# Maximizing Outage Capacity in Rayleigh Fading Channel

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***Abstract*-There are various definitions of wireless channel capacity, depending on whether capacity characterizes the maximum rate averaged over all fading state or maximum constant data rate that can be maintained over all fading states. Between these definitions, outage capacity is a high requested one, since it is a more appropriate definition for delay constrained applications like voice and video. Hence, maximizing this type of capacity would be more concerned. In this paper outage capacity is maximized over cut-off fading states above which transmission should be maintained. The problem is studied for Rayleigh flat fading channel where channel side information (CSI) is known at transmitters and receivers. Optimization problem is solved for both single user and multiuser channel. Numerical results will be shown as the maximum outage capacities and the associated cut-off fading states for either of the channels.**

***Keywords*-Outage Capacity; Rayleigh Fading Channel; Cut-off Fading State**

## I. INTRODUCTION

The growing request for wireless communication makes us concern about its constraints and limitations. Among all the restrictions of wireless communication, channel capacity limits play an important role since they dictate data rate at which information could be transmitted over wireless channels.

In this paper we consider two definitions, which are more in demand: 1. ergodic capacity, and 2. outage capacity (or zero-outage capacity). Ergodic capacity characterizes the achievable rate vectors averaged over all fading states [1]. Outage capacity determines the constant data rate that can be maintained in all fading states subject to a given outage probability [1]. Zero-outage capacity refers to outage capacity with zero outage probability [1]. As mentioned in [1],[2],[3],[6],[7], ergodic capacity isn't an appropriate definition for delay constrained applications, like voice and video. On the other hand, maintaining a constant data-rate even under sever fading states, as in zero outage capacity, dramatically decreases the channel capacity. By suspending transmission in such sever fading states greater capacity could be obtained in the other channel conditions (the case of outage capacity) [1], [2].

This motivates us to pay more attention to outage capacity and its maximization. We consider a Rayleigh fading channel in two cases: 1. single user channel, 2. multiuser channel. In either of these cases it is assumed that the transmitter and all receivers have prefect channel side information (CSI).

As mentioned above outage capacity dictates a constant data rate in non-outage fading states subject to a given outage probability $p_{out}$:

$$p_{out} = p(\gamma < \gamma_0) \tag{1}$$

in which $\gamma_0$ indicates cut-off fading state above which transmission should maintained. It will be shown that outage capacity can be maximized over all possible $\gamma_0$, which means that there is a special cut-off fading state by considering that maximum outage capacity could be achieved on the channel. We study this optimum value of cut-off fading state and its associated outage capacity for both single user and multiuser channels where CSI is known at the transmitter and all receivers.

It is shown that the complexity of finding optimum value of $\gamma_0$ doesn't scale with number of sub-channels in a multiuser channel. Furthermore it will be shown that $(\gamma_0)_{opt}$ only depends on channel statistical characteristics. Numerical results illustrate the maximum outage capacities and their related $(\gamma_0)_{opt}$ .

The rest of this paper is as follows: in section II the system model is explained and then in section III we study single user channel. Multiuser channels are studied in section IV and in section V conclusions are discussed.

## II. SYSTEM MODEL

We consider, for multiuser channel, generally a discrete time Rayleigh block fading channel, in which channel gain $g[i]$ is constant over a block-time $T$ so that channel conditions have enough time to be experienced, with stationary and ergodic time-varying channel gain $\sqrt{g_k[i]}$ , for sub channel $k$ at time $i$ . $(1 \le k \le M)$ . Each sub-channel suffers from AWGN noise with power spectral density $\frac{N_0}{2}$ of $n[i]$ . Let $\bar{P}$ and $B$ denote average transmit power and bandwidth of received signal, respectively. Then we define the SNR of this system as follows:

$$\gamma_{k[i]} = \left(\frac{\bar{P}}{N_0 B}\right) g_k[i] \tag{2}$$

It's obvious that the distribution of $\gamma_{k[i]}$, i.e. fading state, is same as the $g_k[i]$. Therefore, this distribution is exponential, in the case of Rayleigh fading channel, and its pdf is:

$$p(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \tag{3}$$

## III. SINGLE USER CHANNEL

As mentioned in [2], for Rayleigh fading channels zero-outage capacity goes to zero. Therefore outage capacity analyses, in this case, would be more reasonable. The outage capacity is defined as a maximum constant data rate could be maintained in all non-outage states of channel [1]. By this definition, as it mentioned in [1], outage capacity can be defined as:

$$C(p_{out}) = B\, log_2(1+\sigma)p(\gamma > \gamma_0) \tag{4}$$

where $\sigma$ is the constant received SNR and $\gamma_0$ is cut-off fading state above which transmission should be maintained. From [1],$\sigma$ can be defined as:

$$\sigma = {}^{1}\Big/_{E_{\gamma_0}\left[\frac{1}{\gamma}\right]} \tag{5}$$

Where

$$E_{\gamma_0}\left[\frac{1}{\gamma}\right] \triangleq \int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma)\, d\gamma \tag{6}$$

