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A Most-squares Solution for Separating Cars

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Abstract-This paper presents a method for finding the most-squares solution for separating cars, and intends to develop technologies to be able to separate cars from compact and standards based on images taken from the camera. Two major methods are used basically: 1. convex hulls algorithms to obtain car's convex hull region and 2. classifying by separating cars into clusters regarding area and circumference of the regions obtained from car's convex hull. Finally, a numerical example is presented.

Keywords- Separation; Clustering; Cars; Ferry; Linear Manifold for Separation; Convex Hull

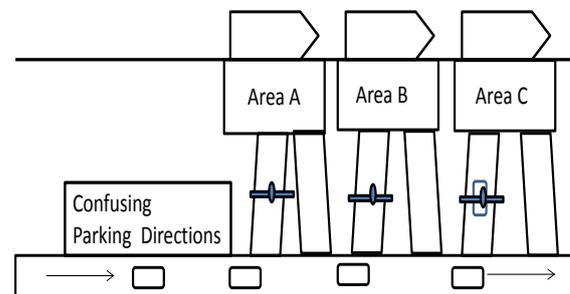
I. INTRODUCTION

There have been various studies that use images to study the clustering of cars, for example, by reading the license plate numbers at toll gates or during bridge crossings [1]. When a license plate is dirty, however, processing is difficult, although the model of the car can be determined. To address this, various methods have been considered, including a probabilistic separation of the background scene, objects, and shadows [2]; measurement of vehicle flow based on images at different points in time [3], and using space-frequency analysis to detect car features that are easily distinguishable [4]. In this study, video images of traffic entering a parking lot were used and then finding a parking space, with one camera at the entrance and several more cameras in the parking lot [5]. Information such as that shown in Fig. 1(a) can be used to determine where cars should be directed, that is, to parking area A, B, or C. Note that since areas A and B are marked full for standard cars, standard cars entering the lot are directed to area C.

This method can also be useful for parking cars on a ferry, as shown in Figs. 1(b)-(d) [6]. More cars can fit on a ferry if they are assessed and divided in this way.

Parking Areas	Size of Cars	No. of Empty Stalls	State
A	Compact	5	Empty
	Standard	0	Full
	Large	20	Empty
B	Standard	0	Full
	Large	10	Empty
C	Standard	15	Empty
	Large	10	Empty

(a) Information about the state (full or empty) of parking areas [5]



(b) Display and camera



(c) Cars in a parking lot [6]



(d) Cars on a ferry [6]

Fig. 1 Use plan of this system

A simulation of taking video images of cars was done from a viaduct on a public highway with the WEB camera (Table 1) aiming at these applications.

After preprocessing the video images, some images of cars were obtained and they were categorized by using a scatter plot. The system was evaluated by measuring the rate at which the types of cars could be identified.

There are various cluster separation studies for items other than cars, and these studies have used various methods, such as one based on overlapping probability distributions, a fuzzy clustering method, a method based on the amount of information amount, and a data recovery approach [7].

In this study, the support vector machine (SVM) method [8] and the subspace method [9] will be considered. In the SVM, for a most-squares solution (for implicit functions) [10], the selected support vector near the separation line will be needed to compute and to derive the straight line that separates clusters of kinds of cars. Obtaining this two-dimensional feature will be used the pseudoinverse operator method [11], which is based on an orthogonal projection theorem [12, 13].

II. IMAGE PROCESSING FOR THE SEPARATION OF CAR MODELS

A. Image Processing Methods

Images of cars were isolated from video images that were obtained by using a web camera (ELECOM UCAM-DLH200HS) that was placed on a bridge (height = 6.2 m) at the entry to a virtual parking lot.

TABLE I MAJOR SPECIFICATION OF WEB CAMERA (ELECOM UCAM-DLH200HS)

Reception Element	1/4 inch CMOS sensor
Pixel Count	200million pixel
Hardware maximum image degree	1600 × 1200
Maximum frame rate	25fps(640 × 480pixel) 12fps(1280 × 720pixel) 7fps(1280 × 960pixel) 5fps(1600 × 1200pixel)
Color	About 1600million color (24bit)
Interface	USB2.0

The cars were sorted by the convex hull method by using the well-known image processing tool Open-CV [14], as shown in Fig. 2. The car images were preprocessed to block out extraneous information, and the resulting images are shown with a black background.

