Statistical Analysis of Wind Speed Profile: A Case Study from Iasi Region, Romania

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***Abstract*- The increased integration of wind power into the electric power systems brings new challenges for effective planning and operation. The Weibull distribution is a widely used distribution, especially for modelling the random variable of wind speed. In the paper, the author presents a comparative analysis of some methods for estimating the Weibull parameters. These methods require historical wind speed data, collected over a certain time interval, to establish the parameters of the wind speed distribution for a particular location. Results for a real-world database, collected from the north-east area of Romania, are presented in a study case.**

***Keywords*- Wind Speed Data; Wind Energy; Probability Density Function; Weibull Distribution**

I. INTRODUCTION

The integration of renewable energy in electric power systems is growing rapidly due to the concerns related to environment and the depleting sources of conventional power generation. Unlike other renewable energy sources, wind power has become competitive with conventional sources of power generation, thus the growth rate of wind power has registered the highest growth among other renewable sources. In accordance with [1], the annual average growth rate of wind power over the last 10 years is about 22%. At the end of 2012, the global installed wind power capacity reached 282.5 GW, from that about 44.8 MW was installed in 2012. In 2012, Romania has doubled its total installed wind power capacity, from 982 MW at end of 2011, to 1905 MW at end of 2012. Wind generation brings a great amount of benefits to power systems, such as the cheaper energy compared with the thermal generation, emission reduction, wind energy is available for large areas, and development of a wind power farm can be implemented relatively easily [2]. Wind generation brings a series of difficulties to traditional power systems, for example, uncontrollability of power generation, the wind generation depends on wind availability, irregularly fluctuating and intermittence of power generation, respectively a poor predictability of the wind generation [3].

The important properties in wind generation are the wind speed frequency and magnitude. The wind power output obtained from the wind is directly proportional to the cube of the wind speed. One of the main characteristics of wind is that it is highly variable and its properties vary from one location to another. Wind speed changes continuously and in order to estimate its speed and frequency values, the statistical approach could be viable method for these estimations. Wind speed probability density function plays an important role in electric power generation applications with wind turbines. The most important requirement for effective wind power planning and operation in power systems is an accurate estimation of wind speed distribution. Investigation of wind power generation should be carefully performed in accordance with the wind speed probabilistic character.

This paper presents a statistical analysis of main characteristics of the wind speed from the region around Iasi, Romania. The wind speed data is measured as hourly average values, being statistically analyzed over one year period of time. The probability density distribution is derived from these values and their distributional parameters are evaluated with different statistical methods.

The paper is organized as follows. In order to evaluate the wind speed distribution, the main issues about the wind speed probability density function, about the dependence of the height and also about the wind direction are presented in Section 2. In Section 3, the main statistical methods used to evaluate the parameters of probability density functions are reported. Furthermore, two statistical tests used in this paper to establish the accuracy of the methods are also reported. In Section 4 a numerical analysis is developed in order to evaluate the statistical parameters, based on the hourly wind speed database collected from the Iasi region. Finally, the main conclusions are given in Section 5.

II. THE WIND SPEED DISTRIBUTION

The available wind energy depends on the wind speed, which is a random variable. The wind-speed values occurrences over a long period of time can be described by its probability distribution function.

A. Wind Speed Probability Density Function

A number of studies have been published in scientific literature related to wind energy, which propose to use a variety of probability density functions (e.g. normal, lognormal, gamma, Rayleigh, Weibull) to describe wind speed distributions [4, 5]. The common conclusion of these studies is that the Weibull distribution with two parameters may be successfully utilized to describe the principle wind speed variation. The Weibull probability density is given by the following expression:

$$f_W(v) = \frac{\beta}{\alpha}\left(\frac{v}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right]. \tag{1}$$

The corresponding cumulative probability function of the Weibull distribution is:

$$F_W(v) = 1 - \exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right], \tag{2}$$

where α (m/s) is the scale parameter and β (dimensionless) is the shape parameter of the Weibull distribution.

