The Calculation and Application for Free Flow Tunnel of Normal Depth and Critical Depth

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***Abstract*- Horseshoe cross-section and circular cross-section are commonly used in water diversion project and water delivery project. It is difficult to calculate hydraulic elements of nonstandard horseshoe shape and circular shape because of their complex section. This article gives the formulas of hydraulic elements and iterative formulas of normal depths and critical depths of horseshoe shape section composed of 6 arcs and 2 kinds of irregular circular shape section by mathematical derivation. And on the basis of the relation with depth, normal depth and critical depth, we can estimate the type of water surface profile. It can be applied in calculation of water surface profile in 81.68 km Qinling tunnel of south water to north project. The formulas of normal depth and critical depth and hydraulic elements can be expected to be used in other similar water delivery project.**

Keywords- Hydraulic elements; Normal depth; Critical depth; Water surface profile

I. INTRODUCTION

Horseshoe cross-section and circular cross-section are commonly used in water diversion project and water delivery project. In the hydraulic calculation, the normal depth and the critical depth are the important parameters in engineering, their application is very frequent and their requirement has high precision. In recent years, many scholars from all over the world have already been doing a lot of researches on the simple sections of channels, such as rectangular, trapezoidal and circular cross-section [1-2]. For example, Wang [2] has studied direct computing method for the critical depth of trapezoidal cross-section. But there is less research for some complex cross-sections used in engineering. Ma and others [3-6] have researched the critical depth of standard horseshoe cross-section; Lv [7] and others have researched the normal depth of standard horseshoe cross-section; Zhang [8] and others have researched the normal depth and the critical depth of circular cross-section; all of the above researches are standard geometrical section, such as Standard I and Standard II horseshoe cross-section composed of 4 arcs. There is rather less research for horseshoe shape section composed of 6 arcs and 2 kinds of irregular circular shape section. Therefore, in this paper, firstly computational formula of the normal depth, the critical depth and hydraulic elements of horseshoe shape composed of 6 arcs and 2 kinds of irregular circular shape will be derived, then using these formulas. The water surface profile of Hanjiang-to-Weihe water project in Shanxi Province can be computed. The results can be expected to be used in other similar projects.

The import of Qinling tunnel of Hanjiang-to-Weihe water project locates in the right bank of the confluence downstream the Sanhekou reservoir 300 m, and its export of northern locates in Huangchigou which is the second embranchment of Weihe river, the design discharge is 70 m^3/s; its total length is 81.68 km; its designed longitudinal slope is 1/2500; the bottom elevations at import and export of the tunnel are 537.17 m and 510.0 m gradually. According to the difference of geological conditions, there are 3 types of cross-section as composite lining section of horseshoe, circular composite lining section and circular minus rough section along whole tunnel, the shapes are shown in Fig. 1 and Fig. 2.

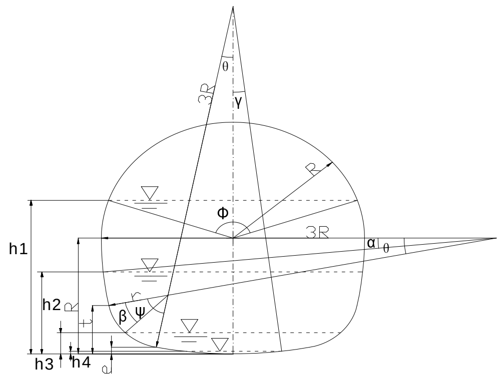
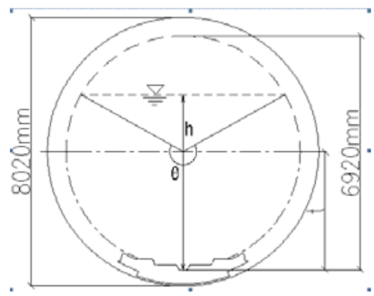
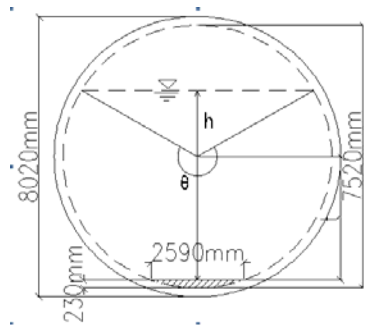


