 3D accelerometer. This has been corrected by using the acceleration ratio of $\mathrm{A}_{\mathrm{y}}$ and $\mathrm{A}_{\mathrm{x}}$ equivalently to calculate the inclination of the z axis to the direction of the x axis approximately, although $\mathrm{A}_{\mathrm{y}}$ and $\mathrm{A}_{\mathrm{x}}$ are not equivalent in the boat.

Using this equation, the inclination and restoration angles were calculated to be $0^{\circ}$ when there was no rotation in the absence of wind.

Fig. 2 shows the explanation of inclination and restoration angles obtained from experimental data as a function of wind speed for a rotor rotation of 5129 rpm and without rotation in case of little inclination of boat pushed by wind. Then, though the boat inclines at the direction of negative further being pushed by wind, the inclination of the boat returns to former by the rotor rotation, and restoration becomes positive direction.


Fig. 2 Static restoration properties by rotor rotation in the case of inclination by wind
In case of which the boat inclines the direction of which push wind originally, it inclines the direction of positive pushed by wind. Moreover, it inclines the direction of positive pushed by the rotor rotation. Then, the rotor returns to the original point.

The Surf function in Matlab was used, and the inclination and restoration angles were plotted as three-dimensional surface plots against the motor voltage and the wind velocity, as shown in Fig. 3. Two points missing from these results were interpolated at the crossing: a planar approximation from which the anchor $d$ is determined from three adjacent points, $a, b$, and c , with $\mathrm{d}=\mathrm{a}+\mathrm{b}-\mathrm{c}$, where c is across from d ; the midpoint of the plane is $\mathrm{m}=(\mathrm{a}+\mathrm{b}) / 2=(\mathrm{c}+\mathrm{d}) / 2$.


Fig. 3 3D surface plots with missing points obtained by interpolation
We note that for the restoration angle, a ridge appears at a motor voltage of 4.5 V . This feature disappears when the least squares method is used to fit the curved surface in two or three dimensions, and thus the pseudoinverse matrix regression method could not be adopted.

## III. KINEMATIC EQUATION FOR ROLLING ANGLE

Rolling of a floating body can be approximated as a second-order linear system, and it is known that a rigid rotating object, such as a top or a rotor, is subject to a gyroscopic moment that causes precession of its rotation axis as shown in Fig. 4 [14].

Although the gyro moment is well known, we show the direction of gyro moment for a rotor boat in Fig. 4.


Fig. 4 Direction of gyro moment for a rotor boat
The swinging turn (precession) phenomenon of the rotation axis happens because the torque of the rotation axis and the gyroscopic moment works simultaneously when all directions are equal resistance like the top and the helicopter.

The stability level of the rotor boat will be presumed to increase because the gyroscopic moment works in the longitudinal direction of the boat in the case of a crosswind so that the reaction moment works from water.

## A. Analysis of Kinematic Equation for Rolling Angle (Complex Resistance of Viscosity)

The restoration due to the ship's geometry is assumed to be proportional to the rolling angle $\theta$, such that the stable state is the starting point for the rotor boat used here. The rotational system's spring constant $k_{\theta}$ and restoration force coefficient $\gamma_{\theta}$ are assumed to depend on the inclination angle $\theta^{\prime}=\theta+\theta_{o}$, as measured from a perpendicular axis. The kinematic equation for the effect of the force of the wind on the inclination angle $\theta$ can be expressed by introducing a complex viscous modulus. However, this equation yields a complex rolling stability. The solution is a complex number in the complex plane:

$$
\begin{align*}
& J_{x} \ddot{\theta}(t)+\left(C_{w}-j \omega_{r} J_{p}\right) \dot{\theta}(t)+k_{\theta} \theta(t)=F_{x}(t) H-\gamma_{\theta} M_{g}(t)  \tag{2}\\
& \text { where } \quad M_{g}=J_{p} \theta_{z}\left|\Omega_{f}\right| \omega_{r}, M_{g}=-J_{p} \theta_{z}\left|\Omega_{b}\right| \omega_{r},  \tag{3}\\
& \qquad J_{x}=m\left(\frac{D_{r}^{2}}{16}+\frac{H_{r}^{2}}{12}\right)+m h^{2} ; J_{p}=m \frac{D_{r}^{2}}{8} \tag{4}
\end{align*}
$$

