Accelerating Bulk Viscous FRW Universe in a Scale-Covariant Theory of Gravitation

Anirudh Pradhan[1], Anand Shankar Dubey[2], Rajeev Kumar Khare[3]

[1]Department of Mathematics, Hindu Post-graduate College, Zamania-232 331, Ghazipur, India

[2, 3]Department of Mathematics, Sam Higginbottom Institute of Agriculture, Technology & Sciences Allahabad-211 007, India

[1]pradhan.anirudh@gmail.com; [2]asdubey77@yahoo.com; [3]drrajeevkhare@gmail.com

***Abstract-* The present study deals with cosmological models of FRW universe in presence of bulk viscous fluid source in the scale-covariant theory of gravitation formulated by Canuto et al. [Phys. Rev. Lett. 39:429, 1977]. A new class of solutions for the modified Einstein's field equations is obtained by considering time dependent deceleration parameter. It is observed that the models are accelerating at present epoch. Some physical aspects of the models are also discussed in this paper.**

Keywords-* *FRW Universe; Variable Deceleration Parameter; Scale-Covariant Theory; Accelerating Models

I. INTRODUCTION

Alternative theories of gravitation have been extensively studied in connection with their cosmological entailment. One such class of theories is the scalar-tensor theories (STT) of gravity. This class of models had renewed interest in recent times for two main reasons. First, the new inflationary scenario as the extended inflation has a scalar field that solves several of problems present in the old theories. Second, string theories and other unified theories contain a scalar field, which plays a similar role to scalar field of the STT. The scalar-tensor theories started with the work Jordan in 1950 [1].

Note-worthy among them are scalar-tensor theories of gravitation formulated by Brans and Dicke [2], Nordtvedt [3], Sen [4], Sen and Dun [5], Wagoner [6] and Saez&Ballester [7]. Another interesting viable alternative to general relativity is the theory proposed by Canuto et al. [8] which is the so-called scale-covariant theory of gravitation. In recent years there has been a considerable interest in this theory. These are theories in which the field equations are invariant not only under groups of transformations, as in general relativity, but also under scale transformations. Dirac revived the more general geometry of Weyl in an effort to extend the invariance of the fundamental equation to changes in scale. The formalism of scale-covariant theories was later developed by Canuto [8] [14]. A particular feature of this theory is that no independent equation for the gauge function Φ exists, and therefore it cannot be determined within the theory. One must go outside the theory in order to specify the form of Φ as well as its relationship with G. External conditions must be provided based on physical considerations, such as Dirac LNH or CMBR. The possibilities that have been considered for gauge function Φ are $\Phi(t) = t^{\varepsilon}$, $\varepsilon = \pm 1, \pm ½$ [9,15].

According to the above scale-covariant theory, Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The components of metric tensor in the two systems of units are related by a conformal transformation/factor

$$\bar{g}_{ij} = \phi^2 (x^k) g_{ij}, \tag{1}$$

where in Latin indices take values 1, 2, 3, 4. The barred quantities refer to gravitational units and un-barred quantities denote to atomic units. In this theory we have a metric and the scalar gauge function Φ, so Canuto attempted to write the field equations in a form such that their form remains invariant under both arbitrary coordinate and arbitrary-scale transformations. The gauge function Φ, $(0 < \Phi < \infty)$ in its most general expression is function of all space-time coordinates. Thus using the conformal transformation of the type given by equation (1), Canuto et al. [8, 9] transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2} g_{ij} R + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\Phi) g_{ij}, \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}\left(\phi\phi^k_{;k} - \phi^k\phi_k\right). \tag{3}$$

Here R_{ij} is the Ricci tensor. R the Ricci scalar, Λ the cosmological 'constant', G the gravitational 'constant' and T_{ij} the energy momentum tensor. A semicolon denotes covariant derivative and Φ_i denotes ordinary derivatives with respect to x_i. A particular feature of this theory is that no independent equation for Φ exists. The speculations that have been considered for gauge function Φ are (Canuto et al. [9])

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\epsilon}, \epsilon = \pm 1, \pm \frac{1}{2}, \tag{4}$$

where t_0 is constant. The form

$$\phi \sim t^{\frac{1}{2}} \tag{5}$$

is the one most favoured to fit observations (Canuto and Goldman, [16, 17]).

