Capacity Precommitment and Three-Stage Duopoly with Complementary Goods

Kazuhiro Ohnishi[#1], Masamichi Kawano[*2]

[#]Institute for Basic Economic Science, Japan

[*]School of Economics, Kwansei Gakuin University, Japan

[1]ohnishi@e.people.or.jp; [2]mkawano@kwansei.ac.jp

***Abstract*-This paper considers a three-stage quantity-setting duopoly model with complementary goods. First, the first-mover firm decides whether or not to make a commitment to capacity. Second, the second-mover firm decides whether or not to make a commitment to capacity. Third, both firms choose their outputs simultaneously and independently. The paper demonstrates that there exist two opposite equilibria, and that at each equilibrium capacity investment is beneficial for both the firms.**

Keywords-Quantity-setting Model; Complementary Goods; Capacity Investment

I. INTRODUCTION

The modern oligopoly theories are essentially a set of different models that have been analyzed. These models include capacity investment, cost-reducing R&D investment, advertising, patent licensing, network competition, and so forth. For example, the possibility of firms using excess capacity to carry out strategic investments was studied by [1]–[3]. This idea is extended to a two-stage model by [7], to a three-stage model by [8]. Spence [2] argues that entry is deterred in an industry when incumbent firms have enough capacity to make a new entrant unprofitable and shows that capacity and other forms of investment are effective entry deterring variable, partly because they are irreversible and represent preemptive commitments to the industry. Dixit [7] presents a two-stage model with strategic capacity investment and shows that an incumbent firm installing excess capacity in the first stage is able to deter the entry of a potential entrant in the second stage. Ware [8] examines the three-stage model in which an incumbent installs its capacity in the first stage, an entrant installs its own capacity in the second stage, and quantity equilibrium is established in the third stage. He concludes that although his three-stage equilibrium is qualitatively similar to Dixit's two-stage equilibrium, it differs in that the strategic advantage available to the first mover is lessened. Kim [14] considers a Stackelberg entry deterrence model with the established firm, as the leader, making a commitment to the output level equal to its pre-entry capacity, and shows that the established firm may hold pre-entry excess capacity under the post-entry game rule of Cournot-Nash. There are many further excellent studies. Most studies then presume that the strategic behavior of firms is quantitative competition through substitute goods in substitutive relationships or price competition through substitute goods in complementary relationships. Therefore, we discuss complementary goods.

We examine a three-stage duopoly model with complementary goods where first-mover and second-mover firms are allowed to choose their capacity levels as a strategic commitment. We consider the following situation. In the first stage, the first-mover firm decides whether or not to make a commitment to capacity. At the end of the first stage, the second-mover firm observes the capacity level of the first-mover firm. In the second stage, the second-mover firm decides whether or not to make a commitment to capacity. At the end of the second stage, the first-mover firm observes the capacity level of the second mover. Neither firm can reduce or dispose of capacity. In the third stage, both firms choose their outputs simultaneously and independently.

The purpose of this paper is to show the equilibrium outcomes of the three-stage quantity-setting duopoly model with complementary goods when first-mover and second-mover firms install the appropriate levels of capacity as a strategic commitment.

The paper will proceed as follows. In Section two, we formulate the three-stage quantity-setting model. Section three discusses the equilibrium outcomes of the model. Section four concludes the paper. All proofs are given in the appendix.

II. THE MODEL

In this model there are two firms, firm 1 and firm 2. For the remainder of this paper, when i and j are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with $i \neq j$. There is no possibility of entry or exit. Firm i 's profit is

$$\pi_i(q_i, q_j) = p_i(q_i, q_j)q_i - m_i q_i, \tag{2}$$

where $p_i : \Re_+^2 \to \Re_+$ is firm i 's inverse demand function, q_i is firm i 's output, and m_i is firm i 's constant marginal cost for output.