The Weibull distribution is one of the most widely used distributions in many technical fields. This distribution has a particular property, namely it does not have a specific characteristic shape, taking the characteristics of other distributions, based on different values of shape parameter, as is shown in Fig. 1.

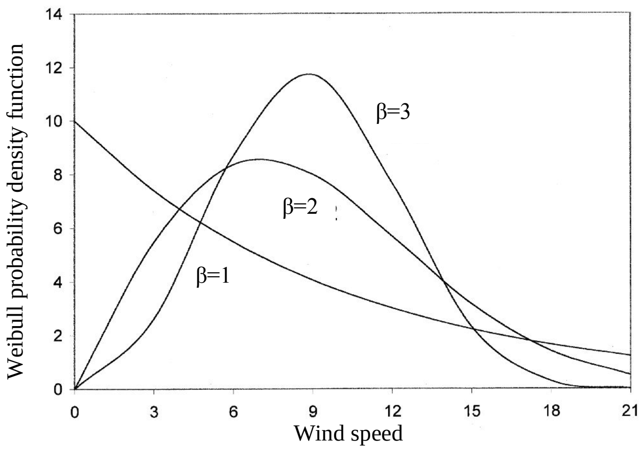

Fig. 1 Weibull probability distribution function with different values of shape parameter

The Weibull distribution becomes a hyperexponential distribution when shape parameter is less than unity and it is a well-known exponential distribution when the shape parameter is equal to 1. The Weibull distribution is a Rayleigh distribution when shape parameter is equal to 2 and becomes a normal distribution when this parameter is equal to 3.4, respectively an approximate normal distribution when β has a close value to 4.

Generally, the scale parameter provides information about the average of the wind speed profile, while the shape parameter provides information about the deviation of the wind speed values around the mean as well as the feature of probability density function. It can be seen that Weibull distribution gets relatively narrower and higher as shape parameter increases. The peak of density function also moves in the direction of higher wind speeds as shape parameter increases. The shape and scale parameters are interconnected through analytical expressions of mean and variance of Weibull probability density function.

B. Variation of Wind Speed with the Height

The wind blows faster at higher altitudes because the influences of the ground surface and the air density have lower values. The most common expressions for wind speed variation with the height use the wind profile power law, which is based on the ground friction coefficient, described by the following equation:

$$v(z)/v(z_r) = (z/z_r)^k \tag{3}$$

In (3), $v(z)$ and $v(z_r)$ are the wind speeds at desired z and registered z_r heights, while k is the friction coefficient, which depends on surface roughness and atmospheric stability [6]. Numerically, it ranges between 0.05 for smooth terrain, and 0.5 for rough terrain, with the most frequently adopted value around 0.14.

In accordance with [7], between the parameters of Weibull distribution for different heights, there are the following relationships:

$$\alpha(z) = \alpha(z_r)\cdot(z/z_r)^k \,;\, \beta(z) = \beta(z_r) \tag{4}$$

Previous relationship is based on the expression of Weibull distribution that it has been changed in accordance with the relationship between the wind speed and height. As can be seen from Eq. (4), the shape parameter is a specific property of wind profile, while the scale parameter may be adjusted, in narrow range, by changing the desired height.

C. Wind Direction Distribution

Generally, the wind blows from different directions, the wind directions could be depicted by the wind rose diagram. A wind rose is a chart which gives a view of how the wind speeds and directions are distributed at a particular location over a specific period of time. It is a very useful representation because a large quantity of data can be summarised in a single plot.

If terrain roughness in all directions is similar, it makes no difference of the wind speeds above to hub of wind turbine, because the turbine yaw system makes the rotor to follow the wind direction. If terrain around the turbine is significantly different with respect to roughness or obstacles, the wind speeds to hub of wind turbine have different values depending on the wind direction distribution, this leading to necessity of developing detailed calculations. Therefore, with exception in critical cases, the fact that the wind comes from different directions is not used, instead all wind is assumed to come from the same direction.

