Numerical Simulation of Glow Discharge in a Magnetic Field Through the Solution of the Boltzmann Equation

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Abstract- **Numerical simulation model of a Direct Current Discharge (DCD) in external magnetic field is used for analysis of behavior of normal DCD at the initial time instants after switching-on of transversal magnetic field. Numerical simulation results are presented for two-dimensional glow discharge at pressure 5 Torr, Emf of power supply of 2 kVolt, and magnetic field induction of B = 0.05 T. The model is based on the diffusion-drift theory of gas discharge consisting of continuity and momentum conservation equations for electron and ion fluids, as well as the Poisson equation for the self-consistent electric field. Feature of our approach is to calculate electron transport coefficients for diffusion-drift model by solving the electron Boltzmann equation. It is shown that the switching-on of transversal magnetic field gives rise to the DCD plasma oscillations. The plasma oscillations are observed also in moving DCD in perpendicular direction to applied electric and magnetic field.**

Keywords- Glow Discharge in a Magnetic Field; Electron Kinetic in a Magnetic Field; Diffusion-drift Model of Glow Discharge in a Magnetic Field

I. INTRODUCTION

Research of fundamental regularities of interaction of gas flows with localized areas of electric discharges is in the center of attention of the modern aerophysics, because increasing of velocities and flying heights of aircrafts gradually displace interests of classical aerodynamics in direction of physical aerodynamics of partially ionized gas flows [1-10]. Experimental and theoretical researches of last years show real features of local control of gas streams for aerospace applications [11].

Developments and applications of computing physical-chemical models of weakly ionized gases are the component parts of the researches. Two groups of such models are generally used: the diffusion-drift model, and the simplified quasi-neutral ambipolar model. These models have allowed to essentially improve understanding of physics of gas discharges in viscous gas streams. Comparison with results of experimental researches has allowed to define areas of applicability of the specified models and to specify such conditions in gas streams at which existing models predict unsatisfactory results [1]. At the same time, using developed models for studying physical phenomena at experimentally studied conditions allows understand some new peculiarities of the physical processes, which were not analyzed earlier.

Our present paper studies an interaction of a glow discharge with external magnetic field, which is of practical interest for aerophysical applications. With developed theory of glow discharge in magnetic field [12], it is shown that at certain conditions in gas discharge gap some “anomalous” behaviors of glow discharge in external magnetic field are observed. Detailed calculations of the glow discharge at earlier time stage after switching-on of external transversal magnetic field show that at the beginning of the process cathode layer drifts in the negative direction of the *x*-axes (see Fig. 1), and only after some microseconds the glow discharge begins to drift in the positive direction of *x*-axes.

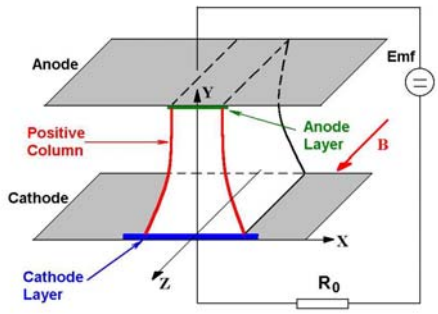


Fig. 1 Schematic of a glow discharge with external transversal magnetic field

The term “anomalous” is used here to stress some peculiarities of the glow discharges in external magnetic field. But, of course, it is shown that there are no any anomalous phenomena – the “anomalous” behavior can be explained by regular classical physics of glow discharges.

II. STATEMENT OF THE PROBLEM AND GOVERNING EQUATIONS

The problem, which is considered in the paper is formulated as following: Let at a some instant of time an external magnetic field is switched-on between two plate electrodes with steady state direct current discharge burning in the normal mode, as it is shown in Fig. 1 (about normal mode of DCD see in [13]). As it was established earlier, for example in [12], if any glow discharge is placed inside transverse magnetic field, then column of such a discharge shifts in direction perpendicular to the applied magnetic and electric fields (in the direction *x*, Fig. 1). Here we will consider in detail temporal evolution of the DCD column at initial instants of time after switching-on the external magnetic field. Data of ionization coefficient and transport properties are calculated by solving the electron Boltzmann equation.

