# An Equivalent Thickness for Buckling Verification of Laminated Glass Panels Under In-Plane Shear Loads

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*Abstract-* Monolithic and laminated glass elements, as known, offer interesting opportunities in the realization of innovative architectures of modern buildings. Nevertheless, similar structural elements are typically brittle and slender, thus frequently subjected to buckling phenomena. In these hypotheses, the paper focuses on the load-carrying behavior of 2-layer and 3-layer simply supported laminated glass panels subjected to in-plane shear loads. Analytical formulations based on the concept of equivalent thickness are presented to describe with accuracy their typical behavior. As shown, predicted critical loads and load-transversal displacement relationships are in good agreement with numerical data obtained by using sophisticated 3D-FE models, as well as simplest but accurate geometrical simplified FE models. According to the suggestions that the Eurocodes give the verification of traditional structural elements, a suitable verification criterion appropriately calibrated to numerical and experimental predictions available in literature is suggested to guarantee the requisites of resistance, serviceability and durability typically imposed in the design of conventional structural systems made of steel, concrete or timber. As a result, the proposed approach could be used in daily practice to perform a suitable and rational buckling verification of such brittle load-bearing elements.

Keywords- Equivalent Thickness; Sandwich Theory; In-Plane Shear Loads; 2 Or 3-Layer Laminated Glass Panels; Buckling Verification; Standardized Buckling Curve

## I. INTRODUCTION

Structural glass elements are frequently adopted as stiffeners for roofs in modern buildings or in the construction of innovative and futuristic architectures. Consequently in-plane or out-of-plane loads could represent the cause of possible instability of these brittle and slender elements [1, 2, 3].

Several authors investigated the buckling behaviour of glass panels or beams in different loading conditions, providing interesting experimental results and sophisticated numerical validations. Belis [4], for example, focused on the out-of-plane bending of laminated glass beams and performed more than 300 tests on laminated glass (LG) beams having various mechanical and geometrical properties. Luible [5] studied the buckling response of compressed columns and panels, as well as beams in out-of-plane bending. Englhardt [6] recently analysed, through experimental and numerical predictions, the buckling response of in-plane compressed monolithic and laminated glass panels. Also Mocibob [7] deeply investigated the buckling response of glass panels supported at top and bottom sides and subjected to in-plane shear forces (e.g. lateral wind acting on a façade of the building and transferred to bracing glass panels by floor slabs), out-of-plane distributed loads (e.g. perpendicular wind) and in-plane compressive forces (e.g. self weight). Through numerical and experimental investigations, Mocibob analysed the buckling response of these glass panels, highlighting the effects of point or linear connections, as well as the interaction of simultaneous loads on their global behaviour. Similarly, Wellershoff and Sedlacek [8] performed tests on monolithic and laminated glass panels simply supported on the four edges and subjected to in-plane shear loads, providing interesting results and considerations. In addition, Wellershoff focused on the use of glass panels in space grid structures and studied analytically, numerically and experimentally the shear buckling response of flat glass panels glued to grid members along the four edges [9]. Further recent experimental investigations on buckled glass elements in various boundary and loading conditions can also be found in [10, 11, 12]. Nevertheless, the knowledge on LG panels behavior under in-plane loads is still limited and with constrained applications.

In this context, it should not be ignored that analytical models existing in literature are in general derived from classical sandwich theory formulations, thus they well apply only to 2-layer composite elements and to specific loading and boundary scenarios [13, 14]. At the same time, it is known that consolidate verification criteria available in literature for buckling verification of traditional structural elements, realized by means of conventional materials as steel, concrete or timber [15, 16, 17], cannot be directly applied to LG elements, because they do not take into account a series of factors typical of glass structures (e.g. influence of production tolerances, initial imperfections, brittle behavior of glass, viscoelastic behavior of thermoplastic interlayers,...).

Based on these assumptions, an analytical approach which requires the contemporary check of maximum stresses, deformations and simultaneous acting loads has recently been proposed by the authors for the buckling verification of LG beams under in-plane compression or out-of-plane bending [18, 19]. Also a new analytical approach, based on the concept of

equivalent thickness, has been proposed for a rational buckling verification of LG panels under in-plane compression [20]. By means of opportunely calibrated correction factors, the model estimates accurately the critical buckling load of these composite panels in various boundary conditions (linear supports, point supports, etc.). In addition, the equivalent thickness approach can be used to simplify numerical simulations, thus to predict with a good level of accuracy their critical load as well as the corresponding load-transversal displacement relationship [20]. As shown in [21], the approach can be rationally applied to the verification of 2 and 3-layer LG panels and columns under compression, by simply calibrating a series of opportune correction factors.

Based on these last considerations, in this paper the same equivalent thickness approach is proposed to perform a rational buckling verification of 2 and 3-layer simply supported LG panels under in-plane shear loads. As shown, predicted critical shear loads and load-transversal displacement relationships are in good agreement with sophisticated numerical simulations. As a result, the buckled response of a simply supported LG panel under in-plane shear can be predicted, for a well-defined temperature and load-time scenario [18, 19, 20, 21]. At the same time, in accordance with the suggestions of Eurocodes 3, 4, 5 [15, 16, 17], a verification criterion is proposed to guarantee the requisites of resistance, serviceability and durability typically imposed by standards in the design of conventional structural systems. As highlighted in the following sections, the proposed verification criterion agrees with numerical results of performed simulations, as well as with experimental data collected in literature, thus it could be used by designers in daily practice.