In recent years there has been a lot of interest in the study of scale covariant theory of gravitation with perfect fluid matter distribution as source. Reddy and Venkateswarlu [18] studied Einstein-Rosen universe in the scale-covariant theory of gravitation. Venkateswarlu and Kumar [19] obtained higher dimensional string cosmologies in scale-covariant theory of gravitation. Reddy [20] also obtained a higher dimensional cosmological model in a scale-covariant theory of gravitation. Recently, Adhav et al. [21], Ram et al. [22] and Belinchón [23] studied the scale-covariant theory of gravitation indifferent context.

Cosmological models with bulk viscosity are important since it has a greater role in getting accelerated expansion of the universe popularly known as inflationary phase. At early stages of the universe when neutrinos decoupling occurred, the matter behaved like viscous fluid. The coefficient of viscosity decreases as the universe expands. Misner [24, 25] examined the effect of viscosity on the evolution of the universe and suggested that the strong dissipation, due to the neutrino viscosity, may considerably reduce the anisotropy of the blackbody radiation. Murphy [26] developed a uniform cosmological model filled with fluid which possesses pressure and bulk viscosity exhibiting the interesting feature that the big-bang type singularity appears in the infinite past. The detail review of bulk viscosity is given in references Singh et al. [27], Pradhan [28, 29, 30]. Recently, Singh and Sorokhaibam [31] and Singh et al. [32] and Singh and Devi [33] have studied Friedmann cosmological models with bulk viscosity in a scale-covariant theory of gravitation.

Motivated by the above discussions, in this paper, we study FRW cosmological models by considering a time dependent deceleration parameter in scale-covariant theory of gravitation proposed by Canuto et al. [8, 9]. The paper has following structure. In Section I, the introduction and the field equations are given. In Section II, the metric and field equations are described. The Section III deals with the exact solution of the field equations and physical behavior of the model. At the end we shall summarize the findings in Section IV. This study is relevant in view of the recent scenario of the accelerating of universe.

II. THE METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and isotropic FRW space-time given by

$$ds^2 = dt^2 - a^2(t)[(1-kr^2)^{-1}dr^2 + r^2\,(d\theta^2 + sin^2\theta d\psi^2)], \tag{6}$$

where a(t) is the scale factor and the curvature constants k are -1,0,+1 for open, flat and closed models of the universe respectively.

The energy-momentum tensor T_j^i for bulk viscousfluid distribution is taken as

$$T_j^i = (\bar{p} + \rho)u^i u_j - \bar{p} g_j^i, \tag{7}$$

together with comoving co-ordinates u^i= (0, 0, 0, 1)and

$$u^i u_i = 1, v^i v_j = 0, \tag{8}$$

$$\bar{p} = p - \xi u^i_{;i}, \tag{9}$$

where u^i is the 4-velocity vector of the cosmic fluid, $\bar{p}$, p and ρ are the effective pressure, isotropic pressure and energy density of matter respectively, ξ is the coefficient of bulk viscosity which is a function of time t. Here ρ, p and Φ are homogeneous function of cosmic time t.

The expansion scalar is given by

$$\theta = u^i_{;i} = 3\frac{\dot{a}}{a}. \tag{10}$$

Using Eqs. (8), (9), (10) and comoving coordinates for the line-element (6), the modified Einstein field equations (2) and (3) lead to the following two independent equations

$$\frac{2\ddot{a}}{a} + \left[\frac{\dot{a}^2 + k}{a^2}\right] - 4\left(\frac{\dot{\phi}\dot{a}}{\phi a}\right) + \left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{2\ddot{\phi}}{\phi} = -G\left(p - 3\xi\frac{\dot{a}}{a}\right), \tag{11}$$

$$3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] - 6\left(\frac{\dot{\phi}\dot{a}}{\phi a}\right) - 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = G\rho. \tag{12}$$

The overhead dots represent time derivative and double dots represent double differentiation with respect to time t.