Direct current discharge is considered in molecular nitrogen between flat electrodes (Fig. 1). The DCD structure in external magnetic field is described by continuity equations for concentration of electrons n_e and positive ions n_i together with the Poisson equations for the electro-static field $\vec{E} = -\mathrm{grad}\,\varphi$, and also by the energy conservation equation for neutral species [12]:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}\left(\mu_e n_e E_{e,x} - \frac{D_e}{1+b_e^2}\frac{\partial n_e}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_e n_e E_{e,y} - \frac{D_e}{1+b_e^2}\frac{\partial n_e}{\partial y}\right) = \alpha\left|\Gamma_e\right| - \beta n_e n_+, \tag{1}$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x}\left(\mu_+ n_+ E_{+,x} - \frac{D_+}{1+b_+^2}\frac{\partial n_+}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_+ n_+ E_{+,y} - \frac{D_+}{1+b_+^2}\frac{\partial n_+}{\partial y}\right) = \alpha\left|\Gamma_e\right| - \beta n_e n_+, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 4\pi e\left(n_e - n_i\right), \tag{3}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + q_J, \tag{4}$$

$$\vec{\Gamma}_e = n_e \vec{V} - D_e \mathrm{grad} n_e - n_e \mu_e \left(\vec{E} + \vec{V} \times \vec{B}\right), \tag{5}$$

$$\vec{\Gamma}_i = n_i \vec{V} - D_i \mathrm{grad} n_i + n_i \mu_i \left(\vec{E} + \vec{V} \times \vec{B}\right), \tag{6}$$

where $\vec{\Gamma}_e, \vec{\Gamma}_i$ are the electron and ion flux densities; $\left|\Gamma_e\right| = \sqrt{\Gamma_{e,x}^2 + \Gamma_{e,y}^2}$; $\vec{B}$ is the external magnetic field induction (its direction is shown in Fig. 1); $q_J = \eta\left(\vec{j}\vec{E}\right)$; $\vec{j} = e\left(\vec{\Gamma}_i - \vec{\Gamma}_e\right)$; $\alpha\left(E\right)$ and β are the ionization and recombination coefficients; μ_e, μ_i are the electron and ion mobilities; D_e, D_i are the electron and ion diffusion coefficients; η is the part of the Joule heat, which is realized in the gas heating;

$$b_e = \frac{\mu_e B_z}{c} = \frac{\omega_e}{\nu_e}, \quad b_+ = \frac{\mu_+ B_z}{c} = \frac{\omega_+}{\nu_{+n}}, \tag{7}$$

are the Hall parameters for electrons and ions;

$$\omega_e = \frac{eB_z}{m_e c} = \frac{eH_z}{m_e c}, \tag{8}$$

is the Larmor frequency of electrons,

$$\omega_+ = \frac{eB_z}{m_+ c} = \frac{eH_z}{m_+ c}, \tag{9}$$

is the Larmor frequency of ions.

Boundary conditions for charged particles and electrical potential are formulated as following:

$$y = 0: \ \frac{\partial n_i}{\partial y} = 0, \ \Gamma_e = \gamma \Gamma_i, \ \varphi = 0, \ T = T_w; \tag{10}$$

$$y = H: \ n_i = 0, \ \frac{\partial n_e}{\partial y} = 0, \ \varphi = \frac{V}{E}, \ T = T_w; \tag{11}$$

$$x = 0: \ \frac{\partial n_e}{\partial x} = \frac{\partial n_i}{\partial x} = \frac{\partial \varphi}{\partial x} = 0, \ \frac{\partial T}{\partial x} = 0; \tag{12}$$

$$x = L: \ \frac{\partial n_e}{\partial x} = \frac{\partial n_i}{\partial x} = \frac{\partial \varphi}{\partial x} = 0, \ \frac{\partial T}{\partial x} = 0; \tag{13}$$

where $T_w = 300$ K is the temperature of the cathode and anode.

A steady state solution for glow discharge at given Emf. of power supply, parameters of external electric circuit and pressure inside the gas discharge gap is used as the initial condition at $t = 0$.

III. CONSTITUTIVE RELATIONSHIPS

It is assumed that transport and thermo-physic properties of neutral particles of the discharge depend on the temperature, therefore:

$$\mu_e\left(p^*\right) = \frac{4.2\times10^5}{p^*}, \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

$$\mu_i\left(p^*\right) = \frac{2280}{p^*}, \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

$$p^* = p\frac{293}{T}, \text{ Torr} \tag{14}$$

$$D_e = \mu_e\left(p^*\right)T_e, \ D_i = \mu_i\left(p^*\right)T,$$

$$c_p = 8.314\frac{7}{2}\frac{1}{M_\Sigma}, \ \frac{\text{J}}{\text{g}\cdot\text{K}},$$