# II. 2-LAYER LAMINATED GLASS PANELS UNDER IN-PLANE SHEAR

## A. Existing Analytical Models for Monolithic Panels

As proposed in Fig. 1, let us consider a flat monolithic glass panel (height *a* and width *b*, thickness *t*; Young's modulus *E*, Poisson's coefficient v) simply supported along the edges and subjected to a shear loading  $N_{xy} = t \tau_{xy} = V$ . The well-known differential equation able to describe its load-carrying behavior is [22]:

$$D\Delta^2 w = 2V \frac{\partial^2 w}{\partial x \partial y},\tag{1}$$

where D and  $\Delta$  are respectively defined as:

$$D = E t^3 / 12 (1 - v^2)$$
<sup>(2)</sup>

the flexural stiffness of the element per unit width b and  $\Delta$  the Laplace's operator.



Fig. 1 Simply supported flat panel subjected to in-plane shear. (a)geometry; (b) deformed configuration

Assuming for the out-of-plane deflection w = w(x, y) of the panel an appropriate form and introducing it in Eq. (1), the resulting critical shear load is defined as:

$$V_{cr}^{(E)} = \frac{\pi^2 D}{b^2} k_{\tau} \,. \tag{3}$$

In Eq. (2), the shear buckling coefficient  $k_{\tau}$  is commonly expressed as a function of the aspect ratio  $\alpha = a/b$  (Fig. 1) and the boundary conditions of the panel. In the case of four sides simply supported elements, for example,  $k_{\tau}$  can be evaluated as [23]:

$$k_{\tau} = \begin{cases} 4.00 + \frac{5.34}{\alpha^2} & \alpha < 1\\ 5.34 + \frac{4.00}{\alpha^2} & \alpha \ge 1 \end{cases}$$
(4)

These formulations can be only applied to monolithic panels under in-plane shear. Nevertheless, with opportune attention, similar considerations can be extended also to the analysis of 2 and 3-layer laminated glass panels under in-plane shear forces.

## B. Existing Analytical Models for 2-Layer Panels

In general, the analysis of a 2-layer LG element is performed by directly taking into account the existing formulations originally proposed for the calculation of sandwich structures. A similar approach often provides accurate results also for laminated glass elements, as checked by Luible for in-plane compressed panels [5]. Nevertheless, the theoretical background of sandwich models does not agree well with the mechanical behavior of LG elements consisting in very thin layers and extremely soft middle films, as highlighted in the following sections. Consequently, particular attention is generally required.

For this purpose, let us examine a 2-layer LG panel (height a, width b, Fig. 1) composed of two monolithic glass sheets (thicknesses  $t_1$  and  $t_2$ ; Young's modulus E, Poisson's ratio v) and a middle interlayer (thickness  $t_{int}$ ; Young's modulus  $E_{int}$ , shear modulus  $G_{int}$ , Poisson's ratio  $v_{int}$ ), simply supported along the four edges.

As noted by Mocibob [7], Kuenzi et al. solved the problem of rectangular isotropic sandwich plates, simply supported or clamped along the edges, subjected to in-plane shear loads [24]. By applying Kuenzi's expression to the examined LG panel, the equilibrium equation of forces and moments due to in-plane shear force V in the deformed configuration is:

$$\frac{D_i}{S_{\text{int}}}\Delta^3 w - \frac{D_{lam}}{D_0 + D_c}\Delta^2 w = 2 V \frac{\partial^2 w}{\partial x \partial y} \left(\frac{\Delta w}{S_{\text{int}}} - \frac{1}{D_0 + D_c}\right),\tag{5}$$

where:

$$D_i = \frac{E(t_1^3 + t_2^3)}{12(1 - v^2)} \tag{6}$$

is the flexural stiffness of the glass sheets around their neutral axes;

$$D_0 = \frac{E(t_1 + t_2 + t_{int})d^2}{2(1 - v^2)}$$
(7)

is the flexural stiffness of the glass sheets around the centroidal axis of the total cross section;

$$d = \frac{t_1}{2} + \frac{t_2}{2} + t_{\text{int}}$$
(8)

is the distance between the centroidal axes of the glass sheets;

$$D_{C} = \frac{E_{\rm int} t_{\rm int}^{3}}{12 \left(1 - v_{\rm int}^{2}\right)}$$
(9)

is the flexural stiffness of the middle interlayer;

$$D_{lam} = D_i + D_0 + D_C \tag{10}$$

represents the flexural stiffness of the LG panel;

$$S_{\rm int} = \frac{G_{\rm int} d^2}{t_{\rm int}} \tag{11}$$

is the shear stiffness of the interlayer.

In this hypothesis, the critical shear load  $V_{cr,lam}^{(E)}$  is:

$$V_{cr,lam}^{(E)} = \frac{\pi^2 D_{lam}}{b^2} k_{\tau,lam},$$
 (12)

where the buckling coefficient  $k_{r,lam}$  is defined by Kuenzi as a function of the aspect ratio  $\alpha$ , the shear stiffness  $S_{int}$  of the adopted interlayer and the applied boundary conditions. For a panel simply supported along the edges its value is:

$$k_{\tau,lam} = \frac{\frac{16}{3} + \frac{4}{\alpha^2}}{1 + \frac{\pi^2 D_{lam}}{b^2 S_{\text{int}}} \left(\frac{13}{3} + \frac{3}{\alpha^2}\right)}.$$
(13)

Similarly, for a clamped panel  $k_{\tau,lam}$  should be estimated as:

$$k_{\tau,lam} = \frac{9 + \frac{17}{3\alpha^2}}{1 + \frac{\pi^2 D_{lam}}{b^2 S_{int}} \left(\frac{23}{3} + \frac{13}{3\alpha^2}\right)}.$$
 (14)

#### C. Equivalent Thickness Approach for 2-Layer LG Panels

An alternative formulation for the analytical evaluation of the critical buckling load of simply supported 2-layer LG panels under in-plane shear can be derived from the simplified approach based on the concept of equivalent thickness originally formulated by Wölfel for the analysis of sandwich structures [25]. The original procedure, actually proposed by the American [26] and Australian Standards (AS1288) for the verification of LG beams, requires the evaluation of a monolithic beam of "effective" thickness with equivalent bending properties to a sandwich beam.