The usual energy momentum conservation relation $T^{ij}_{;j} =$ 0leads to

$$\dot{\rho} + 3\left(p + \rho - 3\xi\frac{\dot{a}}{a}\right)\frac{\dot{a}}{a} = 0. \tag{13}$$

In this paper, we consider a special form of the gauge function as

$$\phi = \kappa t^{\epsilon}, \tag{14}$$

where κ is arbitrary constant. The equation of stateis taken as

$$p = \gamma\rho, \tag{15}$$

where $-1 \leq \gamma \leq 1$.

III. SOLUTIONS OF THE FIELD EQUATIONS

The function a(t) remains undetermined. One may have to apply additional assumption to solve its explicit dependence on ‘t’. We solve the field equations (11)-(12) by considering the deceleration parameter to be time-dependent. We define the deceleration parameter q as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(t), \tag{16}$$

The motivation to choose such time dependent DP is behind the fact that the universe is accelerated expansion at present and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5.Now for a Universe which was decelerating in pastand accelerating at the present time, the DP must show signature flipping (see the Refs. Padmanabhan and Roychowdhury [34], Amendola [35], Riess et al. [36]). So, in general, the DP is not a constant but time variable.

The equation (16) may be rewritten as

$$\frac{\ddot{a}}{a} + b\frac{\dot{a}^2}{a^2} = 0. \tag{17}$$

In order to solve the Eq. (17), we assume b = b(a).It is important to note here that one can assume b = b(t) = b(a(t)), as a is also a time dependent function. It can be done only if there is a one to one correspondences between t and a. But this is only possible when one avoid singularity like big bang or big rip because both t and a are increasing function.

The general solution of Eq. (17) with assumption b = b(a), is given by

$$\int e^{\int \frac{b}{a}da} = t + m, \tag{18}$$

where m is an integrating constant.

One cannot solve Eq. (18) in general as b is variable. So, in order to solve the problem completely, we have to choose $\int \frac{b}{a}da$ in such a mannerthat Eq. (18) be integrable without any loss of generality. Hence we consider

$$\int \frac{b}{a}da = \ln L(a). \tag{19}$$

which does not affect the nature of generality of solution. Hence from Eqs. (18) and (19), we obtain

$$\int L(a)da = t + m. \tag{20}$$

Of course the choice of L(a), in Eq. (19), is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observations, we consider

$$L(a) = \frac{1}{\alpha\sqrt{1+a^2}}, \tag{21}$$

where α is an arbitrary constant. In this case, on integrating, Eq. (19) gives the exact solution

