# Financing Recovery and Reconstruction Project after Natural Disaster: Risk Sharing by Mutual Insurance

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***Abstract*-On March 11 2011, Japan was hit by "the Great East Japan Earthquake", the strongest earthquake to strike the nation in recorded history. Recovery and reconstruction plans necessitate immense expense and time, entailing numerous arguments about the sources of revenues. As described in this paper, mutual insurance among local governments is considered as a decentralized risk sharing system. Comparative statics analysis of a simple two-regional model is conducted, demonstrating the validity of this proposal. Results show that the foundation of a mutual insurance system increases economic welfare. Therefore, to ensure enforcement of the recovery and reconstruction project, it is important to found a mutual insurance system among local governments in advance, and to build a decentralized risk sharing system.**

***Keywords*-Natural Disaster, Risk Sharing, Financing**

## I. INTRODUCTION

On March 11 2011, Japan was hit by "the Great East Japan Earthquake", which was the strongest earthquake to strike that nation in recorded history<1>. Recovery and reconstruction plans have necessitated immense expenditures and time, entailing numerous arguments about the sources of revenues to fund those efforts. This earthquake disaster shares important characteristics with natural disasters occurring throughout the world. Assuming that damage should be compensated impartially by all residents of a nation, then some system serving that function, if it exists, would strengthen the sense of security from the effects of natural disasters. The design of a security system related to natural disasters is an extremely important issue in Japan particularly. Specifically, a proposal to save funds under the title "insurance for recovery and reconstruction", for emergency needs can be considered (e.g. Sakamoto and Hayashiyama [1]). Such a system not only obviates arguments over who pays how much for natural disasters; it also avoids extension of damage and hastens recovery.

As described herein, mutual insurance among local governments is analyzed as a decentralized interregional risk sharing system for national disaster. Persson and Tabellini [2], [3] have discussed such a system by constructing the model of a federation with two region and an immobile population. Mansoorian [4] has extended the model of Persson and Tabellini [2], [3] to imperfect mobile population version on basis of Myers [5], Mansoorian and Myers [6], [7]. These papers are characterized by using a principal agent model between federal authorities and regional ones. Furthermore, Persson and Tabellini focus on political decision making process with various constitutional arrangements, and Mansoorian [4] has drawn the conditions of decentralization of authority risk sharing in consideration of population mobility on the equilibrium. On the other hand, Ihori [8] specified interregional risk sharing by two-person or two-region economy model with pure public goods and a risk of income decreasing for each person, and proved that perfect welfare equalization between individuals is realized by using mutual insurance and public goods provision simultaneously irrespective of differences in income between individuals. This result is based on neutrality theorem in the private provision of public goods by Shibata [9], Warr [10] and Ihori [11].

The purpose of this paper is to analyze the interregional risk sharing system in a regional economy in which income of individual decreases and environmental stock suffers damage by national disaster. The paper is organized as follows. The basic model of mutual insurance is discussed section II. The two-regional model of recovery and reconstruction project with spill-over benefits is discussed in section III and IV. Some concluding remarks are made in section V.

## II. MUTUAL INSURANCE MODEL OF LOCATION SPECIFIC RISK

### A. Basic Model

The model is based on the insurance model described by Ihori [8], and the mutual insurance model for a location specific risk is built. For simplicity, we consider a two-regional economy and two states $A$ and $B$. Assuming the

<1> Around 14:46 on March 11, 2011, a massive earthquake with magnitude of 9.0 struck the Sanriku Coast of Japan. The ensuing tsunami swept across many cities and villages along the Pacific coast of the Tohoku district, causing tremendous human and structural damage. In Miyako City, Iwate Prefecture, tsunami waves of over 8.5 m high were observed. Tokyo also recorded tremors with a seismic intensity of 5-strong, but damage there was modest. On April 7, an earthquake of magnitude 7.1 occurred, with its epicenter off the coast of Miyagi Prefecture. On April 11, a 7.0-magnitude earthquake struck, with its epicenter in the Hamadori area, Fukushima Prefecture. The March 11 earthquake and tsunami engendered emergency situations including failure of reactor-cooling systems in Tokyo Electric Power Company (TEPCO) nuclear power stations in Fukushima Prefecture. The earthquake and tsunami devastated Tohoku and other regions. Damage was inflicted in the Kanto district, as well. The number of deaths was 15,787, the number of injured was 5,932, and the number of missing was 4,059 (September 14, National Police Agency). Although hundreds of thousands of people were housed in shelters in the immediate aftermath, the number of those remaining in shelters by September 9, 2011 was 68,596.

additively separable utility function, then region $i's$ expected utility $W_i(.)$ is given as

$$W_i = (1-\alpha)\ln c_i^A + \alpha \ln c_i^B, \forall i \tag{1}$$

where $c_i^t\,(t = A, B)$ represents private consumption in region $i\,(i=1,2)$; $c^t{}_i$ is subject to uncertainty. In state $A$, which occurs with probability of $1-\alpha$, region $i$ enjoys $c^A{}_i$. In state $B$, which occurs with probability of $\alpha$, regio $i$ cannot enjoy $c^A{}_i$ but can enjoy $c^B{}_i$. $\alpha$ represents the probability of an economically disruptive emergency resulting from a bad state, such as being unemployed.