In accordance with this theoretical model, the level of connection effectively offered by the adopted interlayer can be expressed by means of a shear transfer coefficient  $\Gamma$ , comprised between 0 (*layered limit*) and 1 (*monolithic limit*), defined as:

$$\Gamma = \frac{1}{1 + \pi^2 \beta \frac{E J_s t_{\text{int}}}{G_{\text{in}} t_s^2 \lambda^2}},$$
(15)

where:

$$\lambda = \min(a, b) \tag{16}$$

is a scale factor (minimum dimension of the panel) and the equivalent parameters  $J_s$ ,  $t_{s,1}$ ,  $t_{s,2}$  and  $t_s$  are respectively defined as (Fig. 2):

$$J_{s} = t_{1} t_{s,2}^{2} + t_{2} t_{s,1}^{2}; \qquad (17)$$

$$t_{s,1} = t_s t_1 / (t_1 + t_2); \tag{18}$$

$$t_{s,2} = t_s t_2 / (t_1 + t_2); \tag{19}$$

$$t_s = 0.5 (t_1 + t_2) + t_{\text{int}} \,. \tag{20}$$

In addition,  $\beta$  is a coefficient defined as a function of the specific boundary and loading conditions (in the original formulation,  $\beta = 1$  [25]).



Fig. 2 Schematic view of 2-layer laminated glass panel (cross section)

Based on the original analytical approach, the deformation w of the composite element can be evaluated referring to an equivalent thickness defined as:

$$t_{eq,w} = \sqrt[3]{t_1^3 + t_2^3 + 12 \Gamma J_s} , \qquad (21)$$

in which  $\Gamma$  and  $J_s$  are respectively given by Eq. (15) and Eq. (17).

Similarly, the calculation of the maximum bending stresses in each glass sheet can be performed by taking into account two additional effective thicknesses (one for each glass pane [27]).

In the specific case of simply supported 2-layer LG beams composed of two monolithic glass sheets and a middle interlayer, subjected to an uniformly distributed load q acting orthogonally to their plane, assuming  $\beta = 1$  [25] and evaluating the corresponding equivalent thickness  $t_{eq,w}$  (Eq. (21)), it is possible to accurately describe the associated load – midspan maximum displacement relationship. Undoubtedly, a similar approach only applies to 2-layers LG beams in bending in well-defined conditions of temperature and load duration. Nevertheless, as recently highlighted by Bennison et al. [27], it represents an important simplification for the analysis of similar composite elements. Also Calderone et al. [28] demonstrated that the equivalent thickness formulation adequately calculates stresses and deflections in each glass layer, thus it could represent a useful approach for designers in daily practice.

At the same time, it should not be ignored that Wölfel's formulation was originally proposed for the analysis of sandwich structures characterized by a very soft core and metallic faces. In particular, Wölfel's analytical model has been developed assuming that:

• the external layers are characterized by noticeable axial stiffness but negligible bending stiffness;

• the middle interlayer can be defined only in terms of shear stiffness, whereas its axial and flexural rigidities can be ignored.

Evidently, these assumptions do not apply to 2-layer LG elements. Because of this reason, a series of analytical calculations has been performed by the authors to detect if Wölfel-Bennison's approach can be used for the analysis of LG panels simply supported along the four edges and subjected to in-plane shear forces. Specifically,  $V_{cr,lom}^{(E)}$  has been evaluated, for different LG panels, both using the linear elastic sandwich theory (Eq. (12)) proposed by Kuenzi [24] and the equivalent thickness approach, that is by substituting  $t_{ea,w}$  (Eq. (21), with  $\beta = 1$ ) in Eq. (3) and

$$D_{eq} = E t_{eq,w}^3 / 12 (1 - v^2)$$
(22)

in Eq. (3).

Analytical calculations have been carried out highlighting the effects on  $V_{cr,lam}^{(E)}$  of mechanical and geometrical parameters characterizing a typical 2-layer LG panel, that is the value of  $G_{int}$  ( $10^{-4}$  N/mm<sup>2</sup> <  $G_{int}$  < $10^{4}$  N/mm<sup>2</sup>), the aspect ratio  $\alpha$  ( $1 \le \alpha \le 10$ , with a fixed width b = 1m), the thicknesses of glass sheets and interlayer (6/1.52/6mm, 8/1.52/8mm, 10/1.52/10m m). However, since the aim of this work is to provide useful criteria for the buckling verification of simply supported LG panels under in-plane shear, only the first buckling load was considered in these comparisons (m = 1, Fig. 1).

