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Theoretical Comparison between Alumina and Fiberglass in Saving Heat Portability

Zina A.B. Al -Shadidi

Department of physics, Faculty of Education, Aden University, Aden, Yemen

zabaqer@yahoo.com

***Abstract*-Energy is always exist in surroundings but it cannot be saved. Many sources of renewable energy were discovered, the problem is how to utilize it. One of the most usable energy sources is the solar energy. And one of the most difficulties is the absence of this source in the night, so it must be stored in the form of another kind of energy. This research focus on using the insulation materials in building hot water storage tanks supplied with solar power to save hot water. Theoretical comparison in terms of energy storage portability between the most two common materials are used in building these tanks.**

Matlab and Heat transfer equation used to compute properties for heat insulation and energy loss from the insulating material used to build the tank. Finite difference method used to solve heat equation for two kinds of insulating material (Alumina and fiberglass).

Keywords- Theoretical Study; Heat Equation; Renewable Energy; Matlab

I. INTRODUCTION

Heat transfer between liquid and solid was the subject of many theoretical and practical researches [1-4]. The process of diffusion can be pictured as a drop of dye spreading in a glass of water. The dye consists of a large number of individual particles, each of which repeatedly bounce off of the surrounding water molecules, following an essentially random path. There are so many dye particles that their individual random motions form an essentially deterministic overall pattern as the dye spreads evenly in all directions. In a similar way, the heat energy spreading through random interactions of nearby particles.

In a three-dimensional medium, the heat equation is

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{1}$$

Here T represents the temperature (which is a function of t, x, y, and z), at time t and position (x, y, z) in the medium. The constant k depends on the materials involved where an isotropic material has single value for k in all directions, the anisotropic material has different values according to the direction of motion within the material, which is called *thermal conductivity*. In the case of heat flow in one direction the partial differential equation becomes:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \tag{2}$$

While the time increment of enthalpy within the volume is due to heat input through the frontier plus the energy dissipation in the interior, is represented by the following equation [2].

$$\int_V \rho c\frac{\partial T}{\partial t} dV = -\int_A \bar{q}.\bar{n} dA + \int_V \phi dV \tag{3}$$

Considering that the Heat flux density $\bar{q} = -K\nabla T$, constant material density ρ, constant thermal capacity c, and constant conductivity k. The heat equation will be given by:

$$\rho c\frac{\partial T}{\partial t} = k\nabla^2 T + \phi \tag{4}$$

Where ϕ is the energy release rate per unit volume [2].

In searching for a new composite material that the loss of energy through it is less, the idea of generating a new composite from two commonly used materials to reduce energy loss was suitable. Although directly dealing with the idea cannot be found. Many researches approximates the theoretical picture of this idea. In dealing with thermal characteristics of materials Okamura and Allnatt [5] developed the matrix method for calculations of the isothermal transport coefficients in dilute binary alloys. Then Allnatt and Lidiard [6], concerned with the fundamentals of the theory of the transport of atoms through crystalline solids, such as may result from the existence of gradients of chemical and isotopic composition, electrical potential, stress, temperature, etc.. While Majumdar [7], shows that the heat transport by lattice vibrations in the microscale regime can be analyzed as radiative transport problem. Based on Boltzman transport theory, Muniz [3] analyzed the performance of different

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regularization techniques for an inverse heat condition problem.

Lattimer, Goodrich, Coodack, and Cain [8], discussed an approach for validating and determined properties for the composite materials and models high temperature response.

While Jadhav and Ahirrao [9], studied the heat conduction problem on semi- infinite solid cylinder with heat source. They discussed thermoelastic problem of semi-infinite solid cylinder and obtained the temperature distribution and stresses to analyze particular case with mathematical model, calculations were carried out by using Hankel and Fourier transform technique. Samira Shiri [1], derived a theoretical model for heat convection in a cylindrical vessel.