Region $i's$ budget constraint in each state is given as

$$c_i^A = Y_i - ps_i, \forall i \tag{2}$$

$$c_i^B = (1-\pi_i)Y_i + (1-\pi_i)s_i = c_i^A - \pi_i Y_i + s_i, \forall i \tag{3}$$

where $Y_i$ is the exogenously given income of region $i$. Inequality of $Y_i$ arises from differences in ability. $\pi_i\,(>0)$ denotes the net quantity of resources lost to each yen of private income during contingency unemployment or actual occurrence of a natural disaster: resources are lost from diversion to job search efforts or being cut off because of disruptions in production activities. Consequently, $\pi_i$ is called the penalty ratio. Inequality of $\pi_i$, which is attributed to differences in luck, results in ex-post income differences. $s$ is the return from insurance in the event of an emergency. $p$ denotes the price of insurance, or the premium per yen of insurance coverage. Finally, $ps$ signifies a regional premium paid to the government (an insurance company in the case of private insurance) during state $A$.

Uncertainty is assumed to be restricted to income. The insurance premium paid from region to a supply agent (or the government) in state $B$ is risk free and is not subject to the penalty. Each region is assumed to determine their insurance demand treating exogenous parameters $\alpha$, $\pi$ and insurance price $p$ as given.

The government budget constraint is given as presented below.

$$\frac{\alpha(1-p)}{c_i^A} = \frac{p(1-\alpha)}{c_i^B}, \forall i \tag{4}$$

$$(1-p)c_i^A + pc_i^B = (1-p\pi_i)Y_i, \forall i \tag{5}$$

$$p(s_1+s_2) = \alpha(s_1+s_2) \tag{6}$$

The left-hand side of (6) shows insurance revenue. The right-hand side is the insurance payment in the event of state $B$. Also, $\rho$ is determined to satisfy the balanced budget. In addition, the premium determined by $p=\alpha$ is actuarially called a "Fair Premium". This corresponds to the zero profit condition for the insurance sector.

From (2) and (3) the budget constraint might be rewritten as

$$pc_i^B + \rho c_i^A = (1-p\pi_i)Y_i \tag{7}$$

where $\rho \equiv 1-p$. Also, (3) means that the effective rate of return on insurance premium $ps$ is $(1-p)/p$. The price of insurance $p$ also means the price of consumption in state $B$, whereas $1-p$ means the price of consumption is state $A$. The effective income on the left-hand side of (7) evaluates emergency costs $\pi_i Y_i$ using $p$, the price of consumption in state $B$.

This paper defines expenditure function $E_i(.)$ shown in (8). Furthermore, (9) is materialized from (8). Therefore, a mutual insurance model is expressed as (9) and (10). Here, (9) means that the expected utility of the region $i$ is given as $p$ and $(1-p\pi_i)Y_i$, and that (10) is an equilibrium condition of insurance market. It gives the insurance price $p$. In addition, this is a closed model that does not incorporate demographic shifts or externalities. Therefore, the equilibrium point serves as the Pareto optimum. An important fact is that $s_1$ and $s_2$ in (11) show reverse sign conditions. In the case of $s_1>0\,(s_2=-s_1<0)$, region *1* is an insurance demander and region *2* becomes an insurance supplier. If $Y_1=Y_2$ and $\pi_1=\pi_2$ materialize, then because both regions are equivalent, $W_1=W_2$ will be true and $s_1=s_2$ will be drawn. Moreover, presuming that $Y_1=Y_2$ and $\pi_1>\pi_2$, then insurance demand will occur in both regions.

$$\min_{c_i^A, c_i^B}. E_i(.) \equiv (1-p)c_i^A + pc_i^B, \forall i \quad \text{s.t.}\, W_i \geq \overline{W}_i \tag{8}$$

$$E_i(W_i, \alpha, 1-p, p) = (1-p\pi_i)Y_i, \forall i \tag{9}$$

$$E_i(W_i, \alpha, p) = (1-p\pi_i)Y_i, \forall i \tag{10}$$

$$s_1(W_1, p, \pi_1 Y_1) + s_2(W_2, p, \pi_2 Y_2) = 0 \tag{11}$$

By a variant of Shepherd's Lemma we know the following.

$$c_i^A = \frac{1-\alpha}{1-p}(1-p\pi_i)Y_i, \forall i \tag{12}$$

$$c_i^B = \frac{\alpha}{p}(1-p\pi_i)Y_i, \forall i \tag{13}$$

$$s_i = \frac{1-p-(1-\alpha)(1-p\pi_i)}{p(1-p)}Y_i, \forall i \tag{14}$$

$$\frac{p(1-\pi_2)}{1-\pi_2} > \alpha > \frac{p(1-\pi_1)}{1-\pi_1} \tag{15}$$

If (15) is materialized in (14), then it will be set to $s_1>0>s_2$, with a merit in both regions. Moreover, because $p<1$, (15) means $p>\alpha$. Furthermore, the equilibrium level of (1) and (14) to $p$ is expressed as (16). Therefore, (12) and (13)-(17) can be drawn.