Results presented in Fig. 3, for example, refer to a squared (1 m x 1 m) 2-layer LG panel (8/1.52/8mm) simply supported along the four edges. As known, depending on the effective level of connection offered by the adopted interlayer, that is depending on the value of the shear modulus  $G_{int}$ , the critical shear load of a generic 2-layer LG panel is always comprised between the *layered limit (abs,* no connection between the glass sheets) and *monolithic limit (full,* rigid connection between the glass sheets), which can be rationally estimated as:

• layered limit (abs,  $G_{int} \rightarrow 0$ ):

$$V_{cr,lam}^{(E)} = V_{cr,abs}^{(E)} = \frac{\pi^2 D_{abs}}{b^2} k_{\tau}, \qquad (23)$$

with  $k_{\tau}$  given by Eq. (4) and  $D_{abs}$  by Eq. (6);

• monolithic limit (full,  $G_{int} \rightarrow \infty$ ):

$$V_{cr,lam}^{(E)} = V_{cr,full}^{(E)} = \frac{\pi^2 D_{full}}{b^2} k_{\tau}, \qquad (24)$$

With

$$D_{full} = E t_{full}^{3} / 12 (1 - v^{2})$$
(25)

and

$$t_{full} = t_1 + t_{int} + t_2 \,. \tag{26}$$



Fig. 3 Critical shear load  $V_{cr,lam}^{(E)}$  for 2-layer simply supported laminated glass panels under in-plane shear. Analytical and numerical comparisons ( $\beta$ = 1)

As shown in Fig. 3, the equivalent thickness approach (with  $\beta = 1$ ), allows estimating these *limit* shear buckling loads for the examined 2-layer LG panel. Specifically,  $V_{cr,lam}^{(E)} \rightarrow V_{cr,full}^{(E)}$  in presence of a very stiff interlayer ( $G_{int} > 1000 N/mm^2$ , in this example), whereas  $V_{cr,lam}^{(E)} \rightarrow V_{cr,abs}^{(E)}$  if the connection between glass sheets is soft ( $G_{int} < 0.01N/mm^2$  for the examined LG panel).

Performed analytical comparisons highlighted that also Kuenzi's approach correctly estimates the *full* critical shear load  $V_{cr,full}^{(E)}$  of 2-layer LG panels, but strongly underestimates the effective buckling strength of similar composite elements, especially if the adopted interlayer is extremely soft (in this example,  $V_{cr,lam}^{(E)} \rightarrow 0$  for  $G_{int} < 0.1$ N/mm<sup>2</sup>, Fig. 3).

The intermediate predicted critical loads  $V_{cr,abs}^{(E)} < V_{cr,full}^{(E)}$ , as shown in Fig. 3, do not agree. Nevertheless, no alternative analytical formulations are available in literature to check the accuracy of Kuenzi's approach and the equivalent thickness formulation. Based on these assumptions, numerical simulations were performed to validate both the analytical approaches.

### D. Numerical Validation and Analytical Comparisons for 2-Layer Laminated Glass Panels

An accurate three-dimensional numerical finite element model was developed with the non linear code ABAQUS [29]. In this FE-model (3D + shell), glass sheets (thickness  $t_1 = t_2 = 8$ mm) have been modeled by means of *shell* elements (S4R), whereas PVB-interlayer (thickness  $t_{int} = 1.52$ mm) has been described through 3D-8 node elements (C3D8H, *hybrid formulation, compatible modes*). For the examined LG panel, having dimensions a = 1m x b = 1m, a mesh based on 20 mm x 20 mm module was assumed. Over the depth of the interlayer, two 3D elements have been realized (Fig. 4).



Fig. 4 Detail of 3D+shell FE-model for a simply supported 2-layer laminated glass panel subjected to in-plane shear (ABAQUS)

Accordingly with Luible and Crisinel [30] and previous investigations [20, 21], 3D elements and shell elements were connected together using the same nodes. Moreover, to describe the effective geometry of the examined panel, a section offset  $t_{offset} = 4$ mm from the centroidal axis of each glass sheet was applied to monolithic shell elements. In-plane shear was introduced in the model in the form of shear nodal loads acting on the middle nodes of interlayer. To constrain all four edges of the simply supported LG panel and to avoid possible eccentricities, also boundary conditions have been applied to the central nodes of the interlayer.

Glass has been defined as an isotropic and linear elastic material (Young's modulus  $E = 70000 \text{ N/mm}^2$ , Poisson's ratio v = 0.23, density  $\rho = 2490 \text{ Kg/m}^3$ ). Also the interlayer was considered to behave linear-elastically, assuming a Poisson's ratio  $v_{\text{int}} = 0.49$  and a density  $\rho_{\text{int}} = 660 \text{ Kg/m}^3$  typical of PVB-films [18, 19, 20, 21].

Since the aim of these numerical simulations consisted in validating the proposed analytical approach for the estimation of the critical load  $V_{cr,lam}^{(E)}$ , buckling analyses were performed in ABAQUS to predict the critical load of the examined LG panel in a series of well-defined conditions of temperature and load duration. As a result, the interlayer has been characterized in each of these analyses by a different value of shear modulus  $G_{int}$ , estimated in a pre-established range  $(1 < G_{int} < 1000 \text{ N/mm}^2)$ .

Numerical and analytical results presented in Fig. 5, referred to the squared 8/1.52/8mm panels previously investigated (a = b = 1m), confirm that Kuenzi's formulation constitutes a useful criterion for the evaluation of the critical shear load  $V_{cr,lam}^{(E)}$  of layered "stiff" plates. Nevertheless, this approach cannot be directly applied to the analysis of LG panels typically characterized by the presence of stiff external faces (the glass sheets) bonded together by a soft and thin middle interlayers.



