Effect of Normal Stress and Relative Compaction on Shear Strength Parameters of Cohesionless Soils

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***Abstract*-In most of geotechnical projects linear Mohr-Coulomb envelope is used. In fact, aforementioned envelope is curved for coarse-grained soils and is affected by some factors such as confining pressure and normal stress. The curved strength envelope could be used to study the behavior of deep foundations, earth dams, soil slopes, and the other earth structures in which failure occurs under considerable normal stresses. A deep failure surface may be more critical than a shallow surface when the phenomenon of curved strength envelope is considered, because in fact the internal friction angle decreases with increasing the depth of soil layers and confining pressure. The main purpose of this research is to express the internal friction angle of sands as a function of normal stress and relative compaction. In this study, direct shear tests are performed on air dried and saturated sand samples at different normal stresses to evaluate the variation of internal friction angle with these factors.**

Keywords- Dilatancy; Internal Friction Angle; Normal Stress; Relative Compaction; Stress Level

I. INTRODUCTION

In most geotechnical problems, assessment of shear strength of soil or rock is necessary as they are the most important characteristics of materials. Classical bearing capacity and slope stability analysis problems are examples that their solutions need accurate calculation of shear strength. The most common criterion for calculation of shear strength of materials is linear Mohr-Coulomb that has two components, friction and cohesion. In many practical projects it is assumed that these two components are constant and independent of stress level. Regarding the significance of accurate calculation of shear strength of soils, some researchers have investigated the factors that affect the friction angle. For the first time, Taylor [3] performed triaxial tests on standard Ottawa sand in order to evaluate the effect of normal stress on friction angle and found that for loose state (initial void ratio approximately 0.66) the value of φ decreases from 30° to less than 27° when the confining pressure is increased from 48 kPa to 766 kPa. Similarly, for dense state, φ decreases from approximately 34.5° to about 29° due to the same increase in confining pressure. Many researchers believe that friction angle of coarse-grained soils is composed of two components, basic friction angle and dilatancy, where dilatancy angle is indicator of volume changes of sample during shear and in the case of expansion is considered positive [4, 6, 7, 9]. They mention that dilatancy decreases with increasing normal stress and therefore friction angle decreases. Indraratna [1] explained the effect of normal stress-friction angle relationship on the stability analysis of a rockfill dam and concluded that by considering this effect, the height of rockfill dam can be increased. Maksimovic [5] presented analytical expressions for the nonlinear envelopes in terms of effective stress and classified them in three major groups as power, logarithmic and hyperbolic types and concluded that failure envelope of the hyperbolic type is applicable in wider range of stresses. Hsu [9] offered a constitutive model for studying the uplift behavior of anchors in cohesionless soils in which the effect of normal stress on friction angle was taken into account. Linero [8] performed direct shear tests on rock pile material and concluded that the slope of failure envelope decreases with increasing normal stress.

II. MATERIALS

In this study, three types of sands were used, poorly graded sand from Babolsar (a town in northern Iran) because of numerous projects in that region, standard Ottawa sand in order to compare the results, and a sand with grain-size distribution similar to fine-grained filter of earth dams. Properties and grain-size distribution of these soils are presented in Table 1 and Fig. 1. As can be seen, the main difference between these soils is D_{50}, which implies that Babolsar sand is definitely finer than Ottawa sand and filter. Another important difference is that Babolsar and Ottawa sand are poorly graded while filter is well graded soil.