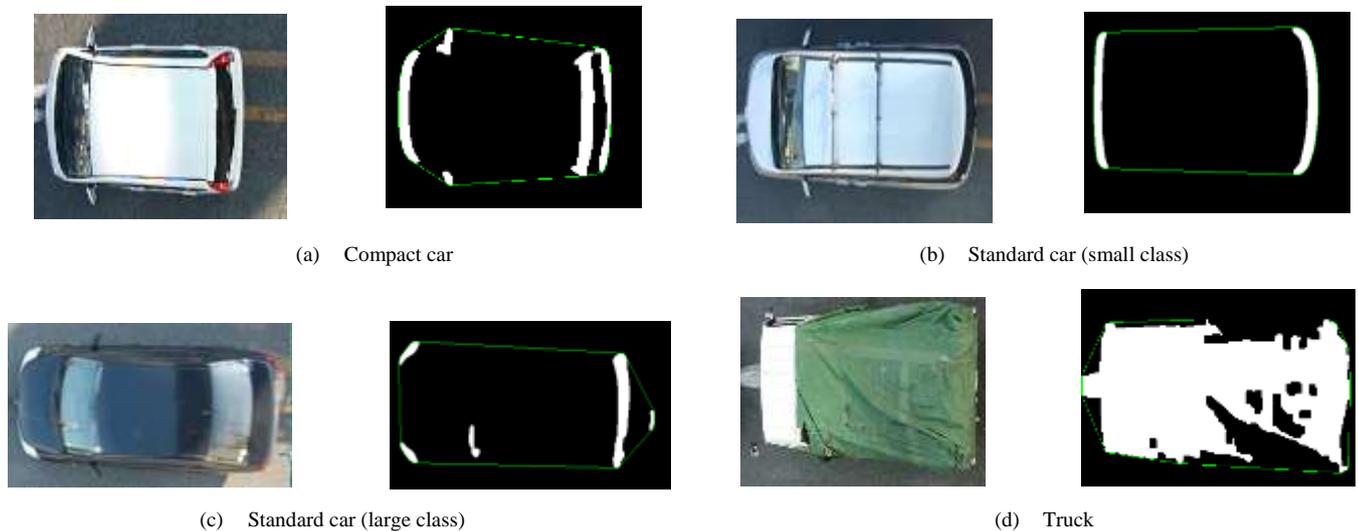


Fig. 2 Capturing the image of a car using the convex hull method

Using the measured features (area and circumference) of the convex hulls, a separation line between the clusters of data were able to quickly obtained in the case of no overlapped cluster. But, the following method was necessary in the case of overlapped cluster for the working.

III. CONCEPT AND RELATIONAL EQUATIONS OF MOST SQUARES METHOD FOR CLUSTERING

The shape of the compact cars, medium-scale cars, and large-scale cars are a similar rectangular figure, then the declines (the rate of area and circumference of the figure (c.f. later Fig. 3) of straight lines connected the points of cars of the same class

are almost same as shown in the feature plane on the area and the circumstance.

Therefore, the car model class cannot be separated by identifying the slope by the fitting of these straight lines by the least square method using a distance of orthogonal shadow in Appendix.

Then, the straight line to which the distance between two points group of each class and a linear manifold was maximum separated by the most square method was obtained.

The method in a general function space to independent variables of implicit functions in the appendix is applied to the problem of a requesting separate straight line in two dimension space.