$$p = \frac{2\alpha}{2-(1-\alpha)(\pi_1+\pi_2)} \tag{16}$$

$$c_i^A - c_i^B = \frac{(1-p\pi_i)(\alpha-p)}{p(1-p)}Y, \forall i \tag{17}$$

*B. Findings from Comparative Statistics*

We can perform comparative statics by which (18) is obtained by differentiating (9) and (11).

$$\begin{bmatrix} \partial E_1(.)/\partial W_1 & 0 & s_1 \\ 0 & \partial E_2(.)/\partial W_2 & s_2 \\ 0 & 0 & \partial s_1/\partial p + \partial s_2/\partial p \end{bmatrix} \begin{bmatrix} dW_1 \\ dW_2 \\ dp \end{bmatrix} = \begin{bmatrix} -pY_1 \\ 0 \\ -Y_1 \end{bmatrix} \tag{18}$$

As a result, (19)-(21) is obtainable. First, (21) shows that the rise of $\pi_i$ which is a loss ratio of region $1$, raises the insurable price. Because $s_2(.)<0$, (20) is set to a positive value, indicating that the economic welfare of region $2$, an insurance supplying district region, has a price effect. However, because (20) is a negative value, the rise of $\pi_i$, the damage rate of incidence of region $1$, brings about income effect from the decreasing expected income $W_2$ of region $2$ increased. Furthermore, regarding region $1$, which is an insurance demand region, the rise of $p$ brings about a negative price effect and strengthens the income effect by the rise of a loss ratio. Therefore, the rise of $\pi_1$, which is a loss ratio of region $1$, decreases the economic welfare of region $1$ and increases the economic welfare of region $2$. This problem decisively raises damage costs occurring in region.

$$\frac{dW_1}{d\pi_1} = -\frac{Y_1 \cdot \partial E_2(.)/\partial W \cdot \left(p(\partial s_1 + \partial s_2)/\partial p - s_1\right)}{\partial E_1(.)/\partial W \cdot \partial E_2(.)/\partial W \cdot (\partial s_1 + \partial s_2)/\partial p} < 0 \tag{19}$$

$$\frac{dW_2}{d\pi_2} = -\frac{Y_1 \cdot s_2 \cdot \partial E_1(.)/\partial W}{\partial E_1(.)/\partial W \cdot \partial E_2(.)/\partial W \cdot (\partial s_1 + \partial s_2)/\partial p} > 0 \tag{20}$$

$$\frac{dp}{d\pi_1} = -\frac{Y_1 \cdot \partial E_1(.)/\partial W \cdot \partial E_2(.)/\partial W}{\partial E_1(.)/\partial W \cdot \partial E_2(.)/\partial W \cdot (\partial s_1 + \partial s_2)/\partial p} > 0 \tag{21}$$

Optimal behavior by regions simplifies the derivation. Especially, (22) means that the level of consumption is completely equilibrated in state $A, B$ in each region by introducing mutual insurance. Furthermore, (23) means that this level of consumption is equal to the individual expected income in each region. Each region should pay the state $B$, i.e. premium money that completely offsets the amount of a loss in case $(s_i < \pi_i Y_i, \forall i)$ of natural, which means not being optimal when requiring case $(s_i > \pi_i Y_i, \forall i)$, where partial compensation of the amount of a loss is necessary and which is superfluous compensation. Furthermore, if the grade of each income level and risk change, then the economic welfare and insurance demand that were evaluated by the expected utility of each region will change. Probably these changes mean that the demand for risk sharing grows. They will serve as a strong basis for promoting an income redistribution policy according to an insurance market.

$$c_i^A = c_i^B = (1-\alpha\pi_i)Y_i, \forall i \tag{22}$$

$$s_i = \pi_i Y_i, \forall i \tag{23}$$

## III. FINANCING BY VOLUNTARILY TAX BURDEN

*A. Model*

To assume the recovery and reconstruction project with spill-over benefits, (1) is transformed and (24) is considered. The recovery and reconstruction project is defined as public goods. It is assumed that an expense burden is based on the voluntary provision of public goods. Assuming the additively separable utility function $V(.)$(=utility portion obtained from private consumption) and $U(.)$ (=utility portion obtained from environmental stock), region $i's$ expected utility $W_i(.)$ is given as

$$W_i = (1-\alpha)\left(V(c_i^A) + U(\gamma_i^A Q^A)\right) + \alpha\left(V(c_i^B) + U(\gamma_i^B Q^B)\right), \forall i \tag{24}$$

where $Q^t$ represents the environmental stock, $\gamma_i^t$ denotes the rate of spill-over to region $i$ of the environmental stock, and where $\gamma_1^t = 1$ and $0 \le \gamma_2^t \le 1$ are assumed. Furthermore, $Q^t$ are forest resources of region $1$. The spill-over benefit can include water purification by forests in region $2$ as an example. Moreover, environmental stocks can be interpreted as environment services or public goods. The concept of the model used for this study is presented in Fig. 1.

