

# The Variation of Primary Energy Factor and Emission Factor of Electricity in Taiwan

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**Abstract-** This paper examines the variation of the primary energy factor and the emission factor of electricity generation from 1982 to 2009 in Taiwan. Results of the analysis show that the major effect of the aggregated primary energy factor of electricity was the generation mix effect. The pure primary energy coefficient effect on the change ratio of the primary energy factor of electricity was positive too, but the effect was limited. The major effect of the aggregated emission factor of electricity was the fossil fuel generation share effect of fossil fuel generation. The emission coefficient effect also had a positive effect on the change of the aggregated emission factor of electricity, but the effect was also limited in scope.

**Keywords-** primary energy use, greenhouse gases emissions, global warming potential (GWP), electricity generation, Logarithmic Mean Divisia Index

## I. INTRODUCTION

Recently, studies of buildings and their effect on climate change showed that building emissions are an important sector of the society in terms of global warming [1, 2]. These studies also showed that emissions from building energy use, especially electricity, were major sources that society must deal with on a daily basis.

One way to stem building emissions, for example, is seen in EU Directive on Energy Performance of Buildings (EPBD, Directive 2002/91/EC) which required member states to develop and introduce energy performance regulations by the year 2006 [3, 4]. The common general framework of the EPBD warrants that the energy performance of a building shall be expressed in a transparent manner and shall include an energy performance indicator and a numeric indicator of primary energy use, based on primary energy factors per energy carrier [3-6].

Moreover, in order to measure energy use and report GHG emissions from building operations, the Common Carbon Metric, proposed by Sustainable Buildings and Climate Initiative (SBCI), includes measures of energy intensity and carbon intensity [7]. Therefore, for each fuel source used, the respective aggregated primary energy factor and the aggregated emission factor of energy carrier of a nation/region will be needed when one hope to evaluate the energy use and emissions of buildings.

Electricity is the major energy source for buildings and industry, which is also a kind of secondary energy with high emission intensity. The aggregated primary energy factor and aggregated emission factor in terms of electricity represents the average situation of electricity generation of a society. Almost 99.3% of the primary energy used in

Taiwan is import. Electricity generation mode adjustments and fossil fuel generation mix adjustments in Taiwan's thermal power and co-generation varied between 1982 and 2009 because of fluctuations in the fuel market [8]. Analyzing the variation in Taiwan's historical primary energy use factor and the emission factor of electricity generation can uncover the characteristics of a changing path, which thus will help Taiwan to face emissions reduction in the future.

In next section, this paper will introduce the estimation method of the primary energy factor and emission factor of electricity. Then the methodology of the Logarithmic Mean Divisia Index (LMDI) will be introduced. Electricity generation information from Taiwan's energy balance sheet from 1982 to 2009 will be used to analyse the supply side aggregated primary energy factor and the aggregated emission factor of electricity. The variations of the primary energy factor and emission factor of electricity will then be discussed. Finally, some concluding remarks will be introduced.

## II. METHODOLOGY

The electricity generation mix of Taiwan includes thermal power, cogeneration, nuclear power, hydro power, and new renewable energy, i.e. photovoltaic and wind energy. The supply side aggregated primary energy factor of electricity can be estimated by summing the primary energy factor of each electricity generation mode (including thermal power, cogeneration, nuclear power, hydro power, and new renewable energy) weighted by its own share. The supply side aggregated emission factor of electricity can be estimated by summing the emission factor of the electricity generation mode weighted by its own share.

### A. The Aggregated Primary Energy Factor of Electricity

If subscript  $i$  denotes the electricity generation mode, the supply side aggregated primary energy factor of electricity can be expressed as

$$f = \sum_i f_i s_i \quad (1)$$

where  $f_i = F_i / E_i$  represents the primary energy input per electricity output in respect to generation mode  $i$ ; and  $s_i = E_i / E$  represents the share of generation mode  $i$  in respect to the total electricity generation.