$$M_\Sigma = 28\frac{\text{g}}{\text{mol}},$$

$$\rho = 1.58\times10^{-5}\frac{M_\Sigma p}{T}, \frac{\text{g}}{\text{cm}^3},$$

$$\lambda = \frac{8.334\cdot10^{-4}}{\sigma^2\Omega^{(2.2)*}}\sqrt{\frac{T}{M_\Sigma}}\left(0.115 + 0.354\frac{c_p M_\Sigma}{\tilde{R}}\right), \ \frac{\text{W}}{\text{cm}\cdot\text{K}},$$

$$\Omega^{(2.2)*} = \frac{1.157}{\left(T^*\right)^{0.1472}}, \ T^* = \frac{T}{(\varepsilon/k)},$$

$$(\varepsilon/k) = 71.4\,\text{K}, \ \sigma = 3.68\,\text{Å},$$

$$\tilde{R} = 8.314\frac{J}{\text{K}\cdot\text{mol}},$$

where p is the undisturbed pressure, $N = 0.954\times10^{19}\frac{p}{T}$ is the concentration of the neutral particles.

Recombination coefficient β and the part of the Joule heat η are taken as constants: $\beta = 2\times10^{-7}$ cm^3/s, η=0.5. Temperature of electrons T_e is predicted by empirical correlation

$$\frac{T_e}{T} = 29.96\ln\left(\frac{E}{p}\right) + 24.64 \tag{15}$$

where T_e is the electron temperature, K; T is the gas temperature, K; E/p is the discharge parameter, $\text{V}/(\text{cm}\cdot\text{Torr})$. Correlation (15) is slightly extrapolated for cathode region, because the value of discharge parameter is out of range of the data [14].

The ionization coefficient is determined as follows (the 1st Townsend's coefficient):

$$\alpha(E) = p^* A_{in} \exp\left[-\frac{B_{in}}{\left(\left|\vec{E}\right|/p^*\right)}\right]\frac{1}{\mathrm{cm}\cdot\mathrm{Torr}}, \tag{16}$$

where $A_{in} = 12\,\frac{1}{\mathrm{cm}\cdot\mathrm{Torr}}$, $B_{in} = 342\,\frac{\mathrm{V}}{\mathrm{cm}\cdot\mathrm{Torr}}$.

Values of the 1st Townsend coefficients and transport coefficients for electrons in external magnetic field are also calculated by solving the Boltzmann electron equation using solver Bolsig+ [15], where the effect of the magnetic field is mimicked by making the electric field oscillate at the Larmor frequency (8). These values are approximated by a function (16) at different ranges of *E/N*. Constants *A* and *B* are presented in Table I.

TABLE I CONSTANTS IN APPROXIMATION (16) OF IONIZATION COEFFICIENTS, WHICH WERE OBTAINED BY SOLVING THE BOLTZMANN ELECTRON EQUATION

	E/N <200Td	$E/N \geq$ 200Td
A, $(\mathrm{cm}\cdot\mathrm{Torr})^{-1}$	4,53	7,91
B, $\mathrm{V}/(\mathrm{cm}\cdot\mathrm{Torr})$	445	498

Dependent transport coefficients in glow discharge on the magnetic field are illustrated in Fig. 2. All parameters of drift-diffusion model decrease in a magnetic field. It is shown that the 1st Townsend coefficient is the fastest changing function of magnetic field and *E/N* [13]. So exactly ionization coefficients have a significant influence upon electrodynamic structure of glow discharge.

A difference between empirical relation (16) end values, obtained by solving the electron Boltzmann equation, is presented in Fig. 3.

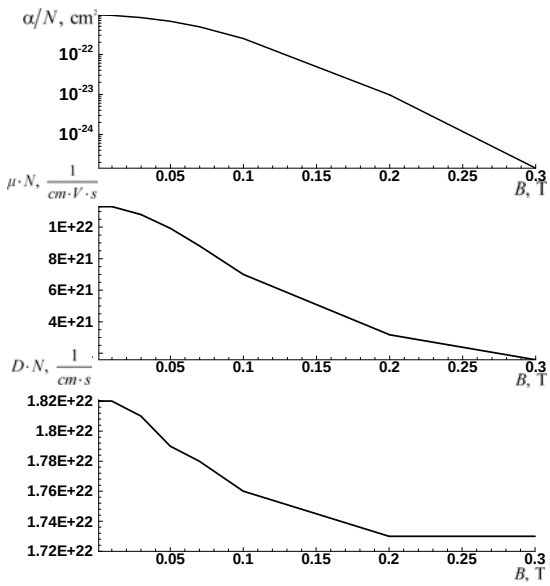


Fig. 2 Values of the ionization coefficients (multiplied by 1/N), mobility (multiplied by N) and diffusion coefficients (multiplied by N) in a magnetic field for E/N=100Td