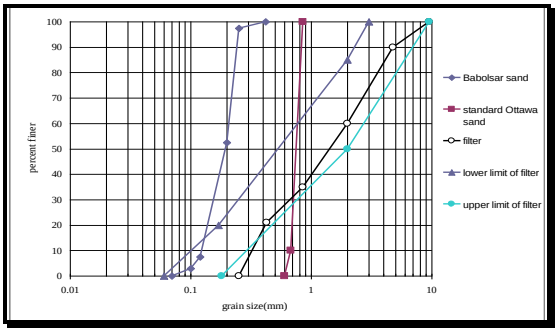

Fig. 1 Grain-size distribution of soils

TABLE 1 PHYSICAL PROPERTIES OF SOILS

Material	G_s	D_{50} **(mm)**	γ_{dmax} **(kN/m^3)**	**USCS Classification**
Babolsar sand	2.772	0.20	16.68	SP
Ottawa sand [2]	2.660	0.74	17.00	SP
filter	2.710	1.50	18.62	SP

III. EXPERIMENTAL PROGRAM

Tests conducted in this study are summarized in Table 2. The shear speeds of tests were 0.50 $\frac{\text{mm}}{\text{min}}$ and 0.25 $\frac{\text{mm}}{\text{min}}$ for air dried and saturated samples, respectively. The air dried samples of Babolsar sand were tested at relative compactions (R_d) of 93% ($\frac{\gamma_{\text{d}}}{\gamma_{\text{dmax}}}$ =0.93), 97%, and 100%, the air dried samples of Ottawa sand were tested just at relative compaction of 93% for the sake of comparing with the technical literature results, the air dried and saturated samples of filter were tested at relative compactions of 93%, 97%, and 100% and all tests were carried out under different normal stresses: 1, 2, 4, 8, and 16 kg/cm^2.

TABLE 2 PERFORMED TESTS ON SOILS

Test	**Standard**	**Parameter**
particle-size analysis	ASTM D 422	grain-size distribution
specific gravity of soil	ASTM D 854	G_s
standard proctor	ASTM D 698	γ_{dmax}
direct shear	ASTM D 3080	shear strength parameters

IV. TEST RESULTS

In this part, results of tests and comparison between them are presented.

A. Failure Envelope

Failure envelopes of Babolsar sand, Ottawa sand, dry filter, and saturated filter at relative compaction of 93% are shown in Figs. 2(a), 2(b), 2(c), and 2(d), respectively, also plots of shear stress against horizontal displacement for these soils at the same relative compaction are shown in Figs. 3(a), 3(b), 3(c), and 3(d), respectively. Failure envelopes of samples at other relative compactions are not presented because this research focuses on variation of internal friction angle and friction coefficient. As can be seen from these figures, the nonlinear regression provided a very well fit to the data and is recommended. Failure envelopes were used to calculate the variation of φ with σ_v. For all soils, polynomial type of degree 2 with zero intercept was selected as nonlinear failure envelope because of higher correlation factors. As can be seen, the curved envelope of Babolsar sand almost goes along with the linear envelope, while for Ottawa sand and filter, the difference between two types of envelopes is obvious. The reason is that with increasing normal stress, the grains of Ottawa sand and filter significantly crush during shear and dilatancy decreases, but for Babolsar sand this phenomenon can be ignored.

Equations resulting from linear and nonlinear regression for tested soils at relative compaction of 93% are summarized in Table 3. In these equations τ and σ_v are shear and normal stresses in kg/cm^2, respectively. Regarding the fact that all materials are completely sandy, intercept of linear envelopes (cohesion) are expected to be approximately zero. In linear regression, the cohesion achieved for Babolsar sand was almost negligible: 0.062 kg/cm^2 and can be accepted, while the cohesion achieved for Ottawa sand and filter were: 0.661 kg/cm^2 and 0.758 kg/cm^2, respectively, which cannot be accepted for sands.

The reason is that at high normal stresses, the end points of linear envelope fall and cause the line to incline to horizontal axis, so the existing line crosses the vertical axis at higher value of cohesion. Failure envelopes of Babolsar sand and filter at other relative compactions are not presented because of space limitations, but the behavior is similar.

