Cosmological Consequences with Time Dependent Λ-Term in Bianchi Type-I Space-Time

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***Abstract*- The Einstein's field equations with variable cosmological "constant" are considered in presence of perfect fluid for a homogeneous and anisotropic Bianchi type-I space-time. Einstein's field equations are exactly solved by considering scale factor $a(t) = te^t$ (Pradhan et al. in Rom. J. Phys. 57: 2012) which yields a time-dependent deceleration parameter (DP), representing models which generate accelerating phase at the present epoch. The cosmological constant Λ is found to be a decreasing function of time and it approaches a small positive value at the present epoch which is corroborated by consequences from recent supernovae Ia observations. From recently developed Statefinder pair, the behaviour of different stages of the evolution of the universe has been studied. The physical and geometric implications of the cosmological models have also been discussed.**

Keywords- Cosmology; Variable Cosmological Term; Perfect Fluid Models; Statefinder Parameter

I. INTRODUCTION

The Einstein field equation has two parameters, the gravitational constant G and the cosmological constant Λ. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein field equation. In theories with a variable Λ-term, one either introduces new terms (involving scalar fields, for instance) in to the left hand side of the Einstein's field equations to cancel the non-zero divergence of Λg_{ij} [1, 2] or interprets Λ as a matter source and moves it to the right hand side of the field equations [3], in which case energy momentum conservation is understood to mean $T^{*ij}_{;j} = 0$, where $T^{*}_{ij} = T_{ij} - (\Lambda/8\pi G)g_{ij}$. It is here that the first assumption that leads to the cosmological constant problem is made. It is that the vacuum has a non-zero energy density. If such a vacuum energy density exists, Lorentz invariance requires that it has the form $T_{\mu\nu} = -\rho g_{\mu\nu}$.This allows defining an effective cosmological constant and a total effective vacuum energy density $\Lambda_{eff} = \Lambda + 8\pi G\rho$ or $\rho_{vac} = \rho + \Lambda/8\pi G$. Note at this point that only the effective cosmological constant, Λ_{eff} , is observable, not Λ, so the latter quantity may be referred to as a 'bare'. The two approaches are of course equivalent for a given theory (Vishwakarma [4]). A dynamic cosmological term $\Lambda(t)$ remains a focal point of interest in modern cosmological theories as it solves the cosmological constant problem in a natural way. For detail discussions, the readers are advised to see the references (Weinberg [5]; Carroll et al. [6]; Croswell [7]; Abdussattar and Vishwakarma [8]; Peebles and Ratra [9]; Lima [10]; Sahni and Starobinsky [11]; Padmanabhan [12, 13]; Singh et al. [14]). There are significant observational evidences that the expansion of the Universe is undergoing late time acceleration [15-22]. This, in other words, amounts to saying that in the context of Einstein's general theory of relativity some sort of dark energy, constant or that varies only slowly with time and space dominates the current composition of cosmos. The origin and nature of such an accelerating field poses a completely open question. Observations (Riess et al. [23]; Knop et al. [24]) of Type Ia Supernovae (SNe) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating. Observations strongly favor a small and positive value of the effective cosmological constant at the present epoch. Among many possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vacuum state of all existing fields in the universe, i.e., $\rho_v = \frac{\Lambda}{8\pi G}$, where Λ is the cosmological constant. However, a constant Λ cannot explain the huge difference between the cosmological constant inferred from observation and the vacuum energy density resulting from quantum field theories. In an attempt to solve this problem, variable Λ was introduced such that Λ was large in the early universe and then decayed with evolution [25]. Cosmological scenarios with a time-varying Λ were proposed by several researchers [26-46]. Since experimental data favour an anisotropic universe and hence it motivates to study models of the universe with anisotropic background space-time structure. The simplest anisotropic models are Bianchi type-I homogeneous models whose spatial sections are flat but the expansion or contraction rate are direction dependent. Motivated by the above discussions, in this paper, we have investigated Bianchi Type-I cosmological models with time dependent deceleration parameter and cosmological Λ-term in presence of perfect fluid. The outline of the paper is as follows. In Sect. II, the metric and basic equations are described. Sect. III deals with the solutions of the field equations. In Sect. IV, the physical and

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geometric aspects of the models have been discussed. In Sect. V, Statefinder diagnostic pair is briefly discussed. Finally, conclusions are summarized in the last Sect. VI.