III. METHODS FOR THE PARAMETERS ESTIMATION

The wind speed distribution is completely determined when its parameters are numerically established. The estimates of the parameters of the Weibull distribution can be found using different estimation methods, which can be classified in graphical methods or analytical methods. The most common analytical methods are maximum likelihood estimator, method of moments and least squares method [8, 9]. Each analytical method, discussed in this paper, has specific criteria which yields estimates that are most suitable in some situations.

The Weibull parameters play a major role in developing a model of electric power wind generator, so it is important that different estimation methods are compared to fit parameters of the Weibull distribution from wind speed database. This paper attempts to find an answer to the question: which method gives the best Weibull parameters estimation? The performance of these methods with the same wind speed database will be analyzed. The relative mean bias error (RMBE) and relative root mean square error (RRMSE) will be used in statistical evaluation of the performance of Weibull parameters evaluation.

A. The Maximum Likelihood Estimator

The Maximum Likelihood Estimator (MLE) is an analytical method, widely applied in engineering and mathematics problems. For our case, for Weibull distribution of wind speed, in accordance with MLE theory, the likelihood function is built as the joint density of the *n* random variables and is a function of the two unknown parameters:

$$L(\alpha,\beta)=\prod_{i=1}^{n} f(v_i)=\prod_{i=1}^{n}\frac{\beta}{\alpha}\left(\frac{v_i}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{v_i}{\alpha}\right)^{\beta}\right], \tag{5}$$

where α and β values can be achieved by using iterative methods or limits method. Last method of parameters evaluation involves taking the partial derivatives of the likelihood function with respect to the parameters, setting the resulting equations equal to zero:

$$\frac{\partial \ln(L)}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\ln v_i-\frac{1}{\alpha}\sum_{i=1}^{n}v_i^{\beta}\cdot\ln v_i=0 \text{ and } \frac{\partial \ln(L)}{\partial \alpha}=-\frac{n}{\alpha}-\frac{1}{\alpha^2}\sum_{i=1}^{n}v_i^{\beta}=0 \tag{6}$$

The values of α and β result from simultaneously solving of both equations.

B. The Method of Moments

The method of moments (MOM) is another analytical method to establish the distribution parameters. If the set of wind data is konwn, the moments of unknown parameters that depend by the two-parameter Weibull distribution will be equalized with the empirical moments.

The analytical expression of mean and the variance of Weibull distributions can be directly calculated from the following equations:

$$M(v)=\alpha\cdot\Gamma(1+1/\beta) \text{ and } D^2(v)=\alpha^2\cdot\left[\Gamma(1+2/\beta)-(\Gamma(1+1/\beta))^2\right] \tag{7}$$

where $\Gamma(\ )$ is the gamma function, while the empirical moments are calculated with following equations:

$$\bar{v}=\sum_{i=1}^{n}\frac{v_i}{n} \text{ and } \sigma^2=\frac{1}{n}\sum_{i=1}^{n}\left(v_i-\bar{v}\right)^2 \tag{8}$$

The β parameter can be got from the coefficient of variation (by dividing the variance on the square mean) and, after that, the α parameter can be established based on first expression from Eq. (7).

C. Least Squares Method

For the estimation of Weibull parameters, the least-squares method (LSM) is extensively used in engineering problems. The method provides a linear relation between the two parameters having as start point the twice logarithms of Weibull cumulative distribution function, as follows:

$$\ln \ln\left[1-\frac{1}{F_w(v)}\right]=\beta \ln(v)-\beta \ln(\alpha) \tag{9}$$

This relationship represents a straight line, expressed as: $Y = a \cdot X + b$, where:

$$Y=\ln \ln\left[1-\frac{1}{F_w(v)}\right],\ X=\ln(v), \text{ and } a=\beta\ ,\ b=-\beta \ln(\alpha) \tag{10}$$

Performing rank regression on Y requires that a straight line to be fitted to a set of data points so that the sum of the squares of the deviations from the points to the line is minimized. Both α and β parameters can be evaluated from coefficients of polynomial linear fitting, using a simple linear regression.