Fig. 5 Critical shear load  $V_{cr,lam}^{(E)}$  for 2-layer simply supported laminated glass panels under in-plane shear. Analytical and numerical comparisons ( $\beta = \beta(\alpha)$ , Eq. (27))

At the same time, it should be noticed that the equivalent thickness approach well applies to the prediction of the shear critical load  $V_{cr,lam}^{(E)}$  of 2-layer simply supported LG panels, but accurate results can be obtained only assuming for the correction factor  $\beta$  a series of appropriately calibrated values [20, 21]. Specifically, parametric numerical simulations and analytical comparisons highlighted that  $V_{cr,lam}^{(E)}$  can be evaluated with Eq. (3), referring to an opportune equivalent thickness  $t_{eq,w}$  (Eq. (21)), by simply assuming in Eq. (15) a series of coefficients  $\beta = \beta(\alpha)$ , numerically calibrated as a function of the aspect ratio  $\alpha$  (Table 1) and well expressed by the fitting curve (Fig. 6):

$$\beta = \frac{5.25}{\alpha^2} + 7.32 \,. \tag{27}$$



table 1 calibrated numerical values for coefficient  $\beta{=}\beta(\alpha)$  2-layer laminated glass panels under in-plane shear

Fig. 6 Coefficient  $\beta = \beta(\alpha)$  for 2-layer laminated glass panels under in-plane shear Calibrated numerical results (Table 1) and fitting curve (Eq. (27))

Calibrated values for  $\beta$  summarized in Table 1 were obtained in this work assuming for the examined LG panels a fixed width *b* and various heights  $a \ge b$ , thus  $\alpha \ge 1$ . However, additional numerical simulations highlighted that the fitting curve of Eq. (27) well applies, with  $k_{\tau}$  given by Eq. (4), also for the analysis of LG elements characterized by an aspect ratio  $\alpha \le 1$  (comparisons were performed in the range  $0.5 \le \alpha \le 1$ ).



Fig. 7 Critical shear stress  $\tau_{cr}$  of monolithic and laminated glass panels Comparison between analytical results (equivalent thickness approach) and experimental/numerical predictions [9]

Further comparisons were performed to check the validity of the proposed approach and the accuracy of the presented correction factors. Comparisons proposed in Fig. 7, for example, refer to experimental tests and numerical predictions performed by Wellershoff [9] on monolithic and laminated squared glass panels characterized by various dimensions (a=b=1.2 m, 1.6 m), nominal thicknesses of glass sheets ( $t_1=t_2=3.85 \text{ mm}$ , 5.85 mm, 7.7 mm, 9.7 mm, 11.7 mm, 14.7 mm) with  $t_{int}=1.52 \text{ mm}$  the thickness of PVB-film, and shear stiffness of interlayer (0.4 N/mm<sup>2</sup> < G<sub>int</sub> < 100 N/mm<sup>2</sup>). Results are proposed in terms of critical shear stress  $\tau_{cr}$ , therefore analytical data were obtained, for each panel, dividing the critical shear load  $V_{cr,lmn}^{(E)}$ 

(Eq. (3), with  $t_{eq,w}$  (Eq. (21)) and  $\beta = 13.15$ , Table 1) for the total thickness of glass sheets.

Also in this circumstance, it is possible to notice that analytical predictions are in good agreement with data available in literature.

#### E. Load-Transversal Displacement Relationship

It is known that studying in a realistic manner the stability problem of a laminated glass panel under in-plane shear loads, the estimation of the critical buckling load  $V_{cr,lam}^{(E)}$  does not constitute a useful criterion to define its ultimate strength, since the

panel, due to possible post-buckled membrane effects, could be able to sustain greater loads than  $V_{cr,lam}^{(E)}$ . In this context, to appropriately analyze the load-carrying behaviour of the composite plate, it is thus important to describe precisely its characteristic load V – transversal displacement w relationship. Generally, this aspect is investigated by means of sophisticated tridimensional numerical models based on the Finite Element Method. Nevertheless, also in this specific circumstance the proposed equivalent thickness approach could constitute a useful criterion for the simplification of numerical modeling and simulation phases.

For this purpose, let us consider the squared (height a=1 m x width b=1 m) simply supported LG panel (8/1.52/8 mm) previously investigated. The panel is affected by an initial imperfection proportional to its first modal shape and characterized by a maximum amplitude  $w_0=a/500$  [7, 20]. As known, the thermoplastic materials typically used to bond together the glass sheets are strongly temperature and load-time dependent, especially PVB-films [20, 31]. Consequently, the effective level of connection offered by the adopted interlayer should be accurately taken into account.

In Fig. 8, the results of incremental static analyses performed in ABAQUS with the 3D+shell FE model are compared. These curves refer to the well-known limit conditions for the examined LG panel (monolithic limit – full – and layered limit – abs) and to a specific condition of temperature and load duration (T=20 °C, 3 seconds of applied load).

Assuming for the interlayer the equivalent shear modulus  $G_{int}$  of a PVB-film subjected to the examined temperature and loading condition ( $G_{int} = 8.06N/mm^2$  [20]), the corresponding load V-transversal displacement w relationship can be suitably obtained, as shown in Fig. 8. Nevertheless, the accurate modeling of a 2-layer LG panel, as well as the execution of static incremental analyses, could request long processing times. At the same time, the presence in the composite element of extremely thin and soft layers (e.g. the interlayer) could compromise the accuracy of results, as well as the convergence of performed simulations [21].



Fig. 8 Load V-transversal displacement w relationship for simply supported2-layer LG panels under in-plane shear. Numerical comparisons (ABAQUS, 3D+shell and Shell-t<sub>eq</sub> FE-models)

Because of these reasons, the proposed equivalent thickness approach could be taken into account in the modeling of a geometrically simplified, but equivalently accurate, *Shell-t<sub>eq</sub>* FE-model consisting in monolithic glass S4R shell elements having a total thickness  $t_{eq,w}$  given by Eq. (21). As shown in Fig. 8, the obtained *V-w* curve is in good agreement with the corresponding curve given by the sophisticated 3D+shell FE-model. In addition, the *Shell-t<sub>eq</sub>* FE-model is quickly implementable and it well applies, in general, for the analysis of LG panels bonded together with very soft and thin interlayers or subjected to various temperature and load-time conditions [20, 21].