$$a(t) = \sinh(\alpha T), \tag{22}$$

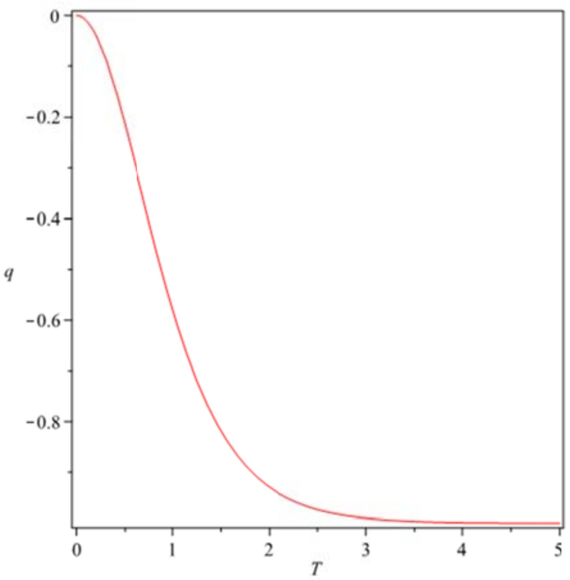

Figure 1 The plot of deceleration parameter q versus T

where T = t + m. We also note that T = 0 and T = ∞ respectively correspond to the proper time t = -m and t = ∞. The relation (22) is recently used by Pradhan et al. [37] in studying dark energy models with anisotropic fluid in Bianchi type-V I_0space-time. Recently, relation (22) is also used by Amirhashchi et al. [38] to study the evolution of dark energy models in a spatially homogeneous and isotropic FRW space-time filled with barotropic fluid and dark energy by considering a time dependent deceleration parameter.

Figure 1 depicts the variation of DP q versus cosmic time T. From the figure, it is observed that the q decreases very rapidly and reaches to value -1, and lastly it remains constant -1 (like de Sitter universe).

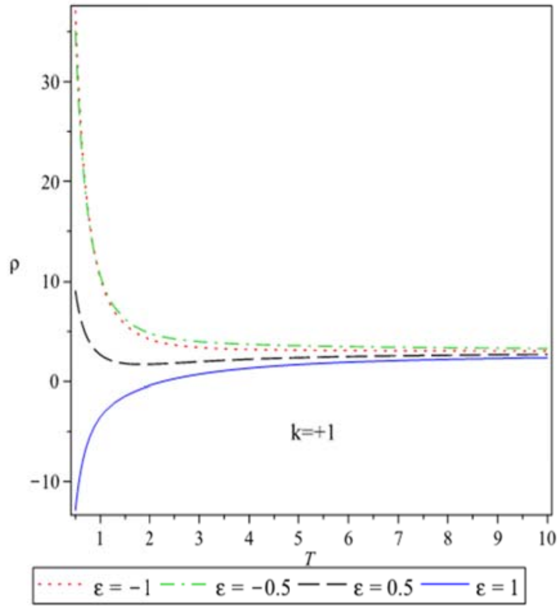

Figure 2 The plot of energy density ρ versus T for closed universe (k = +1). Here G = α =1

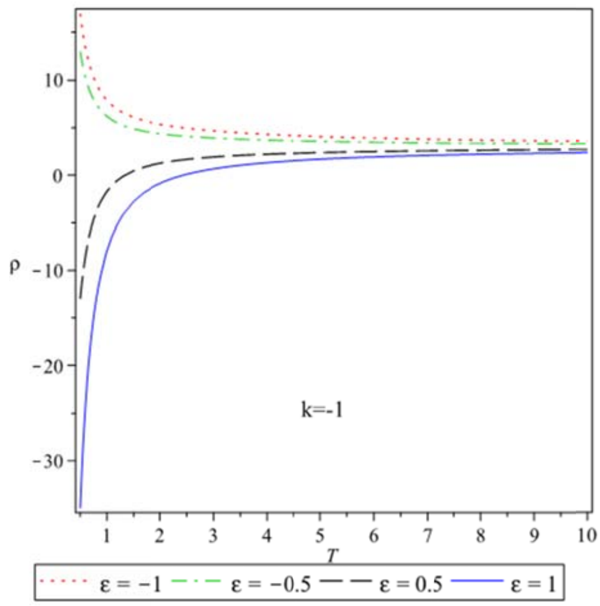

Figure 3 The plot of energy density ρ versus T for open universe (k = -1). Here G = α =1