### B. The Aggregated Emission Factor of Electricity

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The supply side aggregated emission factor can be expressed as

$$e = \sum_i e_i s_i; \quad (2)$$

where  $e_i = C_i / E_i$  represents the emissions per electricity output in respect to each generation mode  $i$ ; and  $s_i = E_i / E$  represents the share of generation mode  $i$  in respect to the total electricity generation.

Hydro power, new renewable energy (like photovoltaic and wind energy), and nuclear power include little, if any fossil fuel emission in their production processes. Thus, one can categorize these generation modes as non-fossil fuel electricity generation. If  $E$  denotes the total electricity generation;  $E_f$  denotes the fossil fuel generation of electricity (including thermal power and co-generation). The emission factor of fossil fuel generation can be expressed as  $e_f = C / E_f$ ; the share of fossil fuel generation can be expressed as  $s_f = E_f / E$ . The aggregated emission factor of electricity can be simplified as follows,

$$e = e_f s_f. \quad (3)$$

The aggregated emission factor of electricity thus can be decomposed as the emission coefficient effect of the fossil fuel generation and the share of fossil fuel generation effect.

### C. Decomposition of the Change Ratio of the Primary Energy Factor of Electricity

The primary energy factor of aggregated electricity generation  $f$  in Eq. 1 can be decomposed into the effects associated with  $f_i$ , and  $s_i$ . If all the variables were treated as a function of time, as time change in the aggregated primary energy factor of electricity is continuous, applying the logarithmic differentiation to both sides of Eq. 1 yields:

$$\frac{df}{d\tau} = \sum_i \frac{df_i}{d\tau} s_i + \sum_i \frac{ds_i}{d\tau} f_i. \quad (4)$$

Eq. 4 shows that the change of the Divisia primary energy factor of electricity was decomposed as

(1) The summation of the primary energy factor component change of each electricity generation mode ( $\sum_i \frac{df_i}{d\tau} s_i$ ), under the condition of the unchanged respective share of the generation mode.

(2) The summation of the share of each generation mode change ( $\sum_i \frac{ds_i}{d\tau} f_i$ ), under the condition of the primary energy factor of the unchanged respective generation mode.

The logarithmic change rate of the Divisia primary energy factor of electricity can be written as follows,

$$\frac{d \ln f}{d\tau} = \sum_i w_i \left( \frac{d \ln f_i}{d\tau} + \frac{d \ln s_i}{d\tau} \right); \quad (5)$$

where  $w_i = \frac{f_i s_i}{\sum_i f_i s_i}$ . (Derivation of Eq. 5 see Appendix 1)

Eq. 5 shows that the logarithmic change rate of the aggregated primary energy factor of total electricity power can be decomposed as the share-weighted average of the logarithmic change rate of the primary energy coefficient and the electricity generation mix [9]. Integrating the logarithmic change rate of the primary energy factor of total electricity power in respect to two periods over  $[t-1, t]$  and rearranging the terms yields:

$$\ln \frac{f_t}{f_{t-1}} = \int_{t-1}^t \sum_i w_i \frac{d \ln f_i}{d\tau} d\tau \times \int_{t-1}^t \sum_i w_i \frac{d \ln s_i}{d\tau} d\tau. \quad (6)$$

In Eq. 6, the primary energy factor of the total electricity power index on the left hand side (LHS) is given by the product of two Divisia integral indexes on the right hand side (RHS). The Divisia integral index is a theoretical model, which was developed in order to provide a discrete approximation formula for empirical study. The following equation provides a hint for a discrete approximation of Eq. 6, which illustrates the changes of the aggregated primary energy factor of electricity.

$$\frac{f_t}{f_{t-1}} = \exp \left( \sum_i w_i(t^*) \ln \frac{f_{i,t}}{f_{i,t-1}} \right) \times \exp \left( \sum_i w_i(t^*) \ln \frac{s_{i,t}}{s_{i,t-1}} \right) \times D_{rd} \quad (7)$$

;  $t^* \in [t-1, t]$ .

The first term on the right hand side (RHS) of Eq. 7 represents the pure primary energy coefficient effect, which shows the change effect of the primary energy coefficient in respect to each generation mode (when the share of each

generation type remains fixed,  $\ln \frac{s_{i,t}}{s_{i,t-1}} = 0$  or  $\frac{s_{i,t}}{s_{i,t-1}} = 1$ ).