II. THE METRIC AND BASIC EQUATIONS

We consider the space time admitting Bianchi Type-I group of motion in the form:

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \tag{1}$$

In general relativity, the Bianchi identities for the Einstein tensor G_{ij} and the vanishing covariant divergence of the energy momentum tensor T_{ij} together with imply that the cosmological term Λ is constant. In theories with a variable Λ-term, one either introduces new terms (involving scalar fields, for instance) into the left-hand- side of the Einstein's field equations to cancel the non-zero divergence of Λg_{ij} (Bergmann [1]; Wagoner [2]) or interprets Λ as a matter source and moves it to the right-hand-side of the field equation (Zeldovich [3]), in which case energy momentum conservation is understood to mean:

$$T_{ij} - \left(\frac{\Lambda}{8\pi G}\right) g_{ij} = 0. \tag{2}$$

Of course, the two approaches are equivalent for a given theory (Overduin [48]). Here we follow the later approach and assume that the cosmic matter is represented by the energy momentum tensor of perfect fluid augmented with the Λ-term as:

$$T_{ij} = (\rho + p)u_i u_j + \left(p - \frac{\Lambda}{8\pi G}\right) g_{ij}, \tag{3}$$

Together with a perfect gas equation of state:

$$p = \gamma\rho, \quad 0 \le \gamma \le 1, \tag{4}$$

where ρ, p are the energy density, thermo dynamical pressure and u_i is the four-velocity vector of the fluid comporting the relation.

$$u_i u^i = -1. \tag{5}$$

The Einstein's field equations $read\ \ as$:

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G T_{ij}\,. \tag{6}$$

For the Metric (1) and Energy-Momentum Tensor (3) in comoving coordinate system, the Field Equation (6) proceeds a set of four independent equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda, \tag{7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -8\pi G p + \Lambda, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = 8\pi G \rho + \Lambda. \tag{10}$$

Here, and also in what fallows, a dot designates ordinary differentiation with respect to 't'.

The energy conservation equation $T^{ij}_{j} = 0$, leads to the fallowing expression:

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{\Lambda}}{8\pi G} = 0, \tag{11}$$

which is a consequence of the Field Equations (7)–(10).

We define the fallowing parameters to be used in solving Einstein's field equations for the Metric (1).

The average scale factor of Bianchi Type-I Model (1) is defined as:

$$a = (ABC)^{\frac{1}{3}}. \tag{12}$$

A volume scale factor V is given by

$$V = a^3 = ABC\,. \tag{13}$$

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In analogy with FRW universe, we also define the generalized Hubble parameter H as:

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\,(H_1 + H_2 + H_3), \tag{14}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively. Here, and also in what follows, a dot indicates ordinary differentiation with respect to ‘ t’.

Further, the deceleration parameter ‘q’ is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}\ . \tag{15}$$

We introduce the kinematical quantities such as expansion scalar (θ), shear scalar (σ^2) and anisotropy parameter (A_m), defined as follows:

$$\theta = u^i_{;i}\,, \tag{16}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}\,, \tag{17}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{18}$$

where $u^i = (0, 0, 0, 1)$ is the matter four-velocity vector and

$$\sigma_{ij} = \frac{1}{2}\left(u_{i\,;\,\alpha}P^{\alpha}_{j} + u_{j\,;\,\alpha}P^{\alpha}_{i}\right) - \frac{1}{3}\theta P_{ij}\,. \tag{19}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j\ . \tag{20}$$

These dynamical scalars, in Bianchi Type-I, have the forms

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{21}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}\ . \tag{22}$$

III. SOLUTIONS OF THE FIELD EQUATIONS

Subtracting (7) from (8) and integrating, we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC} = k_1 a^{-3}\,, \tag{23}$$

where k_1 is constant of integration. Again integrating (23), we get

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right)\,, \tag{24}$$

where d_1 is an integrating constant.

Similarly subtracting (7) and (8) from (9), and continuing as above we get two more relations;

$$\frac{A}{C} = d_2\ \exp\left(k_2 \int \frac{dt}{a^3}\right), \tag{25}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right)\,, \tag{26}$$

where d_2, d_3, k_2 and k_3 are constants of integration.