D. Statistical Tests Analysis

The relative mean bias error (RMBE) and the relative root mean square error (RRMSE) have been used in statistical evaluation of the performance of the Weibull distribution. These statistical tests are based on the following expressions:

$$RMBE(\%)=\frac{1}{N}\sum_{i=1}^{N}\left(v_i-v_i^*\right)\Big/\frac{1}{N}\sum_{i=1}^{N}\left(v_i^*\right)\ \cdot 100 \tag{11}$$

$$RRMSE(\%)=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(v_i-v_i^*\right)^2}\Big/\frac{1}{N}\sum_{i=1}^{N}\left(v_i^*\right)\cdot 100 \tag{12}$$

where v_i is the ith actual data, v_i^* is the ith predicted data with the Weibull distribution, N is the number of observations. The RMBE and RRMSE provide information about the model's performance, lower values being desirable [9, 10]. Therefore, the best distribution function can be selected according to the lowest values of RMBE and RRMSE.

IV. CASE STUDY FROM IASI REGION

In the present case study, the wind potential of the region is statistically analysed based on one year hourly measurements. The wind speed database behind these studies has been recorded from the north-east area of Romania, namely from Iasi region. The region of interest is located at 47°10' north latitude, 27°37' east longitude and 80 m above the sea level. The recorded data covers the period between 1th January 2010 and 31th December 2010. The measurements available in the original database are characterized by one hour acquisition intervals, the hourly average value being recorded.

The wind speed values collected at anemometer height (10 m above the ground) have been involved in an adjustment with the height of wind turbine. In the paper, the Eq. (3) has been used to evaluate the wind speed values at the hub wind turbine height (100 m), considering the same terrain roughness around to wind turbine and 0.14 as the friction coefficient. Fig. 2 shows the wind speed values available at hub wind turbine height (100 m).

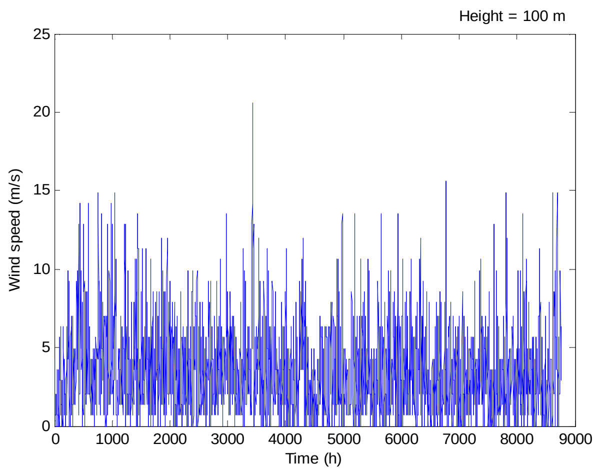

Fig. 2 The hourly speed database collected on Iasi region

The variation of wind speed with the direction has also been studied. The wind rose is a graphical representation of the wind directions and speeds, over the period of time at the specific location. To create a wind rose, average wind direction and wind speed values have been sorted by wind direction so that the percentage of time that the wind was blowing from each direction has been determined. Typically, the wind direction data is sorted into twelve equal arc segments, 30° each segment, in preparation for plotting a circular graph in which the radius of each of the twelve segments represents the percentage of time that the wind blew from each of the twelve 30° direction segments. Fig. 3 shows the wind rose diagram of the wind speed database at 100 m height. As can be seen, the main direction of the wind follows the north-south direction. Considering the same roughness of terrain around wind turbine and taking into account that the rotor follows the wind direction, it can be stated that the wind speed values are not affected by the wind direction.