Although the environmental stock suffers damage in state $B$ of disaster, the recovery and reconstruction project is assumed to restore it (25). In addition, $\eta(0 \le \eta \le 1)$ is the loss ratio and $G$ denotes public goods, which show the environmental stock as restored by the recovery and reconstruction project. If $G > \eta Q^A$, then $Q^B > Q^A$ it is set, meaning that the environmental stock improves from state $A$. Furthermore, we assume (26) as $\eta = 1$, which shows the state in which $Q^A$ suffered destructive damage.

$$Q^B = (1-\eta)Q^A + G \tag{25}$$

$$Q^B = G \tag{26}$$

Consequently, (27) is obtainable from the formulization presented above.

$$W_i = (1-\alpha)\left(V\left(c_i^A\right) + U\left(\gamma_i^A Q^A\right)\right) + \alpha\left(V\left(c_i^B\right) + U\left(\gamma_i^B G\right)\right), \forall i \quad (27)$$

The recovery and reconstruction project expenses assume that funds are provided by voluntary tax burdens imposed on both regions. Therefore, (28) will be realized if the voluntary tax burden of region $i$ is set to $g_i$. Furthermore, if this tax burden is assumed irrespective of states $A, B$, then the budget constraint of region for states $A, B$ can be expressed, respectively, by (29) and (30). Therefore, (31) and (32) can be drawn.

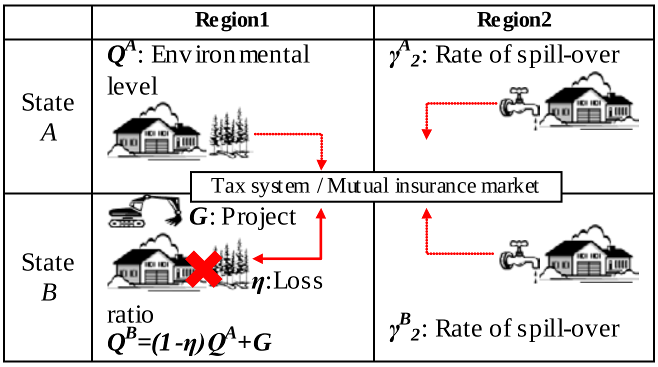


Fig. 1 Concept of Recovery and Reconstruction Project with Spill-Over Benefits

$$G = g_1 + g_2 \quad (28)$$

$$c_i^A = Y_i - g_i, \forall i \quad (29)$$

$$c_i^B = (1-\pi_i)Y_i - g_i = c_i^A - \pi_i Y_i, \forall i \quad (30)$$

$$W_i = (1-\alpha)\left(V\left(c_i^A\right) + U\left(\gamma_i^A Q^A\right)\right) + \alpha\left(V\left(c_i^A - \pi_i Y_i\right) + U\left(\gamma_i^B G\right)\right), \forall i \quad (31)$$

$$c_i^A + G = Y_i + g_j, i \neq j, \forall i, j \quad (32)$$

The following expenditure functions are defined as presented below.

$$\min_{c^A{}_i, G}. E_i(\cdot) \equiv c_i^A + G, \forall i$$
$$\text{s.t. } (1-\alpha)\left(V(c_i^A) + U(\gamma_i^A Q^A)\right) + \alpha\left(V(c_i^A - \pi_i Y_i) + U(\gamma_i^B G)\right) \geq \overline{W}_i \quad (33)$$

Region $i$ makes tax burden $g_j$ of region $j(\neq i)$ as given. When its tax burden $g_i$ shall be determined, economic welfare $W_i$, which can be attained as a result, will satisfy (34).

$$Y_i + g_j = E_i\left(W_i, \pi_i Y_i, \gamma_i^A, \gamma_i^B\right), \forall i, j \quad (34)$$

In Nash equilibrium, because public goods demand becomes equal between regions, (35) is materialized and $G_i(W_i, \pi_i Y_i, \gamma_i^A, \gamma_i^B)$ becomes the optimal solution of (33). Moreover, (36) will be obtained if (34) is substituted for (28). Therefore, the economic welfare $W_i$ in the Nash equilibrium must satisfy (35) and (36).

$$G_1(W_1, \pi_1 Y_1, \gamma_1^A, \gamma_1^B) = G_2(W_2, \pi_2 Y_2, \gamma_2^A, \gamma_2^B) \quad (35)$$

$$Y_1 + Y_2 + G_1(W_1, \pi_1 Y_1, \gamma_1^A, \gamma_1^B) = E_1(W_1, \pi_1 Y_1, \gamma_1^A, \gamma_1^B) + E_2(W_2, \pi_2 Y_2, \gamma_2^A, \gamma_2^B) \quad (36)$$

## *B. Findings from Comparative Statics*

### *1) Expenditure Function*

The first order conditions for comparative statics about the optimal solution of (33) are set to (37) and (38). Furthermore, (39) is obtained by differentiating (37) and (38). Additionally, it omits $f'(x) = df(x)/dx$, $f''(x) = d^2 f(x)/dx^2$.