The timing of the game runs as follows. In the first stage, firm 1 is allowed to install capacity k_1. At the end of the first stage, firm 2 observes the value of k_1. In the second stage, firm 2 is allowed to install capacity k_2. At the end of the second stage, firm 1 observes the value of k_2. Neither firm can reduce or dispose of capacity. In the third stage, both firms decide their outputs (q_1 and q_2) simultaneously and independently.

Therefore, firm i 's profit changes as follows:

$$\hat{\pi}_i(k_i, q_i, q_j) = \begin{cases} \pi_i(q_i, q_j) & \text{if } q_i \geq k_i, \\ \pi_i(q_i, q_j) + (q_i - k_i)r_i & \text{if } q_i \leq k_i, \end{cases} \tag{1}$$

where $r_i \in (0, m_i]$ is firm i 's constant cost per unit of capacity. If firm i installs capacity k_i, then the cost of $r_i k_i$ sinks. Hence, firm i 's marginal cost exhibits a discontinuity at $q_i = k_i$.

Now, the following assumptions are made.

Assumption 1. p_i is twice continuously differentiable with bounded derivatives, $\partial p_i / \partial q_i < 0$ (downward-sloping demand), and $\partial p_i / \partial q_j > 0$ (complementary goods).

Assumption 2. $\partial^2 \pi_i / \partial q_i q_j > 0$ (strategic complementarity[3]).

Assumption 3. $\partial^2 \pi_i / \partial q_i^2 + \partial^2 \pi_i / \partial q_i \partial q_j < 0$ (stability).

Assumption 1 ensures a smooth demand function. Assumption 2 means that firm i 's marginal profit with respect to its own output goes up with firm j 's output. Assumption 3 means that the own effects dominate the cross effects.

Given q_j, firm i maximizes its profit with respect to q_i. If firm i 's marginal cost for output is constantly equal to m_i, then its Cournot reaction function is defined by

$$R_i(q_j) = \arg\max_{\{q_i \geq 0\}} \pi_i(q_i, q_j), \tag{3}$$

and if firm i 's marginal cost for output is constantly equal to $m_i - r_i$, then its Cournot reaction function is defined by

$$\overline{R}_i(q_j) = \arg\max_{\{q_i \geq 0\}} [\pi_i(q_i, q_j) + r_i q_i]. \tag{4}$$

Therefore, if firm i installs k_i, then its best response changes as follows:

$$\hat{R}_i(k_i, q_j) = \begin{cases} R_i(q_j) & \text{if } q_i > k_i, \\ k_i & \text{if } q_i = k_i, \\ \overline{R}_i(q_j) & \text{if } q_i < k_i. \end{cases} \tag{5}$$

Firm i aims to maximize its profit with respect to its own output level, given the output level of firm j. The equilibrium must satisfy the following conditions: If firm i does not install k_i, then the first-order condition is

$$p_i + \frac{\partial p_i}{\partial q_i} q_i - m_i = 0, \tag{6}$$

and the second-order condition is

$$2\frac{\partial p_i}{\partial q_i} + \frac{\partial^2 p_i}{\partial q_i^2} < 0. \tag{7}$$

If firm i installs k_i and reduces its marginal cost, then the first-order condition is

$$p_i + \frac{\partial p_i}{\partial q_i} q_i - m_i + r_i = 0, \tag{8}$$

and the second-order condition is (7). Furthermore, we have

$$R_i{}'(q_j) = \overline{R}_i{}'(q_j) = -\frac{\partial p_i / \partial q_j + (\partial^2 p_i / \partial q_i \partial q_j) q_i}{2(\partial p_i / \partial q_i) + (\partial^2 p_i / \partial q_i^2) q_i}. \tag{9}$$

From assumption 2, $\partial p_i / \partial q_j + (\partial^2 p_i / \partial q_i \partial q_j) q_i$ is positive; that is, both $R_i(q_j)$ and $\overline{R}_i(q_j)$ are upward sloping.