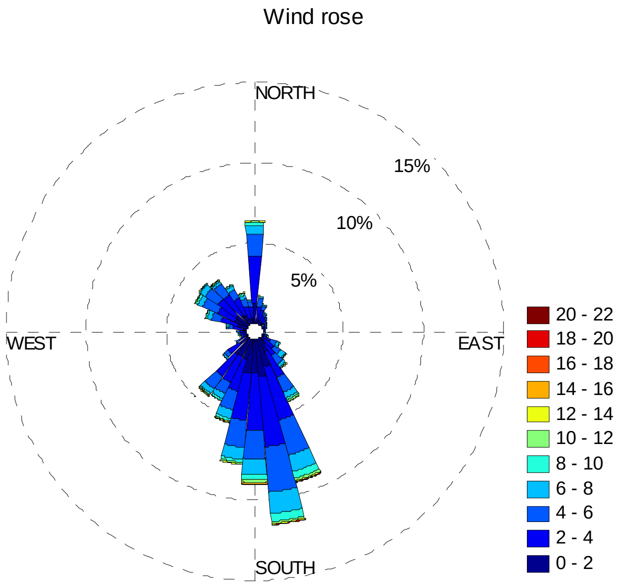

Fig. 3 The wind rose associated to the wind database

Based on these measurements, the parameters of Weibull distribution that approximate the real database of wind speed frequency have been estimated. In order to compare the methods described earlier, a Matlab® program has been developed to evaluate the Weibull parameters, based on previous methods and same wind speed database.

To evaluate the performance of these methods, the relative mean bias error (RMBE) and relative root mean square error (RRMSE) have been used to evaluate the accuracy of estimated probability density function to real distribution. The RMBE and RRMSE are statistical tests widely used to evaluate the difference between values provided by an estimated probability density function and the real values of the database distribution. The Table 1 shows the Weibull parameters for analyzed database, the scale and shape parameters being determined using previous methods reported in Section 3.

TABLE 1 WEIBULL PARAMETERS FOR WHOLE YEAR

Parameters /statistical tests	**Method**		
	MLE	**MOM**	**LSM**
Scale parameter α	3.9788	3.9876	3.6466
Shape parameter β	1.8687	1.8767	1.6424
RMBE	0.3438	0.5310	-0.8379
RRMSE	1.0868	1.3991	1.6910

As can be seen, the parameters of Weibull distribution evaluated through above three methods are very close, the scale parameter lies between 3.6466 and 3.9876 m/s, while the shape parameter lies between 1.6424 and 1.8767. Likewise, in the last two lines of table are shown the values of RMBE and RRMSE for each used method. It is found that MLE is superior in accuracy and has a smaller error compared with the MOM and LMS methods. Furthermore, for this analysis, it seems that the LMS method is the least accurate method.

Furthermore, in order to find if the diurnal variation of wind speed has a significant difference, the diurnal variations of the wind speed have been further studied. To this end, the wind speed database has been divided into two sets of wind speed values. For simplicity in this case, day-time is defined between 7 AM and 7 PM, and night-time is defined as 7 PM to 7 AM. These two databases have been separately analysed, a typical diurnal representation of the wind speed is shown in Figs. 4 and 5.