Fig. 1 Different water depths of horseshoe shape composed of 6 arcs



(a) Circular composite lining section



(b) Circular minus rough section

Fig. 2 Tow kinds of irregular circular shape section

II. HYDRAULIC ELEMENTS OF CROSS-SECTION

The horseshoe cross-section is composed of 6 circular arcs, the radius of bottom arch and hollows arch is three times longer than top arch, the radius of underside arch is as 0.444 times as that of top arch, and the hydraulic elements of cross-section varies with different depths, shown in Fig. 1 and Table I. In Fig. 1, R is radius of top arch; $3R$ is radius of bottom arch and hollows arch; r is radius of underside arch; 2θ is central angle of bottom arch; θ (=0.199rad) is central angle of hollows arch; ψ(=5.89θ) is central angle of underside arch; Φ is central angle of water surface; α and β are central angle of hollows arch and underside arch of cross-section; γ is half as large as central angle of bottom arch; angular unit is radian.

TABLE I HYDRAULIC ELEMENTS OF HORSESHOE CROSS SECTION

The range of Water depth	$R \le h < 2R$ $0 < \phi \le \pi$	$t \le h < R$ $0 < \alpha \le \theta$	$e \le h < t$ $0 < \beta \le \psi$	$0 \le h < e$ $0 \le \gamma < \theta$
The flow area A(m^2)	$R^2(3.3635 + 0.5\sin\phi - \frac{\phi}{2})$	$R^2[1.7927 - 9\alpha - \frac{9}{2}\sin(2\alpha) + 12\sin\alpha]$	$R^2(0.314 + 0.197\beta - 0.479\sin\beta - 0.0967\cos\beta - 0.0565\cos 2\beta - 0.0831\sin 2\beta)$	$R^2(9\gamma - 4.5\sin(2\gamma))$
Wetted perimeter χ(m)	$R(6.5720 - \phi)$	$R(-6\alpha + 3.4304)$	$R(2.2356 - 0.8876\beta)$	$6R\gamma$
The width of water h(m)	$R\left[1 + \cos(\frac{\phi}{2})\right]$	$R(1 - 3\sin\alpha)$	$R[0.4943 - 0.4438\sin(\beta + \theta)]$	$3R(1 - \cos\gamma)$
The depth of water B(m)	$2R\sin(\frac{\phi}{2})$	$2R(3\cos\alpha - 2)$	$R(0.9093 + 0.4350\cos\beta - 0.08780\sin\beta)$	$6R\sin\gamma$

The bottoms of circular composite lining section and minus rough section are not circular arc, but rather irregular curve and cushion ply of precast-block, as shown in Fig. 2. In Fig. 2, φ is central angle of wetted perimeter of cross-section, its unit is radian.

The hydraulic elements of horseshoe shape and circular shape under different circumstances can be deduced, as shown in Table I and Table II. In Table II, A' is the area of cushion ply at the bottom of circular tunnel; χ' is the difference between circular arc and subtense; h' is height of precast-block of circular tunnel; d is diameter of circular section.

TABLE II HYDRAULIC ELEMENTS OF CIRCULAR CROSS SECTION

The range of Water depth	**The flow area A(m^2)**	**Wetted perimeter χ(m)**	**The width of water h(m)**	**The depth of water B(m)**
$0 < h < d$	$\frac{1}{8}d^2(\varphi - \sin\varphi) - A'$	$\frac{1}{2}\varphi d - \chi'$	$\frac{1}{2}d\left(1 - \cos\frac{\varphi}{2}\right) - h'$	$d\sin\frac{\varphi}{2}$

III. THE NORMAL DEPTH AND THE CRITICAL DEPTH OF DIFFERENT CROSS-SECTION

The normal depth of channel uniform flow can be calculated by Chezy formula [10]:

$$Q = \frac{1}{n}\frac{A^{\frac{5}{3}}}{\chi^{\frac{2}{3}}}J^{\frac{1}{2}} \tag{1}$$

The flow of open channel has the slow flow, the rapids and the critical flow. The critical flow corresponding to the flow of open channel is minimum flow phenomenon of specific energy. The critical depth can be calculated by the critical flow equation [10]:

$$\frac{\alpha_K Q^2}{g} = \frac{A_K^3}{B_K} \tag{2}$$

Where Q (m^3/s) is discharge; A (m^2) is section area; n is roughness of channel; J is longitudinal slope of the channel; χ (m) is wetted perimeter; A_K (m^2) is flow area of cross-section under the critical flow; B_K (m) is water width of the critical flow; we used g=9.8 m/s^2, which is gravity acceleration; we used α_K=1.0, which is correction coefficient of kinetic energy.