We consider only pure-strategy subgame perfect equilibria. Since inverse demand is defined only for non-negative outputs, it is ensured that all outputs obtained in equilibrium are non-negative.

III. EQUILIBRIUM OUTCOMES

In this section, we begin by presenting the following three lemmas.

Lemma 1. Firm i 's profit-maximizing output is higher when it installs k_i than when it does not.

Lemma 2. If firm i installs capacity k_i and an equilibrium is achieved, then in equilibrium $q_i = k_i$.

Lemma 3. Suppose the quantity-setting game with no capacity installed. Then firm i 's Stackelberg leader output is higher than its Cournot output.

Lemma 1 means that if firm i installs capacity in advance of production, then its optimal output increases. Lemma 2 means that in equilibrium firm i does not install extra capacity. We assume that if firm i unilaterally installs excess capacity, then its reaction function shifts at least up to its Stackelberg leader solution.

We now discuss the equilibrium outcomes of strategic complements in which goods are complements. If firm i increases its output, firm j 's amount of demand increases because of complementary goods. That is, increasing firm i 's output leads firm j to increase its output. This states that the quantity-setting game with complementary goods makes firms strategic complements. We will discuss the following three types.

Type 1: $\pi_1^L > \pi_1^F$.

Type 2: $\pi_1^L < \pi_1^F$.

Type 3: $\pi_1^L = \pi_1^F$.

Here, L denotes the Stackelberg point where firm 1 is the leader, and F the Stackelberg point where firm 1 is the follower. We will discuss these types in order.

Type 1

Fig. 1 depicts both firms' reaction curves for the quantity-setting model with complementary goods. R_i is firm i 's reaction curve when the marginal cost for output is constantly equal to m_i. Both the firms' reaction curves are upward sloping because of strategic complements. If neither firm installs capacity, then the equilibrium is at N.

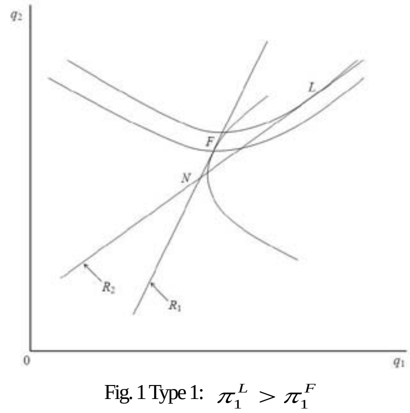

Fig. 1 Type 1: $\pi_1^L > \pi_1^F$

We now consider the case that each firm is allowed to install capacity. In the first stage, firm 1 installs k_1^L up to the level equal to L because its profit is the highest at L on R_2. Then firm 1's reaction curve becomes the kinked bold lines drawn in Fig. 2. $\bar{R}_1$ is firm 1's reaction curve when the marginal cost for output is constantly equal to $m_1 - r_1$. In the second stage, firm 2 can install k_2. Firm 2's profit is the highest at L on the feasible segment ALB. Therefore, firm 2 does not install excess capacity. In the third stage, both firms' outputs and profits are decided in a Cournot fashion, and the equilibrium occurs at L.

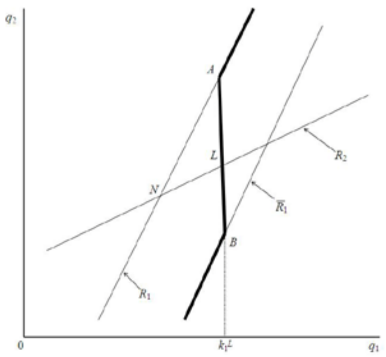

Fig. 2 Type 1 equilibrium

Type 2

Type 2 is illustrated in Fig. 3 and Fig. 4. Firm 1's profit is higher when it is the leader than when it is the follower, as shown in Fig. 3. We consider the case that each firm is allowed to install capacity. In the first stage, firm 1 does not install excess capacity. In the second stage, firm 2 installs k_2^F up to the level equal to F because its profit is the highest at F on R_1. Then firm 2's reaction curve becomes the kinked bold lines drawn in Fig. 4. $\bar{R}_2$ is firm 2's reaction curve when the marginal cost for output is constantly equal to $m_2 - r_2$. In this type, though firm 1 acts as leader, the equilibrium occurs at F.