B. Variation of Internal Friction Angle with Normal Stress and Relative Compaction

Using nonlinear failure envelopes, values of friction angle were calculated at each normal stress and relative compaction and then two variable functions of φ were defined in terms of σ_v and R_d, which are tabulated in Table 4. In these equations φ is friction angle in degrees, R_d is relative compaction (decimal), and σ_v is normal stress in kg/cm^2. These equations were plotted and are shown for Babolsar sand, dry filter, and saturated filter in Figs. 4, 5, and 6, respectively. It can be seen from these figures that internal friction angle decreases with increasing normal stress and decreasing relative compaction. For instance, at relative compaction of 93%, friction angles of Babolsar sand, dry filter, and saturated filter decreased 2.6˚(from 38.1˚ to 35.5˚), 18.5˚(from 49.9˚ to 31.4˚), and 19.8˚(from 49.3˚ to 29.5˚), respectively, as the normal stress increased from 1 kg/cm^2 to 16 kg/cm^2.

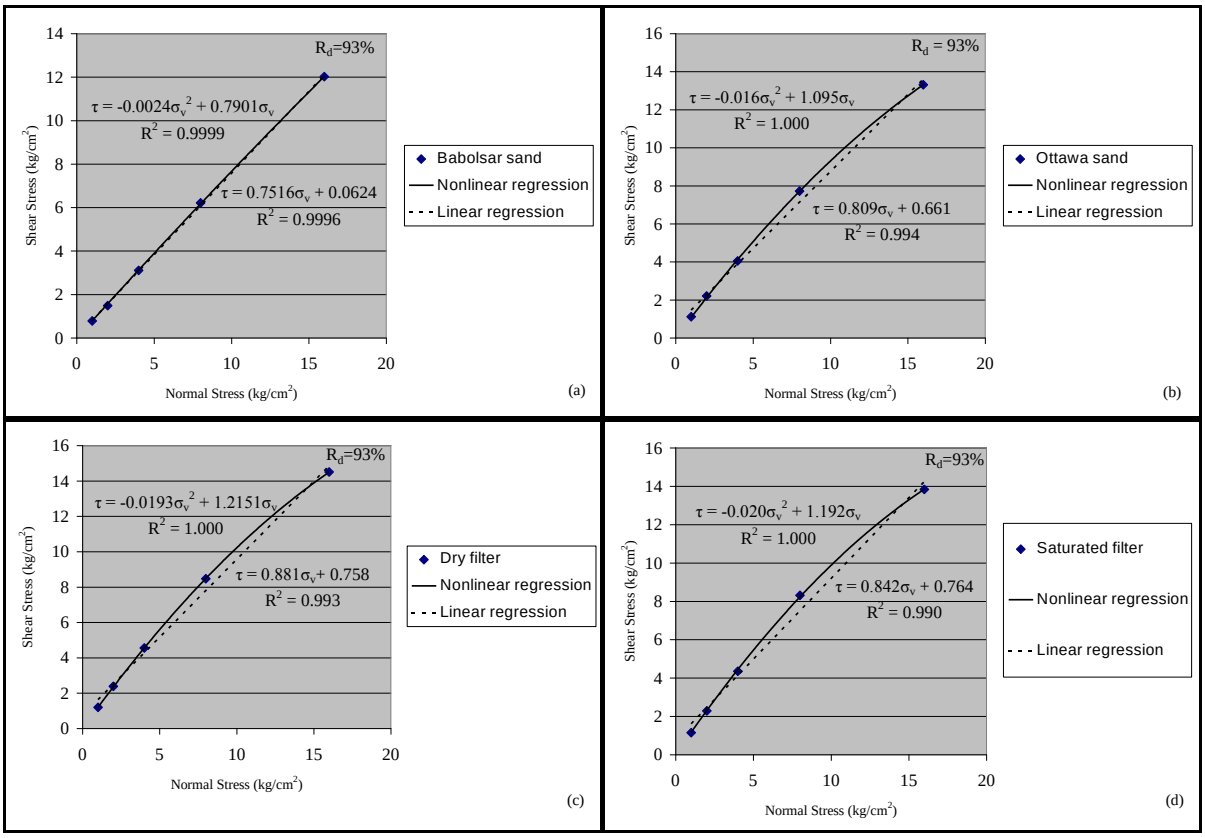

Fig. 2 Linear and nonlinear regression of failure envelope for (a) Babolsar sand (b) Ottawa sand (c) dry filter (d) saturated filter