The second term on the RHS of Eq. 7 represents as the generation mix effect, which shows the change effect of the electricity generation share in respect to each generation mode (when the primary energy coefficient of each generation type remains fixed,  $\ln \frac{f_{i,t}}{f_{i,t-1}} = 0$  or  $\frac{f_{i,t}}{f_{i,t-1}} = 1$ ).

The remaining component on the RHS of Eq. 7,  $D_{rd}$ , is the residual term.

Theoretically, the Divisia index will change based on the weighting along time change. In practice, a discrete approximation of the weighting will be needed. Ang (2005) and related studies showed that logarithmic mean weighting with perfect decomposition property is better than

arithmetical mean weighting, as it includes a comparative advantage when facing the zero-value problem [10-14].

This paper uses logarithmic mean weighting as follows,

$$w_k(t^*) \equiv \frac{L(w_{k,t}, w_{k,t-1})}{\sum_i L(w_{i,t}, w_{i,t-1})}; \quad (8)$$

where  $w_i = \frac{F_i}{F}$ . (For the derivation sees Appendix 2)

#### D. Decomposition of the Change Ratio of the Emission Factor of Electricity

The aggregated emission factor of electricity can be decomposed as the emission coefficient effect and the share effect of fossil fuel generation. The change ratio of the aggregated emission factor of electricity from period  $t-1$  to period  $t$  can be written as follows,

$$\frac{e_t}{e_{t-1}} = \frac{e_{f,t}}{e_{f,t-1}} \bullet \frac{s_{f,t}}{s_{f,t-1}}; \quad (9)$$

where  $e_{f,t}$  denotes the emission coefficient of the fossil fuel used in period  $t$ ;  $s_{f,t}$  denotes the share effect of the fossil fuel used in period  $t$ . Eq. 9 rewrites the change ratio of the aggregated emission factor of electricity by multiplying the change ratio of the emission coefficient of fossil fuel generation and the change ratio of the share of fossil fuel generation. Following the additional property of the logarithmic growth rate, the aggregated emission factor of electricity from period  $t-1$  to period  $t$  can be written as follows,

$$\ln \frac{e_t}{e_{t-1}} = \ln \frac{e_{f,t}}{e_{f,t-1}} + \ln \frac{s_{f,t}}{s_{f,t-1}}. \quad (10)$$

Eq. 10 shows that the logarithmic growth rate of the aggregated emission factor of electricity can be decomposed completely in term of the emission coefficient effect and the share effect.

### III. DATA AND ANALYSIS

This paper uses the constant unit heat values of hydro power, nuclear power, and new-renewable energies, as provided by the Bureau of Energy, Taiwan (ROC) to calculate the primary energy input of electricity generation. By using the data from the Taiwan energy balance sheet [8], the emission factor for each fuel recommended in the 2006 IPCC Guidelines for GHG Inventories [15], and the weighting of each greenhouse gas in respect to the 100 year horizon recommended by the IPCC 2007 AR4 [16], one can estimate the heat value input, electricity generation, and emissions of thermal power and co-generation in Taiwan from 1982 to 2009. By combining heat value, electricity generation, and emissions information of other electricity generation modes,

as well as Eq. 1 and Eq. 2, the aggregated primary energy factor and emission factor of electricity can be attained.

#### A. The Aggregated Primary Energy Factor and Emission Factor of Electricity

Fig. 1 shows an estimation of Taiwan's aggregated primary energy factor of electricity from 1982 to 2009. Fig. 2 shows an estimation of Taiwan's aggregated emission factor of electricity from 1982 to 2009. Table 1 shows that the mean value of the supply side aggregated primary energy factor of electricity ( $f$ ) between 1982 and 2009 was 2.63; with a range of 2.55-2.71. The standard deviation of the aggregated primary energy factor of electricity was 0.05; the respective coefficient of variation was 1.9%.