In which $p(\gamma)$ indicates pdf of $\gamma$ and, in Rayleigh fading case, is exponential. That is:

$$p(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \tag{7}$$

It's obvious that $C(p_{out})$ is a function of two variables: 1. $\gamma_0$: cut-off fading state 2.$\bar{\gamma}$: expectation of channel SNR. Hence, this definition of capacity can be optimized over these two variables. However, we optimize the capacity over all possible $\gamma_0$ and consider $\bar{\gamma}$ as a parameter of capacity function. It means that:

$$C = max_{\gamma_0} B log_2(1+\sigma) p(\gamma > \gamma_0) \tag{8}$$

It is obvious that if we differentiate $C$ with respect to $\gamma_0$ , to obtain the optimum value of that ,this optimum value is obtained as a function of $\bar{\gamma}$.hence it can be said that the optimum cut-off fading state is a function of channel statistical characteristics that, in this case, is expectation of channel SNR.

We prove the existence of this optimum cut-off fading state in Rayleigh fading channel as bellow and to find its distinct value numerical solutions could be established.

For Rayleigh fading channel, $\gamma_0$ increases as the value of $p(\gamma > \gamma_0)$ decreases. On the other hand, by increasing the$\gamma_0$, transmission should be suspended in more conditions, and therefore higher data rate is achieved in non-outage states and it yields to increase the logarithmic term of capacity:

$$B log_2\left(1 + \frac{1}{E_{\gamma_0}\left[\frac{1}{\gamma}\right]}\right)$$

Hence the outage capacity function has an extreme point and our simulations show that the extreme point has a maximum value in function in this case.

Existence of extreme point in outage capacity function could be proved mathematically as follows:

We consider this function to be as:

$$C(\gamma_0) = f(\gamma_0) g(\gamma_0) \tag{9}$$

Where

$$f(\gamma_0) = B log_2\left(1 + \frac{1}{E_{\gamma_0}\left[\frac{1}{\gamma}\right]}\right) \tag{10}$$

And

$$g(\gamma_0) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \tag{11}$$

From differentiation theory we have:

$$\frac{\partial c}{\partial \gamma_0} = g(\gamma_0)\frac{\partial f}{\partial \gamma_0} + f(\gamma_0)\frac{\partial g}{\partial \gamma_0} \tag{12}$$

The value of $f(\gamma_0)$, $g(\gamma_0)$ and $\frac{\partial f}{\partial \gamma_0}$ will be greater than zero when all $\gamma_0$ but $\frac{\partial g}{\partial \gamma_0}$ gets negative values while $\frac{\partial c}{\partial \gamma_0}$ reaches zero with certain value of $\gamma_0$.

In Fig. [1] simulation results are shown for $\bar{\gamma} = 10\, dB$. In this figure normalized outage capacity about channel bandwidth is plotted versus $\gamma_0$.our numerical results show that by decreasing the cut-off fading state, the outage capacity increases to an extreme point. After that it begins to decrease. It means that outage capacity in the case of Rayleigh fading channel could be maximized over all possible cut-off fading states. On the other hand, if we adjust the transmitter to transmit a power by which the optimum cut-off fading state could be guaranteed, the maximum data rate in the channel would be achieved.

It is important to notice that receiving a constant SNR at the receiver tantamount to design a transmission scheme can invert channel variations optimally. Further researches can be done to investigate these schemes based on the results of this paper.

These results are tabulated in Table [1].The effect of $\bar{\gamma}$ will be analyzed for multiuser channel where each sub-channel experience different $\bar{\gamma}$.

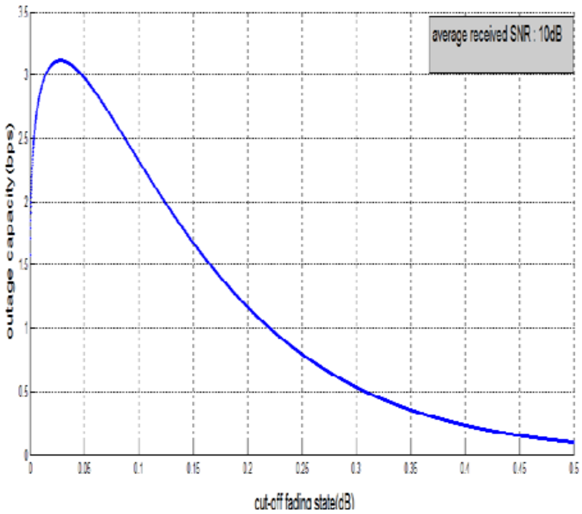


Figure 1 Normalized outage capacity of single user channel as function of cut-off fading state for an average SNR of 10dB