First, slope a that the sum of distance between a linear manifold $y = ax + b$ that passes the point (x_{1p}, x_{2p}) and the point of N piece (x_i, y_i) in two dimensions becomes extrem places is obtained as follows.

$$b = x_{2p} - ax_{1p} \quad (1)$$

$$d_i = \sqrt{\frac{\{ax_{1i} - (x_{2i} - x_{2p} + ax_{1p})\}^2}{a^2 + 1}} \quad (2)$$

$$J = \sum_{i=1}^N d_i^2 = \sum_{i=1}^N \frac{\{ax_{1i} - (x_{2i} - x_{2p} + ax_{1p})\}^2}{a^2 + 1} \quad (3)$$

This evaluation function is differentiated, the stationary equation is set up, the variable x is replaced with a difference from the street point X , and they simplify as follows.

$$\sum_{i=1}^N (aX_{1i} - X_{2i})(X_{1i} + aX_{2i}) = 0 \quad (4)$$

$$a^2 \sum_{i=1}^N X_{1i} X_{2i} + a \sum_{i=1}^N (X_{1i}^2 - X_{2i}^2) - \sum_{i=1}^N X_{1i} X_{2i} = 0 \quad (5)$$

$$Aa^2 + Ba - A = 0 \quad (6)$$

where

$$A = \sum_{i=1}^N X_{1i} X_{2i}, B = \sum_{i=1}^N (X_{1i}^2 - X_{2i}^2) \quad (7)$$

$$a = \frac{-B \pm \sqrt{B^2 + 4A^2}}{2A} \quad (8)$$

One of the straight lines with two slopes is minimum extreme, and the other is maxim extreme. The former will become the line by the least squares method, and the latter will become the line by most squares method. In the many kinds of literature on cluster separating method, the ordinary least squares method on the dependent variable of explicit function has been used; they are called to regressive cluster separating method for obtaining a regressive straight line of a cluster like a line [7]. Here, the most squares method is necessary for the reason mentioned before.

Second, let obtain the slope a and the intercept b of a straight line that distance from points of N pieces become extreme minimum. Assuming $(x_p, y_p) = (0, b)$ in the above equations because explicit functions pass to $(0, b)$, the differential on the slope a is same as the above equation and then differentiating on b , the slope a is obtained as follows.

$$J = \sum_{i=1}^N d_i^2 = \sum_{i=1}^N \frac{\{ax_{1i} - (x_{2i} - b)\}^2}{a^2 + 1} \quad (9)$$

$$\frac{\partial J}{\partial b} = \sum_{i=1}^N \frac{2\{ax_{1i} - (x_{2i} - b)\}}{a^2 + 1} = 0 \quad (10)$$

$$aY_1 - Y_2 + b = 0$$

$$a = \frac{b - Y_2}{Y_1} = \frac{-B \pm \sqrt{B^2 + 4A^2}}{2A} \quad (11)$$

where

$$A = \sum_{i=1}^N x_{1i}(x_{2i} - b), \quad (12)$$

$$B = \sum_{i=1}^N \{x_{1i}^2 - (x_{2i} - b)^2\} \quad (13)$$

If these equations are solved on b , the each straight line of both the least squares solution and the most squares solution on the distance of difference as the evaluation function is obtained.

The straight line obtained by the least square solution is a so-called regression line. The straight line obtained by most squares solution becomes a candidate of the cluster separating line if the clusters of the problem are able to separate if the problem is mix one of which classes have twin peaks. If the problem has one class with the single peak with the data of N pieces and it is not able to separate, then the solution is not the candidate of cluster separation but only most squares solution of the points of N pieces.

The third problem is a selection method on the points of N pieces (it is called support vector later) of a separating straight line neighborhood. The separating cluster problem is trivial, and the support vector which distance from a separating straight line is short are easy to select when there is a pause between two classes scattered data in the feature plane.

As for the selection of the total N piece, the assumption that the in halves level from two classes means appropriate is plausible assuming the principle of tug-of-war. Finally, it has to select the support vector placed as becoming a tie on the tug-of-war because the tug is pulled to one with strong power with a lot of numbers.

There are various researches [7] on some cases even when two cluster distributions are not pause and overlapped. In the case of this car model clustering problem, the clusters become indistinguishable if they are mixes because the feature of clusters are similar and have an almost same slope in the data scattering plane. This case is a problem of requesting the minimum area between so-called twin peak histograms; some devices are necessary to obtain the solution for selecting the neighborhood points vector (support vector). For examples, above mentioned tug-of-war tie method or improvement of separating by transformation of variable.