Results of the estimation in Table 1 also shows that the mean value of the supply side aggregated emission factor of electricity ( $e$ ) between 1982 and 2009 was 0.545 kg-CO<sub>2</sub>e/kWh; with a range 0.311-0.680 kg-CO<sub>2</sub>e/kWh. The standard deviation of the aggregated emission factor of electricity was 0.114; the respective coefficient of variation was 21%. The relative variation of the aggregated emission factor of electricity was larger than that of the primary energy factor of electricity between 1982 and 2009.

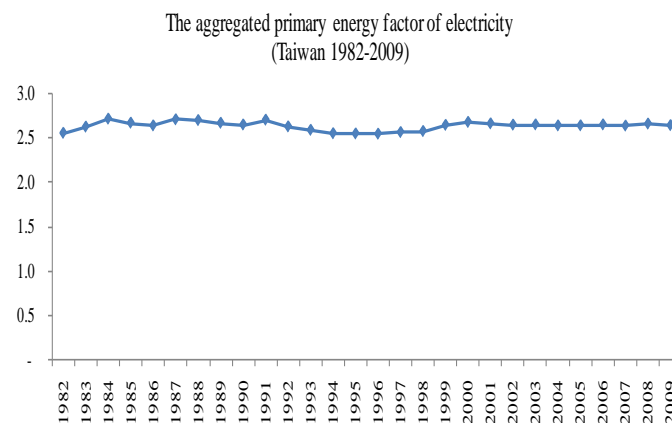


Fig. 1 The aggregated primary energy factor of electricity (Taiwan 1982-2009)

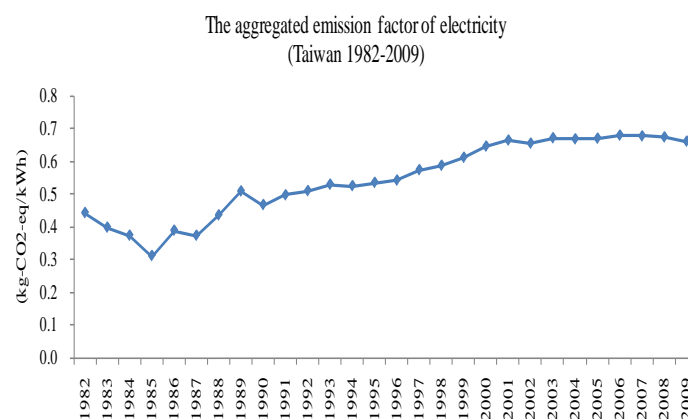


Fig. 2 The aggregated emission factor of electricity (Taiwan 1982-2009)

TABLE I  
THE AGGREGATED PRIMARY ENERGY FACTOR AND THE EMISSION FACTOR OF  
ELECTRICITY IN TAIWAN (1982-2009)

(Supply side)	Mean	Stdev	C.V.	Range
The aggregated primary energy factor of electricity	2.63	0.05	1.9%	2.55-2.71
The aggregated emission factor of electricity (kg-CO <sub>2</sub> e/kWh)	0.545	0.114	21%	0.311-0.680

#### A. The Variation Of the Aggregated Primary Energy Factor of Electricity

The aggregated emission factor of electricity can be decomposed as the pure primary energy coefficient effect and the generation mix effect. Fig. 3 shows the results of the factorial decomposition in regard to the yearly change ratio of the aggregated primary energy factor of electricity in Taiwan between 1982 and 2009.

Considering the magnitude and direction of the effect concurrently, results of the analysis show that the generation mix effect was the major effect of the yearly change in aggregated primary energy factor of electricity from 1982 to 1994. However, the major effect of the yearly change in the aggregated primary energy factor of electricity between 1994 and 2009 was the pure primary energy coefficient effect.

In regard to the major effect, the yearly change in the aggregated primary energy factor of electricity between 1982 and 2009 (27 sets of yearly change ratios in total) included 17 sets of the major effect with a pure primary energy coefficient effect. Additionally, the pure primary energy coefficient effect had the largest magnitude in absolute value as well as the same direction with the yearly change ratio of aggregated primary energy factor of the electricity. Thus this effect emerged as the major effect of the yearly change of

factors. The remaining 10 sets of the major effect set that represents the yearly change of aggregated primary energy factor between 1982 and 2009 featured the generation mix effect.