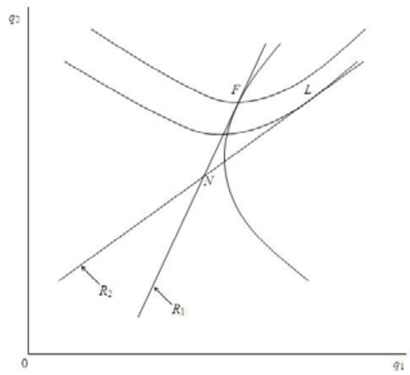

Fig. 3 Type 2: $\pi_1^L < \pi_1^F$

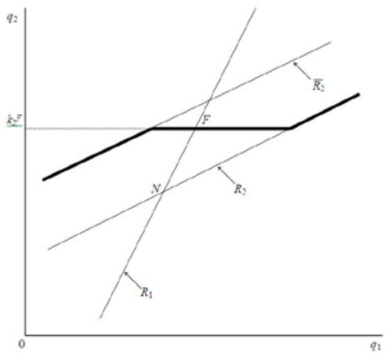

Fig. 4 Type 2 equilibrium

Type 3

Type 3 is depicted in Fig. 5. If firm 1 installs k_1^L up to the level equal to L in the first stage, then L is achieved as equilibrium. On the other hand, if firm 1 does not install excess capacity in the first stage, then firm 2 installs k_2^F up to the level equal to F in the second stage and F is achieved as an equilibrium.

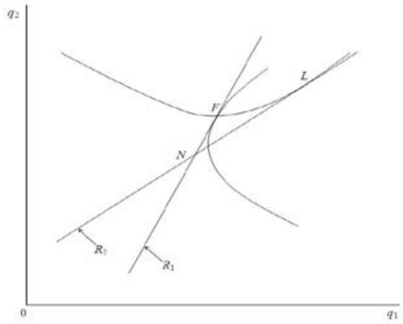

Fig. 5 Type 3: $\pi_1^L = \pi_1^F$

The main result of this study is described by the following propositions.

Proposition 1. (i) There exist subgame perfect equilibria that occur at points L and F. (ii)At each equilibrium, both the firms earn higher profits than in the Cournot game with no capacity.

Proposition 1 implies that capacity investment is beneficial for both the firms in the three-stage quantity-setting model of strategic complements where goods are complements.

IV. CONCLUSION

We have discussed the equilibrium outcomes of the three-stage quantity-setting duopoly competition with complementary goods. We have demonstrated that there exist two opposite equilibria, that is, the first-mover's Stackelberg leader equilibrium and the second-mover's Stackelberg leader equilibrium. In addition, we have shown that in each kind of equilibrium, capacity investment is beneficial for both the firms.

APPENDIX

We begin by proving the lemmas.

Proof of Lemma 1

Suppose that firm i unilaterally installs capacity. From (1) and (2), we see that capacity investment will never increase the marginal cost of the firm. In the second equation of (2), the first-order condition for firm i is (8). Here, where the sign of r_i is positive. To satisfy (8), $p_i + (\partial p_i / \partial q_i) q_i - m_i$ must be negative. Thus, firm i 's optimal output is higher when its marginal cost is $m_i - r_i$ than when its marginal cost is m_i. Q.E.D.

Proof of Lemma 2

First, consider the possibility that $q_i < k_i$ in equilibrium. From (1) and (2), firm i 's profit is $p_i(q_i, q_j) q_i - m_i + (q_i - k_i) r_i$. If $q_i < k_i$, then firm i installs extra capacity. That is, firm i can increase its profit by reducing k_i, and the equilibrium does not change in $q_i \le k_i$. Hence, $q_i < k_i$ does not result in equilibrium.