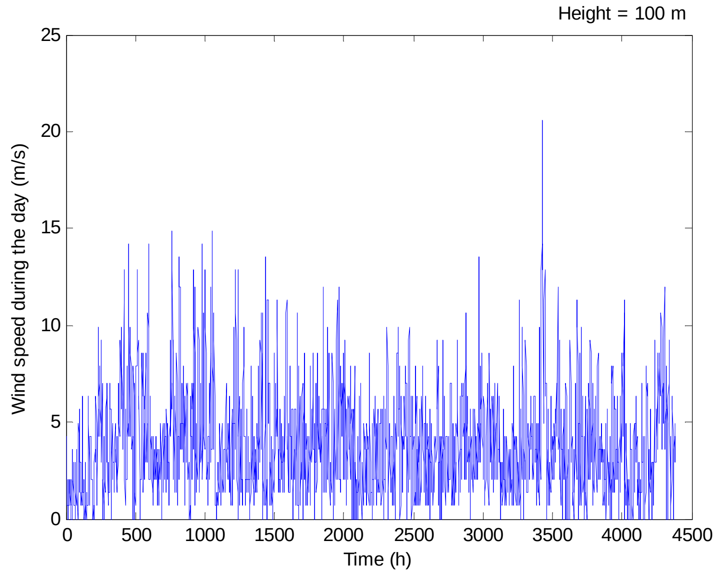

Fig. 4 The diurnal speed database (day-time)

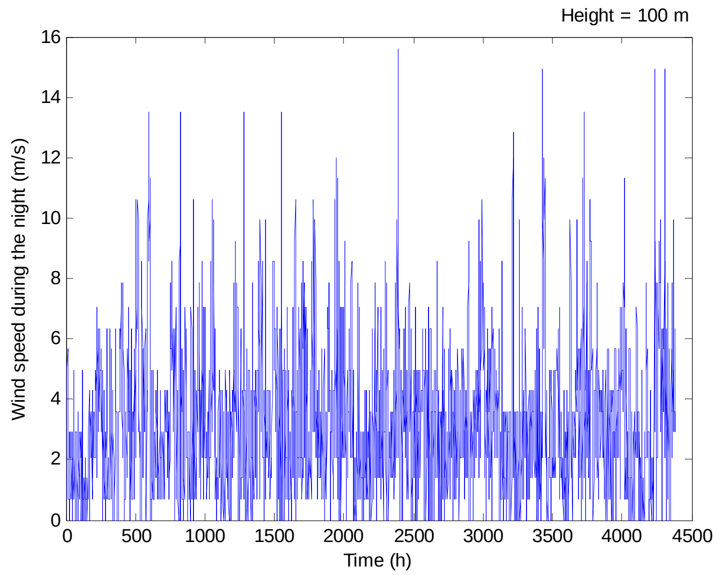

Fig. 5 The diurnal speed database (night-time)

A similar analysis of the diurnal wind direction has been conducted, the wind rose diagrams for both sets of wind speed, over the day and night, being depicted in Figs. 6 and 7.

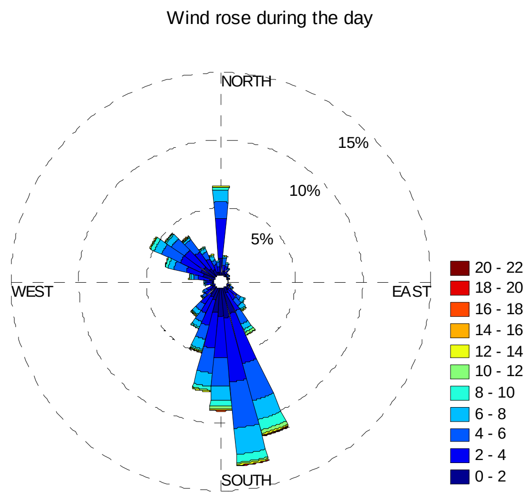

Fig. 6 The wind rose associated to the diurnal wind database (day-time)

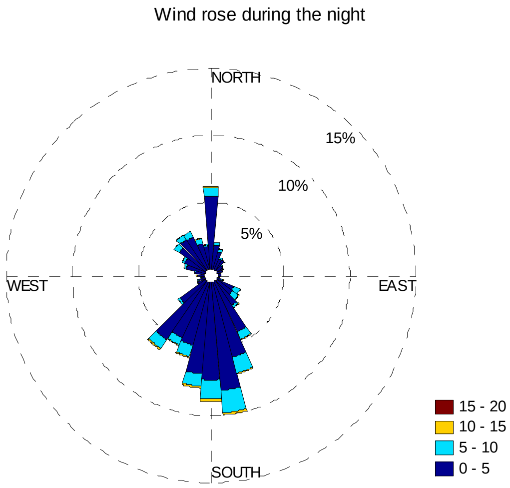

Fig. 7 The wind rose associated to the diurnal wind database (night-time)