Fig. 4 shows that the change ratio of the aggregated primary energy factor of electricity from 1982 to 2009 was 1.033 (base year: 1982). The major source of the positive effect was the generation mix effect (the change ratio of the effect was 1.020). The pure primary energy coefficient effect also had a positive effect on the aggregated emission factor of electricity, but the effect was small (the change ratio of the effect was 1.008).

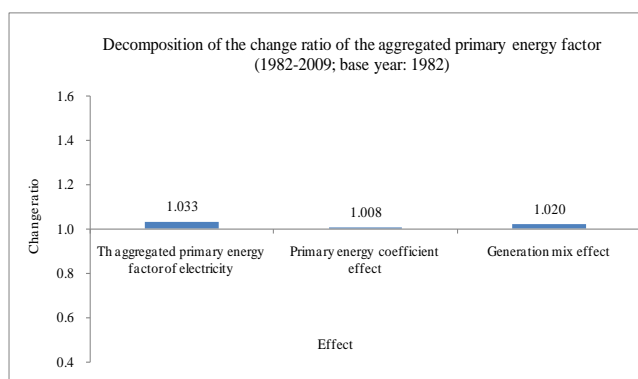


Fig. 4 Decomposition of the change ratio of the aggregated primary energy factor of electricity (Taiwan 1982-2009; Base year: 1982)

Finally, the annual logarithmic growth rate of the aggregated emission factor of electricity during this period was 0.12%, which can be broken down 0.03% (came from the growth of primary energy coefficient effect), and 0.08% (came from the generation mix effect).

#### B. The Variation of the Aggregated Emission Factor of Electricity

The aggregated emission factor of electricity can be decomposed as the complete emission coefficient effect and

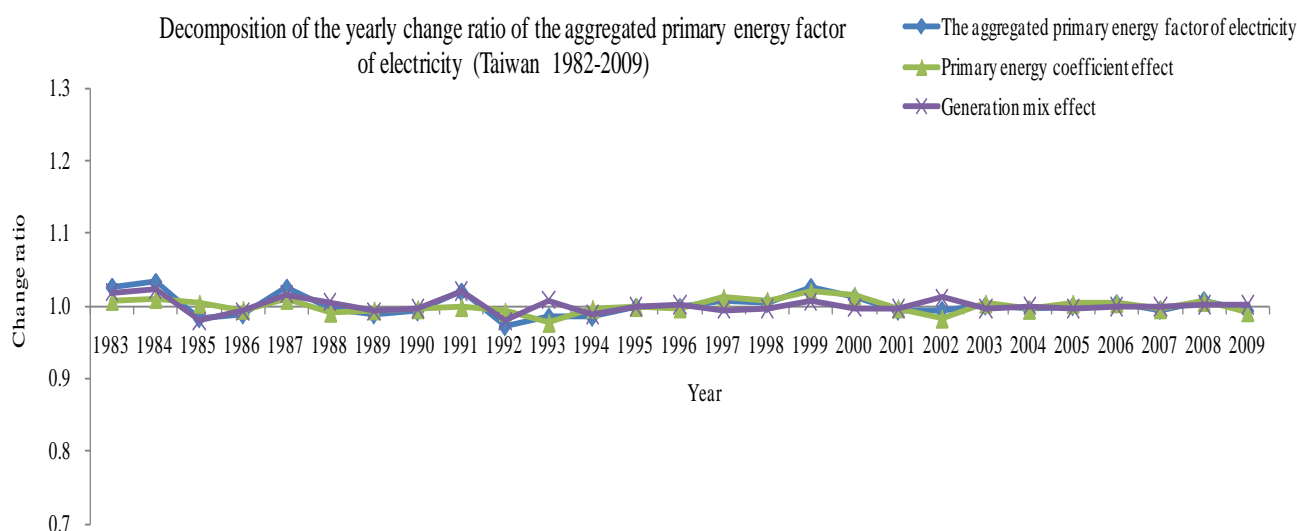


Fig. 3 Decomposition of the yearly change ratio of the aggregated primary energy factor of electricity (Taiwan 1982-2009)