Everet [10], studied the natural convection from a heat general cylinder for dry cask storage applications, he focused on evaluating the fundamental physics associated with the buoyancy-induced flow around a confined heated horizontal cylinder. Numerical models were derived and analyzed using the commercial FLUENT computational fluid dynamics software.

In this work, 1D heat equation (one dimension heat equation) is used to analyze thermal properties of some insulating solid materials and thermal diffusion equation for the most commonly used insulating material (Alumina & Fiberglass), paving the way to calculate the amount of lost energy in a water storage tanks which were used in manufacturing the solar heater systems in each kind (Alumina & fiberglass) tank.

II. SOLVING THERMAL CONDUCTION EQUATION FOR ANISOTROPIC MATERIALS

The heat transfer equation represents the energy balance for heat transfer through the material. In the case of steady state heat generation $\frac{\partial T}{\partial t} = 0$, therefore

$$k\frac{d^2T}{dx^2} + \phi = 0 \tag{5}$$

Let $\phi = \dot{q}$, where $\dot{q}$ represents the steady state heat generation in X- direction [11, 12].

$$k\frac{d^2T}{dx^2} + \dot{q} = 0 \tag{6}$$

The solution for the above equation will be:

$$T_x = \frac{-\dot{q}x^2}{2k_1} + Ax + B \tag{7}$$

Which represents the temperature flow in x-direction.

Likewise for y-direction:

$$T_y = \frac{-\dot{q}y^2}{2k_1} + Cy + D \tag{8}$$

Where T_y is the temperature flow in y-direction.

Were A, B, C, and D in equation (7) and (8) are constants.

On the other hand the spatial temperature profile given by (9),

$$T = T(x, t_0) \tag{9}$$

For fixed $t = t_0$,

$$T(x,t) = \frac{4T_0}{\pi} e^{-\pi^2 t_0} \sin(\pi x) \qquad t \geq {}^{1}/_{\pi^2} \tag{10}$$

Where $T(x, t)$ is the temperature at time t a distance x along the heat path.

III. A FINITE-DIFFERENCE SOLUTION

To solve this partial differential equation *initial conditions* of the form $u(x, 0) = f(x)$ are needed, where $f(x)$ gives the temperature distribution in the material at time 0, and boundary conditions at the endpoints of the material, at x = a and $x = b$, the boundary conditions where $T(a,t) = T_a$ and $T(b,t) = T_b$, which correspond to the temperature being held steady at values T_a and T_b at the two end points. *Finite difference* method was the chosen method to assist in finding the heat distribution way in the solid material.

To find a general solution for equation (3) in anisotropic solid materials a boundary condition must be set

$$\begin{aligned} &T_t = T_{xx} + T_{yy} \qquad 0 < x < 1 \qquad 0 < y < Y \qquad t > 0 \\ &T(x, y, t = 0) = T_o(x, y) \qquad 0 < x < 1 \qquad 0 < y < Y \end{aligned} \tag{11}$$

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$$T(0,y,t) = g_0(y,t)\ ,\quad T(1,y,t) = g_1(y,t)\ ,\qquad 0 < y \le Y \qquad t \ge 0$$
$$T(x,0,t) = g_2(y,t)\ ,\quad T(x,Y,t) = g_0(x,t)\ ,\qquad 0 < x \le 1 \qquad t \ge 0$$

Domain with a two-dimensional grid will be suitable to show the heat distribution through solid material. The step size in the x and y directions assumes to be equal to h. The time discretized with a step size f. Then the dimensions will be (x=mh, y=lh, t=nf),

$$0 \le m \le M = \frac{1}{h} \qquad 0 \le l \le L = \le \frac{Y}{h} \qquad 0 \le n \le N = \frac{t_{max}}{f} \tag{12}$$

The solution on the above grid is:

$$T_{ml}^{n} = u(mh, lh, nf), \qquad 0 \le \le m \le M\ , \quad 0 \le l \le L,\ \ 0 \le n \le N \tag{13}$$

As long as this grid is rectangular, the array of values T_{ml}^{n} for each given n can be represented as $(M+1)\times(L+1)$.