A. The Normal Depth and Critical Depth of Horseshoe Cross-section

1) The Normal Depth:

According to Fig. 1 and Table I, the flow discharges (Q_{R0}, Q_{t0}, Q_{e0}) of three special water depth (R, t, e) can be computed by Eq. 1 as shown in Table III. Actual flow discharge and its normal depth and the discharge of the three special water depth may have the following four cases: ① if $Q \ge Q_{R0}$, $h_0 \ge R$; ② if $Q_{t0} \le Q < Q_{R0}$, $t \le h_0 < R$; ③ if $Q_{e0} \le Q < Q_{t0}$, $e \le h_0 < t$; ④ if $0 \le Q < Q_{e0}$, $0 \le h_0 < e$. So we can judge the location of the normal depth of the flow by comparing actual discharge with Q_{R0}, Q_{t0}, Q_{e0}, then the formula of the normal depth can be given by applying the corresponding hydraulic elements and Eq. 1.

TABLE III FLOW FORMULA FOR 3 SPECIAL POSITION OF THE NORMAL DEPTH IN HORSESHOE CROSS SECTION

The normal depth (m)	$h_0 = R$	$h_0 = t$	$h_0 = e$
The Corresponding flow (m^3/s)	$Q_{R0} = 1.1631\frac{R^{\frac{8}{3}}J^{\frac{1}{2}}}{n}$	$Q_{t0} = 0.2701\frac{R^{\frac{8}{3}}J^{\frac{1}{2}}}{n}$	$Q_{e0} = 0.005\frac{R^{\frac{8}{3}}J^{\frac{1}{2}}}{n}$

According to material of the project, the slope of tunnel J=1/2500, roughness n=0.014, the radius of top arch R=3.38 m and the flow Q=70 m^3/s, we can get the discharge at three special normal depths (R, t, e) by the formula listed in Table III as Q_{R0}=42.75 m^3/s, Q_{t0}=9.93 m^3/s, Q_{e0}=0.202 m^3/s. Because of $Q>Q_{R0}$, $h_0>R$. Substituting A, χ obtained under the condition of $h_0>R$ into Eq. 1, we can get iterative formula as:

$$\Phi^{(j+1)} = 2\times\left[3.3635+0.5\sin\Phi^{(j)}-\left(\frac{nQ}{\sqrt{J}}\right)^{0.6}\frac{\left(6.5720-\Phi^{(j)}\right)^{0.4}}{R^{1.6}}\right]$$

(initial value $\Phi=\pi$) (3)

Where variable superscript ($j+1$ or j) is the number of iterations; the meaning of other symbols are the same as above.

We can get Φ=2.223 rad, taking it into water depth formula of $h=R\left[1+\cos\left(\frac{\Phi}{2}\right)\right]$, the normal depth, h_0, of the horseshoe cross-section can be determined as 4.879 m.

2) The Critical Depth:

The process of calculating critical depth is similar to that of normal depth. Discharge formula of Q_{RK}, Q_{tK}, Q_{eK} at three special water depth (R, t, e) can be derived as shown in Table 4 by Eq. 2. There are four cases between the actual discharge Q and that of three special water depths: ① if $Q\ge Q_{RK}$, $h_K\ge R$; ② if $Q_{tK}\le Q<Q_{RK}$, $t\le h_K<R$; ③ if $Q_{eK}\le Q<Q_{tK}$, $e\le h_K<t$; ④ if $0\le Q<Q_{eK}$, $0\le h_K<e$. So we can judge the location of the critical depth of the flow by comparing actual flow and Q_{RK}, Q_{tK}, Q_{eK}, and then the formula of the critical depth can be given by applying the corresponding hydraulic elements and Eq. 2.