From these contexts, it is plausible as a visible and qualitative image that the followings should be already-known in prior as an assumption to apply the most squares method for the cluster separation problem.

- 1) The treated data have multi-peak, there is a valley between two classes, and the clusters can separate in the valley.
- 2) The necessary data of total N pieces (support vector) can be selected from separating two classes to be able to apply the tug-of-war tie method.

It seems that it may be necessary to compose many sides of the manifold which is the minimum cutting set to cut the linear separation manifold between two adjacent classes by each intersection though it is difficult to image similar for the multi-dimensional multi class problem as well as two dimensions.

IV. NUMERICAL EXPERIMENTS FOR CLUSTERING OF CAR MODELS

A. Separation between Compact Cars and Standard Cars

Based on the results of previous investigations, a clustering method used a separation hyper-plane to divide the data was used. This hyper-plane was obtained from the most-squares solution for the support vector, and the clusters are separated by the least-squares regression lines, which had different slopes and were solution of the lumped data.

The least-squares method was not able to divide compact and standard cars, as shown in the following figure, since the difference in the slopes of the regression lines was not sufficient.

Therefore, the support vector was chosen, and to divide the data were attempted by using the most-squares solution.

The slope of the line was obtained as that of the most-squares solution for the support vector data. The y intercept was determined by performing a random search for the sum of the maximum distance, simultaneously with calculating the regression line. The support vector was selected from the mean value of each characteristic (including all data within nine standard deviations). The results were shown in Fig. 3. The short dashed line indicates the separation line, and the solid line is the regression line.

In the figure, data are plotted in the area-circumference parameter plane. These data are a case of characteristic plane with coming in succession.

Here, the x -axis indicates the area and the y -axis are the circumference in Fig. 3.

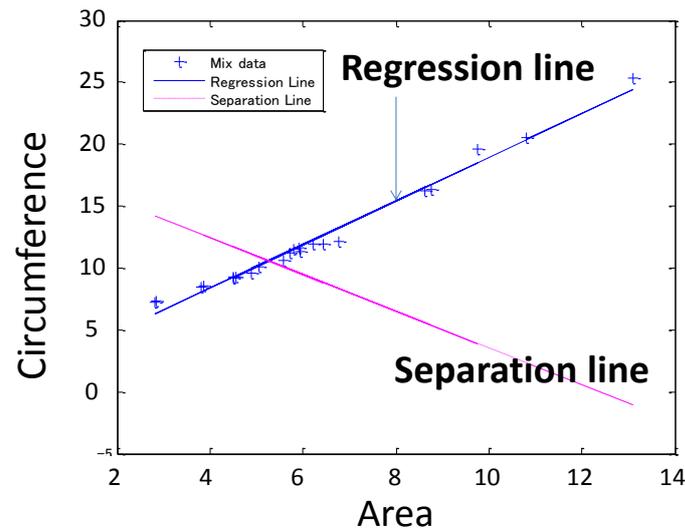


Fig. 3 A separation line and a regression line for the convex hull image dataset for compact and standard cars

V. CONCLUSIONS

The motivation for this research is to make parking more efficient by sending cars to empty spaces, and this is accomplished by using images of the cars to cluster them into size categories and then directing them to the appropriate locations. The sizes were then able to be separated into clusters by using the most-squares method, even though the least-squares method was unable to accomplish this. The most-squares method must be used in more clustering area as same as the least-squares method.

Projection was started onto the hyper-plane and then used the least-squares method, after a literature search and some initial experimentation. A general set of stationary equations was obtained from the evaluation function; these were constructed from the sum of squares of the distances from N points to a linear manifold that passes through a particular point. The orthogonal projection and the point were obtained from the data by uniting the SVM method and the subspace method. The most-squares solution was computed for the support vector nearest to the separation line, and the straight line that separated the clusters of different kinds of cars was obtained by the most-squares method and the pseudoinverse operator. The least-squares and most-squares solutions for this stationary equation are useful for separating the clusters in this dataset. The slope and y -intercept of the least-squares solution (regression line) and the most-squares solution represent a separation of two-dimensional features, as was shown in the example to which this method was applied.