The Weibull parameters analytically calculated for the available data using the methods given in Section 3 are presented in Table 2. It is seen from the table that the scale factor varies between 3.9183 and 4.1987 m/s, the shape factor ranges from 1.7631 to 1.9155 for the wind speeds during the day, while during the night, the scale factor varies between 3.5523 and 3.6997 m/s, and the shape factor ranges from 1.5542 to 1.7620.

TABLE 2 WEIBULL PARAMETERS FOR DIURNAL DATABASE

	Parameters / statistical tests	**Method**		
		MLE	**MOM**	**LSM**
Day - time	Scale parameter α	4.1288	4.1987	3.9183
	Shape parameter β	1.8975	1.9155	1.7631
	RMBE	0.6140	0.7268	-0.9390
	RRMSE	1.1600	1.2660	1.6603
Night - time	Scale parameter α	3.6793	3.6997	3.5523
	Shape parameter β	1.6796	1.7620	1.5542
	RMBE	0.7629	0.9665	-1.2718
	RRMSE	1.3682	1.2900	1.9136

In Table 2 the RMBE and RRMSE values are also calculated. As can be seen, the scale and shape parameters, from the whole database and from diurnal value, have the best estimation when the MLE is used.

In order to compare the estimation methods using different sample sizes of database, the above methods have been applied for the wind speed values from the months from each of the four seasons. The average seasonal Weibull parameters are presented in Table 3.

TABLE 3 WEIBULL PARAMETERS FOR SEASONS

Season	**Parameters / statistical tests**	**Method**		
		MLE	**MOM**	**LSM**
Spring	Scale parameter, α	3.6447	3.3856	3.6324
	Shape parameter, β	1.8502	1.8367	1.8777
	RMBE	0.1800	0.1953	-0.2083
	RRMSE	1.3511	1.4322	1.8451
Summer	Scale parameter, α	2.7708	2.8906	3.0591
	Shape parameter, β	1.9701	1.9075	1.9158
	RMBE	0.2276	0.2639	-0.2891
	RRMSE	1.2514	1.4129	1.8633
Autumn	Scale parameter, α	3.1094	3.1594	3.5774
	Shape parameter, β	2.0348	1.9924	2.1058
	RMBE	0.2402	0.2618	-0.2684
	RRMSE	1.2214	1.3945	1.7412
Winter	Scale parameter, α	4.4682	4.1037	4.5159
	Shape parameter, β	2.1584	2.1605	2.0556
	RMBE	0.1695	0.1884	-0.2458
	RRMSE	1.1855	1.3868	1.7015

As can been observed, the scale and shape parameters, from the whole database and from seasonal values, have the best estimation in case of MLE. Thus, it can be summarised that the MLE is the best method used to estimate the parameters for the two-parameter Weibull distributions taking into consideration the RMBE and RRMSE as measurements of comparison, while the LMS method is the least accurate method.

V. CONCLUSIONS

In practice, it is very important to describe the variation of wind speeds for optimal design of the wind generation systems. The wind variation for a typical site is usually described using the Weibull distribution. Therefore, it is very important to know the best method for parameters evaluation, with minimal errors. This study has been developed to compare the results of three methods of parameters estimation, for the same database.

In the present study, hourly wind speed data of Iasi has been statistically analyzed. The probability density distributions have been derived from this database and the distributional parameters have been evaluated. It has been shown, from computational results, that method which gives the lowest values of statistical tests is the MLE, in both cases, for whole year database and for diurnal values. However, from accuracy viewpoint, the MOM and LSM of fitting Weibull function were also good methods because the methods give the close parameters as MLE and, more over, Matlab package contains functions and tools that estimate the parameters and confidence intervals for Weibull data.

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