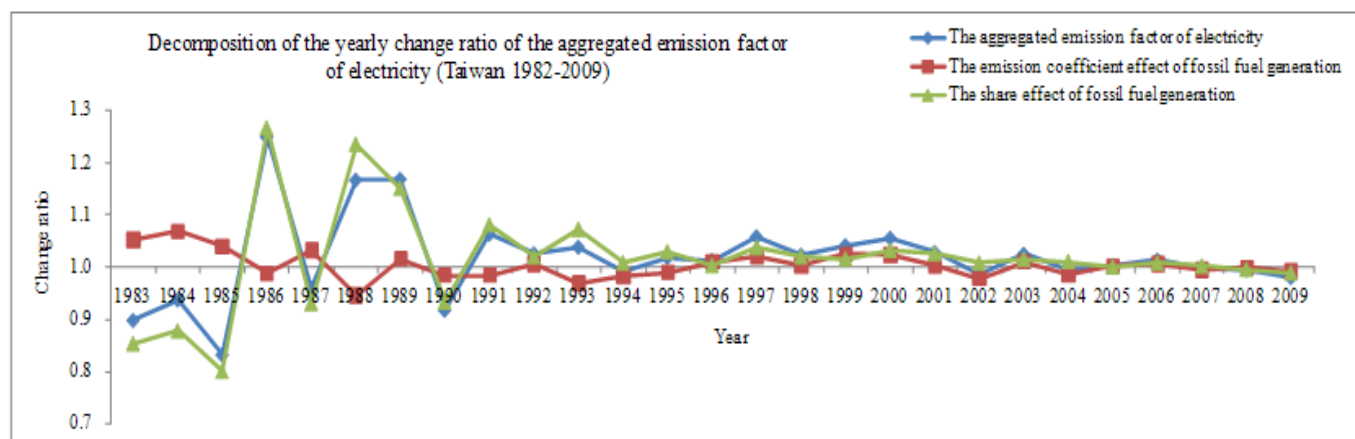


Fig. 5 Decomposition of the yearly change ratio of the aggregated emission factor of electricity (Taiwan 1982-2009)

the share effect of fossil fuel generation. Fig. 5 shows the results of the factorial decomposition respective to the yearly change ratio of the aggregated emission factor of electricity in Taiwan from 1982 to 2009.

After considering the magnitude and direction of the effect concurrently, results of the analysis show that all the major effects of the yearly change in emission factor are a result of the share effect of fossil fuel generation from 1982 to 1993 (in 11 sets of yearly change ratio in total). From 1993 to 2009 (in 16 sets of yearly change ratio in total), 9 sets of the major effect of the yearly change in emission factor showed the share effect of fossil fuel generation while 7 sets of the major effect of the yearly change in the aggregated emission factor of electricity showed the emission coefficient effect of fossil fuel generation.

Fig. 6 shows that the change ratio of the aggregated emission factor of electricity from 1982 to 2009 was 1.495 (base year: 1982). The major source of the positive effect was the share effect of fossil fuel generation (the change ratio of the effect was 1.349). The emission coefficient effect of the fossil fuel generation also had a positive effect on the aggregated emission factor, but the effect was small (the change ratio of the effect was 1.108).

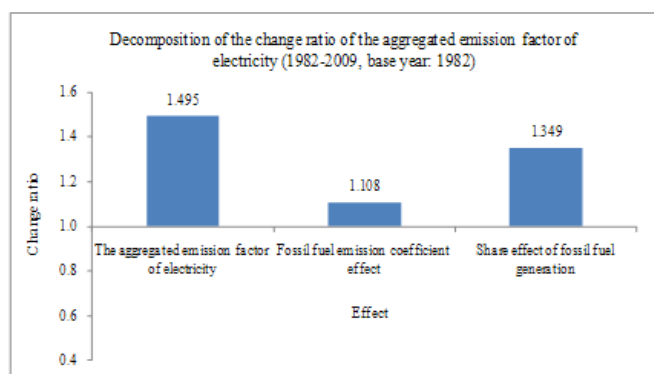


Fig. 6 Decomposition of the change ratio of the aggregated emission factor of electricity (Taiwan 1982-2009; Base year: 1982)

Finally, the annual logarithmic growth rate of the aggregated emission factor of electricity during this period was 1.49%, which can be broken down into 0.38% (came from the growth of emission coefficient effect) and 1.11% (came from the share effect).