$$(1-\alpha)V'\left(c_i^A\right) + \alpha V''\left(c_i^A - \pi_i Y_i\right) - \alpha\gamma_i^B U'\left(\gamma_i^B G\right) = 0, \forall i \quad (37)$$

$$(1-\alpha)V(c_i^A) + \alpha V(c_i^A - \pi Y_i) + (1-\alpha)U(\gamma_i^A Q^A) + \alpha U(\gamma_i^B G) - W_i = 0, \forall i \quad (38)$$

$$\begin{bmatrix} (1-\alpha)V''\left(c_i^A\right) + \alpha V''\left(c_i^A - \pi_i Y_i\right) & -\alpha(\gamma_i^B)^2 U''\left(\gamma_i^B G\right) \\ (1-\alpha)V''\left(c_i^A\right) + \alpha V''\left(c_i^A - \pi_i Y_i\right) & \alpha\gamma_i^B U'\left(\gamma_i^B G\right) \end{bmatrix} \begin{bmatrix} dc_i^A \\ dG \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} dW_i + \begin{bmatrix} \alpha V'\left(c_i^A - \pi_i Y_i\right) \\ \alpha V'\left(c_i^A - \pi_i Y_i\right) \end{bmatrix} d(\pi_i Y_i) + \begin{bmatrix} 0 \\ -(1-\alpha)Q_i^A U'\left(\gamma_i^B G\right) \end{bmatrix} d\gamma_i^A + \begin{bmatrix} \alpha\gamma_i^B G U'\left(\gamma_i^B G\right) \\ -\alpha G U'\left(\gamma_i^B G\right) \end{bmatrix} d\gamma_i^B, \forall i \quad (39)$$

Result of such comparative statics is shown as (40)-(52).

$$\Delta_i \equiv \alpha\gamma_i^B U'\left(\gamma_i^B G\right)\{(1-\alpha)V''\left(c_i^A\right) + \alpha V''\left(c_i^A - \pi_i Y_i\right)\} + \alpha(\gamma_i^B)^2 U''\left(\gamma_i^B G\right) \cdot \left((1-\alpha)V'\left(c_i^A\right) + \alpha V'\left(c_i^A - \pi_i Y_i\right)\right) < 0, \forall i \quad (40)$$

$$\frac{\partial c_i^A}{\partial W_i} = \frac{\alpha(\gamma_i^B)^2 U''\left(\gamma_i^B G\right)}{\Delta_i} > 0, \forall i \quad (41)$$

$$\frac{\partial c_i^A}{\partial(\pi_i Y_i)} = \frac{\left[\alpha^2\gamma_i^B V''\left(c_i^A - \pi_i Y_i\right)U'\left(\gamma_i^B G\right) + (\alpha\gamma_i^B)^2 V'\left(c_i^A - \pi_i Y_i\right)U''\left(\gamma_i^B G\right)\right]}{\Delta_i} > 0, \forall i \quad (42)$$

$$\frac{\partial c_i^A}{\partial \gamma^A{}_i} = -\frac{\alpha(1-\alpha)(\gamma_i^B)^2 Q^A{}_i \cdot U'\left(\gamma_i^B G\right)U''\left(\gamma_i^B G\right)}{\Delta_i} < 0, \forall i \quad (43)$$

$$\frac{\partial c_i^A}{\partial \gamma_i^B}=\frac{\begin{bmatrix}(\alpha\gamma_i^B)^2 GU'\left(\gamma_i^B G\right)U''\left(\gamma_i^B G\right)\\ -(\alpha\gamma_i^B)^2 GU'\left(\gamma_i^B G\right)U''\left(\gamma_i^B G\right)\end{bmatrix}}{\Delta_i}=0,\ \forall i \tag{44}$$

$$\frac{\partial G}{\partial W_i}=\frac{(1-\alpha)V''\left(c_i^A\right)+\alpha V''\left(c_i^A-\pi_i Y_i\right)}{\Delta_i}>0,\ \forall i \tag{45}$$

$$\frac{\partial G}{\partial(\pi_i Y_i)}=\frac{\left[\alpha(1-\alpha)\begin{pmatrix}V'\left(c_i^A-\pi_i Y_i\right)V''\left(c_i^A\right)\\ -V''\left(c_i^A-\pi_i Y_i\right)V'\left(c_i^A\right)\end{pmatrix}\right]}{\Delta_i} \tag{46}$$

$$>0,\ \forall i$$

$$\frac{dG}{d\gamma_i^A}=-\frac{\begin{bmatrix}(1-\alpha)QU'\left(\gamma_i^A Q^A\right)\\ \cdot\left[(1-\alpha)V''\left(c_i^A\right)+\alpha V''\left(c_i^A-\pi_i Y_i\right)\right]\end{bmatrix}}{\Delta_i} \tag{47}$$

$$<0,\ \forall i$$

$$\frac{dG}{d\gamma_i^B}=-\frac{\begin{bmatrix}\alpha GU'\left(\gamma_i^B G\right)\\ \cdot\left((1-\alpha)V''\left(c_i^A\right)+\alpha V''\left(c^A-\pi Y\right)\right)\\ +\alpha\gamma_i^B GU''\left(\gamma_i^B G\right)\\ \cdot\left((1-\alpha)V'\left(c_i^A\right)+\alpha V'\left(c_i^A-\pi_i Y_i\right)\right)\end{bmatrix}}{\Delta_i} \tag{48}$$