TABLE 1.NORMALIZED OUTAGE CAPACITIES FOR SOME CUT-OFF FADING STATE FOR AN AVERAGE RECEIVED SNR OF 10DB

| Cut-off fading state(dB) | Normalized Outage capacity(bps/Hz) |
|---|---|
| -5.4 | 0.5895 |
| -7.1 | 1.2010 |
| -8.5 | 1.7620 |
| -15.5 | 3.1132 |
| -22.8 | 2.6482 |

## IV. MULTIUSER CHANNEL

We consider a Rayleigh block fading broadcast channel which consists of a transmitter and $M$ receivers. In broadcast channel as a single user channel, a constant data rate is expected at each sub-channel thus a constant SNR should be met at each receiver which is indicated by $\sigma_i$ in this paper. The values of constant received SNR for each sub-channel respectively depends on expected values of SNR($\overline{\gamma_i}$ ,$1 \leq i \leq M$) and cutoff fading state associated with the sub-channel ($\gamma_{0i}$ , $1 \leq i \leq M$), as shown in (5),(6),(7). Since $\gamma_{0i}$ for sub-channel $i$ depends only on $\overline{\gamma_i}$ , as discussed above, the optimum value of that could be obtained independently for sub-channels, where each sub-channel experiences independent power constraints. As a result the complexity of finding optimum values of $\gamma_{0i}$ doesn't scale with the number of sub-channels and it can reduce computational load in practical systems.

For simulating, we consider $\overline{\gamma_i}$ as a parameter of capacity function and numerical results have been obtained. It is shown in Fig. [2], as a simulation result, since $\overline{\gamma_i}$ decreases, the $\gamma_{0i}$ (cut-off fading state at which the maximum outage capacity is achieved) also decreases. It has been shown that, by increasing the $\overline{\gamma_i}$ the values of maximum capacity decrease at first and then it begins to increase. However, it is obvious that maximum outage capacity doesn't fall from some minimum values. These results are shown numerically in Table [2].

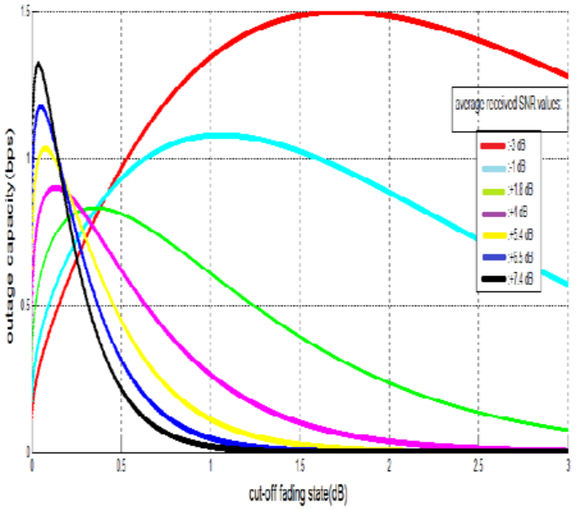


Figure 2 Normalized outage capacity as function of cut-off fading state for some value of average received SNR

TABLE 2.NORMALIZED MAXIMUM OUTAGE CAPACITIES AND THEIR RELATED AVERAGE RECEIVED SNR AND OPTIMUM CUT-OFF FADING STATE

| Average received SNR(dB) | Optimum cut-off fading state(dB) | Normalized Maximum outage capacity(bps/Hz) |
|---|---|---|
| -3 | 2.4 | 1.4955 |
| -1 | 0.2 | 1.0769 |
| +1.8 | -4.5 | 0.8287 |
| +4 | -8.8 | 0.8981 |
| +5.4 | -11.2 | 1.0315 |
| +6.5 | -13.0 | 1.1742 |
| +7.4 | -14.3 | 1.3174 |

## V. CONCLUSION

Outage capacity of Rayleigh fading channel was studied. It was assumed that CSI is known at transmitters and receivers. Optimization problem was solved for both single user and multiuser channels. We showed that by designing a special transmission scheme, maximum outage capacity of the channel could be obtained. Furthermore it was shown that this optimization problem can be solved for multiuser channels without scaling the process complexity with number of channel users. The maximum value and optimum parameters of outage capacity were analyzed.

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