From the last equations group

$$T_t = \frac{T_{m,l}^{n+1} - T_{m,l}^{n}}{f} = \frac{\delta_t T_{m,l}^{n}}{f}$$
$$T_{xx} = \frac{T_{m+1,l}^{n} - 2T_{m,l}^{n} + T_{m-1,l}^{n}}{h^2} = \frac{\delta_x^2 T_{m,l}^{n}}{h^2} \tag{14}$$
$$T_{yy} = \frac{T_{m,l+1}^{n} - 2T_{m,l}^{n} + T_{m,l-1}^{n}}{h^2} = \frac{\delta_y^2 T_{m,l}^{n}}{h^2}$$

Assuming that $T_t = T_{xx} + T_{yy}$

$$T_m^{n+1} = T_{m,l}^{n} + r\left(\delta_x^2 T_{m,l}^{n} + \delta_y^2 T_{m,l}^{n}\right) = (1 + r\delta_n^2 + r\delta_y^2)T_{m,l}^{n} \tag{15}$$

The above equation used to draw the temperature as a function of the x, and y dimensions for the Alumina specimen as shown in Fig. 1 and for fiberglass as shown in Fig. 2.

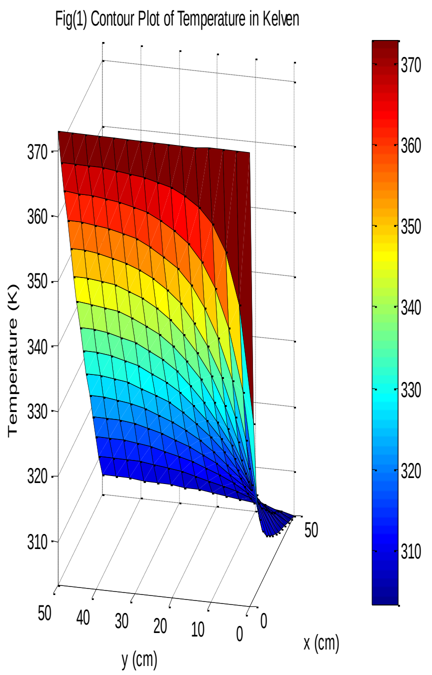

Fig. 1 The temperature distribution in two dimensions for Alumina using Finite difference method

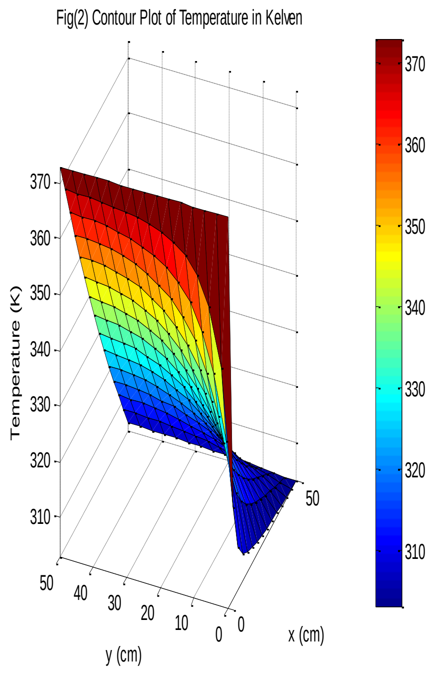

Fig. 2 The temperature distribution in two dimensions for Fiberglass using Finite difference method

Figs. 1 and 2 show the variation of Temperature (ranged from room temperature to 370K) as a function of x and y coordinates. The numerical approach has been used assuming a certain range of x and y coordinates and its subdivisions, initial and final temperature, and material enthalpy were required to calculate the temperature distribution shape through the material, these informations were obtained using wrobel, Pawlal, and Gmuzia [13] and Frederick [14]. In these figures the colors show the temperature distribution through material body, red color represents the highest temperature down to the blue one which represents the lowest temperature. On the x-y domain the colored curve can be obviously seen which represents the heat

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diffusion path. These shapes refer to the anisotropic heat properties for the materials used, which means that it has different heat conductivity along each direction.