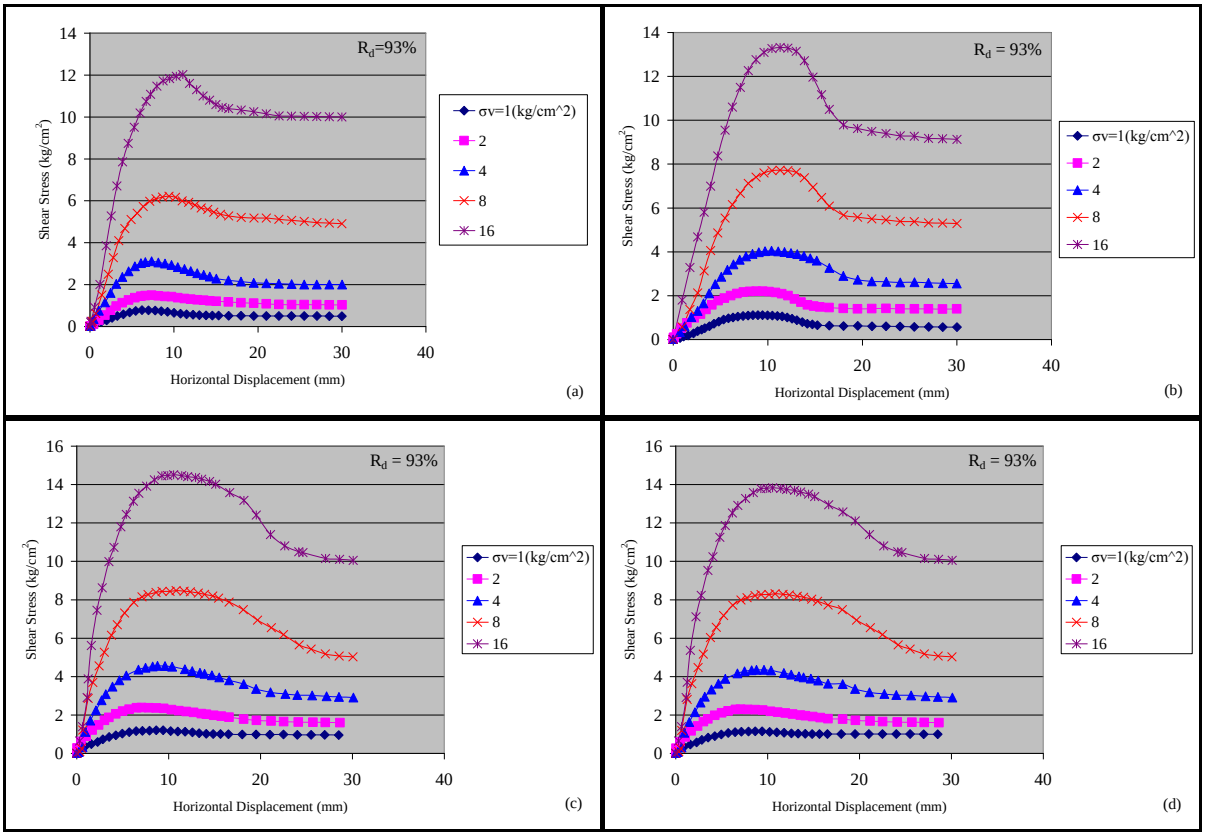

Fig. 3 Shear stress versus horizontal displacement for (a) Babolsar sand (b) Ottawa sand (c) dry filter (d) saturated filter