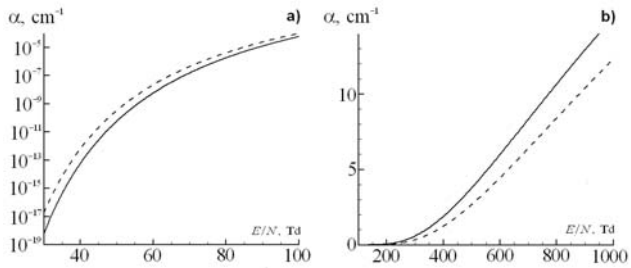


Fig. 3 Ionization coefficient data (the 1st Townsend coefficient) for electrons in magnetic field B=0.05T, calculated by solving the Boltzmann equation (dashed lines) and empirical relation (16) (solid lines). Data is presented for values of E/N, inherent for positive column (a) and for values of E/N inherent for cathode layer of Glow Discharge (b)

Equations (1)–(6) are supplemented with the equation for an external circuit, which is written for a stationary current as

$$\mathrm{E} = V + IR_0, \tag{17}$$

where V is the voltage on the electrodes; I is the discharge current; E is the Emf in power supply, and R_0 is the external resistance (this case corresponds to one sectioned cathode)

IV. NUMERICAL SIMULATION RESULTS

Calculations were performing for the following initial data: pressure of gas (N_2) is $p = 5$ Torr, Emf of a power supply is $E = 2$ kV, resistance of an external electric circuit is $R_0 = 300$ kOhm; the distance between flat surfaces of the cathode and the anode is $H = 2$ cm, and length of the flat channel is $L = 6$ cm (see Fig. 1). An induction of external magnetic field guided along an axis *z*, changed in a range $B_z = 0.05 \div 0.1$ T. Calculations at $B_z = 0.1$ T showed similar results in comparison with the case of $B_z = 0.05$ T, therefore only results corresponding to $B_z = 0.05$ T are presented here. All calculations were executed counting upon unity of length (1 cm) along an axis *z*.

Initial conditions of the calculations were formed by the following: at an initial instant a quasineutral plasma cloud was situated above the cathode at $x_0 = 3$ cm.

Concentration of charged particles was assumed equal to $n_0 = 10^{11}$ cm^{-3}. Formation of a glow discharge existing in a mode of normal current density was observed after process beginning at $15 \div 25$ μs (at absence of a magnetic field). Obtained stationary solution for the glow discharge in two-dimensional flat geometry was just used as initial condition for solution of the problem on dynamics of a glow discharge in transversal magnetic field. It should be stressed that in used numerical simulation procedure the Boltzmann equation was integrated in each node of calculation domain depending on current values *E/N*.