From Eqs. (24)-(26), the metric functions can be obtained explicitly as

$$A(t) = l_1\, a\ \exp\left(m_1 \int \frac{dt}{a^3}\right)\,, \tag{27}$$

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$$B(t) = l_2\, a \ \exp\left(m_2 \int \frac{dt}{a^3}\right), \tag{28}$$

$$C(t) = l_3\, a \ \exp\left(m_3 \int \frac{dt}{a^3}\right), \tag{29}$$

where

$$l_1 = \sqrt[3]{d_1 d_2}\ , l_2 = \sqrt[3]{d_1^{-1} d_2}\ , l_3 = \sqrt[3]{(d_1 d_2)^{-1}}$$

$$3m_1 = k_1 + k_2\ , 3m_2 = k_3 - k_1\ , \qquad 3m_3 = -(k_2 + k_3)\ ,$$

where the constants m_1, m_2, m_3, and l_1, l_2, l_3, satisfying the relations

$$m_1 + m_2 + m_3 = 0, \qquad l_1 l_2 l_3 = 1. \tag{30}$$

It is clear from Eqs. (27)-(29) that once we get the value of average scale factor ‘a’, we can easily calculate the metric functions A, B, C.

The understanding of the global evolution of the observationally amenable universe, mathematically encoded in the dynamics of its scale factor *a*, is of utmost importance in explaining practically all cosmological phenomena. One of the most intriguing aspects of this evolution is the recently established late-time transition from a decelerated to an accelerating regime of the expansion of the Universe. Therefore, following Pradhan et al. [49], we take following *ansatz* for the scale factor, where increase in term of time evolution is

$$a(t) = te^t\ . \tag{31}$$

The motivation to choose such time dependent DP is behind the fact that the universe is accelerated expansion at present as observed in recent observations of Type Ia supernova (Riess et al. [18]; Perlmutter et al. [16]; Tonry et al. [50]; Riess et al. [23]; Clocchiatti et al. [51] and CMB anisotropies (Bennett et al. [52]; de Bernardis et al. [53]; Hanany et al. [54]) and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping (see the Refs. Padmanabhan and Roychowdhury [55], Amendola [56]). So, in general, the DP is not a constant but time variable. The motivation to choose such scale factor (31) yields a time dependent DP.

Using Eq. (31) into (27)-(29), we get the fallowing expression for scale factors:

$$A(t) = l_1\,(te^t)\ \exp[m_1 \textstyle\int (te^t)^{-3} dt]\ , \tag{32}$$

$$B(t) = l_2\,(te^t)\ \exp[m_2 \textstyle\int (te^t)^{-3} dt]\ , \tag{33}$$

$$C(t) = l_3\,(te^t)\ \exp[m_3 \textstyle\int (te^t)^{-3} dt]\ . \tag{34}$$

Hence the geometry of the universe (1) is reduced to

$$ds^2 = -dt^2 + l_1^2\,(te^t)^2 \exp\left[2m_1 \int (te^t)^{-3} dt\right] dx^2 +$$
$$l_2^2\,(te^t)^2 \exp\left[2m_2 \int (te^t)^{-3} dt\right] dy^2 +$$
$$l_3^2\,(te^t)^2 \exp[2m_3 \textstyle\int (te^t)^{-3} dt]\ dz^2. \tag{35}$$

From Eqs. (32)- (34), we obtain

$$\frac{\dot{A}}{A} = \left(\frac{t+1}{t}\right) + m_1\,(te^t)^{-3}\ ,$$
$$\frac{\dot{B}}{B} = \left(\frac{t+1}{t}\right) + m_2\,(te^t)^{-3}\ ,$$
$$\frac{\dot{C}}{C} = \left(\frac{t+1}{t}\right) + m_3\,(te^t)^{-3}\ , \tag{36}$$

and

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$$\frac{\ddot{A}}{A} = \left(\frac{t+1}{t}\right)^2 + m_1 (te^t)^{-6} - m_1 \left(\frac{t+1}{t}\right)(te^t)^{-3} - \frac{1}{t^2},$$
$$\frac{\ddot{B}}{B} = \left(\frac{t+1}{t}\right)^2 + m_2 (te^t)^{-6} - m_2 \left(\frac{t+1}{t}\right)(te^t)^{-3} - \frac{1}{t^2},$$
$$\frac{\ddot{C}}{C} = \left(\frac{t+1}{t}\right)^2 + m_3 (te^t)^{-6} - m_3 \left(\frac{t+1}{t}\right)(te^t)^{-3} - \frac{1}{t^2}. \tag{37}$$