TABLE 3 ENVELOPE EQUATIONS FOR LINEAR AND NONLINEAR REGRESSION

Soil	Linear Regression	Nonlinear Regression
Babolsar sand	$\tau = 0.752\sigma_v + 0.062$ (R^2=0.9996)	$\tau = -0.002\sigma_v^2 + 0.790\sigma_v$ (R^2=0.9999)
Ottawa sand	$\tau = 0.809\sigma_v + 0.661$ (R^2=0.994)	$\tau = -0.016\sigma_v^2 + 1.095\sigma_v$ (R^2=1.000)
dry filter	$\tau = 0.881\sigma_v + 0.758$ (R^2=0.993)	$\tau = -0.019\sigma_v^2 + 1.215\sigma_v$ (R^2=1.000)
saturated filter	$\tau = 0.842\sigma_v + 0.764$ (R^2=0.990)	$\tau = -0.020\sigma_v^2 + 1.192\sigma_v$ (R^2=1.000)

In other words, due to 15 times increase in σ_v, friction angles of Babolsar sand, dry filter, and saturated filter decreased about 7%, 37%, and 40%, respectively. Also, at normal stress of 1 kg/cm^2, friction angles of Babolsar sand, dry filter, and saturated filter increased 5˚ (from 38.1˚ to 43.1˚), 2.8˚ (from 49.9˚ to 52.7˚), and 2.8˚ (from 49.3˚ to 52.1˚), respectively, as the relative compaction increased from 93% to 100%. It should be noted that aforementioned function cannot be achieved for Ottawa sand, because this soil was only tested at relative compaction of 93%.

TABLE 4 TWO VARIABLE FUNCTIONS OF φ IN TERMS OF σ_v AND R_D

Soil	Function
Babolsar sand	$\varphi = (-7.857R_d^2 + 13.67R_d - 6.101)\sigma_v + (72.24R_d - 28.87)$
dry filter	$\varphi = (-1.309R_d^2 + 2.413R_d - 1.134)\sigma_v^2 + (-5.119R_d^2 + 9.551R_d - 5.309)\sigma_v + (69.04R_d^2 - 94.69R_d + 78.84)$
saturated filter	$\varphi = (-0.595R_d^2 + 1.006R_d - 0.445)\sigma_v^2 + (-7.142R_d^2 + 13.37R_d - 7.161)\sigma_v + (-82.14R_d^2 + 197.8R_d - 62.97)$