TABLE IV FLOW FORMULA FOR 3 SPECIAL POSITION OF THE CRITICAL DEPTH IN HORSESHOE CROSS SECTION

The critical depth (m)	$h_K=R$	$h_K=t$	$h_K=e$
The Corresponding flow (m^3/s)	$Q_{RK}=1.6973\sqrt{gR^5}$	$Q_{tK}=0.3638\sqrt{gR}$	$Q_{eK}=0.00942\sqrt{gR}$

According to the date of the project, we can get Q_{RK}, Q_{tK} and Q_{eK} at three critical depths (R, t, e) as 115.60 m^3/s, 23.92 m^3/s and 0.619 m^3/s by using the formula listed in Table IV. Because of $Q_{tK}<Q<Q_{RK}$, $t<h_K<R$. Substituting A, χ obtained under the condition of $t\le h_K<R$ into Eq. 2, we can get following formula:

$$\alpha^{(j+1)}=\frac{1}{9}\left[1.7927-\frac{9}{2}\sin(2\alpha^{(j)})+12\sin\alpha^{(j)}-\sqrt[3]{\frac{2Q^2(3\cos\alpha^{(j)}-2)}{gR^5}}\right]$$

(initial value $\alpha=\theta$) (4)

We can get α=0.081 rad. Substituting α into Eq. 2 and the corresponding hydraulic elements in Table I, the critical depth h_K of the horseshoe cross-section can be obtained as 2.56 m.

B. The Normal Depth and Critical Depth of Irregular Circular Cross-section

Substituting the hydraulic elements of circular cross-section in Table II into Eq. 1 and Eq. 2, we can get:

$$f_0(\varphi)=\left(\frac{nQ}{\sqrt{J}}\right)^{0.6}\left(\frac{1}{2}\varphi d-\chi_0\right)^{0.4}-\frac{1}{8}d^2(1-\cos\varphi)+A' \quad (5)$$

$$f_K(\varphi)=\theta-8\left(\frac{Q^2}{gd^5}\sin\frac{\varphi}{2}\right)^{\frac{1}{3}}-8\frac{A'}{d^2}-\sin\varphi \quad (6)$$

Let $\left(\frac{nQ}{\sqrt{J}}\right)^{0.6}=M$, $\frac{Q^2}{gd^5}=N$, taking them into the Eq. 5 and Eq. 6, and applying Newton iterative formula, we can get:

The equation of uniform flow:

$$\varphi_0^{(j+1)}=\varphi_0^{(j)}-\frac{M\left(\frac{1}{2}\varphi_0^{(j)}d-\chi'\right)^{0.4}-\frac{1}{8}d^2\left(\varphi_0^{(j)}-\sin\varphi_0^{(j)}\right)+A'}{0.2dM\left(\frac{1}{2}\varphi_0^{(j)}d-\chi'\right)^{-0.6}-\frac{1}{8}d^2\left(1-\cos\varphi_0^{(j)}\right)} \quad (7)$$

The equation of critical flow:

$$\varphi_K^{(j+1)}=\varphi_K^{(j)}-\frac{\varphi_K^{(j)}-8\left(N\sin\frac{\varphi_K^{(j)}}{2}\right)^{\frac{1}{3}}-8\frac{A'}{d^2}-\sin\varphi_K^{j}}{1-\frac{4}{3}N^{\frac{1}{3}}\left(\sin\frac{\varphi_K^{(j)}}{2}\right)^{-\frac{2}{3}}\cos\frac{\varphi_K^{(j)}}{2}-\cos\varphi_K^{(j)}} \quad (8)$$

The meaning of symbols from Eq. 7 and Eq. 8 are the same as above.

The circular cross-section of the project has circular composite lining section and circular minus rough section. The diameter of the former is 6.92 m, A'=0.096 m^2, χ'= -0.304 m, h'=0 m; the diameter of the latter is 7.52 m, A'=0.400 m^2, h'=-0.054 m, h'=0.23 m; the values of n, Q, i are the same as above. Taking them into Eq. 7 and Eq. 8, we get the angle of φ_0 and φ_K are 5.237 rad and 4.578 rad, separately at the normal depth and the critical depth of circular composite lining section, and φ_0 and φ_K are 4.578 rad and 2.671 rad, separately at the normal depth and the critical depth of circular minus rough section. Taking them into the formula of $h=\frac{1}{2}d\left(1-\cos\frac{\varphi}{2}\right)-h'$, we get the normal depths which are 2.701 m and 2.671 m of circular composite lining section and minus rough section, and the critical depths which are 5.237 m and 5.783 m.