IV. SOME PHYSICAL AND GEOMETRIC PROPERTIES

Substituting (36)- (37) into Eqs. (9) and (10), and solving with (4), we get the expressions for pressure (p), energy density (ρ) and cosmological term Λ for the model (35) as

$$p = \frac{\gamma}{8\pi G(1+\gamma)}\left[\frac{2}{t^2} - \beta_2 (te^t)^{-6}\right], \tag{38}$$

$$\rho = \frac{1}{8\pi G(1+\gamma)}\left[\frac{2}{t^2} - \beta_2 (te^t)^{-6}\right], \tag{39}$$

$$\Lambda = 3\left(\frac{t+1}{t}\right)^2 - \frac{2}{(1+\gamma)t^2} + \frac{\beta_3}{(1+\gamma)}(te^t)^{-6}, \tag{40}$$

where

$$\beta = m_1^2 + m_2^2 + m_3^2,$$
$$\beta_2 = m_1^2 + m_2^2 - m_3(m_1 + m_2),$$
$$\beta_3 = m_1^2 + m_2^2 + m_1 m_2 + \gamma(m_1 m_2 + m_2 m_3 + m_3 m_1). \tag{41}$$

From above relations (38)-(40), we can obtain four types physically relevant models:

- When $\gamma = 0$, we obtain empty model.
- When $\gamma = \frac{1}{3}$, we obtain radiating dominated model.
- When $\gamma = -1$, we have the degenerate vacuum or false vacuum or ρ vacuum model (Cho [57]).
- When $\gamma = 1$, the fluid distribution correspond with the equation of state $\rho = p$ which is known as Zeldovich fluid or stiff fluid model (Zeldovich [3]).

From Eq. (39), it is observed that the energy density ρ is a decreasing function of time and $\rho > 0$ always. The energy density has been graphed versus time in Fig. 1 for $\gamma = 0; \frac{1}{3}; 1$. It is evident that the energy density remains positive in all three types of model. However, it decreases more sharply with the cosmic time in Zeldovich universe, comparing to radiating dominated and empty fluid universes.

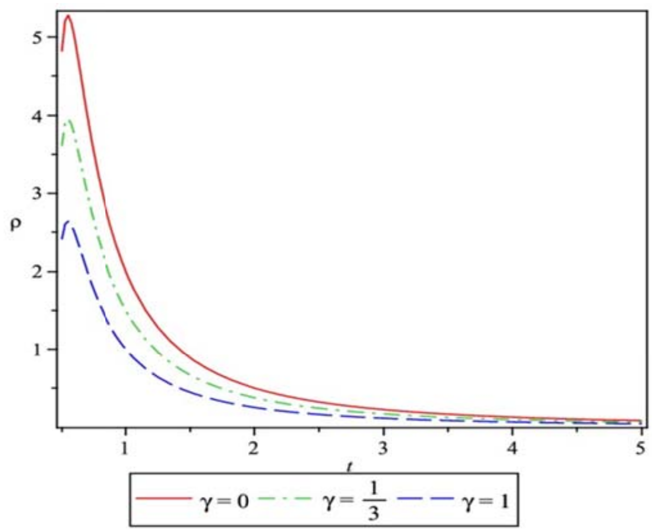

Figure 1 The plot of energy density ρ versus t
Here $8\pi G = 1$, $\beta_2 = 1$

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Figure 2 is the plots of cosmological term Λ versus time for $\gamma = 0; \frac{1}{3}; 1$. In all three types of models, we observe that Λ is decreasing function of time t and it approaches a small positive value at late time (i.e. at present epoch). However, it decreases more sharply with the cosmic time in empty universe, compare to radiating dominated and stiff fluid universes. Recent cosmological observations (Garnavich et al. [20, 21]; Perlmutter et al. [15-17]; Riess et al. [18, 19]; Schmidt et al. [22]) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda\left({G\hbar}/{c^3}\right) = \mathbf{1}0^{-123}$. These observations on magnitude and red-shift of Type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, the nature of Λ in our derived model is supported by recent observations.