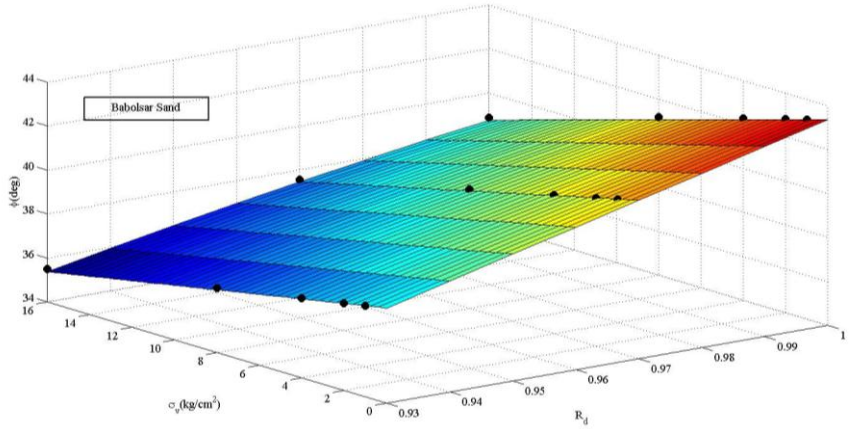

Fig. 4 Dependence of φ on σ_v and R_d for Babolsar sand

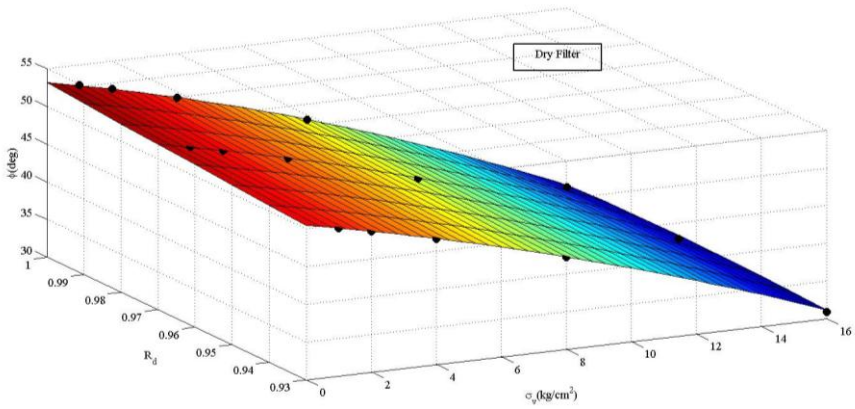

Fig. 5 Dependence of φ on σ_v and R_d for dry filter

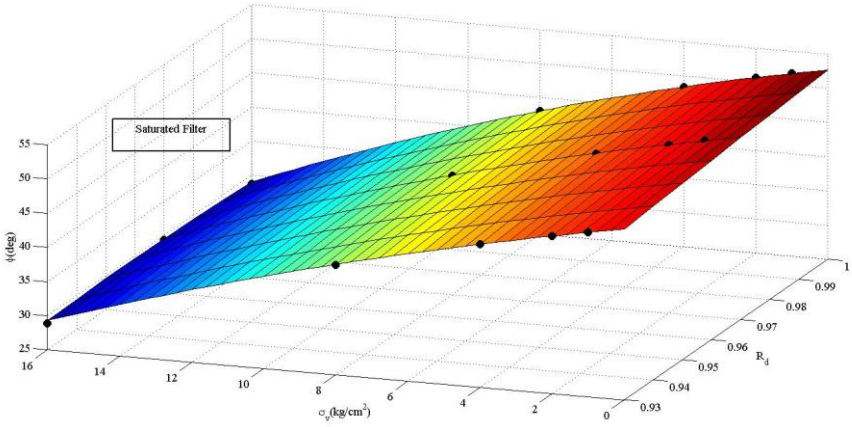

Fig. 6 Dependence of φ on σ_v and R_d for saturated filter

C. Variation of Friction Coefficient with Normal Stress and Relative Compaction

Friction coefficient (μ) is an important mechanical parameter which can control the mechanism of shear failure and equals tanφ. Two variable functions of μ were defined in terms of σ_v and R_d, which are tabulated in Table 5. In these equations μ is friction coefficient, R_d is relative compaction (decimal), and σ_v is normal stress in kg/cm^2. These equations were plotted and are shown for Babolsar sand, dry filter, and saturated filter in Figs. 7, 8, and 9, respectively. It can be seen from these figures that friction coefficient decreases with increasing normal stress and decreasing relative compaction. For instance, at relative compaction of 93%, friction coefficients of Babolsar sand, dry filter, and saturated filter decreased 0.072 (from 0.785 to 0.713), 0.579 (from 1.177 to 0.598), and 0.600 (from 1.152 to 0.552), respectively, as the normal stress increased from 1 kg/cm^2 to 16 kg/cm^2. In other words, due to 15 times increase in σ_v, friction coefficients of Babolsar sand, dry filter, and saturated filter decreased about 9%, 49%, and 52%, respectively. Also, at normal stress of 1 kg/cm^2, friction coefficients of Babolsar sand, dry filter, and saturated filter increased 0.15 (from 0.785 to 0.935), 0.118 (from 1.177 to 1.295), and 0.117 (from 1.152 to 1.269), respectively, as the relative compaction increased from 93% to 100%.