#### IV. COMMENTS ON THE VARIATION IN THE EMISSIONS FACTOR AND THE PRIMARY ENERGY FACTOR OF ELECTRICITY

The electricity generation mix, and in particular the share of fuel input of fossil fuel generation, (including thermal power and co-generation) changed dramatically between 1982 and 2009, following fluctuations in the fuel market. Thus, the primary energy factor and the emission factor of thermal power and cogeneration changes, and as a result, the aggregated primary energy factor and the aggregated emission factor of electricity changes each year as well.

Examining the electricity generation evolution of each generation mode of Taiwan, Fig. 7 shows that the share of nuclear power increased in the early stage from 1982 to 1987, while the share of thermal power increased sharply after 1987. As a result, the major effects of the yearly change in the aggregated primary energy factor of electricity from 1982 to 1994 were the generation mix effects, though the directionality did differ in certain instances.

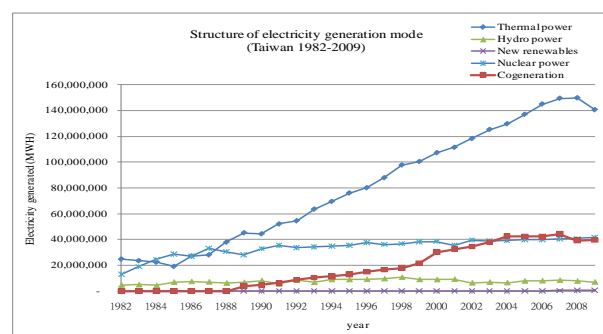


Fig. 7 Evolution of electricity generation modes of Taiwan during 1982-2009

Whenever the share of thermal power reached a rather high plateau in the latter stage from 1994 to 2009, the major effect of the yearly change in the aggregated primary energy factor of electricity shifted to the pure primary energy coefficient effect. Thus, the change of the pure primary energy coefficient reflected the combined generation efficiency of thermal power, which had a greater effect on the variation of the aggregated primary energy factor of electricity.

In terms of generation mode, nuclear power growth occurred quite rapidly in the early stage, from 1982 to 1985. During that period, the share of non-fossil fuel generation increased sharply; the share of fossil fuel generation decreased as follows. Thus the share of nuclear power and the share of fossil fuel generation were almost complementary during the period. The high share of nuclear power results in an aggregated emission factor of electricity in 1985 to be the lowest in the period from 1982 to 2009.

Nuclear power generation increase slightly from 1985 to 2009, but thermal power and cogeneration expanded more rapidly and in greater degree. Rapid expansion of fossil fuel generation resulted in a high share change in fossil fuel generation during the period, increasing in a peak of 80% in 2007. The high share of fossil fuel generation resulted in the aggregated emission factor in 2007 to be the highest one in the period from 1982 to 2009. Thus the major effects of the yearly change of emission factor of electricity during the whole study period, 1982-2009, were the comprehensive share effects of fossil fuel generation.

In regard to the aggregated emission factor of electricity, the high escalation rate of emission factors of electricity from 1985 to 2009 came from the constant increase in the share fossil fuel generation. This also implies that if we hope to lower target emissions, depreciation of the emission factor of electricity will be necessary in the future. Thus, exclusive of nuclear power, the development of new-renewables will be the key to a low-carbon society.