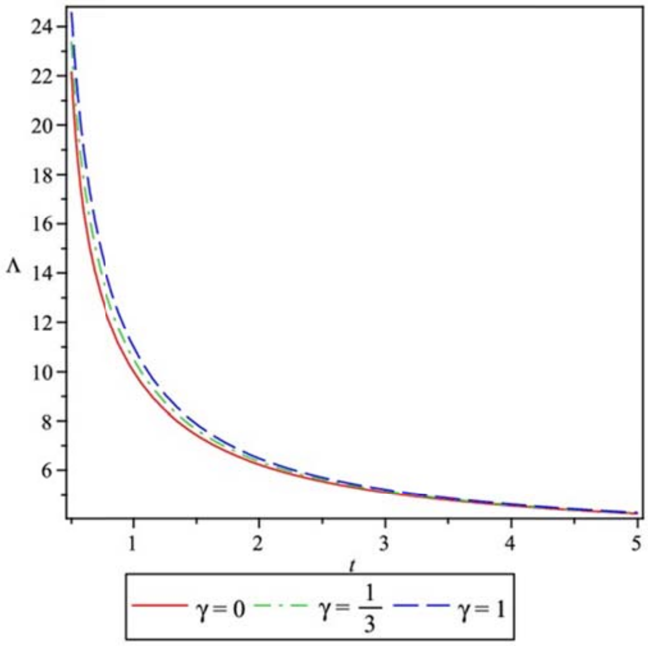

Figure 2 The plot of cosmological constant Λ versus t
Here $\beta_3 = 1$

The physical parameters such as spatial volume (V), Hubble parameter (H), expansion scalar (θ), shear scalar (σ) and anisotropy parameter (A_m) for the Model (41) are given by

$$V = (t\, e^t)^3\,, \tag{42}$$

$$\theta = 3H = 3\left(\frac{t+1}{t}\right), \tag{43}$$

$$q = -1 + \frac{1}{(1+t)^2}, \tag{44}$$

$$\sigma^2 = \frac{1}{2}\beta(t\, e^t)^{-6}\,, \tag{45}$$

$$A_m = \frac{1}{3}\frac{\beta t^2}{(1+t)^2}\,(t\, e^t)^{-6}\,. \tag{46}$$

From Eqs. (42) and (43), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario. From Eqs. (32)- (34), we observe that the spatial scale factors are zero at the initial epoch $t = 0$ and hence the model has a point type singularity (MacCallum [58]). We observe that proper volume increases with time.

The dynamics of the mean anisotropic parameter depends on the Constant β. From Eq. (46), we observe that at late time when $t \to \infty$, $A_m \to 0$. Thus, our model has transition from initial anisotropy to isotropy at present epoch which is in good harmony with current observations. Figure 3 depicts the variation of anisotropic parameter (A_m) versus cosmic time t. From the figure, we observe that A_m decreases with time and tends to zero as $t \to \infty$. Thus, the observed isotropy of the universe can be achieved in our model at present epoch.

DOI: 10.5963/JBAP0202003

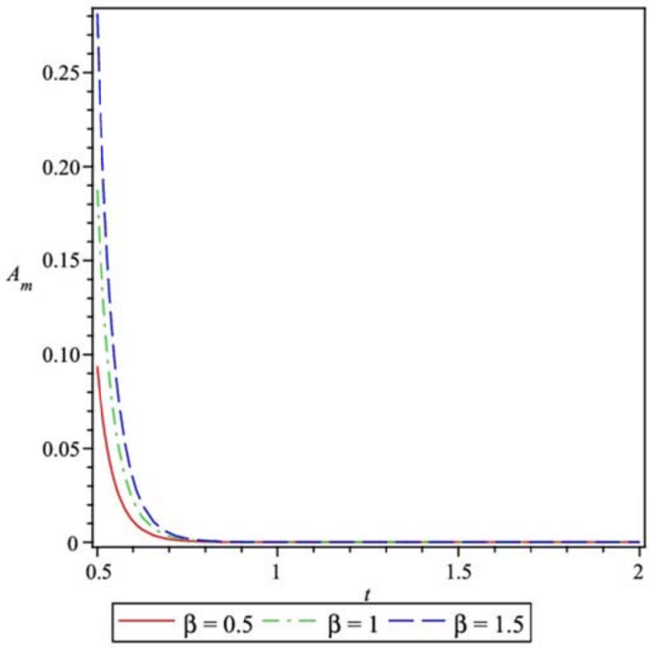

Figure 3 The plot of anisotropy parameter A_m versus t
Here β = 1

From Eq. (44), we observe that $q > 0$ for $t < \sqrt{2} - 1$ and $q < 0$ for $t > \sqrt{2} - 1$. Figure 4 graphs the deceleration parameter (q) versus time which gives the behavior of q. We observe that q decreases very rapidly approaching to -1 and then after it remains Constant -1 (like de Sitter universe).