TABLE 5 TWO VARIABLE FUNCTIONS OF μ IN TERMS OF σ_v AND R_d

Soil	**Function**
Babolsar sand	$\mu = (-0.238R_d{}^2 + 0.402R_d - 0.173)\sigma_v + (2.195R_d - 1.252)$
dry filter	$\mu = (-0.476R_d{}^2 + 0.804R_d - 0.374)\sigma_v + (5R_d{}^2 - 7.85R_d + 4.191)$
saturated filter	$\mu = (-0.476R_d{}^2 + 0.804R_d - 0.376)\sigma_v + (-2.142R_d{}^2 + 5.921R_d - 2.461)$

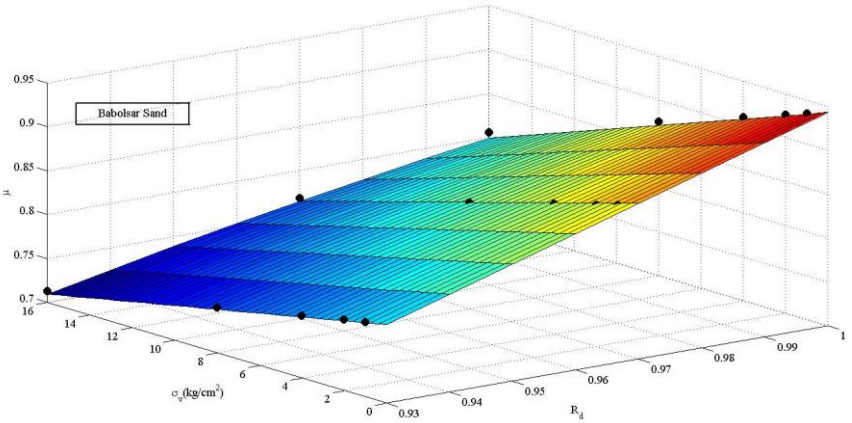

Fig. 7 Dependence of μ on σ_v and R_d for Babolsar sand

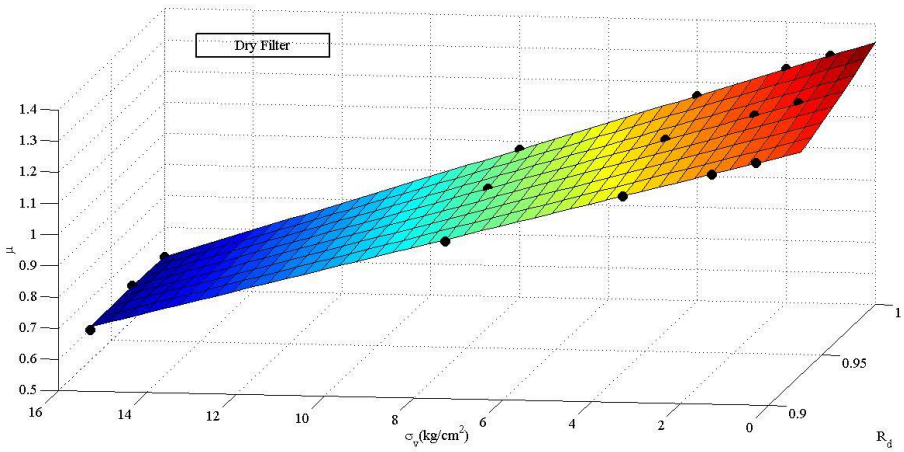

Fig. 8 Dependence of μ on σ_v and R_d for dry filter

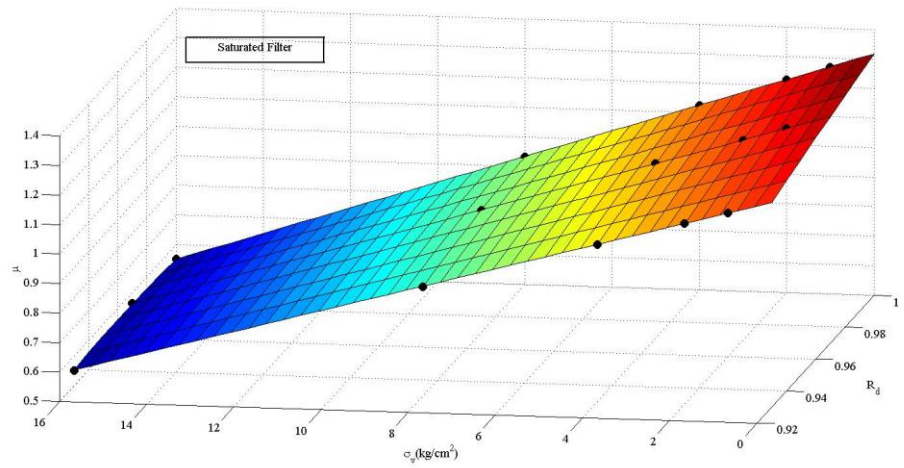

Fig. 9 Dependence of μ on σ_v and R_d for saturated filter

V. CONCLUSION

In this study, effect of normal stress and relative compaction on internal friction angles and friction coefficients of Babolsar sand, Ottawa sand, and sand with grain-size distribution similar to fine-grained filter of earth dams was investigated. Using test results, following conclusions can be made:

- Internal friction angle decreases with increasing normal stress. For instance, at relative compaction of 93%, friction angles of Babolsar sand, dry filter, and saturated filter decreased 2.6˚ (from 38.1˚ to 35.5˚), 18.5˚ (from 49.9˚ to 31.4˚), and 19.8˚ (from 49.3˚ to 29.5˚), respectively, as the normal stress increased from 1 kg/cm^2 to 16 kg/cm^2. In other words, due to 15 times increase in σ_v, friction angles of Babolsar sand, dry filter, and saturated filter decreased about 7%, 37%, and 40%, respectively.
- Friction coefficient decreases with increasing normal stress. For instance, at relative compaction of 93%, friction coefficients of Babolsar sand, dry filter, and saturated filter decreased 0.072 (from 0.785 to 0.713), 0.579 (from 1.177 to 0.598), and 0.600 (from 1.152 to 0.552), respectively, as the normal stress increased from 1 kg/cm^2 to 16 kg/cm^2. In other words, due to 15 times increase in σ_v, friction coefficients of Babolsar sand, dry filter, and saturated filter decreased about 9%, 49%, and 52%, respectively.