In general, the insulating material used in building storage systems is porous where the liquid penetrates through solid material and thus transmission and loss of heat energy occurs through the solid material into the outer perimeter. Therefore, volumetric heat loss will be regarded. And for all the above reasons the energy heat loss was calculated for each axis individually, then the volumetric heat loss was calculated.

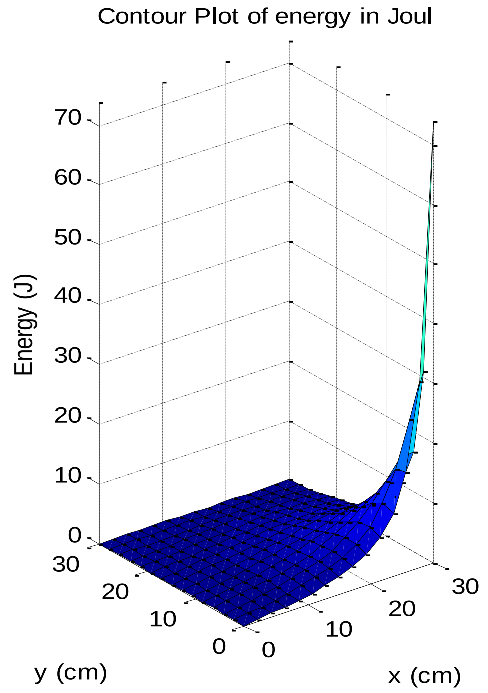

Fig. 3 Heat distribution in 2- dimentions

Fig. 3 shows the heat energy distribution according to equation (10). It shows that the heat energy will have a special shape so that the energy through the specimen have a steady increasing state until reaching the highest energy zone, in other words, the energy spread through the material firmly within a specific range of temperatures. Therefore, keeping work within this range of heat increments will be suitable to avoid other thermal effects resulting from irregular thermal distribution of heat within the material because of the anisotropic characteristics for the solid material.

IV. ENERGY BALANCE IN SOLID THERMAL STORAGE MATERIAL

According to energy conservation law, the energy must obey the following equation

$$\dot{E}_g + \dot{E}_{in} - \dot{E}_{out} = \dot{E} \tag{16}$$

Where:

$$\dot{\mathrm{E}} = \text{rate of energy change within element}$$

$$\dot{\mathrm{E}}_{\mathrm{in}} = \text{rate of energy added to element}$$

$$\dot{\mathrm{E}}_{\mathrm{g}} = \text{energy generation rate per unit volume}$$

$$\dot{\mathrm{E}}_{\mathrm{out}} = \text{rate of energy removed from the element}$$

To calculate the heat loss from the system to the ambient regarding the heat transfer area between solid & fluid per unit length of tank denoted as A_{loss} and the overall heat transfer coefficient U_{loss}.

The total loss in energy can be expressed by the following equation [1]:

$$(\rho_{sf} C_{p,s} \frac{\partial T_s}{\partial t} = k_s \frac{\partial^2 T_s}{\partial x^2} - \frac{\epsilon}{1-\epsilon} h_{s/V}\left(T_f - T_s\right) - U_{loss/V}\left(T_f - T_\infty\right)) \tag{17}$$

Where $U_{loss/V}$ describes the radial heat loss by unit volume due to conduction through the insulation, this parameter depends on the ambient temperature, insulation thickness, and material properties of the insulation. ρ_{sf} Describes the solid material density relative to the liquid density, $C_{p,s}$ represents the heat capacity of the solid material at constant pressure, k_s is the