#### V. CONCLUSIONS

This paper uses the energy balance sheet of the ROC to analyse the variations of the aggregated primary energy factor and the aggregated emission factor of electricity generation in Taiwan from 1982 to 2009. The research results show that the relative variation of the aggregated emission factor of electricity was larger than that of the primary energy factor of electricity between 1982 and 2009. The major component of the change ratio of the aggregated primary energy factor of electricity from 1982 to 2009 was the generation mix effect. The pure primary energy coefficient effect of the electricity generation on the change ratio of the aggregated primary energy factor of electricity was positive too, but the effect was small. In regard to the analysis of the emission factor, the aggregated emission factor of electricity can be decomposed as the emission coefficient effect and the share effect of fossil fuel generation. The research results show that the share effect of fossil fuel generation was the major effect of the change ratio of the emission factor from 1982 to 2009. The emission coefficients of fossil fuel generation also had a positive effect on the aggregated emission factor of electricity, but the effect was also limited.

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#### APPENDIX 1: DERIVATION OF EQ. 5

Multiply  $\frac{1}{f}$  both sides of the Eq. 4, we attain,

$$\begin{aligned}
 [A1] \quad \frac{df}{d\tau} \frac{1}{f} &= \sum_i \frac{df_i}{d\tau} \frac{s_i}{\sum_i f_i s_i} + \sum_i \frac{ds_i}{d\tau} \frac{f_i}{\sum_i f_i s_i} \\
 &= \sum_i \frac{df_i/d\tau}{f_i} \frac{f_i s_i}{\sum_i f_i s_i} + \sum_i \frac{ds_i/d\tau}{s_i} \frac{f_i s_i}{\sum_i f_i s_i} \\
 \therefore \frac{d \ln f_i(\tau)}{d\tau} &= \frac{1}{f_i(\tau)} f_i'(\tau), \\
 \frac{d \ln s_i(\tau)}{d\tau} &= \frac{1}{s_i(\tau)} s_i'(\tau); \\
 \therefore \frac{d \ln f_i}{d\tau} &= \frac{df_i/d\tau}{f_i}, \text{ and } \frac{d \ln s_i}{d\tau} = \frac{ds_i/d\tau}{s_i}.
 \end{aligned}$$

Let  $w_i$  denotes the weighting  $\frac{f_i s_i}{\sum_i f_i s_i}$ . The Eq. [A2]

can be rewritten as follows,

$$\begin{aligned}
 \text{[A2]} \quad \frac{df}{d\tau} \frac{1}{f} &= \sum_i \frac{d \ln f_i}{d\tau} \frac{f_i s_i}{\sum_i f_i s_i} + \sum_i \frac{d \ln s_i}{d\tau} \frac{f_i s_i}{\sum_i f_i s_i} \\
 &= \sum_i w_i \frac{d \ln f_i}{d\tau} + \sum_i w_i \frac{d \ln s_i}{d\tau} \\
 &= \sum_i w_i \left( \frac{d \ln f_i}{d\tau} + \frac{d \ln s_i}{d\tau} \right).
 \end{aligned}$$

The logarithmic change rate of Divisia primary energy factor of electricity can be expressed as  $\frac{d \ln f}{d\tau} = \frac{df}{d\tau} \frac{1}{f}$ , so

Eq. [A2] can be re-written as follows,

$$\text{[A3]} \quad \frac{d \ln f}{d\tau} = \sum_i w_i \left( \frac{d \ln f_i}{d\tau} + \frac{d \ln s_i}{d\tau} \right).$$

APPENDIX 2: DERIVATION OF WEIGHTING FACTOR OF ELECTRICITY GENERATION MODE I USED IN EQ. 8

Based on definition of primary energy input per electricity output respect to each generation mode,  $f_i = F_i / E_i$ ; and share of each generation mode,  $s_i = E_i / E_f$ ; we attain

$$f_i s_i = \frac{F_i}{E_i} \frac{E_i}{E_f} = \frac{F_i}{E_f}$$

and  $\sum_i f_i s_i = \sum_i \frac{F_i}{E_i} \frac{E_i}{E_f} = \frac{\sum_i F_i}{E_f} = \frac{F}{E_f}$ . So the weighting factor of generation mode  $i$  used in Eq. 8,  $w_i$ , can be written as follows,

$$\text{[A4]} \quad w_i = \frac{f_i s_i}{\sum_i f_i s_i} = \frac{F_i / E_f}{F / E_f} = \frac{F_i}{F}.$$