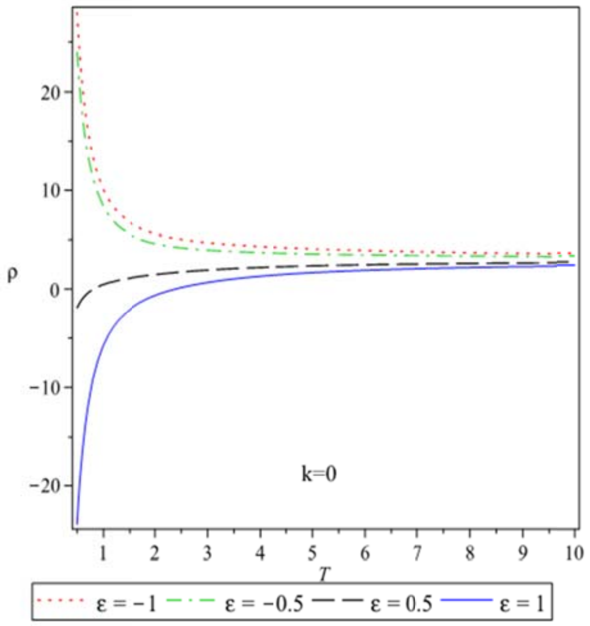

Figure 4 The plot of energy density ρ versus T for flat universe (k = 0). Here G = α =1

Using Eqs. (14), (15) and (22) in (11) - (12), we obtain the expressions of energy density (ρ), pressure (p), and coefficient of bulk viscosity (ξ) as

$$\rho = \frac{3}{G}\left[\frac{\alpha^2}{tanh^2(\alpha T)} + \frac{k}{sinh^2(\alpha T)} - \frac{2\alpha\epsilon}{ttanh(\alpha T)} - \left(\frac{\epsilon}{t}\right)^2\right], \tag{23}$$

$$p = \frac{3\gamma}{G}\left[\frac{\alpha^2}{tanh^2(\alpha T)} + \frac{k}{sinh^2(\alpha T)} - \frac{2\alpha\epsilon}{ttanh(\alpha T)} - \left(\frac{\epsilon}{t}\right)^2\right], \tag{24}$$

$$\xi = \frac{3}{3GH}\left[2\alpha^2 + \frac{(1+3\gamma)\alpha^2}{tanh^2(\alpha T)} + \frac{(1+3\gamma)k}{sinh^2(\alpha T)} - \frac{2(2+3\gamma)\alpha\epsilon}{ttanh(\alpha T)} + \frac{\epsilon\{2-(1+3\gamma)\epsilon\}}{t^2}\right], \tag{25}$$

The expressions for the Hubble parameter H, scalar of expansion θ, the deceleration parameter and the spatial volume for the derived model are given by

$$\theta = 3H = 3\alpha \coth(\alpha T), \tag{26}$$

$$q = -tanh^2(\alpha T), \tag{27}$$

$$V = sinh^3(\alpha T). \tag{28}$$

From Eq. (28), we observe that the spatial volume is zero at T = 0 and it increases with the increase of time. This shows that the universe starts evolving with zero volume at T = 0 and expands with cosmic time T. From Eq. (27), it can be seen that the kinematical parameters H and θ diverge at the initial singularity. There is a point type singularity (MacCallum [42]) at T = 0 in the models. We found that the shear scalar σ is zero. Hence we find $\lim_{t\to 0}\frac{\sigma}{\theta} = 0$. This relation confirms the space-time is isotropic.

We discuss the following three interesting physical cosmological models:

Case I: Radiating Dominated Solution (γ = 1/3)

For γ = 1/3, the disordered radiation corresponds with equation of state ρ = 3p. The physical parameters in terms of cosmic time 'T' have the following expressions:

$$\rho = \frac{3}{G}\left[\frac{\alpha^2}{tanh^2(\alpha T)} + \frac{k}{sinh^2(\alpha T)} - \frac{2\alpha\epsilon}{ttanh(\alpha T)} - \left(\frac{\epsilon}{t}\right)^2\right], \tag{29}$$

$$p = \frac{1}{G}\left[\frac{\alpha^2}{tanh^2(\alpha T)} + \frac{k}{sinh^2(\alpha T)} - \frac{2\alpha\epsilon}{ttanh(\alpha T)} - \left(\frac{\epsilon}{t}\right)^2\right], \tag{30}$$

$$\xi = \frac{1}{3GH}\left[2\alpha^2 + \frac{2\alpha^2}{tanh^2(\alpha T)} + \frac{2k}{sinh^2(\alpha T)} - \frac{6\alpha\epsilon}{ttanh(\alpha T)} + \frac{2\epsilon(1-\epsilon)}{t^2}\right]. \tag{31}$$