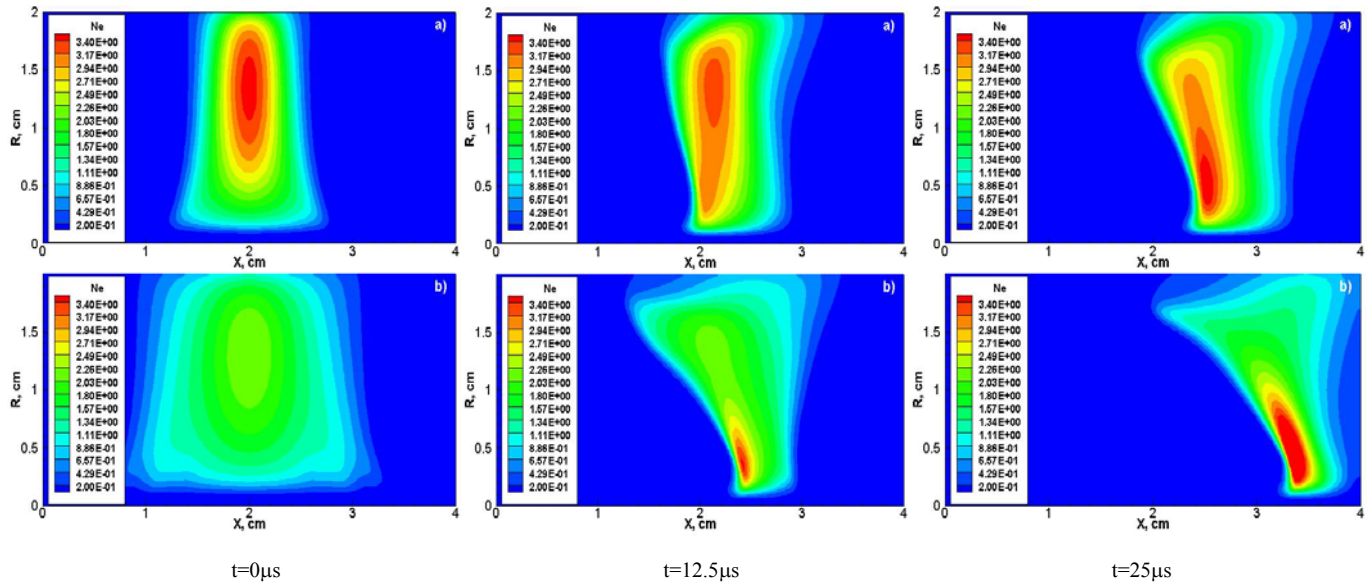
The inductivity of the magnetic field exerts influence on the global structure of the glow discharge. Fig. 4, 5 shows electron and ion contours in the glow discharge at $p = 5$ Torr and $E = 2$ kV at consecutive instants after the magnetic field of $B_z = 0.05$ T is applied. These are transient configurations of the direct glow discharge.

The specified unsteady solution is presented in Fig. 4. Electron and ion concentrations in the glow discharge are shown in Fig. 4. Near electrode regions of the glow discharge (the cathode and anode regions) and also positive column are well visible in these figures.

In all cases presented, the discharge column moves continuously along electrode surfaces. First of all, it is revealed that a transverse magnetic field shifts discharge path from the initial position. Comparing numerical simulations for different magnetic fields B, one can conclude that velocity of the discharge drifts perpendicular to applied magnetic field and is proportional to the value of B. The average velocity of such drift in the case of $B_z = 0.05$ T equals to $u_x = 8.1 \times 10^4$ cm/s. Due to the nature of ambipolar mechanism, these drift velocities are much less than electronic drift velocities but greater than the ionic drift velocities.

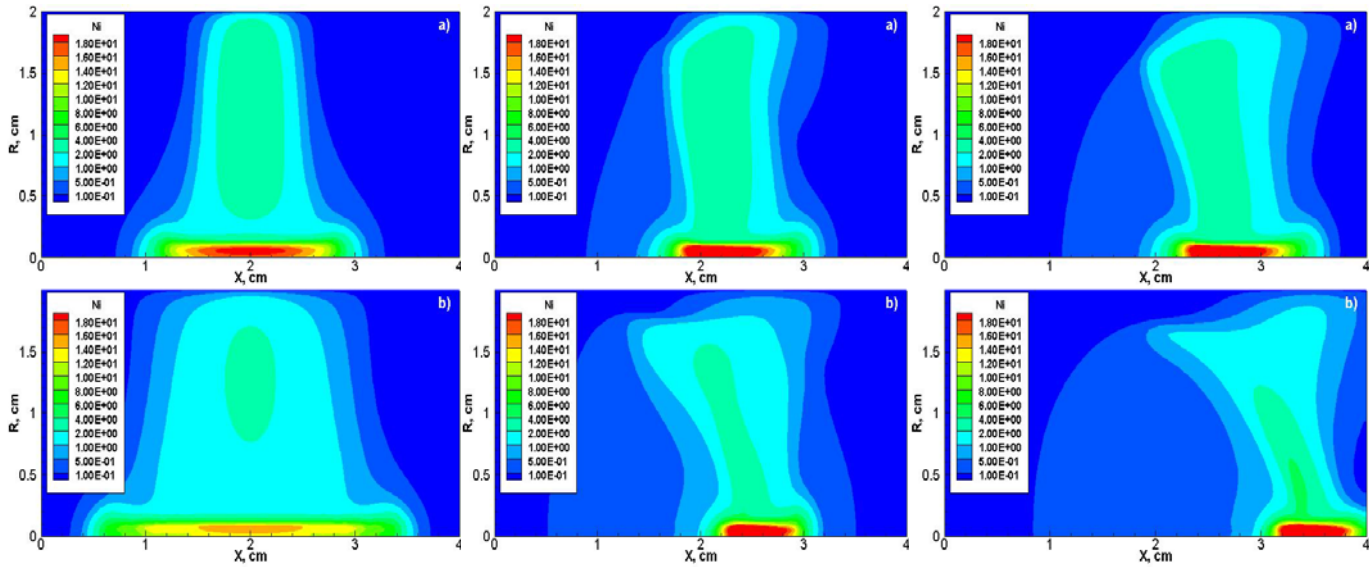
Let us consider regularity of evolution of the glow discharge plasma configuration after magnetic field switching-on. Initial configuration of electrons and ions concentration is shown in Fig. 4, 5 (*t*=0). Because electrons are much more mobile in crossed electric and magnetic field than ions, at the first nanosecond they begin to move in a direction of an *x*-axis. The electrons close to anode the large distance they shift at positive *x*-direction. But as the electrons shift from the initial steady-state configuration, a polarization field increases. Therefore from some instant of time they begin to shift in opposite direction. After that one can observe oscillation of electronic concentration near to anode, which will continue even after beginning of movement of the glow discharge column. Intensity of an electric field near to cathode is much more strong (one can say that there is a potential well near to cathode for electrons), therefore it is more difficult for these electrons to take part in oscillation mentioned above and stimulated by magnetic force. Note that for considering configuration of electric and magnetic field, the glow discharge begins to move in positive *x*-direction in full conformity with classical electrodynamic lows. This movement is distinctly seen at *t*>1 microsecond.