- Internal friction angle increases with increasing relative compaction. For instance, at normal stress of 1 kg/cm^2, friction angles of Babolsar sand, dry filter, and saturated filter increased 5˚ (from 38.1˚ to 43.1˚), 2.8˚ (from 49.9˚ to 52.7˚), and 2.8˚ (from 49.3˚ to 52.1˚), respectively, as the relative compaction increased from 93% to 100%.
- Friction coefficient increases with increasing relative compaction. For instance, at normal stress of 1 kg/cm^2, friction coefficients of Babolsar sand, dry filter, and saturated filter increased 0.15 (from 0.785 to 0.935), 0.118 (from 1.177 to 1.295), and 0.117 (from 1.152 to 1.269), respectively, as the relative compaction increased from 93% to 100%.
- The difference between curved envelope and linear envelope increased with increasing D_{50} (mean particle diameter).

- With increasing D_{50}, decrease in internal friction angle became more noticeable, because grain crushing increased and dilatancy became more limited.
- Saturating the specimens caused the nonlinear envelopes to have more curvature, so, decrease in internal friction angles and friction coefficients increased. It also caused the linear envelopes to have smaller φ and larger c values compared with linear envelopes of air dried specimens.
- Diagrams of internal friction angle versus normal stress were linear with good approximation and on this basis; diagrams of friction coefficient versus normal stress were also linear.
- Proposed two variable functions can be used to estimate the internal friction angle and friction coefficient by substituting the values of normal stress and relative compaction and therefore no additional test is needed.
- It is better to use curved envelope instead of linear envelope for sands, because it provides a much better fit to the data.

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