#### III. BUCKLING VERIFICATION OF 2-LAYER LAMINATED GLASS PANELS UNDER IN-PLANE SHEAR

### A. Traditional Verification Criterion

In accordance with the Limit State design approach, the buckling verification of a flat 2-layer LG panel under in-plane shear, simply supported along its edges, could be reasonably developed by contemporarily considering two different conditions, respectively referred to requisites of deformability and durability [18, 19, 20, 21].

Concerning the deformability, the maximum transversal displacement  $w_{max}$  of the panel should be limited as a function of its aspect ratio, by posing for example the condition:

$$w_{\max} \le \frac{a}{k},$$
 (28)

with *a* the height of the panel and *k* an appropriate coefficient to be defined for the specific limit state considered. Based on results of a series of performed numerical and analytical simulations, the coefficient *k* should be assumed equal to k = 300. Nevertheless, a reasonable check of the maximum deformation for the composite element should be carried out taking into account also an initial imperfection proportional to the first modal shape of the panel, having a maximum amplitude  $w_{0,tot}$  defined as:

$$w_{0,tot} = w_{0,imp} + w_{0,e} + w_{0,b} \tag{29}$$

and representative of:

• initial imperfections, as for example geometrical tolerances in product standards, structural imperfections due to fabrication, residual stresses ( $w_{0,imp}$ );

- possible load eccentricities  $(w_{0,e})$ ;
- possible boundary eccentricities  $(W_{0,h})$ .

As suggested by Mocibob [7], a minimum amplitude  $w_{0,\min} = a/1000$  should be taken into account for the initial imperfection. In this context, as proposed in the previous section, the maximum deformability of a generic LG panel could be rationally checked by taking into account a geometrically simplified but precise *Shell-t<sub>eq</sub>* FE-model.

Contemporarily, the design shear load  $V_{Ed}$  should be compared with the buckling resistant value  $V_{b,Rd}$  of the composite panel, defined as:

$$V_{Ed} \le V_{b,Rd} = \frac{V_{cr,lam}^{(E)}}{\gamma_{M1}},\tag{30}$$

where  $V_{cr,lam}^{(E)}$  is the critical buckling load, obtained from Eq. (3) by posing  $t = t_{eq,w}$  (Eq. (21) and  $\beta = \beta(\alpha)$ , Eq. (27)), and  $\gamma_{M1}$  is an appropriate safety coefficient. Based on numerical simulations and experimental tests, Wellershoff and Sedlacek suggest for the safety factor a value  $\gamma_{M1} = 1.40$  [8].

### B. Alternative Verification Criterion

As known, for the buckling verification of traditional structural elements made of steel, concrete or timber, consolidate verification criteria are available in literature. The Eurocode 3 [15], for example, estimates the buckling resistance of compressed steel members by taking into account a series of appropriate buckling curves, opportunely calibrated to take into account the effects in their buckled response of possible initial imperfections of different amplitude, as well as different residual stresses.

The suggestions of Eurocode 3 cannot be directly applied to the buckling verification of 2-layer LG panels under in-plane shear. However, since extremely practical, similar buckling curves could be opportunely calibrated and used to predict the buckling resistance of 2-layer simply supported laminated glass panels subjected to in-plane shear loads and affected by initial imperfections or boundary/loading eccentricities.

The main advantage of buckling curves proposed by Eurocodes, in fact, consists in their theoretical background. In them, as originally proposed by Ayrton-Perry [32] for the analysis of geometrically imperfect columns loaded by uniform compressive loads, the initial imperfections, as well as other effects (residual stresses, possible eccentricities) are efficiently described through an equivalent initial sine-shape imperfection, that is through a generalized imperfection factor  $\eta$ . In the

specific case of a compressed monolithic column affected by a sine-shape imperfection of maximum amplitude  $w_0$ , for example, the generalized non-dimensional imperfection factor proposed by Ayrton and Perry is defined as [32]:

$$\eta = w_0 \frac{A}{W},\tag{31}$$

where A and W respectively individuate the total cross-section area and the elastic resistant modulus of the examined column.

- As a result, the governing parameters able to describe the buckling resistance of an imperfect compressed column are:
- the normalized slenderness of the examined column

$$\overline{\lambda} = \sqrt{\frac{A\sigma_{Rk}}{N_{cr}^{(E)}}},$$
(32)

with  $\sigma_{_{Rk}}$  the characteristic tensile strength of glass and  $N_{_{cr}}^{^{(E)}}$  the Euler's critical buckling load, and

a buckling reduction factor

$$\chi = \frac{N_{cr}}{A\sigma_{Rk}},\tag{33}$$

defined as a function of the axial load  $N_{cr}$  associated to the reaching in the mid-span cross section of the tensile strength  $\sigma_{Rk}$ .

In this context, the buckling verification of a 2-layer simply supported LG panel subjected to in-plane shear could be still performed by satisfying the condition given by Eq. (30), in which the design buckling resistance  $V_{b,Rd}$  of the layered panel could be expressed as:

$$V_{b,Rd} = \chi A \tau_{Rd} , \qquad (34)$$

with  $\chi$  an opportune reduction factor, and:

$$A = b(t_1 + t_2), (35)$$

the glass resistant cross-section area of the composite panel. In Eq. (34), conservatively, the design shear strength  $\tau_{Rd}$  of glass should be assumed equal to the design tensile strength  $\sigma_{Rd}$ , as recommended by Wellershoff and Sedlaceck [8].