where, the positive root $\Omega_{f}$ indicates clockwise (CW) rotation, the negative root $\Omega_{b}$ indicates counterclockwise (CCW) rotation, there is forward and backward rotation, $J_{x}\left[\mathrm{kgm}^{2}\right]$ is the moment of inertia along the rolling rotational axis of the rotor ship, $J_{d}$ $\left[\mathrm{kgm}^{2}\right]$ is the moment of inertia along the horizontal axis that passes through the center of gravity of the rotor, $J_{p}\left[\mathrm{kgm}^{2}\right]$ is the polar moment of inertia along the vertical axis that passes through the center of gravity of the rotor, $m$ [ kg ] is the mass of the rotor, $h[\mathrm{~m}]$ is the distance from the rolling rotational axis of the rotor ship to the horizontal axis that passes through the center of gravity of the rotor, $D_{r}[\mathrm{~m}]$ is the diameter of the rotor, $H_{r}[\mathrm{~m}]$ is the height of the rotor, $C_{w}$ is the viscous modulus of the air, $\omega_{r}[\mathrm{rad} / \mathrm{s}]$ is the angular velocity of the rotor, $k_{\theta}[\mathrm{N} / \mathrm{rad}]$ is the spring constant of the rotational system due to the water's buoyancy, $F[\mathrm{~N}]$ is the force acting at the center of the wind force around the rolling axis ( $x$ axis), $H[\mathrm{~m}]$ is the distance from the rolling rotational axis of the rotor ship to the center of the wind force around the rolling axis ( $x$ axis) of the rotor, $M_{g}[\mathrm{kgm}$ ] is the gyroscopic moment generated by inclining the rotor rotation axis, and $\gamma_{\theta}$ is the restoration force coefficient that is the ratio of the gyroscopic moment to the restoring moment of the hull.

The other basic physical parameters are shown as Table 1.
TABLE 1 THE OTHER BASIC PHYSICAL PARAMETERS

| Item | Value | Item | Value |
| :---: | :---: | :---: | :---: |
| $M$ | 2.36 kg | $\mathrm{~J}_{\mathrm{x}}$ | $0.0313 \mathrm{kgm}^{2}$ |
| $H$ | 0.147 m | m | 0.886 kg |
| $\mathrm{D}_{\mathrm{r}}$ | 0.1 m | $\mathrm{H}_{\mathrm{r}}$ | 0.1 m |

## B. Estimation of Precession Frequency (Root of the Characteristic Equation)

In Eq. (2), we assume that $C_{w}=0$, since the air resistance is low. If we introduce a gyroscopic factor, $\xi=J_{p} / J_{x}$, and rearrange the equation, we obtain the following second-order differential equation with complex coefficients from Eq. (2) divided by $J_{x}$.

$$
\begin{gather*}
\ddot{\theta}-j \xi \omega_{r} \dot{\theta}+\omega_{n}^{2} \theta=0  \tag{5}\\
\omega_{n}=\left(k_{\theta} / J_{x}\right)^{0.5} \tag{6}
\end{gather*}
$$

The precession frequency of the rotor due to the gyroscopic moment can be obtained from the roots of the above equation, as follows.

$$
\begin{align*}
& \Omega^{2}-\omega_{r} \xi \Omega=\omega_{n}^{2} \\
& \Omega=\frac{\omega_{r} \xi \pm \sqrt{\omega_{r}^{2} \xi^{2}+4 \omega_{n}^{2}}}{2} \tag{7}
\end{align*}
$$

Fig. 5 shows the relation among the wind direction, the direction of rotation of the rotor, the direction in which the rotor advances, and the direction of the precession of the axis of rotation. The right-hand figure shows the wind coming from the left, and the precession is CW (clockwise) and forward. The left-hand figure shows the wind coming from the right, and the precession is CCW (counterclockwise) and backward. $\Omega_{f}$ is forward angular speed, and $\Omega_{b}$ is backward angular speed. Fig. 6 shows an analysis of the results in the form of $A_{x}-A_{y}$ traces, with a detailed enlargement of the 3D acceleration data. When the wind is from the left, the rotation of the axis is CW , or forward angular speed ( $\Omega_{f}=228 \mathrm{rad} / \mathrm{s}$ ), and the $A_{x}-A_{y}$ traces are circles; a detailed enlargement of the 3D acceleration data obtained at the front deck is shown in Fig. 6.


Fig. 5 Direction of swinging of rotation axis


Fig. 6 Ax-Ay plane acceleration $\left[\mathrm{m} / \mathrm{s}^{2}\right]$ measured by 3 D accelerometer $\# 81$ at bows

When the wind comes from the right, the rotation of the axis is CCW, or backward angular speed ( $\Omega_{b}=419 \mathrm{rad} / \mathrm{s}$ ), and the left figure is similar to a collapsed oval (like front view of an elephant). We found that very small amounts of precession can cause larger changes in the direction in which the boat proceeds. Thus, we can conclude that this model is valid and correct, although it predicts a very slight vibration that we were unable to perceive visually with the boat model.

## C. Identification of Angle, Restoration Angles for the Inclination of Roll and 3D Plots

Changes in the inclination angle are determined by variations in gravitational acceleration, as measured by an accelerometer. We also want to know the coefficient of rigidity of the water in relation to the roll stability of the hull, the coefficient of the restoration force that changes with the gyroscopic precession of the rotor rotation, and the restoration force due to the buoyancy of the ship.