Next, consider the possibility that $q_i > k_i$ in equilibrium. From (1), (2) and (5), we see that firm i has to incur the full marginal costs of producing any given quantity. It is impossible for firm i to change its output in equilibrium because such a strategy is not credible. That is, if $q_i > k_i$, capacity investment cannot function as a strategic commitment. Q.E.D.

Proof of Lemma 3

Firm i selects q_i, and firm j selects q_j after observing q_i. When firm i is the Stackelberg leader, it maximizes its profit $\pi_i(q_i, R_j(q_i))$ with respect to q_i. Therefore, firm i's Stackelberg leader output satisfies the first-order condition:

$$p_i + \frac{\partial p_i}{\partial q_i} q_i - m_i + \frac{\partial p_i}{\partial q_j} q_i \frac{\partial R_j}{\partial q_i} = 0 . \qquad (10)$$

From $\partial p_i / \partial q_j > 0$ and $\partial R_j / \partial q_i > 0$, to satisfy (10), $p_i + (\partial p_i / \partial q_i) q_i - m_i$ must be negative. Thus Lemma 3 follows. Q.E.D.

We now prove Proposition 1.

Proof of Proposition 1

We first consider Type 1. (i) Firm 1's Stackelberg leader output exceeds its Cournot output (Lemma 3). Furthermore, $\pi_1(q_1, q_2) = p_1(q_1, q_2)q_1 - m_1 q_1$ is continuous and concave in q_1. $R_2(q_1)$ gives firm 2's profit-maximizing output for each output of firm 1. In R_2, π_1 is highest at L, and the further the point on R_2 gets from L, the more π_1 decreases. By Lemmas 1 and 2, if firm 1 installs k_1, then its profit is higher than in the Cournot equilibrium with no capacity installed. Therefore, suppose that firm 1 installs k_1 up to the level equal to L in the first stage. Firm 1's reaction function has a flat segment at L level (see expression (5) and Fig. 2). $R_2(q_1)$ gives firm 2's profit-maximizing output for each output of firm 1. Furthermore, $\pi_2(q_1, q_2) = p_2(q_1, q_2)q_2 - m_2 q_2$ is continuous and concave in q_2. Firm 2's capacity investment decreases its profit and thus it does not install excess capacity. Our equilibrium concept is subgame perfection, and all information is common knowledge. Thus, firm 1's Stackelberg leader solution is achieved as equilibrium.

(ii) At equilibrium, firm 1 unilaterally makes a commitment to capacity. From the preceding discussion, it is obvious that firm 1's equilibrium profit exceeds its Cournot profit with no capacity installed.

We prove that at equilibrium firm 2's profit is higher than in the Cournot equilibrium with no capacity installed. Firm 1 installs capacity in the first stage, and firm 2 does not install capacity in the second stage. Firm 1's profit-maximizing output is higher when firm 1 installs k_1 than when it does not (Lemma 1). Since $\partial \pi_2 / \partial q_1 = (\partial p_2 / \partial q_1) q_2 > 0$, increasing q_1 increases π_2, and given q_2.

Second, we consider Type 2. (i) Firm 1's Stackelberg leader output is higher than its Cournot output (Lemma 3). Furthermore, $\pi_1(q_1, q_2) = p_1(q_1, q_2)q_1 - m_1 q_1$ is continuous and concave in q_1. In R_2, π_1 is highest at L. Our equilibrium concept is subgame perfection, and all information is common knowledge. Hence, firm 1 knows that its Stackelberg follower profit exceeds its Stackelberg leader profit. Therefore, firm 1 does not install excess capacity. Firm 2's Stackelberg leader output is higher than its Cournot output (Lemma 3). Furthermore, $\pi_2(q_1, q_2) = p_2(q_1, q_2)q_2 - m_2 q_2$ is continuous and concave in q_2. In R_1, π_2 is highest at F, and the further the point on R_1 gets from F, the more π_2 decreases. By Lemmas 1 and 2, if firm 2 installs k_2, then its profit is higher than in the Cournot equilibrium with no capacity installed. Thus, firm 2 installs k_2 up to the level equal to F in the second stage, and firm 2's Stackelberg leader solution is achieved as equilibrium.