In addition, the Eurocode 3 suggests for the reduction factor  $\chi$  the expression:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}^2}}, \text{ with } \chi \le 1$$
(36)

and:

$$\Phi = 0.5 \left[1 + \alpha_{inp} (\overline{\lambda} - \alpha_0) + \overline{\lambda}^2\right], \tag{37}$$

$$\overline{\lambda} = \sqrt{\frac{A\tau_{Rk}}{V_{cr}^{(E)}}}$$
 the slenderness of the composite panel, (38)

 $\tau_{Rk}$  the characteristic shear strength of glass ( $\tau_{Rk} = \sigma_{Rk}$  [8]),

 $\alpha_{imp}$  and  $\alpha_0$  appropriate imperfection coefficients.

Specifically, the coefficient  $\alpha_0$  individuates the values of slenderness  $\overline{\lambda}$  associated to a reduction factor equal to  $\chi = 1$  (thus  $\chi = 1$  for  $\overline{\lambda} \le \alpha_0$ ). At the same time, the coefficient  $\alpha_{imp}$  is representative of the maximum allowable imperfection for the panel. In this work, the imperfection factors  $\alpha_{imp} = 0.49$  and  $\alpha_0 = 0.50$  have been appropriately calibrated on the basis of numerical (ABAQUS) and experimental data available in literature for simply supported monolithic or laminated glass panels under in-plane shear. Specifically, numerical results have been obtained by performing in ABAQUS (*3D+shell* FE-model) a series of static incremental analyses on 2-layer simply supported LG panels under in-plane shear characterized by different geometrical properties. An initial imperfection proportional to the first modal shape of the examined panels, having maximum

amplitude  $w_0 = a/1000$ , was taken into account [7]. Additional numerical comparisons allowed noticing that a similar limitation (Eq. (34), with  $\alpha_{imp} = 0.49$  and  $\alpha_0 = 0.50$ ) approximately coincides, for  $\overline{\lambda} > 1.10$ , with the assumption of k = 300 in Eq. (28) or  $\gamma_{M1} = 1.40$  in Eq. (30). In the same Figure, additional numerical data are proposed for monolithic and laminated simply supported glass panels [9]. Finally, experimental results obtained by Wellershoff by performing tests performed on glass panels are taken into account [9].



Fig. 9 Buckling "EC Curve" for the verification of simply supported 2-layer laminated glass panels under in-plane shear Comparisons with numerical (ABAQUS) and experimental data [9]

As a result, data collected in Fig. 9 to validate the proposed verification approach are in good agreement with the obtained buckling curve (called "EC Curve").

## IV. 3-LAYER LAMINATED GLASS PANELS UNDER IN-PLANE SHEAR

As known, special applications in modern architectures require the use of 3-layer (or even more) laminated glass elements. A similar technological solution guarantees an important increasing of resistance to high impact impulsive loads and significant anti-theft functions. Nevertheless, existing analytical approaches generally apply only to 2-layer LG elements, as well as to specific loading and boundary scenarios. At the same time, numerical simulations performed on similar structural elements typically require onerous modeling and computing times.

Based on these assumptions, additional analytical calculations were performed to extend the proposed equivalent thickness approach to the analysis of 3-layer LG panels under in-plane shear. In this specific context, only simply supported panels were examined. However, a similar approach could be easily applied to LG elements with various boundary and loading conditions, as for example recently proposed for LG columns and panels under in-plane compression [20, 21]. As a result, noticeable simplifications could be provided in the investigation of the buckled response of 2 and 3 (or more) layer laminated glass elements. At the same time, a generalized unified buckling verification approach could be assumed for them.

For this purpose, let us consider a 3-layer simply supported LG panel obtained by assembling 3 glass sheets (thickness  $t_1$  for the external faces and thickness  $t_2$  for the middle sheet) and two interlayers of thickness  $t_{int}$ , as proposed in Fig. 10.



Fig. 10 Schematic view of 3-layer laminated glass panel (cross section)

Based on simple geometrical considerations, the critical shear load  $V_{cr,lam}^{(E)}$  can be still estimated by means of Eq. (12), assuming for the equivalent thickness the expression [21]:

$$t_{eq,w} = \sqrt[3]{2t_1^3 + t_2^3 + 12 \Gamma J_s}, \qquad (39)$$

where  $\Gamma$  and  $\lambda$  are respectively given by Eq. (15) and Eq. (16). In addition, with reference to Fig. 10, the following parameters should be taken into account [21]:

$$J_{s} = 2t_{1} t_{s,1}^{2}; (40)$$

$$t_{s,1} = 0.5t_1 + t_{\text{int}} + 0.5t_2;$$
(41)

$$t_s = t_1 + 2t_{\rm int} + t_2. \tag{42}$$

Finally, the correction factor  $\beta$  can be estimated by means of Eq. (27), as previously done for the analysis of 2-layer LG panels.

Also in this circumstance, depending on the shear stiffness of the adopted interlayer, the predicted critical shear load  $V_{cr,lam}^{(E)}$  (Eq. (3), with  $t_{eq,w}$  given by Eq. (39)) is always comprised between the limit values of buckling load  $V_{cr,abs}^{(E)}$  ( $G_{int} \rightarrow 0$ ) and  $V_{cr,abl}^{(E)}$  ( $G_{int} \rightarrow \infty$ ), given respectively by Eq. (23) and Eq. (24), where:

$$D_{abs} = \frac{E\left(2t_1^3 + t_2^3\right)}{12\left(1 - \nu^2\right)} \tag{43}$$

is the *layered* flexural stiffness of the 3-layer LG panel and  $D_{full}$  (Eq. (25)), with

$$t_{full} = 2t_1 + 2t_{int} + t_2, \tag{44}$$

is the monolithic flexural stiffness.