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conductivity for the solid material. For the temperature symbols (T) the subscript s denotes the solid, the subscript f denotes the fluid, and ∞ denotes the ambient temperature, $h_{s/V}$ represents the volumetric heat transfer coefficient (it has been calculated according to Nellis and Klein method) [15]. Finally ϵ is the void fraction which is the fluid volume relative to the total volume of the tank:

$$\epsilon = \frac{V_f}{V_{total}} \quad (18)$$

Equation (17) contains two terms of heat loss, which is the last two terms. This equation means that the energy change due to the temperature gradient with respect to time equals the energy due to divergence of temperature plus the volumetric heat loss. This loss is a combination of two terms in this equation, and as long as the heat flux at the boundary is proportional to the temperature difference with the environmental, the boundary condition will be of (Robin) type.

$$\frac{\partial T_s}{\partial t} = \frac{k_s}{\rho_{sf} C_{p,s}} \frac{\partial^2 T_s}{\partial x^2} - \frac{h}{\rho C_p} \frac{\epsilon}{1-\epsilon} \left(T_f - T_s\right) - \frac{U_{loss}}{\rho C_p} \left(T_f - T_\infty\right) \quad (19)$$

At equilibrium temperature of the solid T will be equal to (T_∞). And ($T_f - T_\infty$) will be represented as (ΔT):

$$\frac{\partial T_s}{\partial t} = \frac{k_s}{\rho_{sf} C_{p,s}} \frac{\partial^2 T_s}{\partial x^2} - \frac{h}{\rho C_p} \frac{\epsilon}{1-\epsilon} (\Delta T) - \frac{U_{loss}}{\rho C_p} (\Delta T) \quad (20)$$