The constant $k_{\theta}$ in the above kinematic equation is obtained from the steady-state inclination angle $\theta_{z}$ when the rotor is not rotating.

$$
\begin{equation*}
k_{\theta}=\frac{F_{x} H}{\theta_{z}} \tag{8}
\end{equation*}
$$

The above equation needs to be modified to include the inclination angle $\theta^{\prime}$ from the vertical axis, because the effect is changed if the initial inclination angle changes, even if the wind speed does not change. The dependency of the above equation on $\theta$ was found to be $k_{\theta^{\prime}}=\min \left(37.5 / \theta^{\prime}, 7.39\right)$; this was determined from the data for wind from the left, using the least squares method.

In a similar way, the coefficient of the restoration force can be obtained from the steady-state inclination angle of rotation at a constant rate of revolution as a function of the inclination angle, as follows.

$$
\begin{align*}
& \gamma_{\theta}= \pm \frac{k_{\theta} \theta_{z}-F_{x} H}{J_{p}\left|\Omega_{f, b}\right| \omega_{r}}  \tag{9}\\
& F_{x}=P\left(f_{D}\left(\omega_{r}\right)\left(u / u_{o}\right)^{2}\left(S / S_{o}\right)-F_{\text {dead }}\right) \tag{10}
\end{align*}
$$

Here, the + sign indicates forward motion, and the - sign indicates backward motion. Eq. (10) can be approximated by a second-order equation to include the effect of changes in wind speed. Notation of P is a function generating a force in the direction of x -axis $F_{x}$. Notation of $f_{D}$ is drag force to the rotor, u is wind speed, $u_{o}$ is nominal wind speed, S is surface area for drag force, So is nominal surface area, and $F_{\text {dead }}$ is non-effective force.

In a preliminary experiment, the coefficient for the restoration force was obtained from the above equation, for a constant rate of revolution of the rotor. The results indicate that the coefficient of the restoration force increases as the inclination angle increases. We assume that the reason for this is that the distance between the center of gravity and the center of buoyancy increases as the inclination angle increases, and thus the moment of the restoration force also increases. The reason for the high sensitivity to slight changes in inclination angle is the design of the hull.

## IV.DISCUSSION AND CONSIDERATIONS

The restoration angle increases as the voltage of the motor increases, and it reaches a maximum at 4.5 V , it then decreases at 6 V . This tendency increases as the wind speed increases. The reason why the restoration force has an optimal revolution rate of $5200-5300 \mathrm{rpm}$ is that the drag force becomes larger as the revolution rate increases; the same thing happens with the lift force [5]. Also, at a high rate of revolution, the drag force overwhelms the restoration force. Thus, not only the lift force but also the drag force depends on the revolution rate, and this dependency increases as the wind speed increases.

A voltage of 12 V to the $\mathrm{DC} / \mathrm{DC}$ converter is significantly larger than the maximum usage voltage of 6 V , and the output voltage does not change even if the battery voltage decreases, as it does when trying to supply more than 9 V . Thus, this power supply system is suitable for use in an experiment in which the revolution rate of the rotor is varied over a long time period.

Although a conventional real-coefficient rolling boat model is sufficient for investigating roll stability, the rolling boat model with a complex viscosity coefficient is more effective because precession can be actually observed. However, the influence of the complex coefficient is very small in the case of a rectangular boat.

If the center of gravity is raised enough that the constant $k_{\theta}$ becomes negative, the boat may capsize, since the origin of the system is changed from a stable node to a saddle point in the case of an autonomous system with real coefficients. A boat should be balanced in such a way that a small inclination angle will maintain a low center of gravity; this can be accomplished with ballast.

## V. CONCLUSION

We conducted an experiment to identify the parameters related to roll stability of a model boat, where one rotor was subjected to wind from both the port (left) and starboard (right) sides. The inclination angle was measured with a 3D
accelerometer, and we confirmed the validity of this model from the minute swinging (precession) movement with an Ax-Ay traces analysis.

We found that the coefficient of rigidity and the restoration force coefficient were changed by interaction between the water and the boat, and this depended nonlinearly on the roll rotation angle of the boat. This is thought to be due to the design of the hull.

By varying the rotational speed and the restoration angle of the rotor boat in a crosswind, we determined the best rotational speed for a given restoration angle.

We believe that this is due to the dependence of both the restoration force and the rotor drag on the rotational speed and the gyroscopic precession.

The rotating rotor not only assists in propelling a boat, through the Magnus effect, it also acts as an active stabilizing component that improves the roll stability of yachts and boats based on gyroscopic precession.

Our data includes the influences of waves and the tension of the string, and thus it is important to verify our results in a system free from these influences; in particular, it should be noted that the boat in the small pool was held horizontal with a string.

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