$$<0,\ \forall i$$

$$\frac{\partial E_i(.)}{\partial W_i}=\frac{1}{(1-\alpha)V'\left(c_i^A\right)+\alpha V'\left(c_i^A-\pi_i Y_i\right)}>0,\ \forall i \tag{49}$$

$$\frac{\partial E_i(.)}{\partial(\pi_i Y_i)}=\frac{\alpha V'\left(c_i^A-\pi_i Y_i\right)}{(1-\alpha)V'\left(c_i^A\right)+\alpha V'\left(c_i^A-\pi_i Y_i\right)}>0,\ \forall i \tag{50}$$

$$\frac{\partial E_i(.)}{\partial \gamma_i^A}=-\frac{(1-\alpha)Q^A U'\left(\gamma_i^A Q_i^A\right)}{(1-\alpha)V'\left(c_i^A\right)+\alpha V'\left(c_i^A-\pi_i Y_i\right)}<0,\ \forall i \tag{51}$$

$$\frac{\partial E_i(.)}{\partial \gamma_i^B}=-\frac{\alpha GU'\left(\gamma_i^B G\right)}{(1-\alpha)V'\left(c_i^A\right)+\alpha V'\left(c_i^A-\pi_i Y_i\right)}<0,\ \forall i \tag{52}$$

TABLE 1 RESULT OF COMPARATIVE STATICS OF EXPENDITURE FUNCTION

| | $W_i$ | $\pi_i Y_i$ | $\gamma_i^A$ | $\gamma_i^B$ |
|---|---|---|---|---|
| $c_i^A$ | + | + | − | 0 |
| $g_i$ | − | − | + | 0 |
| $G$ | + | + | − | − |
| $E_i(.)$ | + | + | − | − |

The result of the comparative statics of the expenditure function can be adjusted as shown in Table 1. However, sign conditions must be regarded carefully under the assumption of Constant Relative Risk Aversion (CRRA).

*2) Nash Equilibrium*

$\gamma_1^t=1$ and $0\le\gamma_2^t\le 1$ are assumed. The conditional expression of economic welfare in the Nash equilibrium are rewritten respectively as (53) and (54). Moreover, (55) is obtained by differentiating these equations.

$$G_1(W_1,\pi_1 Y_1,1,1)=G_2(W_2,\pi_2 Y_2,\gamma_2^A,\gamma_2^B) \tag{53}$$

$$\begin{aligned}&Y_1+Y_2+G_1(W_1,\pi_1 Y_1,1,1)\\ &\quad=E_1(W_1,\pi_1 Y_1,1,1)+E_2(W_2,\pi_2 Y_2,\gamma_2^A,\gamma_2^B)\end{aligned} \tag{54}$$

$$\begin{aligned}&\begin{bmatrix}-\dfrac{\partial c_1^A}{\partial W_1} & -\dfrac{\partial E_2(.)}{\partial W_2}\\ \dfrac{\partial G_1}{\partial W_1} & -\dfrac{\partial G_2}{\partial W_2}\end{bmatrix}\begin{bmatrix}dW_1\\ dW_2\end{bmatrix}=\begin{bmatrix}\dfrac{\partial E_2(.)}{\partial \gamma_2^A}\\ \dfrac{\partial G_2}{\partial \gamma_2^A}\end{bmatrix}d\gamma_2^A\\ &\quad+\begin{bmatrix}\dfrac{\partial E_2(.)}{\partial \gamma_2^B}\\ \dfrac{\partial G_2}{\partial \gamma_2^B}\end{bmatrix}d\gamma^B{}_2+\begin{bmatrix}\dfrac{\partial c_1^A}{\partial(\pi_1 Y_1)}\\ -\dfrac{\partial G_1}{\partial(\pi_1 Y_1)}\end{bmatrix}d(\pi_1 Y_1)\end{aligned} \tag{55}$$

The results of the comparative statics are shown as (56)-(60).

$$\frac{\partial W_1}{\partial(\pi_1 Y_1)}=\frac{-\dfrac{\partial G_2}{\partial W_2}\dfrac{\partial c_1^A}{\partial(\pi_1 Y_1)}-\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_1}{\partial(\pi_1 Y_1)}}{\dfrac{\partial c_1^A}{\partial W_1}\dfrac{\partial G_2}{\partial W_2}+\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_1}{\partial W_1}}<0 \tag{56}$$

$$\frac{\partial W_1}{\partial \gamma_2^A}=\frac{-\dfrac{\partial G_2}{\partial W_2}\dfrac{\partial E_2(.)}{\partial \gamma_2^A}+\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_2}{\partial \gamma_2^A}}{\dfrac{\partial c_1^A}{\partial W_1}\dfrac{\partial G_2}{\partial W_2}+\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_1}{\partial W_1}}=0 \tag{57}$$

$$\frac{\partial W_2}{\partial \gamma_2^A}=\frac{-\dfrac{\partial c_1^A}{\partial W_1}\dfrac{\partial G_2}{\partial \gamma_2^A}-\dfrac{\partial E_2(.)}{\partial \gamma_2^A}\dfrac{\partial G_1}{\partial W_1}}{\dfrac{\partial c_1^A}{\partial W_1}\dfrac{\partial G_2}{\partial W_2}+\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_1}{\partial W_1}}>0 \tag{58}$$