Case II: False Vacuum Model (γ = -1)

When γ = -1, we have the degenerate vacuum or false vacuum or ρ vacuum (Cho [39]). Mohanty and Pradhan [40], Singh et al. [32] and Singh and Devi [33] have investigated such type of problem for viscous isotropic scenarios. The physical parameters in terms of cosmic time 'T' have the following expressions:

$$\rho = -p = \frac{3}{G}\left[\frac{\alpha^2}{tanh^2(\alpha T)} + \frac{k}{sinh^2(\alpha T)} - \frac{2\alpha\epsilon}{ttanh(\alpha T)} - \left(\frac{\epsilon}{t}\right)^2\right], \tag{32}$$

$$\xi = \frac{1}{3GH}\left[2\alpha^2 - \frac{2\alpha^2}{tanh^2(\alpha T)} - \frac{2k}{sinh^2(\alpha T)} + \frac{2\alpha\epsilon}{ttanh(\alpha T)} + \frac{2\epsilon(1+\epsilon)}{t^2}\right]. \tag{33}$$

Case III: Stiff Fluid Model (γ = 1)

For γ = 1, the fluid distribution corresponds with the equation of state ρ = p which is known as Zeldovich fluid or stiff fluid model (Zeldovich [41]; Barrow [42]). The physical parameters in terms of cosmic time 'T' have the following expressions:

$$\rho = p = \frac{3}{G}\left[\frac{\alpha^2}{tanh^2(\alpha T)} + \frac{k}{sinh^2(\alpha T)} - \frac{2\alpha\epsilon}{ttanh(\alpha T)} - \left(\frac{\epsilon}{t}\right)^2\right], \tag{34}$$

$$\xi = \frac{1}{3GH}\left[2\alpha^2 + \frac{4\alpha^2}{tanh^2(\alpha T)} + \frac{4k}{sinh^2(\alpha T)} - \frac{10\alpha\epsilon}{t tanh(\alpha T)} + \frac{2\epsilon(1-2\epsilon)}{t^2}\right]. \tag{35}$$

Figure 2 depicts the variation of energy density ρ verses cosmic time T for closed universe (k = +1). From this figure, we observe that for ε = -1, ± 0.5, ρ is a positive decreasing function of time and ultimately it approaches to zero at late time. For ε = 1, ρ is negative in at early time and also an increasing function of time approaching to a small positive constant near zero at late time.

Figure 3 depicts the variation of energy density ρ verses cosmic time T for open universe (k = -1).From Fig. 3, we observe that for ε = -1, 0.5, ρ is a positive decreasing function of time and ultimately it approaches to zero at late time. For ε = 0.5, 1, ρ is negative in at early time and also an increasing function of time approaching to a small positive constant near zero at late time.

Figure 4 graphs the variation of energy density ρ verses cosmic time T for flat universe (k = 0). The nature of energy density for flat universe is found to be similar as for open universe. Thus, in our study for negative values of ε = -1, & -0.5, we obtain physically viable models of the universe for all three open, flat and closed universe.

Figures 5 graphs the variation of the coefficient of bulk viscosity ξ versus cosmic time T for γ = -1in all three open, flat and closed universes. From this figure, we conclude that the case is not physically feasible.