Since no existing analytical formulations can be used to validate the accuracy of the proposed equivalent thickness approach, also in this specific circumstance parametric buckling analyses were performed in ABAQUS to check the accuracy of the model.

#### A. Numerical Validation for 3-Layer Laminated Glass Panels

At first, a further 3D+shell FE-model was developed in ABAQUS to estimate the critical load of 3-layer LG panels. In it, the middle glass sheet (thickness  $t_2$ ) and the interlayers (thickness  $t_{int}$ ) were described through 3D-8 node elements, whereas the external glass sheets were modelled in the form of monolithic S4R shell elements (thickness  $t_1$ ) with offset ( $t_{offsel} = t_1/2$ ). Also in this specific circumstance, a second *Shell-t<sub>eq</sub>* FE-model consisting in monolithic S4R shell glass elements of thickness  $t_{eq,w}$  (Eq. (39)) was taken into account for further comparisons.

Buckling analyses were performed on both the FE-models to check the accuracy of the presented analytical approach. Several geometrical and mechanical parameters were taken into account in performing simulations, as for example the shear stiffness of the adopted interlayers ( $10^{-4}$  N/mm<sup>2</sup> <  $G_{int}$  <  $10^{4}$  N/mm<sup>2</sup>), the aspect ratio of the panels ( $1 \le \alpha \le 10$ , with b = 1m), the thicknesses of each layer (6/1.52/6/1.52/6 mm, 6/0.76/6/0.76/6 mm, 6/0.38/6/0.38/6 mm).

The main numerical results are proposed in Figs. 11-13 and compared with analytical predictions (Eq. (3), with  $t_{eq,w}$  given by Eq. (39)). As shown, analytical critical loads are in good agreement with numerical results obtained by the sophisticated 3D+shell FE-model. Clearly, the 3D+shell FE-model is the more accurate, but the modeling of the LG panel and the performance of buckling analyses generally require rather long processing time. Furthermore, as noticed in previous works [19, 20], the 3D+shell FE-model could overestimate the real critical buckling load  $V_{cr,lam}^{(E)}$  in presence of extremely soft thermoplastic films (Figs. 11-13,  $G_{int} < 10^1 N/mm^2$ ), or could present convergence problems in performing simulations.

These aspects should not be ignored, especially in the verification of LG panels assembled with PVB-films or subjected to elevated temperatures, as well as long-term loads, and particular attention should be dedicated to their modeling and analysis. In contrast, the *Shell-t<sub>eq</sub>* FE-model can be quickly implemented and buckling analyses can be performed in a very short time, thus a similar modeling approach could represent a major simplification in the verification LG panels. Moreover, the *Shell-t<sub>eq</sub>* FE-model has no convergence problems associated with the presence of extremely thin layers or very soft films, as would happen by using for example *multilayer* shell models [21].

In conclusion, based on the proposed comparisons, the equivalent thickness approach could represent a useful expedient to be used in the analysis of 2 and 3-layer simply supported LG panels under in-plane shear. The buckling verification of 3-layer LG panels, also in this specific context, could be rationally performed by contemporarily satisfying the deformability and resistance conditions expressed by Eq. (28) and Eq. (30). Equivalently, by simply replacing in Eq. (35) the glass cross-section area A of the examined 3-layer LG panel, the buckling "EC Curve" proposed in Fig. 8 should be taken into account in performing a suitable verification.



Fig. 11 Critical shear load  $V_{cr,lam}^{(E)}$  for 3-layer simply supported laminated glass panels under in-plane shear (6/1.52/6/1.52/6 mm, *a*= 1 m x *b*= 1 m). Analytical and numerical comparisons ( $\beta = \beta(\alpha)$ , Eq. (27))



Fig. 12 Critical shear load  $V_{cr,lam}^{(E)}$  for 3-layer simply supported laminated glass panels under in-plane shear (6/0.76/6/0.76/6 mm, a = 1 m x b = 1 m). Analytical and numerical comparisons ( $\beta = \beta(\alpha)$ , Eq. (27))



Fig. 13 Critical shear load  $V_{cr,lam}^{(E)}$  for 3-layer simply supported laminated glass panels under in-plane shear (6/0.38/6/0.38/6 mm, a = 1 m x b = 1 m). Analytical and numerical comparisons ( $\beta = \beta(\alpha)$ , Eq. (27))

## V. CONCLUSIONS

Because of their characteristic high slenderness, LG panels subjected to in-plane shear can be affected by stability problems. Existing analytical formulations derived from the theory of sandwich panels in different boundary and loading conditions, due to the hypotheses they have been formulated on, frequently cannot be directly applied to LG panels.

Because of these reasons, an analytical model, based on the concept of equivalent thickness, is proposed for the analysis and verification of simply supported 2 and 3-layer LG panels. As shown, the proposed approach allows designers to simply and realistically evaluate their critical buckling load, by taking into account the effective level of connection associated to well-defined temperature and load-time conditions. As highlighted by the proposed numerical validations, the presented analytical formulation provides realistic results In addition, numerical computation can be simplified by realizing geometrical simplified but still accurate *monolithic shell* models having a glass cross-section of equivalent thickness.

In this way, according to the State Limit approach, a rational buckling verification of laminated glass panels under in-plane shear could be carried out by contemporary satisfying deformability and resistance criteria. Equivalently, as usually suggested by the Eurocodes for the buckling verification of conventional structural elements made of steel, concrete or timber, a buckling "EC curve" opportunely calibrated to numerical and experimental data could be taken into account in performing an extremely suitable and rational buckling verification.

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