$$\frac{\partial W_1}{\partial \gamma_2^B}=\frac{-\dfrac{\partial G_2}{\partial W_2}\dfrac{\partial E_2}{\partial \gamma_2^B}+\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_2}{\partial \gamma_2^B}}{\dfrac{\partial c_1^A}{\partial W_1}\dfrac{\partial G_2}{\partial W_2}+\dfrac{\partial E_2(.)}{\partial W_2}\dfrac{\partial G_1}{\partial W_1}}<0 \tag{59}$$

$$\frac{\partial W_2}{\partial \gamma_2^B}=\frac{-\frac{\partial c_1^A}{\partial W_1}\frac{\partial G_2}{\partial \gamma_2^B}-\frac{\partial E_2(\cdot)}{\partial \gamma_2^B}\frac{\partial G_1}{\partial W_1}}{\frac{\partial c_1^A}{\partial W_1}\frac{\partial G_2}{\partial W_2}+\frac{\partial E_2(\cdot)}{\partial W_2}\frac{\partial G_1}{\partial W_1}}>0 \tag{60}$$

Results of the comparative statics of Nash equilibrium can be adjusted as shown in Table 2. First, in the model assumed here, it was clarified from (i) in Table 2 that the rate of spill-over $\gamma_2^A$ of state $A$ (usual) does not affect the voluntary tax burden $g_i$ or provision of public goods $G$. Moreover, as for (ii), the voluntary tax burden of region $2$ is reduced, and the rate $\gamma_2^B$ of spill-over of state $B$ (disaster) enjoys spill-over benefits in being high. However, the voluntary tax burden of region $1$ becomes high. The reason is that the rate of spill-over must be high for the free rider effect to increase. The economic welfare of region $1$ falls and that of region $2$ increases when carrying out recovery and reconstruction project by the voluntary tax burden. Therefore, the economic welfare can be checked by (iii). Moreover, the influences of the gap of economic welfare between regions can be assessed.

TABLE 2 RESULT OF COMPARATIVE STATICS OF NASH EQUILIBRIUM

| | $\pi_1 Y_1$ | $\gamma_2^A$ | $\gamma_2^B$ |
|---|---|---|---|
| $W_1$ | $-$ | 0 | $-$ (iii) |
| $W_2$ | $\gtreqless$ | $+$ | $+$ (iii) |
| $c_1^A$ | $\gtreqless$ | 0 | $-$ |
| $c_2^A$ | $\gtreqless$ | 0 | $+$ |
| $g_1$ | $\gtreqless$ | 0 | $+$ (ii) |
| $g_2$ | $\gtreqless$ | 0 | $-$ (ii) |
| $G$ | $\gtreqless$ | 0 (i) | $-$ |

The following are understood when (35) is considered and result of comparative statics are examined. First, (61) is realized from (35). In this model, $\pi_1 Y_1 \geq \pi_2 Y_2$ is generally materialized from the assumptions that region $1$ suffers a great deal of damage. However, the size relation of economic welfare which can be set in this case becomes indefinite. It can be expressed as (62).

$$\text{if } \pi_1 Y_1 = \pi_2 Y_2 \text{ then } W_1 > W_2 \tag{61}$$

$$\text{if } \pi_1 Y_1 \geq \pi_2 Y_2 \text{ then } W_1 \gtreqless W_2 \tag{62}$$

## IV. FINANCING BY VOLUNTARILY TAX BURDEN AND MUTUAL INSURANCE

### A. *Model*

The preceding section clarified the economic welfare of region to which public goods are supplied with spill-over benefits. A case is assumed in which the working expenses of the recovery and reconstruction project is covered by the mutual insurance between local governments in addition to the voluntary tax burden. Actually, (65) can be drawn if $s_i$ is eliminated from (63) and (64), and if (28) is used.

$$c_i^A = Y_i - ps_i - g_i,\ \forall i \tag{63}$$

$$\begin{aligned} c_i^B &= (1-\pi_i)Y_i - ps_i + s_i - g_i \\ &= (Y_i - ps_i - g_i) - \pi_i Y_i + s_i = c_i^A - \pi_i Y_i + s_i,\ \forall i \end{aligned} \tag{64}$$

$$(1-p)c_i^A + pc_i^B + G = (1-p\pi_i)Y_i + g_j,\ \forall i, j \tag{65}$$

Furthermore, if we are cautious about (6), which are conditions of a fair premium, the expenditure function shown in (66) is defined. $c^A{}_i = c^B{}_i$ is obtained from (67)-(70), which are the first order conditions of (66), and $c_i^A, G$ which minimizes the expenditure, is expressed by (71) and (72). $\lambda_i$ denotes Lagrange's multiplier.