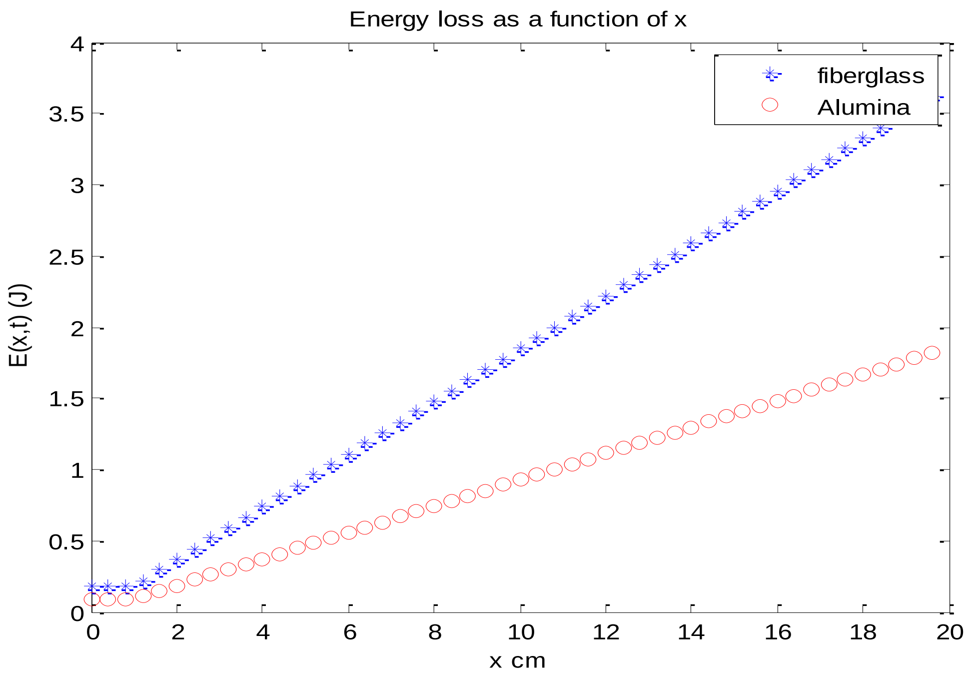

Fig. 4 The heat distribution throw the solid material

Fig. 4 plotted by Matlab according to the last equation depending on the Robin type boundary conditions, and assuming only the last two terms are the loss terms. Fig. 4 shows that Alumina has lower temperature gradient than fiberglass, higher thermal resistivity. From this figure it can be realized that the alumina line is located under fiberglass curve and it has a lower slope or lower temperature gradient because of the lower heat transfer coefficient (high thermal insulation). Notice that the thermal resistance depends on the nature of the material (thermal conductivity k, and the geometry of the body), Higher specific heat means that heat arriving at a layer is stored rather than transmitted to the next layer which happen in Alumina more than in fiberglass and hence higher thermal conductivity allows quicker propagation of heat into the body. The higher diffusivity owned by Alumina, but still there is some energy loss. The aim here is to reduce this energy loss to the least. For this purpose a combination of both Alumina and Fiberglass was proposed in the construction of reservoirs.

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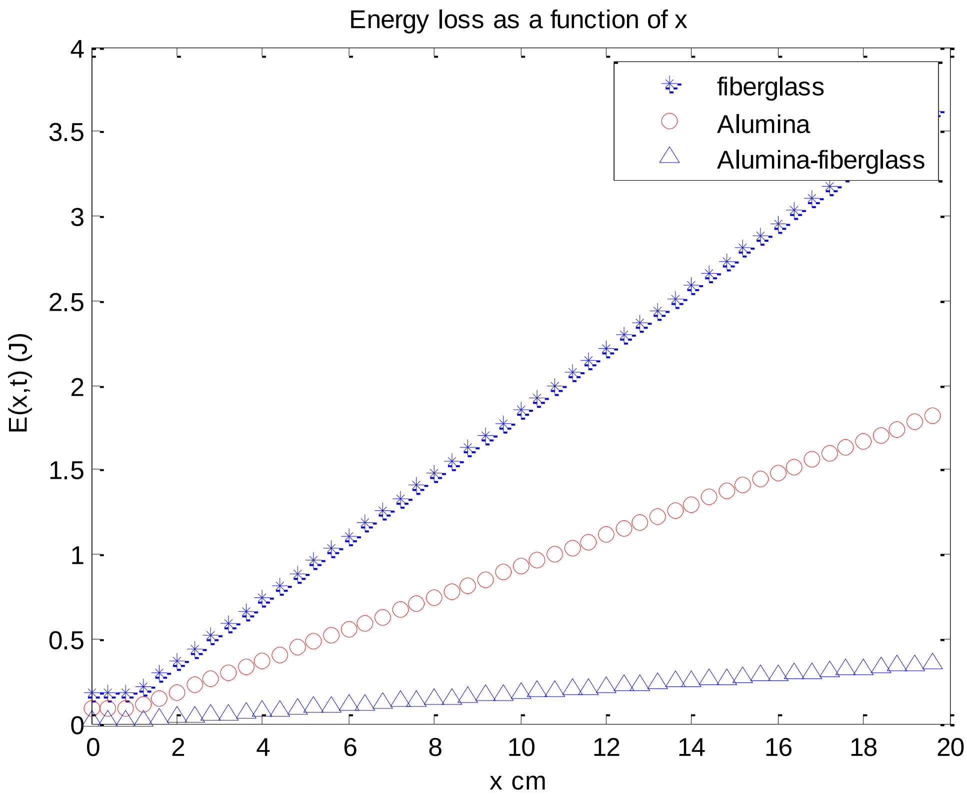

Fig. 5 The temperature gradient comparison between Alumina, fiberglass, and the combination of both of them

In Fig. 5, the combination of Alumina and fiberglass gives perfect heat insulation, and lower energy loss. For a cylindrical tank

$$\mathrm{A_s} = 2\pi \mathrm{r_o L}, V = \pi r_o^2 L \tag{21}$$

Under steady conditions, the heat generated within the inner cylinder must equal to the heat conducted through the outer surface of the inner cylinder.

$$kA_r \frac{dT}{dr} = \dot{q} V_r \tag{22}$$

$$k(2\pi r L)\frac{dT}{dr} = \dot{q}(\pi r^2 L) \tag{23}$$

$$dT = \frac{\dot{q}}{2k} r dr \tag{24}$$

Integrating from $r = 0$, where $T(0) = T_0$.