IV. THE CALCULATION OF WATER SURFACE PROFILE

It is primary mission in the design of overland free flow tunnel to determine the normal depth, the critical depth, the depth of transition section, and the depth at back water and drop down water. For some cross-section with simple geometry, e.g. rectangular and trapezoidal, several useful calculation methods have been put forward. At present, Excel tool has been widely used due to its higher precision and efficiency. In this paper, by means of Excel tool, spread sheet for calculating the water surface profile was created.

The method of piecewise trial was used to calculate water surface profile. Firstly total length of the tunnel needs to be divided into several subsections. Then using the data of the known section (downstream), water level was determined by Eq. 9.

$$\Delta s = \frac{\Delta E_s}{i - \overline{J}} = \frac{E_{sd} - E_{su}}{i - \overline{J}} \tag{9}$$

In which, E_s is specific energy in cross-section, can be expressed as $E_s = h + \frac{\alpha v^2}{2g} = h + \frac{\alpha}{2g}\left(\frac{Q}{A}\right)^2$; $\overline{J}$ is mean value of hydraulic gradient, can be expressed as $\overline{J} = \frac{1}{2}(J_d + J_u)$; J is hydraulic slope, can be expressed $J = \frac{v^2}{C^2 R}$; C is Chezy coefficient, $C = \frac{1}{n} R^{\frac{1}{6}}$; R is hydraulic radius, $R = \frac{A}{\chi}$; ΔE is the difference between E_{sd} and E_{su}; E_{sd} and E_{su} are specific energies in downstream and upstream cross-section respectively; i denotes channel slope; v denotes average velocity; the meaning of other symbols are the same as above.

Since total length of the tunnel and the depth in the lowest section are known, we can divide total length S into n equal intervals called sub-section. In the lowest sub-section, because downstream depth is known, and upstream (inlet) depth can be assumed, then, by substituting them into Eq. 9, ΔS can be obtained. If the value of ΔS equals to the length of the nth sub-section, we can enter to calculate the (n-1)th sub-section; if not, a new value of water depth needs to be reassumed, until ΔS equals to the length of the nth sub-section, repeating as the above in this manner for each sub-section at last, water depth of the tunnel inlet can be gained finally.

The type of water surface profile must be judged before calculation is carried on. It is well known that flow regime can be classified by comparing h_0 and h_K. If h_0 is smaller or larger than h_K, the flow regime is supercritical or subcritical flow; if h_0=h_K, the flow is critical flow. Because the uniform flow water depth, h_0, in each section is greater than the critical depth, h_K, namely, h_0(=4.879 m)>h_K(=2.56 m) in horseshoe cross-section, h_0(=5.237 m)>h_K(=2.701 m) in circular composite lining section, and h_0(=4.578 m)>h_K(=2.671 m) in circular minus rough section, therefore, the flow regime in whole tunnel can be judged as subcritical flow.

Fig. 3 is the results of water surface profile of horseshoe cross-section. In Fig. 3, it is shown that the water surface profile is back water cure, and cross-section ponds with the result obtained from qualitative analysis, the ratio of headroom calculated is greater than 15%. It illustrates that the design of the tunnel satisfies the specification's demand.

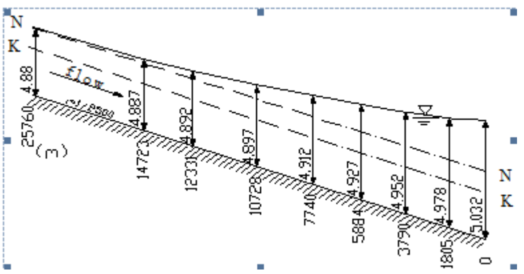


Fig. 3 The water surface profile from import to export of horseshoe cross section

V. CONCLUSIONS

It is of great importance for water delivery project to calculate water surface profile of the tunnel in this paper. Hydraulic elements of same typical cross-sections like horseshoe shape composed of 6 arcs and 2 kinds of irregular circular shape were proposed by mathematical derivation. Then the iterative formula for resolving normal depth and critical depth of the flow were derived. Finally, using these formulas obtained, the normal depth and critical depth of Qinling tunnel were calculated. Furthermore, with the help of Excel tool and hydraulic theory, the water surface profile of the tunnel was computed. Result gained in this paper is expected to be used in other similar water delivery project.

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