$$\begin{aligned} &\min_{c_i^A, c_i^B, G} E_i(\cdot) \equiv (1-\alpha)c_i^A + \alpha c_i^B + G,\ \forall i \\ &\text{s.t. } (1-\alpha)\left(V(c_i^A) + U(\eta_i^A Q^A)\right) \\ &\qquad + \alpha\left(V(c_i^B) + U(\eta_i^B G)\right) \geq \overline{W_i} \end{aligned} \tag{66}$$

$$(1-\alpha) - \lambda_i (1-\alpha) V'(c_i^A) = 0,\ \forall i \tag{67}$$

$$\alpha - \lambda_i \alpha V'(c_i^B) = 0,\ \forall i \tag{68}$$

$$1 - \lambda_i \alpha \gamma_i^B U'(\gamma_i^B G) = 0,\ \forall i \tag{69}$$

$$\begin{aligned} &(1-\alpha)\left(V(c_i^A) + U(\gamma_i^A Q^A)\right) \\ &\quad + \alpha\left(V(c_i^B) + U(\gamma_i^B G)\right) - W_i = 0,\ \forall i \end{aligned} \tag{70}$$

$$V'(c_i^A) - \alpha\gamma_i^B U'(\gamma_i^B G) = 0,\ \forall i \tag{71}$$

$$V(c_i^A) + (1-\alpha)U(\gamma_i^A Q^A) + \alpha U(\gamma_i^B G) - W_i = 0,\ \forall i \tag{72}$$

### B. *Findings from Comparative Statics*

(73) is obtained by differentiating (71) and (72).

$$\begin{aligned} &\begin{bmatrix} V''(c_i^A) & -\alpha(\gamma_i^B)^2 U''(\gamma_i^B G) \\ V'(c_i^A) & \alpha\gamma_i^B U'(\gamma_i^B G) \end{bmatrix} \begin{bmatrix} dc_i^A \\ dG \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} dW_i + \begin{bmatrix} 0 \\ -(1-\alpha)Q^A U'(\gamma_i^A Q^A) \end{bmatrix} d\gamma_i^A \\ &\quad + \begin{bmatrix} \alpha\gamma_i^B G U''(\gamma_i^B G) \\ -\alpha G U'(\gamma_i^B G) \end{bmatrix} d\gamma_i^B,\ \forall i \end{aligned} \tag{73}$$

The results of such comparative statics are shown as (74)-(76).

$$\begin{aligned} \frac{\partial G}{\partial W_i} &= \frac{V''(c_i^A)}{\alpha\gamma_i^B U'(\gamma_i^B G) V''(c_i^A) + \alpha(\gamma_i^B)^2 U''(\gamma_i^B G) V'(c_i^A)} \\ &> 0,\ \forall i \end{aligned} \tag{74}$$

$$\frac{\partial G}{\partial \gamma_i^A} = -\frac{(1-\alpha)QU'(\gamma_i^A Q)V''(c_i^A)}{\alpha\gamma_i^B U'(\gamma_i^B G)V''(c_i^A) + \alpha(\gamma_i^B)^2 U''(\gamma_i^B G)V'(c_i^A)} < 0, \forall i \quad (75)$$

$$\frac{\partial G}{\partial \gamma_i^B} = -\frac{\alpha GU'(\gamma_i^B G)V''(c_i^A) + \alpha\gamma_i^B GU''(\gamma_i^B G)V'(c_i^A)}{\alpha\gamma_i^B U'(\gamma_i^B G)V''(c_i^A) + \alpha(\gamma_i^B)^2 U''(\gamma_i^B G)V'(c_i^A)} < 0, \forall i \quad (76)$$

In the Nash equilibrium, because public goods demand is equal between regions (77) and (78) which are obtainable as shown below.

$$(1-\alpha\pi_1)Y_1 + (1-\alpha\pi_2)Y_2 + G(W_1,1,1) = E(W_1,1,1) + E(W_2,\gamma_2^A,\gamma_2^B) \quad (77)$$

$$G(W_1,1,1) = G(W_2,\gamma_2^A,\gamma_2^B) \quad (78)$$

As described in this paper, because it is assumed that they are $\gamma_1^t = 1$ and $0 \le \gamma_2^t \le 1$, if the comparative statics analysis result is used, then $W_1 \ge W_2$ will be drawn from (78). The case in which the source of revenue supply was conducted through mutual insurance between regions in addition to a voluntary tax burden was analyzed, presuming a recovery and reconstruction project with spill-over benefits. Results show clearly that the economic welfare of region in which such recovery and reconstruction project are conducted is improved. At some prior stage, i.e. in usual circumstances, a mutual insurance system is founded between local governments, underscoring the extreme importance of building a decentralized risk sharing system.

## V. CONCLUSIONS

This paper extracts a point of argument related to recovery and reconstruction project after a natural disaster. We present one proposal for risk management by local governments, with the foundation of an insurance market in mind. As the method of financing for recovery and reconstruction project, a mutual insurance market among local governments and a system incorporating regional risk sharing is proposed. The origin of the analyses presented in the paper is reservation of a local resident's economic welfare level. It is based on the idea that it should be compensated: some income redistribution policy should be performed. Although the motive of insurance subscription is to create an insurance function of a risk transfer and distribution, it is difficult for a policyholder to judge the fairness of a premium. Therefore, after eliminating information asymmetry, it is necessary to determine a policyholder's amount of payment acceptance based on exact judgment.

Finally, there are various directions in which this paper could be extended. It would be fruitful to consider the possibility of capital accumulation of risk-sharing with population mobility.

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