Convex hulls of compact and standard cars were extracted from the image data, and the area and circumferences were plotted. The usefulness of this method was shown by obtaining a regression line and another line that separated the clusters.

The spatial meaning and the way to derive a separation of data into clusters is not clear in the conventional SVM method. When the slopes of two regression lines are similar, as in this example, it is not possible to use them to separate clusters.

It is necessary to use the method in this paper in conjunction with the conventional least-squares method to obtain a generalized cluster separation method.

It is expected that there will be many applications for this method.

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APPENDIX

A. Distance to Orthogonal Shadow

When an arbitrary vector $x \in R^n$ is decomposed into the sum of an orthogonal shadow $\hat{x} \in L$ on a subspace L and a leg $\tilde{x} \in L^\perp$ on an orthogonal complement L^\perp , $x = \hat{x} + \tilde{x}$, then there is a symmetric matrix P such that $\hat{x} = Px$, $\tilde{x} = (I - P)x$. Such a P is called an orthogonal shadow operation on a subspace L , and $(I - P)$ an orthogonal shadow operation are called on an orthogonal complement L^\perp .

Now, if let X be the matrix for which the row vectors are the basis vectors for the subspace L , then the equation $\tilde{x}^T X = 0$ is obtained because \tilde{x} is orthogonal to all the basis vectors of L .

The general solution of this equation can be written using the pseudoinverse matrix X^+ , as follows.

$$\begin{aligned} X^T \tilde{x} &= 0 \\ \tilde{x} &= X^{T+} 0 + (I - X^{T+} X^T) x \\ \text{where } X^{T+} &= (X X^T)^{-1} X \end{aligned} \quad (A1)$$

This is called the Penrose solution [11]. Here, y is a vector with the same dimension as that of \tilde{x} . Using the properties of symmetry and the pseudoinverse, the orthogonal shadow operation P can be decomposed as follows.

$$P = X X^+ \quad (A2)$$

The distance d from an arbitrary vector to the orthogonal projection on the subspace L is defined from Eq. (A1) and Eq. (A2) as follows.

$$d = \|\tilde{x}\| = \|(I - P)x\| \quad (A3)$$

The distance d_p from x to a linear manifold $L + x_p$ through x_p can be written as follows if x_p is the parallel moving amount from the origin.

$$d_p = \|(I - P)(x - x_p)\| \quad (A4)$$

Let an evaluation function be a sum of distances from N points of $(x_i; i=1, \dots, N)$ to a linear manifold $L + x_p$ through x_p

$$J_p = \sum_{i=1}^N d_{i,p}^2 = \sum_{i=1}^N \|(I-P)(x_i - x_p)\|^2 = \sum_{i=1}^N (x_i - x_p)^T (I-P)^2 (x_i - x_p) \quad (A5)$$

Then, the variation equations of the above evaluation (A5) are obtained as functions of the independent parameters P and x_p , and these can be used to find the stationary solutions.

$$\begin{aligned} \frac{\partial J_p}{\partial P} &= 2 \sum_{i=1}^N (x_i - x_p)^T P (x_i - x_p) - 2 \sum_{i=1}^N (x_i - x_p)^T (x_i - x_p) = 0 \\ \frac{\partial J_p}{\partial x_p^T} &= 2 \sum_{i=1}^N (P-I)^2 (x_i - x_p) = 0 \end{aligned} \quad (A6)$$

These can then be rearranged and used the properties of matrices to obtain the following set of stationary equations.

$$\begin{aligned} \text{tr}\{X^T (P-I)X\} &= 0 \\ (P-I)^2 Xet &= 0 \\ \text{where } X &= [x_1 - x_p, \dots, x_i - x_p, \dots, x_N - x_p], et = [1, 1, \dots, 1]^T \end{aligned} \quad (A7)$$

When these equations are solved for P and x_p , the linear manifolds $L + x_p$ as stationary solutions are obtained; these may contain a least-squares solution and a most-squares solution.

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