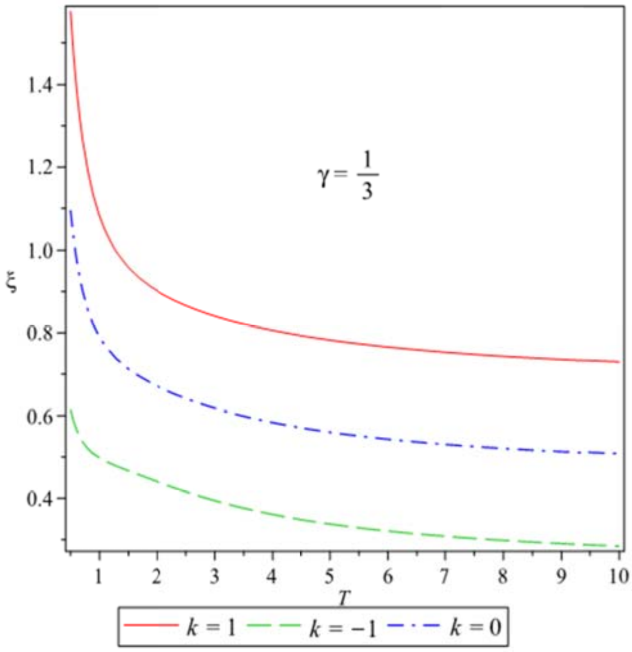

Figure 5 The plot of bulk viscous coefficient ξ versus T for false vacuum (γ= -1). Here G = α =1, ε = -1

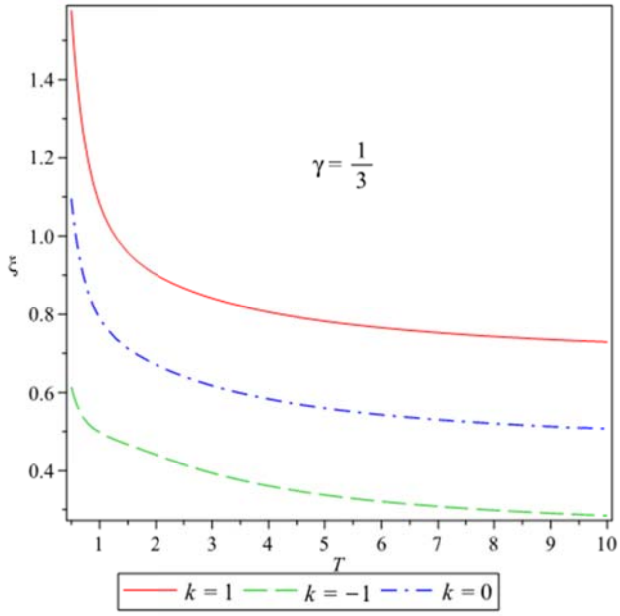

Figure 6 The plot of bulk viscous coefficient ξ versus T for radiating dominated model (γ= 1/3). Here G= α=1, ε = -1

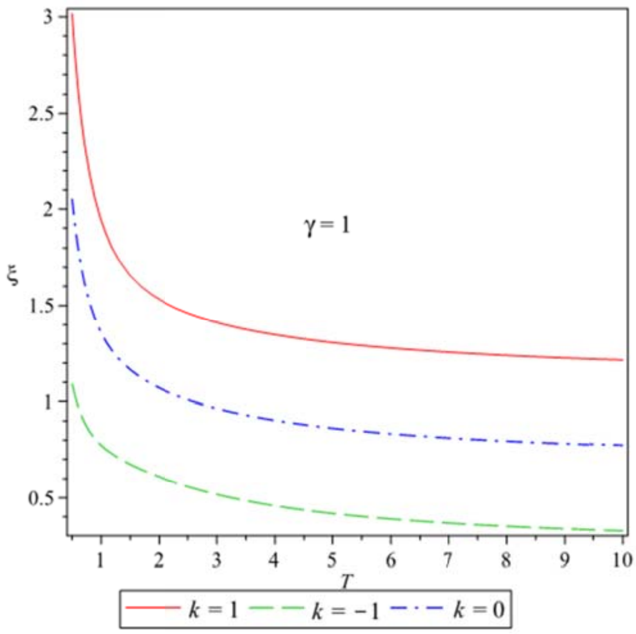

Figure 7 The plot of bulk viscous coefficient ξ versus T for stiff fluid model (γ= 1/3). Here G= α=1, ε = -1

Figures 6 and 7 depict the variation of the co-efficient of bulk viscosity ξ versus cosmic time T in radiating dominated and stiff fluid models of the universe respectively. From these figures, we observe that ξ is a decreasing function of time for all three open, flat and closed models of the universe as expected.