To $\mathrm{r} = \mathrm{r_o}$, where $\mathrm{T(r_o)} = \mathrm{T_s}$

$$\Delta T_{max} = T_o - T_s = \frac{\dot{q} r^2}{4K} \tag{25}$$

The heat loss $(\dot{q})$ is represented by the last two terms from the right hand side of equation (16), then:

$$\dot{q} = (\frac{\epsilon}{1-\epsilon} h + U)\Delta T \tag{26}$$

From equation (23)

$$\frac{4k}{r_o^2} = \left(\frac{\epsilon}{1-\epsilon} h + U\right) \tag{27}$$

$$U = \frac{4k}{r_o^2} - \frac{\epsilon}{1-\epsilon} h \tag{28}$$

The last equation shows the dependence of the lost energy on the properties of the solid material which is heat conductivity, enthalpy, density, In addition to the geometry of the tank represented by the void fraction (∈) and the internal diameter for the cylindrical tank (r_o) in the last equation.

According to equation (28), the energy loss equation will be:

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$$U = \frac{4k}{r_o^2} - \frac{{v_f}/{v_T}}{1-{v_f}/{v_T}} h \quad (29)$$

$$U = \frac{4k}{r_o^2} - \frac{v_f}{v_T - v_f} h \quad (30)$$

$$U = \frac{4k}{r_o^2} - \frac{\pi r_o^2 L}{\pi r^2 L - \pi L r_o^2} h \quad (31)$$

$$U = \frac{4k}{r_o^2} - \frac{r_o^2}{\pi (r^2 - r_o^2)} h \quad (32)$$

According to the above equation, the thermal energy loss calculated for the Alumina equals to (0.886392 J/m^3 .s) while its equal to (0.961592 J/m^3 .s) for fiber glass. The composite consist of these two kinds of insulating materials reduces the heat loss to (0.160029 J/m^3 .s), for the case of the new composite, the effective thermal conductivity were calculated according to the Lewis-Nielsen model [16]. The effective thermal conductivity calculated assuming that the (length to diameter) for the fiberglass rods is large, and that the glass fibers are oriented in three dimensions.

V. CONCLUSIONS

This study carried out a numerical study (FDM) of thermal behavior for thermal storage tanks. Two kinds of materials were considered in this study to realize that the energy spreads through the material firmly within a specific range of temperatures, the work was done within a specific range of temperature to avoid the anisotropic characteristics effects for these materials. Thermal diffusion equation is used to calculate thermal energy loss in a water storage tanks which were used in manufacturing the solar heater systems in each kind (Alumina & fiberglass) tank.

For the problem of thermal energy loss, the purpose was to calculate the energy loss using different kinds of material, the results were as follows:

1- For a reservoir building, the Alumina is preferred from fiberglass because Alumina has lower temperature gradient and higher thermal resistance leading to a lower energy loss.

2- In order to reduce the energy loss, a two layer composite of both Alumina and fiberglass was proposed in the construction of reservoirs.

3- For a cylindrical tank, the loss in energy will be as presented in equation (32). The energy loss calculation was (0.886392 J/m^3 .s) for Alumina, while its equal to (0.961592 J/m^3 .s) for fiber glass.

4- Building tanks from combination of the two insulating materials (Alumina & Fiber glass) will be more energy saving, where the composite consist of these two kinds of insulating materials reduces the heat loss to (0.160029 J/m^3 .s).

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