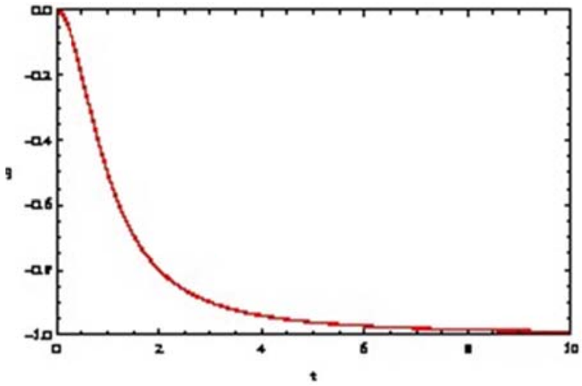

Figure 4 The variation of DP q vs t

It is important to note here that $\lim_{t\to 0}\left(\frac{\rho}{\theta^2}\right)$ spreads out to be constant. Therefore, the model of the universe goes up homogeneity and matter is dynamically negligible near the origin. This is in good agreement with the result already given by Collins [59].

V. STATEFINDER DIAGNOSTIC

Over the past few decades, a simple cosmological model called the lambda cold dark matter (ΛCDM) model has emerged as the best fit to the current observation data. ΛCDM model has emerged as the best fit to the current observational data. ΛCDM stands for cosmological constant current expansion of space against the attractive (collapsing) effects of gravity. In order to explain the cosmic acceleration a form of negative pressure matter called dark energy was suggested. For a current review, see Magaña et al. [60].

Sahni et al. [61] and Alam et al. [62] have introduced a pair of parameters $\{r,\ s\}$, called state finder parameters. In fact, trajectories in the $\{r,\ s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behavior. The state finder parameters can effectively differentiate between different forms of dark energy and provide simple diagnosis regarding whether a particular model fits into the basic observational data. The above state finder diagnostic pair has the fallowing form:

DOI: 10.5963/JBAP0202003

$$r = 1 + 3\,\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \;\; and \;\; s = \frac{r-1}{3\left(q-\frac{1}{2}\right)}. \tag{47}$$

For our model, the parameters $\{r,\ s\}$ can be explicitly written in terms of t as

$$r = 1 - \frac{3t+1}{(3t+1)^3}\ ,\quad s = \frac{2(3t+1)}{3(t+1)(3t^2+6t+1)}. \tag{48}$$

From Eq. (48), we observe that 's' is negative when $r \geq 1$. We also observe that the universe starts from an asymptotic Einstein static era ($r \to \infty,\ s \to -\infty$) and goes to the ΛCDM model ($r = 1,\ s = 0$).

VI. CONCLUSIONS

In this paper, a class of cosmological models is presented with variable cosmological term Λ in spatially homogeneous and anisotropic Bianchi Type-I space-time in the presence of a perfect fluid. To find the deterministic solution, we have considered a scale factor $a(t) = te^t$ which yields a time dependent deceleration parameter so that the universe is accelerating at present epoch which is corroborated from the recent supernovae Ia observation (Riess et al. [18, 19]; Perlmutter et al. [16]; Tonry et al. [50]; Riess et al. [23]; Clocchiatti et al. [51]). The parameters H, θ, and σ diverge at the initial singularity. There is a Point Type singularity (MacCallum [58]) at $t = 0$ in the model. The rate of expansion slows down and finally tends to zero as $t \to 0$. The pressure, energy density and cosmological term Λ become negligible whereas the scale factors and spatial volume become infinitely large as $t \to \infty$, which would give essentially an empty universe.

The main features of the models are as follows:

- The models are based on exact solutions of the Einstein's field equations for the anisotropic Bianchi-I space-time filled with perfect fluid with variable Λ-term.
- The model represents expanding, shearing and non-rotating universe.
- The nature of decaying vacuum energy density $\Lambda(t)$ in our derived models is supported by recent cosmological observations. These observations on magnitude and red-shift of Type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term.
- In literature it is a plebeian practice to consider constant deceleration parameter. Now for a Universe which was decelerating in past and accelerating at present epoch, the DP must show signature flipping as already discussed in Section II. Therefore, our consideration of scale factor which provides a time dependent DP to be variable is physically justified. Our derived model is accelerating at present epoch (Fig. 4)
- We observe that the evolution of the universe starts from asymptotic Einstein static era ($r \to \infty;\ s \to -\infty$) and approaches to ΛCDM model ($r = 1;\ s = 0$). So, from the Statefinder parameter $\{r,\ s\}$ the behaviour of different stages of the evolution of the universe has been generated.

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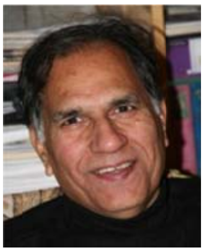

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