IV. CONCLUSIONS

In this paper we have studied a spatially homogeneous and isotropic FRW space-time within the framework of the scalar-tensor theory of gravitation proposed by Canuto et al. [8]. To find the deterministic solution, we have considered a time dependent deceleration parameter which yields a scale factor as $a(t) = \sinh(\alpha T)$. In this case, it is observed that as $T\rightarrow\infty$, $q = -1$. This is the case of de Sitter universe. For $T\rightarrow 0$, $q = 0$. This shows that in the early stage the universe was decelerating whereas the universe is accelerating at present epoch which is corroborated from the recent supernovae Ia observation (Riess et al. [44]; Perlmutter et al. [45]; Tonry et al. [46]; Riess et al. [47]; Clocchiatti et al.[48]). The parameter H_i, H, θ, and σ diverge at the initial singularity. There is a Point Type singularity (MacCallum [43]) at $T = 0$ in the model. The rate of expansion slows down and finally tends to zero as $T \rightarrow 0$. The pressure, energy density and scalar field become negligible whereas the scale factors and spatial volume become infinitely large as $T \rightarrow \infty$, which would give essentially an empty universe.

The model represents expanding, shearing and non-rotating universe. In literature we can get the solutions of the field equations in scalar-tensor theory of gravitation formulated by Canuto et al. [8] by using a constant deceleration parameter. So the solutions presented in this paper are new and different from other author's solutions. Our solutions maybe useful for better understanding of the evolution ofthe universe in FRW universe within the frameworkof Canuto et al.'s scalar-tensor theory of gravitation.

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Dr. Anirudh Pradhan born on 13th January, 1952 in the village Sultanpur of Ghazipur district in Uttar Pradesh. He got educated at University of Gorakhpur where from he obtained M. Sc. & Ph. D. degrees in the subject of Mathematics. He worked for his Ph. D. on Einstein's General Theory of Relativity under the supervision of Prof. K. B. Lal. He is working for his D. Sc. Degree on "Observed Variation in the Cosmological Constant Lambda: A Critique of Theory and Applications". Presently he is Associate Professor and Head of Mathematics in Hindu Post-graduate College, Zamania, Ghazipur. He has been the acting Principal (2001-02) and presently Vice-Principal of the College. Dr. Pradhan has worked as a Visiting Professor in the University of Zululand, South Africa (1995), Yasouj University, Iran (2006). He also worked as a Visiting Scientist to Institute for Mathematical Research (INSPEM), University of Putra, Malaysia (2010, 2011) and JINR Laboratory of Information Technologies, Dubna, Moscow Region, Russia (2010). The main field of Dr. Pradhan is Classical Relativity, Cosmology and Astrophysics. He has published more than 145 research papers in internationally reputed scientific journals with high impact factors. Dr. Pradhan is also a Visiting Associate of two nationally reputed autonomous institutes of higher research learning (i) Inter-University Centre for Astronomy & Astrophysics (IUCAA), Pune (2002-2013) and (ii) Institute of Mathematical Sciences (IMSc.), Chennai (Madras) (2009-2012). Apart from the above visiting assignments, Dr. Pradhan also visited different institutes/universities in India & abroad to address academic meetings, seminar and to take part in international conferences.

Anand Shankar Dubey did his M.Sc. (Mathematics) in 2003 from Allahabad University, Allahabad, India. He is doing PhD from Sam Higginbottom Institute Agriculture, Technology & Sciences (Deemed University), Allahabad, India.

Dr. Rajeev Kumar Khare is presently Professor in Department of Mathematics & Statistics, Sam Higginbottom Institute of Agriculture, Technology & Sciences, Allahabad. He has published nearly twenty research papers in national & international journals.