The proof of (ii) is omitted, since it is essentially the same as the proof of (ii) of Type 1.

Third, we consider Type 3. In this type, if firm 1 installs capacity in the first stage, then firm 2 does not. On the other hand, if firm 1 does not install capacity in the first stage, then firm 2 does. The proof of Type 3 is omitted, since it is essentially the same as the proofs of Types 1 and 2. Q.E.D.

REFERENCES

[1] Wenders, J. T., 1971. Excess capacity as a barrier to entry. Journal of Industrial Economics 20, 14-19.

[2] Spence, A. M., 1977. Entry, capacity, investment and oligopolistic pricing. Bell Journal of Economics 8, 534-544.

[3] Dixit, A. K., 1979. A model of duopoly suggesting a theory of entry barrier. Bell Journal of Economics 10, 20-32.

[4] Tirole, J., 1988. The Theory of Industrial Organization. MIT Press, Cambridge MA.

[5] Gilbert, R. J., 1989. Mobility barriers and the value of incumbency. In: Schmalensee, R., Willig, R. D. (Eds.), Handbook of Industrial Organization, Vol. 1. Elsevier Science Publishers, Amsterdam, pp. 475-535.

[6] Waldman, D. E., Jensen, E. J., 2006. Industrial Organization: Theory and Practice, 3rd edition. Addison Wesley, Boston MA.

[7] Dixit, A. K., 1980. The role of investment in entry-deterrence. Economic Journal 90, 95-106.

[8] Ware, R., 1984. Sunk costs and strategic commitment: a proposed three-stage equilibrium. Economic Journal 94, 370-378.

[9] Ohnishi, K., 2009. Strategic commitment and three-stage games with labor-managed and profit-maximizing firms. Finnish Economic Papers 22, 63-74.

[10] Ireland, N. J., Law, P. J., 1982. The Economics of Labour-managed Enterprises. Croom Helm, London.

[11] Stephan, F. H. (Ed.), 1982. The Performance of Labour-managed Firms. Macmillan Press, London.

[12] Bonin, J. P., Putterman, L., 1987. Economics of Cooperation and the Labor-managed Economy. Harwood Academic Publisher, Chur, Switzerland.

[13] Putterman, L., 2008. Labour-managed firms. In: Durlauf, S. N., Blume, L. E. (Eds.), The New Palgrave Dictionary of Economics, Vol. 4. Palgrave Macmillan, Basingstoke, Hampshire, pp. 791-795.

[14] Kim, H., 1996. Strategic excess capacity and first-mover advantage under variable demand. Seoul Journal of Economics 9, 105-122.

[15] Bulow, J. I., Geanakoplos, J. D., Klemperer, P. D., 1985. Multimarket oligopoly: strategic substitutes and complements. Journal of Political Economy 93, 488-511.

Kazuhiro Ohnishi received his Ph.D. in economics from Osaka University, his M.A. in economics from Okayama University, and his B.A. in sociology from Kwansei Gakuin University. He is a researcher at the Institute for Basic Economic Science, Japan. His research interests include industrial organization, microeconomics, new institutional economics, international economics, public economics, game theory, urban economics, and law and economics.

Prof. Dr. Masamichi Kawano received his doctoral degree from University of Tsukuba, and is currently affiliated with Kwansei Gakuin University (Japan). His research interests include industrial organization, dynamic economics, international economics, and regional economics. His research has been published in Metroecomica, Economic Studies Quarterly, Studies in Regional Science, and in other academic journals.