Ions are much more inert in comparison with electrons, therefore they actually do not feel magnetic field at the initial stage of the process, and their configuration actually remains invariable, with the exception of two spatial regions (near to anode and near to cathode). Near to anode one can observe increasing of ion concentrations in positive *x*-direction. It can be explained by additional ionization of neutral particles by increased numbers of electrons. Near to cathode one can observe decreasing of ions concentrations in positive *x*-direction. From the first point of view, it seems that the ions cloud is shifted in negative *x*-direction. But actually it is observed only some decreasing of ionization processes due to electrons gone the region under applied magnetic field. Some oscillations in ions concentrations at the beginning of the process ($t \le 1$ microsecond) are explained exceptionally by oscillation of electronic clouds. Nevertheless, after *t*~1 microsecond one can observe beginning of movement of the glow discharge column in positive *x*-direction.



t=0μs t=12.5μs t=25μs

Fig. 4 Electron contours (in 10^9 cm^{-3}) in the gas discharge gap at time moments t = 0, 12.5, 25 μs: with values of the ionization coefficients calculated by empirical relation (16) (a) and the electron Boltzmann equation solver [15] (b)



t=0μs t=12.5μs t=25μs

Fig. 5 Ion contours (in 10^{-9} cm^{-3}) in the gas discharge gap at time moments t = 0, 12.5, 25 μs: with values of the ionization coefficients calculated by empirical relation (16) (a) and using the electron Boltzmann equation solver [15] (b)

Usage of the electron Boltzmann equation to obtain ionization coefficient leads to growth of cathode and anode spots, decrease in concentration of electrons and ions in the center of glow discharge. Velocity of moving discharge in magnetic field also increases significantly.

It is worth to mention that analogous two- and three-dimension models of glow discharge in external magnetic field have existed already [16-20]. But empirical relations for transport coefficients of glow discharge plasma are used in these papers. Using the electron transport coefficients obtained by the solution of the Boltzmann equation will allow us in future to take into account heating effects in glow discharges in view of vibrational excitation of molecular gas and to consider the three-dimensional problems.

V. CONCLUSIONS

A theory and two-dimensional numerical simulations for modeling the electrodynamic structure of the glow discharges with a magnetic field are presented. With using developed two-dimensional model of glow discharge in a magnetic field is shown that switching-on transversal magnetic field results in origination of oscillation of electronic and ions concentration inside the column of glow discharge and in cathode and anode regions. It is also shown that mentioned oscillations are kept during the process of glow discharge movement in external transversal magnetic field.

All two-dimensional calculations of the glow discharge structure in nitrogen have been performed under various kinetic parameters of drift-diffusion model. The computed results exhibit good agreement with the classic theory of von Engel and Steenbeck.

The comparison of numerical simulation results obtained in the paper with those obtained by empirical coefficients of the diffusion-drift model demonstrates acceptable agreement. But in the both considered cases the Joule heat was assumed identical. Using of the electron energy distribution function obtained by integration of the Boltzmann equation will allow us in the future to take into account a real part of the Joule energy which goes to gas heating.

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